Information Spillovers and Performance Persistence in Private Equity Partnerships*

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Abstract

We present a simple model that rationalizes performance persistence in private equity partnerships. In contrast to the model for mutual funds of Berk and Green (2004), the learning in our model pertains to profitability associated with an emerging sector or an innovative trading strategy, rather than ability specific to the fund manager. As a result of potential information spillovers, which would increase competition in the sector if informed investors were to partner with non-incumbent managers, incumbent managers will let informed investors benefit from increases in estimated profitability following high realized returns in the sector.

1 Introduction

Private equity partnerships, such as venture capital funds, buyout funds, and hedge funds, have been shown to exhibit persistence in the abnormal performance they generate for investors (see, e.g., Kaplan and Schoar 2005, Phalippou and Gottschalg 2008, Fung, et al 2008). Mutual funds, in contrast, show little performance persistence. The persistence that is evident in mutual fund performance is concentrated in the worst performing funds (see Carhart 1997, Berk and Tonks 2008), where it appears to be largely attributable to inattention by investors in those funds. Such an explanation for persistence in performance for private equity is inconsistent with the nature of the investor base, which consists of institutions and wealthy, relatively sophisticated, individuals. It is also at odds with the facts. As Kaplan and Schoar (2005) show, private-equity flows chase performance, just as mutual-fund flows do. Nevertheless, performance is persistent for funds that perform well, and are thus able to attract new flows.

An explanation for the sensitivity of mutual-fund flows to performance, despite the lack of persistence in performance, is offered by Berk and Green (2004). In that model, investors learn about heterogeneous ability through past returns, but there are decreasing returns to scale in deploying those abilities. In light of this explanation for the behavior of mutual funds, private equity partnerships present a puzzle. If flows respond to learning about private equity returns, as they appear to do, why do managers not expand the fund or raise their fees to capture the rents going forward? In their paper about the performance of private equity partnerships, Kaplan and Schoar (2005) write:

> If, indeed, the persistence results are driven by heterogeneity in [General Partner, hereafter the GP] skill and limited scalability of human capital, it remains puzzling that these returns to superior skills are not appropriated by the scarce input (i.e., the GP) in the form of higher fees.

In this paper, we rationalize performance persistence in private equity partnerships. Our model is based on evident differences in the institutional setting between mutual funds and private equity funds. Private equity partnerships provide financing in a wide range of settings, from venture capital in emerging industries, to hedge funds developing trading strategies, to buyout funds focusing on distressed companies. In all these settings, however, information accumulated by an experienced fund manager is likely to be valuable to competitors contemplating entry into a *sector* (e.g., an emerging industry or a new trading strategy) where an incumbent fund operates.

Private equity funds have another common feature, despite the wide range of investment activities they engage in. They are *private*. They are organized as limited partnerships with finite lives, and solicit funds from large, "qualified" investors. This frees them from the elaborate disclosure requirements and oversight mutual funds and publicly traded corporations are subject to. The common choice of organizational form is an endogenous response. A concern that disclosure and oversight, and the associated leakage of information, would erode their ability to generate rents is a natural place to look for a common, primitive determinant of this choice.

Private equity partnerships are widely reported to be secretive about their strategies and return histories. They have been known to exclude investors from later funds who disclose such information to third parties. For example, in 2003 the University of California appealed a lower court decision forcing it to disclose past performance data about private equity funds it had invested endowment funds in. A press release (see `http://www.universityofcalifornia.edu/news/article/9917`) issued by the University stated:

> The University contends that the IRRs are trade secrets and therefore exempt from disclosure under the California Public Records Act and also that disclosure of such confidential information will result in the University being excluded from future investment in these funds. Private equity funds have achieved the best return of all the University's investments...

More recently, on January 15, 2008, the front page of the *Wall Street Journal* reported on the large profits John Paulson's hedge fund earned betting against the housing market, *and* on the success of one of his former investors, then backed by two investment banks, following a similar strategy:

> It was the spring of 2006, and Mr. Paulson, seeking investors for a new fund, gave Mr. Greene a peek at his plan. Mr. Greene didn't wait for the fund to open. He beat his friend to the punch by doing the same complex mortgage-market trade on his own.

Such examples are consistent with the description of the private equity industry in Phalippou (2007):

> Past performance is top secret. Some funds have fund-raising prospectuses that cannot be photocopied; some funds refuse money from any LP that might publicly declare the fund's performance. ... Nonetheless, LPs require wide-ranging information to monitor the performance of the fund and meet with their general partners on a regular basis to discuss the progress of the portfolio.

The concerns evidenced by private equity funds for confidentiality suggest that what investors learn from past returns in the private equity arena is not limited to ability or talent unique to the manager, as assumed by Berk and Green (2004) for the mutual fund industry, or in models of "soft information" applied to private equity as in Hochberg, Ljungqvist and Vissing-Jørgensen (2007). Investors (and managers) may also be learning about the profitability of emerging sectors of the economy or of innovative trading strategies, as emphasized by Sorensen (2008). This information, if known to others, would attract imitation and competition. In such an environment, approaching new investors and convincing them to invest in a follow-on fund is likely to require disclosure and lead to information spillovers that reduce profits to the incumbent general partner and limited partners.

Our model considers this possibility in a setting similar to that of Berk and Green (2004). As in their paper, both managers and investors learn about the profitability of the fund through past performance. Future profitability in Berk and Green (2004) depends negatively on assets under management, due to decreasing returns to scale. This is also the case in our model, but in addition the investments made in the sector by other partnerships reduce profitability to incumbents going forward.

In the model, there is an infinite number of potential LPs, whereas the number of potential GPs is finite. In dealing with investors, the GP makes the first take-it-or-leave-it offer, consistent with the GP's abilities or skills being the ultimately scarce resource. The critical question is why high expected performance going forward should increase the outside option, or reservation price, of the LPs in deciding whether to accept or reject the offer. To illustrate the intuition, we first fix the number of potential GPs exogenously. We later illustrate how the set of potential GPs can be determined endogenously through a fixed cost of entry.

We assume that any party with information useful in estimating future returns credibly and fully discloses it to outsiders when soliciting their participation. The reservation price of an LP being solicited by a GP is determined by the LP's ability to approach new potential GPs and disclose expected future performance in the sector in which the LP has been investing. Each such disclosure to a new GP, if expected profits are positive, adds a competitor in that sector, and thus reduces potential profits for the incumbents.

At each stage of the game, the reservation price of an LP dealing with an offer from a GP is determined by the LP's expected payoff from approaching a new GP and making him an offer. The reservation price of the new GP, responding to an offer from an LP, is determined by his ability to disclose information to, and solicit capital from, a new LP. Thus, the expanding set of competitors that results from the search for alternative partners acts like a discount factor in an alternating-offer bargaining game.

We formulate this game recursively, and solve for the expected payoffs of the various parties as functions of the number of GPs currently informed and competing in the sector, the number of GPs who could potentially enter the sector, and the estimated profitability of the sector. We then examine conditions under which secrecy is an equilibrium, and the incumbent LP agrees to continue as a partner in the follow-on fund. Since the reservation price of the LP is increasing in the expected returns in the sector going forward, his share of those profits will be as well. Returns to investors will persist across generations of follow-on funds organized by the same general partner.

Our focus on the consequences of information spillovers leads us to abstract from many obviously important features of the contracting environment for private equity funds. We ignore asymmetric information and moral hazard. As a result, investment is "first-best". The form of the contract between managers and investors is irrelevant, as is the timing of flows and payouts within the life of a fund. A period in our model corresponds to the life of a fund, consistent with the empirical evidence on performance persistence.

Our intent is not to minimize the importance of these considerations. Staged financing through the life of a given fund, commitment of personal wealth to funds by general partners, and the form of the partners' compensation contracts, all seem to be responses to asymmetric information and moral hazard. Indeed, these features of the environment have been extensively studied in the theoretical literature.[1] Our focus is on returns across generations of follow-on funds, rather than

[1] See Axelson, Strömberg, and Weisbach (2007) for a recent treatment, and the references therein.

contracting over the life of a given fund. If a general partner has positive information about future performance that can be disclosed credibly to investors, why should he not raise fees to the point that investors earn a competitive expected return going forward? Our model provides a simple answer to this question.

An alternative explanation for persistence in private equity returns is offered in contemporaneous work by Hochberg, Ljungqvist and Vissing-Jørgensen (2007). It is based on the acquisition of "soft information" about the GP's abilities by incumbent LPs, who then hold up the GP, as in models of relationship banking such as Sharpe (1990), Rajan (1992), and von Thadden (2004). The information LPs gain through experience with a GP is assumed to be costly or impossible for the GP to communicate directly to potential investors. Our model is aimed at the same set of facts, but relies on a completely different mechanism. Both mechanisms may well be at work in the private equity setting. There surely is learning about attributes specific to the manager through experience, as in Hochberg, Ljungqvist and Vissing-Jørgensen (2007), and these attributes might well be difficult to credibly communicate to outsiders. This may, indeed, contribute to the persistence in returns. Asymmetric information also implies that a decision by incumbent LPs not to participate in follow-on funds conveys bad news about the manager to other investors, which seems quite plausible.

Our approach, however, may better capture some other features of the environment. The concerns for confidentiality evident in private equity are at odds with a soft-information story. Absent other frictions, with soft information GPs would pre-commit to disclosure if they could do so. Instead, they appear to go to some lengths to avoid such disclosure. A second difference is that under soft information, the outside option of the investor (i.e., bank or LP) is a competitive return. Thus, if the GP could make a take-it-or-leave-it offer to the LP, it would be accepted, the GP would collect all the rents, and the persistence would disappear. Instead, in the typical soft-information setting, the bargaining game changes between the intial and the intermediate date. In the banking models, it is the customer who makes a take-it-or-leave-it offer. In Hochberg, Ljungqvist and Vissing-Jørgensen (2007) the GP and LP play an alternating offer bargaining game at the intermediate date, thus sharing the surplus an the persistence in returns. Our approach is to directly model the LPs' outside option, so that the division of rents is determined by the relative scarcity of the resources the parties bring to the transaction. The GP continues to collect most of the rents associated with follow-on funds, because the abilities of the GPs are ultimately the resource that is in scarce supply.

Our model produces other behaviors consistent with empirical findings on private equity. For example, Lerner and Schoar (2004) document continuity in the composition of limited partners across funds raised by the same general partners. Kaplan and Schoar (2005) and Phalippou and Gottschalg (2008) show there is a positive association between fund size and returns to investors, and that returns are higher for investors in follow-on funds than in earlier funds.

We calibrate the model to the unconditional moments of the cross section of funds, such as the probability of raising a follow-on fund, and the average fund return before fees. We then ask if the model can reproduce the persistence in returns observed in the data, and simultaneously match the behaviors implied when the contracts between GP and LP follow the ubiquitous 2/20 rule used by most funds.[2] We find that the model can match the observed point estimates of persistence under the theoretical sharing rule or under the 2/20 rule, but not for both. The 2/20 rule does not allow the GP's return to be quantitatively as responsive to past performance as our theory predicts. When parameterizing our model such that a 2/20 rule would generate an empirically sensible level of performance persistence, our theoretical optimal sharing rule generates a persistence in returns to LPs that is lower than the empirical estimate, but that is still economically significant.

The paper is organized as follows. The next section describes the setting and solves for the optimal investment policy in a given sector. Section 3 models the outside option of each agent and solves for the division of rents between the parties. In Section 4 we derive the model's predictions in terms of secrecy, returns to investors, and fund flows. The robustness of some of these predictions is discussed in Section 5. Section 6 endogenizes the number of potential general partners using a fixed cost of entry. This leads to an interplay between expected future profitability and relative bargaining power, which may produce interesting dynamics when embedded in a dynamic model of entry. Section 7 endogenizes entry in an initial period, and uses simulation to evaluate the

[2]Under the standard compensation scheme in private equity partnerships, which we call the "2/20 rule", GPs charge investors an annual management fee of 2 percent of assets under management and a carried interest of 20 percent of profits, adjusted for a hurdle rate, when these profits are positive.

quantitative realism of the model's predictions. The last section summarizes and concludes. Proofs are relegated to the Appendix section.

2 The Setting

There are two types of risk-neutral agents: potential general partners and potential limited partners. The M general partners (GPs) have access to investment opportunities, but no capital. Funds must be obtained from one of the countably infinite identical limited partners (LPs), who have capital but lack the knowledge, networks, time, or experience to independently identify and exploit profitable investment opportunities. For the moment, we take M as exogenously given, but we will later show that a fixed cost of entry can be used, with some added notational complexity, to determine this quantity endogenously. This will tie relative bargaining power to expected returns for the sector as a whole, producing a richer set of dependencies between past returns and expected performance.

The GP makes a take-it-or-leave-it partnership offer to an LP to raise investment funds. The offer is such that the GP collects the highest possible expected profit from the partnership, subject to satisfying the LP's participation constraint. In this sense, our model is similar to the classic text-book descriptions of corporate financing. The firm acts as a Stackleberg leader in its dealings with competitive financial markets, and under first-best collects the net present value of any investment opportunities, while investors simply earn competitive returns. As Berk and Green (2004) make clear, in such an environment learning should lead flows to respond to past performance, but there is no reason that performance should persist going forward.

Each project (and its financing) lasts only one period and is continuously scalable. For a given project, the ith GP invests a positive amount Q_i in order to maximize expected profits. Because of diseconomies of scale, the cost of finding good projects in a given sector (i.e., emerging industry, new trading strategy, etc.) increases in Q_i much as in Berk and Green (2004). In our setting, however, the costs also increase in the total funds invested in the sector by all private equity partnerships, as denoted by $\overline{Q} \equiv \sum_j Q_j$. The cost function facing GP_i is $\frac{C}{2}Q_i\overline{Q}$. This specification is somewhat familiar because it reduces to the quadratic cost function $\frac{C}{2}Q_i^2$ when the private equity partnership faces no competition, and by adding up the funds committed by all the partnerships in a sector, we obtain the quadratic cost function $\frac{C}{2}\overline{Q}^2$. For simplicity, the competitive return/discount rate is set to zero.

Most of the analysis in our model involves the dealings between an incumbent GP and LP considering a follow-on fund. At this point, they have observed returns on previous investments in the sector and are deciding whether to continue their partnership, and if so, how to split the expected profits from follow-on funds. This point in time can be viewed as an intermediate date in a dynamic model, where in a previous period or periods, the incumbent GP and LP have invested in the sector together. In Section 7, when we calibrate the model and explore its quantitative properties, we explicitly employ a three-date, two period model, where follow-on funds are formed at the intermediate date.

Investing in a follow-on fund will produce a realized return, before accounting for the diseconomies of scale, of $\phi + \epsilon$, where ϕ is the expected profitability of the sector, given the information accumulated in previous funds, and ϵ is a regression error with $E(\epsilon \mid \phi) = 0$. We assume that ϕ will be positively correlated with past performance, as is natural if there is learning. In Berk and Green (2004), for example, ϕ is the posterior expectation of a constant mean resulting from Bayesian updating. The specific form of the correlation with past performance is not important for our results. The question facing us is why expected returns to outside investors should depend positively on ϕ, which would imply performance persistence.

A partnership's realized profit in the sector depends on this return, the size of the investment, and the costs linked to diseconomies of scale. Specifically, the partnership's realized profit follows the stochastic process:

$$Q_i[\phi + \epsilon] - \frac{C}{2} Q_i \overline{Q}. \tag{1}$$

For a given estimate, ϕ, of the profitability of the sector, each partnership i will choose to invest an amount Q_i that maximizes expected profit:

$$Q_i \phi - \frac{C}{2} Q_i \sum_{j=1}^{N} Q_j. \tag{2}$$

In a symmetric Nash equilibrium, each partnership's optimal investment will be:

$$Q^*(\phi, N) = \frac{2\phi}{C}\frac{1}{N+1}, \tag{3}$$

and its optimal expected profit, which we denote $\Pi(\phi, N)$, will be:

$$\Pi(\phi, N) = \frac{2\phi^2}{C}\frac{1}{(N+1)^2}. \tag{4}$$

To have $Q^*(\phi, N) > 0$, we need $\phi > 0$. Otherwise, the sector is not expected to provide any abnormal return, even on the first dollar invested. We focus at this point on $\phi > 0$, which is the interesting case.

Equations (2)-(4) immediately give an expression for returns gross of fees generated by a partnership within a period:

$$\begin{aligned} R(\phi, N) &= \frac{\Pi(\phi, N)}{Q^*(\phi, N)} \\ &= \phi - \frac{C}{2}\overline{Q} \\ &= \frac{\phi}{N+1}. \end{aligned} \tag{5}$$

3 Information Spillovers

GPs in our model are of two sorts. Incumbent GPs have experience in the sector, and through this experience enter a period knowing ϕ. Non-incumbent, potential GPs have the general expertise to enter and compete in the sector, but lack specific knowledge of its potential profitability.

When any informed agent solicits a potential partner, we assume he discloses, fully and credibly, his information about past returns, or equivalently, ϕ. We abstract from the possibility a GP would commit fraud or mislead investors through incomplete disclosure, and assume no agent would agree to partner with someone absent full and credible disclosure. We can view the first LP approached by an incumbent GP as the investor who has partnered with the GP in the previous fund, but since the GP cannot operate without disclosing ϕ to an LP first, it is not essential that the LP has previous experience. As we will see, it is in an incumbent GP's interest to minimize the number of informed parties, so if he has partnered with a particular LP in the past, he would approach that LP first.

Once disclosure is made to an LP, information is symmetric between the two parties. Similarly, there is no moral hazard in the model. The GP will invest to maximize expected profit (or Net Present Value). The functional form of the contract will therefore be indeterminate, given the shares of the value created accruing to the two parties.[3] That is, once disclosure is made we are in a Miller and Modigliani world without contracting frictions except for the possibility of information spillovers. In such an environment, if information in past returns pertains to ability that is unique to the manager, as in Berk and Green (2004), investors will simply earn their reservation expected return of zero going forward. If the possibility of information spillovers alters their reservation price in a direction that reflects the information conveyed by past performance, then we would expect performance to persist.

Consider an incumbent GP and LP with a shared knowledge of ϕ based on the realized returns on a past investment. This information could be fully and credibly disclosed to other, potential, GPs (in finite supply), or LPs (in infinite supply). If the incumbent LP terminates the partnership with his initial GP, the "rejected" GP will be able to solicit a new and so far uninformed LP to invest with during the next period. Similarly, the incumbent LP, with the intent of forming a new partnership, will be able to bring the expected returns available in the sector to the attention of a new GP, by disclosing the information in ϕ.

Because of these disclosures, partnership offers made to new GPs or LPs expand the set of informed agents and competitors in the sector. In the event that they are not satisfied with the profit sharing rule offered by the party they are bargaining with, these disclosures provide solicited agents with outside options. The incumbent GP and LP will nonetheless have an incentive to continue their partnership together in a subsequent period, because involving new partners in the sector would increase competition and reduce the profit earned by each partnership.[4]

[3]The homogeneity in contract terms and organizational form across funds might be viewed as inconsistent with our model, which implies the share of value created varies with the GP's history and competitive conditions. Phalippou (2007) and Metrick and Yasuda (2007), however, argue that, while GP's fees appear homogenous in certain respects, numerous differences in less transparent contract terms, such as catch-up clauses, hurdle returns, and carry timing, make the effective profit sharing rules heterogenous across funds.

[4]Our derivation of endogenous reservation prices as a result of outside opportunities the potential partners have in bargaining with others has some similarities to the bargaining problems considered by Duffie, Gârleanu and Pedersen (2007) in over-the-counter markets.

Consider the bargaining problem between a general partner who previously invested in a sector and the limited partner who provided him with the funds. All agents who participated in the sector in a previous fund know ϕ (> 0). The general partner solicits capital for a new fund from the limited partner through a take-it-or-leave-it offer. The limited partner's reservation price is determined by his opportunity to inform a non-incumbent GP of ϕ, and make him a take-it-or-leave-it offer. In turn, the new GP, who is now informed about the profit opportunities in the sector, can share that information with a new LP, and make him a take-it-or-leave-it offer, and so on. We assume there are a finite number, M, of potential GPs with the skill and expertise needed to operate in the sector, and an infinite number of potential LPs. As we will show, this implies the GPs have more bargaining power in this game, since they are supplying the resource that, ultimately, is the scarce one.

An incumbent GP, then, makes a take-it-or-leave-it offer to his original LP that consists of a share of the expected profits $\Pi(\phi, N)$. Since the participation constraint of an agent, whether LP or GP, in a sector with N competing partnerships depends on the profits he could make in the sector competing with $N + 1$ partnerships, we need to use backward induction to solve for participation constraints.

Denote as:

$V(\phi, N)$ Given ϕ, the expected payoff to a GP making a take-it-or-leave-it offer to an LP, when there are currently N active and informed GPs competing in the market.

$W(\phi, N)$ Given ϕ, the expected payoff to an informed LP, if there are currently $N - 1$ informed GPs, and the LP reveals ϕ to a non-incumbent GP through a take-it-or-leave-it offer.

If there are N GPs currently informed and active in the sector, then profits in a symmetric equilibrium are given by equation (4). An incumbent GP, then, with a successful offer to an LP, whether incumbent or newly informed by the GP, will earn this profit less what he offered the LP. The lowest offer that will succeed must pay the LP what he would obtain if he declined, and sought another non-incumbent GP as a partner, $W(\phi, N + 1)$. Therefore:

$$V(\phi, N) = \Pi(\phi, N) - W(\phi, N + 1). \tag{6}$$

To obtain an expression for $W(\phi, N+1)$, consider the LP who rejects the original GP's offer and seeks a new partner. The LP informs this new, $N+1^{th}$, GP of ϕ, and makes a take-it-or-leave-it offer. This offer must be at least as large as what the new GP could obtain, making an offer to a new LP, rationally anticipating that the rejected LP, who is now informed, will seek yet another GP, resulting in $N+2$ competitors. The lowest successful offer then gives the LP the following expected payoff:

$$W(\phi, N+1) = \Pi(\phi, N+1) - V(\phi, N+2). \tag{7}$$

Substituting recursively then gives us the following expressions for the expected payoffs for the LP and GP:

$$V(\phi, N) = \Pi(\phi, N) - \Pi(\phi, N+1) + V(\phi, N+2), \tag{8}$$

and

$$W(\phi, N) = \Pi(\phi, N) - \Pi(\phi, N+1) + W(\phi, N+2) \tag{9}$$

Solving the above system requires terminal values for the expected payoff functions. Recall the total number of potential GPs is M. When there are already M GPs informed and competing in the sector, then the LPs have no outside option beyond a competitive financial return, which is assumed to be zero. They will accept any offer from a GP that pays them this expected return, so:

$$W(\phi, M+1) = 0, \tag{10}$$

and

$$\begin{aligned} V(\phi, M) &= \Pi(\phi, M) \\ &= \frac{2\phi^2}{C}\frac{1}{(M+1)^2}. \end{aligned} \tag{11}$$

Suppose, then, that there are $M-1$ GPs currently competing in the sector. An offer made by a GP to an LP will be accepted if the LP gets more than he would making an offer to the M^{th} GP, rationally anticipating that the rejected GP, who is informed about ϕ, will seek yet another LP, resulting in M competitors. But once the M^{th} and final GP is informed he can make an offer to one of the remaining LPs, who have only the competitive outside option. Therefore:

$$W(\phi, M) = 0, \tag{12}$$

and

$$\begin{aligned} V(\phi, M-1) &= \Pi(\phi, M-1) \\ &= \frac{2\phi^2}{C}\frac{1}{M^2}. \end{aligned} \tag{13}$$

The critical players in the system are the GPs and LPs when there are $M-2$ competitors. At this point the LP has bargaining power. An LP approaching the $M-1^{th}$ GP can make an offer that will be accepted if it exceeds the profit he would earn seeking a new partner, anticipating the rejected LP will also seek a new partner, so that the $M-1^{th}$ GP will end up competing with M other GPs. Therefore:

$$W(\phi, M-1) = \frac{2\phi^2}{C}\left(\frac{1}{M^2} - \frac{1}{(M+1)^2}\right) > 0. \tag{14}$$

Then, an offer made by a GP to an LP will be accepted if the LP gets more than $W(\phi, M-1)$. Therefore:

$$V(\phi, M-2) = \frac{2\phi^2}{C}\left(\frac{1}{(M-1)^2} - \frac{1}{M^2} + \frac{1}{(M+1)^2}\right), \tag{15}$$

which is larger than $V(\phi, M-1)$ when $\phi > 0$.

By iterating these steps, we can solve for the general form of the payoff functions $V(\phi, N)$ and $W(\phi, N)$. If $N < M$, then:

$$V(\phi, N) = \frac{2\phi^2}{C}\sum_{j=1}^{A(N,M)}\frac{(-1)^{j+1}}{(N+j)^2}, \tag{16}$$

and

$$W(\phi, N) = \frac{2\phi^2}{C}\sum_{j=1}^{B(N,M)}\frac{(-1)^{j+1}}{(N+j)^2}. \tag{17}$$

where $A(N, M) = M-N$ and $B(N, M) = M-N+1$, when $M-N$ is odd, and $A(N, M) = M-N+1$ and $B(N, M) = M-N$, when $M-N$ is even.

4 Model Predictions: Secrecy, Return Persistence, and Fund Flows

In this section we derive predictions from our model in terms of information secrecy, return persistence, and fund flows for the interesting scenario where investment in the sector is optimal, i.e., $\phi > 0$. To facilitate these derivations, we first present Lemma 1.

Lemma 1 *Let* $F(N, N+K) = \sum_{j=1}^{K}(-1)^{j+1} f(N+j)$ *where* $K \in \mathcal{N}^{++}$, $N \in \mathcal{N}^{++}$, *and* $f(\cdot)$ *is a function satisfying* $f(N) > f(N+1) > ... > f(N+K-1) > f(N+K) \geq 0$. *Then* $F(N, N+K) > 0$.

Proof: See Appendix for all proofs.$\blacksquare$

We now establish that each agent who can make a take-it-or-leave-it offer at any point will act in equilibrium to avoid the information spillovers that would result from soliciting outside potential partners. To make "secrecy" optimal, we must show both parties obtain higher expected payoffs with less competition. The next proposition accomplishes this.

Proposition 1 *The payoff functions,* $V(\phi, N)$ *and* $W(\phi, N)$, *are weakly decreasing in* N, *and are strictly decreasing in* N *for* $N < M$.

This result implies that an agent making a take-it-or-leave-it offer will always find it optimal to satisfy his initial partner's participation constraint, keeping the sector's estimated profitability as secret as possible. The benefits from keeping the competition to a minimum outweigh the potential benefits of reducing the solicited partner's bargaining power by increasing the number of informed agents in the sector. That is, secrecy is an equilibrium.

Consequently, the first offer an incumbent GP makes to his initial LP will be good enough to ensure that it is accepted. An incumbent LP will therefore expect to receive from a follow-on fund a payoff of:

$$W(\phi, N+1) = \frac{2\phi^2}{C} \sum_{j=1}^{B(N+1,M)} \frac{(-1)^{j+1}}{(N+1+j)^2}. \tag{18}$$

Expected returns to LPs are then straightforward to calculate. The LP contributes capital $Q^*(\phi, N)$ and received an expected payoff of $W(\phi, N+1)$. From equations (3) and (17), the per dollar expected return is:

$$\frac{W(\phi, N+1)}{Q^*(\phi, N)} = \phi \sum_{j=1}^{B(N+1,M)} (-1)^{j+1} \frac{N+1}{(N+1+j)^2}. \tag{19}$$

Proposition 2 *The expected payoff $W(\phi, N+1)$ and return $\frac{W(\phi,N+1)}{Q*(\phi,N)}$ to a LP are positive and increasing in ϕ when $\phi > 0$ and strictly positive and strictly increasing in ϕ when $N < M - 1$.*

This result establishes that the outside option for an LP is positive and increasing in ϕ. Any successful offer by an incumbent GP to an LP, whether new or incumbent, will pay positive expected profits that are increasing in estimated future profitability. Thus, as long as expected sector profitability increases with past returns, we have established that expected returns to LPs will show persistence for follow-on funds.

In our model, fund flows will follow performance, as long as expected future sector profitability, ϕ, increases with past returns.

Proposition 3 *Aggregate flows committed to a sector:*

1. *increase in estimated future profitability, ϕ,*
2. *increase in the degree of competition, N, and*
3. *are more responsive to estimated profitability when there is more competition.*

Flows respond to higher expected sector profitability for the same reasons they do in Berk and Green (2004). While marginal costs rise with fund size, and with aggregate industry flows, the higher expected returns compensate investors for these increased costs.

Increased competition leads partnerships to invest more aggressively because with more competitors, each fund internalizes less of their impact on aggregate flows, and the resulting increase in costs. By the same logic, the model predicts that flows are more responsive to performance the more competitive the sector.

The set of possible partners for a GP in our model is unlimited, while the number of potential partners for an LP is finite. As a result, the GPs in our model always have more bargaining power and command a greater share of the profits. They are in, relatively, scarce supply. The following result formalizes the intuitive link between this relative scarcity and the share of value created accruing to both partners. Recall that $V(\phi, N)$ is the expected payoff to a GP, with N active incumbents, while $W(\phi, N+1)$ is the expected profit that accrues to an LP. When there are M active partnerships, the LP's only outside options is a competitive return of zero, so the difference between his payoff and that of the GP is the value of the partnership. The proof of the proposition shows by a recursive argument that this difference is a lower bound for the difference between the expected profits to the GP and LP for any N.

Proposition 4 *For any N, the difference between the payoff functions for the GP and LP, $V(\phi, N) - W(\phi, N)$ is constant, and $V(\phi, N) > W(\phi, N+1)$.*

The impact of increased competition on the relative bargaining power of the GP and the LP is more complex. In Figure 1 we plot the share of total profits accruing to the GP, $\frac{V(\phi,N)}{\Pi(\phi,N)}$, and the LP, $\frac{W(\phi,N+1)}{\Pi(\phi,N)}$. Evidently, these are not monotonic in the degree of competition. The LP's relative bargaining power is tied both to the number of potential GPs available, and to the extent to which profits dissipate with additional competition in the sector. These quantities are changing at different rates, creating the non-monotonic relationship evident in the figure.

Figure 2 depicts the relationship between the degree of competition, N, the expected return to the partnership as a whole, and the expected return to the LP. The expected return to the LP decreases monotonically. As N increases the expected payoff to the LP decreases. The capital committed also falls, but not by enough since the partnership fails to internalize the impact on aggregate profits, and this problem becomes more severe with more competitors.

Kaplan and Schoar (2005) and Phalippou and Gottschalg (2008) find that future performance increases with fund size and is lower for first-time funds. The model suggests these outcomes when viewed as applying to the cross section of funds. If we fix N and M—that is, control for the bargaining power of the LPs—the covariance between the profits shared with the LP and the quantity of money invested is:

$$\begin{aligned} cov\left(W(\phi, N+1), Q^*(\phi, N)\right) &= cov\left(\frac{2\phi^2}{C}\sum_{j=1}^{B(N+1,M)}\frac{(-1)^{j+1}}{(N+1+j)^2}, \frac{2\phi}{C}\frac{1}{N+1}\right) \\ &= cov\left(\phi^2, \phi\right)\frac{4}{C^2(N+1)}\sum_{j=1}^{B(N+1,M)}\frac{(-1)^{j+1}}{(N+1+j)^2}. \end{aligned} \tag{20}$$

We showed in Proposition 2 that $\sum_{j=1}^{B(N+1,M)}\frac{(-1)^{j+1}}{(N+1+j)^2}$ is positive. We know that $cov(\phi^2, \phi)$ is positive for $\phi > 0$. Hence, if we control for N and M, then the cross-sectional variations in Q^* will help predict the expected profit the LP will collect from follow-on funds.

The same derivation can be made for the expected return to the LP:

$$\begin{aligned} cov\left(\frac{W(\phi, N+1)}{Q^*(\phi, N)}, Q^*(\phi, N)\right) &= cov\left(\phi\sum_{j=1}^{B(N+1,M)}(-1)^{j+1}\frac{N+1}{(N+1+j)^2}, \frac{2\phi}{C}\frac{1}{N+1}\right) \\ &= var(\phi)\frac{2}{C}\sum_{j=1}^{B(N+1,M)}(-1)^{j+1}\frac{1}{(N+1+j)^2}. \end{aligned} \tag{21}$$

The logic behind the model also suggests expected returns to LPs should be lower for first-time funds. The GP in the model makes the first take-it-or-leave-it offer. In the absence of information spillovers the LP would earn a competitive return of zero. The LP in a follow-on fund expects to collect $W(\phi, N+1)$ if $\phi > 0$ and zero otherwise. Accordingly, in the initial fund the GP could, in principle, offer the LP a negative expected profit equal to $-E\left[W(\phi, N+1)\cdot I(\phi>0)\right]$ and the LP's ex-ante participation constraint would bind. The expected profit on the first project will therefore be smaller than the expected profit on the following project, as observed in the data. Evidently, this relationship also holds for expected returns, given the signs of each term. It seems unlikely, of course, that LPs will enter initial private equity partnerships without some disclosure of proprietary information to them, but as long as *more* such information accrues to them through experience in the sector or with the strategies pursued by the GP, we would expect the LP's bargaining power to increase across generations of funds.

5 Robustness to Variations in the Model's Assumptions

So far, we have kept our model as frictionless as possible in order to maximize its tractability and transparency. For example, our model is a game of disclosure, where all information is costlessly and credibly communicated from incumbent partners to outsiders when they decide to disclose. This allows us to abstract from the complications of a bargaining game under asymmetric information, where offers from incumbents would signal private information. We now briefly consider the robustness of the model's implications with respect to variations in the assumptions.

Three central elements are at work in our model:

1. Potential expected returns going forward must be correlated with past returns. This plays the same role in our model as in Berk and Green (2004).

2. The capacity to generate excess returns must be something the incumbent LP can, to some extent, take with him on defecting from an existing partnership. This, along with the first item above, ensures the LP's outside option increases in past returns.

3. There must be frictional costs of some sort that dissipate rents when the LP defects. These costs ensure secrecy is an equilibrium, and that the GP, as first mover, captures a greater share of the available surplus.

The simplifying assumptions in our model serve to make the interaction between these three elements particularly stark. The LP is assumed to be able to fully communicate information acquired through past participation and thus completely replicate with an outside GP whatever he could achieve with the incumbent GP (item 2). None of the ability to create expected returns is specific to the manager. The only cost to going to an outsider is increased competition (item 3). This cost acts like an endogenously determined, non-constant discount factor in a repeated offer game, steadily diminishing the shared surplus as the set of informed parties grows.

If we were to allow for noisy transmission of information to outsiders, asymmetric information in dealing with outsiders, or additional skills unique to the incumbent GP, it would certainly complicate the model. It does not, however, appear likely to reverse or overturn the model's implications, as long as the three elements described above are still present, as they seem likely to be in the institutional setting under consideration. Since the GP is making the first take-it-or-leave-it offer, different assumptions about the profits, their origins, or their transferability are likely to affect the expected surplus the GP is collecting, but not the predictions that secrecy is optimal or that incumbent LPs collect rents that increase with past returns.

6 Endogenous Entry at Intermediate Date

The analysis to this point takes the number of potential GPs as fixed and finite, which imparts a bargaining advantage to them in their dealings with LPs. In this section we illustrate how the set of potential GPs at the intermediate date can be endogenously determined through a fixed cost of entry. As the number of competing GPs increases, per-partnership profits fall. If there is a fixed cost for non-incumbent GPs to enter, this will limit the set of potential, non-incumbent GPs. That limit will, in turn, reflect the expected profitability of the sector, ϕ, leading to a dependence between past returns and the relative bargaining power of the two parties. Nevertheless, the functions describing the division of rents between the GP and the LP retain the same form, with a few notational complications.

Suppose that non-incumbent GPs face a fixed cost of k upon entering the sector. The timing of events is as follows. First, the new GP receives information about ϕ from an LP, along with an offered sharing rule. Next, the GP makes the entry decision, and either incurs the fixed cost k or walks away and receives a payoff of zero. Third, the GP can reject the offer from the initial LP, and solicit financing from a new LP while disclosing ϕ. Thus, the entry cost is naturally interpreted as effort or expenditure required for a new GP to reach a point where he can effectively solicit funding in the sector in question.

Once the entry cost is paid, symmetric equilibria determining profits and investment in the industry are the same as before. If there are $N-1$ informed GPs competing in the sector, then upon learning the value of ϕ a new GP will enter only if:

$$\begin{aligned} \Pi(\phi, N) - k &= \frac{2\phi^2}{C}\frac{1}{(N+1)^2} - k \\ &\geq 0. \end{aligned} \tag{22}$$

The maximum number of potential entrants is given by the $M^*(\phi)$ that exhausts the profitability of the industry. Thus, $M^*(\phi)$ is the maximum integer M such that:

$$\frac{2\phi^2}{C}\frac{1}{(M+1)^2} - k \geq 0 \tag{23}$$

or equivalently, such that:

$$M \leq \phi\sqrt{\frac{2}{kC}} - 1. \tag{24}$$

Now suppose an informed GP makes an offer to his former LP when there are N informed GPs in the sector. The GP will have to offer the LP at least $W(\phi, N+1)$ and will make:

$$V(\phi, N) = \Pi(\phi, N) - W(\phi, N+1). \tag{25}$$

If the LP rejects the offer, he knows that the informed GP has already paid the entry cost, hence will enter as long as $\phi > 0$. The LP will have to make an offer to an uninformed GP of at least $V(\phi, N+2) - k$, the new GP's expected payoff if he were to reject the offer. Thus:

$$\begin{aligned} W(\phi, N+1) &= \Pi(\phi, N+1) - k - [V(\phi, N+2) - k] \\ &= \Pi(\phi, N+1) - V(\phi, N+2). \end{aligned} \tag{26}$$

In order for the LP to have an outside option, he needs to ensure that entering the sector with an uninformed GP, given that the GP who made him the earlier offer will also enter the sector, promises positive expected profits, that is:

$$\Pi(\phi, N+1) \geq k. \tag{27}$$

At $M^*(\phi)$, the maximum number of partnerships that keeps the sector profitable, the informed LP will not be able to convince an uninformed GP to enter a partnership with him because the $M^*(\phi)+1^{th}$ partnership will not be profitable. Therefore:

$$W(\phi, M^*(\phi)+1) = 0. \tag{28}$$

An informed GP at $M^*(\phi)$ is sure that no uninformed GP will ever pay k to enter the sector, hence the informed GP has all the bargaining power at $M^*(\phi)$. Thus:

$$V(\phi, M^*(\phi)) = \Pi(\phi, M^*(\phi)), \tag{29}$$

which is greater or equal to k, by definition of $M^*(\phi)$.

Similarly, if an informed LP was to reject a GP offer at $M^*(\phi)-1$ and make an offer to a $M^*(\phi)^{th}$ GP, this previously uninformed GP would know that by rejecting the LP's offer, the rejected LP would be unable to find a $M^*(\phi)+1^{th}$ GP. Hence:

$$W(\phi, M^*(\phi)) = 0, \tag{30}$$

and

$$V(\phi, M^*(\phi)-1) = \Pi(\phi, M^*(\phi)-1). \tag{31}$$

By recursion, we can find the exact same value functions $V(\cdot,\cdot)$ and $W(\cdot,\cdot)$ as in our model with M, except that $V(\cdot,\cdot)$ represents the payoff to an incumbent GP and $V(\cdot,\cdot)-k$ represents the payoff to a non-incumbent GP.

Figures 3 and 4 show how the relative bargaining power and expected returns vary with expected sector profitability, ϕ, when there is one incumbent partnership. As Figure 3 illustrates, the LP has no bargaining power when expected profits are low, because the fixed cost would deter entry by other potential partners. Any new entrant would dissipate profits to a point where the entry cost could not be recovered. (This point corresponds to $N = M$ in the model without endogenous entry.) As expected profits rise, bargaining power for the LP increases, but at a decreasing rate. The increase in the number of potential GPs who could enter rises, but each new potential entrant dissipates industry profits less. The increase in bargaining power as sector profitability rises translates into higher expected returns for the LP, as evident in Figure 4. Both the rents available, and the LP's ability to extract them increase with ϕ.

7 Endogenous Entry at Initial Date with a Calibrated Example

This section describes how we can endogenize entry at an initial date, before any learning has occurred, and through simulation generate a cross section of partnership returns with which to examine the quantitative properties of the model.

There are two periods, and three dates, $t = 0, 1, 2$. The entry cost at the beginning of the initial period is k_0, which we assume to be greater than the entry cost at the intermediate date, k_1. The latter corresponds to the cost of entry in Section 6. When entering the sector at the intermediate date, the GP has been informed by the LP about the sector. The fact that the LP has already invested, through a competing GP, in the sector makes the subsequent entry with a non-incumbent GP less costly or difficult.

The return to the sector from $t = 0$ to $t = 1$ (on the first dollar invested) is $r_1 = \rho + \epsilon_1$, where ϵ_1 is distributed normally with mean zero and variance σ^2. The expected return ρ is unkown to both managers and investors, who have shared priors that it is normally distributed with mean ϕ_0 and variance η^2. The information set of investors at the initial date is the number of partnerships active in the sector. Incumbent partners observe their payoffs over the initial period, and since strategies are common knowledge they can infer r_1. They will then update as Bayesians to estimate the expected profitability going forward as:

$$\phi_1 = \left(\frac{\frac{1}{\eta^2}}{\frac{1}{\eta^2} + \frac{1}{\sigma^2}} \right) \phi_0 + \left(\frac{\frac{1}{\sigma^2}}{\frac{1}{\eta^2} + \frac{1}{\sigma^2}} \right) r_1. \qquad (32)$$

Given this estimate of the sector's profitability, we can calculate the number of GPs who, if informed, would enter the sector at the initial date. This is $M^*(\phi_1)$, the maximum integer such that:

$$M \leq \phi_1 \sqrt{\frac{2}{kC}} - 1. \tag{33}$$

Given an estimate of the sector's profitability, ϕ_0, which is known by every agent, the number of partnerships $N^*(\phi_0)$ to enter in the first period will be the maximum integer N such that:

$$\Pi(\phi_0, N) + E\left[\Pi(\phi_1, N) \cdot I(\phi_1 > 0)\right] - k_0 \geq 0. \tag{34}$$

Notice that if ϕ_1 is sufficiently small compared to ϕ_0, then $M^*(\phi_1) \leq N^*(\phi_0)$ and the LP owns no bargaining power at the intermediate date. How small ϕ_1 has to be for that situation to occur depends on how the cost of entry at the intermediate date, k_1, compares with the cost of entry at the initial date, k_0. We would expect k_0, the cost of entering a sector in the first period, without special information about the sector, to be greater than k_1, the cost of entering the sector with the information shared by an incumbent LP.

The second term in the equilibrium entry condition, (23), is the expectation of a nonlinear function of ϕ_1, and does not admit a closed form solution. It is straightforward to calculate numerically, however, and also allows us to solve numerically for $N^*(\phi_0)$. We can then simulate two periods of returns for a large cross section of artificial funds, and in this way evaluate the model's ability to quantitatively capture salient empirical facts about private equity investing.

We simulate a cross section of 50,000 private equity partnerships. Each partnership invests in a fund, learns about the profitability of the sector, and then decides whether to invest in a follow-on fund. Each simulated fund is a representative of its own "sector." All funds are identical initially, and therefore operate in sectors with the same number $N^*(\phi_0)$ of competitive funds. First-period returns are drawn independently, expectations are updated, $M^*(\phi_1)$ is determined for each fund, and the expected profits are split according to the bargaining model between the LP and GP. We then draw returns again to determine realized profits for those partnerships that survived and were sufficiently profitable to launch follow-on funds.

The artificial cross section of all first-generation funds and the surviving second-generation funds is used to compare the simulated outcomes to unconditional moments from past empirical studies about venture capital and buyout funds. We set parameter values to approximate these unconditional moments. To evaluate the model, we ask how well it captures the facts regarding fund returns across initial and follow-on funds, and how closely we can match the model's predictions to those achieved with a 2/20 sharing rule.

Our model predicts how expected profits are shared between GPs and LPs, but the sharing contract in terms of realized profits is indeterminate. In the data, a particular contract predominates. Under the so-called "2/20 rule" GPs charge investors an annual management fee of 2 percent of assets under management and a carried interest of 20 percent of profits, adjusted for a hurdle rate, when these adjusted profits are positive. We assume that the hurdle rate is a close proxy for the normal return a fund should generate. Hence, fund returns in excess of the the hurdle rate are considered as abnormal, and correspond to the excess returns in our model. One question we ask, therefore, is how close the 2/20 contract comes to implementing the optimal solutions from our model in the simulated data. That is, does the division of surplus using this specific contract approximate, both unconditionally and conditional on previous returns, the division of surplus implied by our model.

Using simulations, Metrick and Yasuda (2007) estimate the present value of management fees to be paid over the lifetime of 238 private equity funds. The average fund in their simulation has management fees worth around 12 percent of the fund's assets under management. Therefore as our benchmark fee structure for the lifetime of a fund, we use a "12/20" rule, which is basically a "2/20" rule, but accumulated over ten years, discounted in present value terms, and adjusted for idiosyncrasies in private equity compensation contracts. To anticipate our results, we find that we can match the unconditional moments and observed levels of persistence for the theoretical model, or for the model when a 2/20 rule is applied rather than the optimal sharing rule, but not simultaneously for both.

The parameter values used in our simulations are presented in the table below. Given that most funds have lifetimes lasting ten years, each period in our model is assumed to last ten years.

Keep in mind that the measure of expected profitability, ϕ_0, applies only to the first dollar invested. Average realized profitability will be much lower.

Table 1: Parameter values used in our simulation

	Symbol	Value
Diseconomy-of-scale cost	C	0.0016
Entry cost at initial date ($M)	k_0	60
Entry cost at intermediate date ($M)	k_1	25
Unconditional mean of profitability	ϕ_0	0.5
S.D. of profitability	η	1
S.D. of idiosyncratic noise	σ	0.1

Assuming normally distributed excess returns simplifies the learning mechanism in our simulation. It has the disadvantages of producing excess returns that are smaller than -100% with positive probability. Keep in mind, however, that this applies to realized *excess* returns. The frequency with which limited liability would be violated would be much lower for realized total return, though of course it is still possible. The probability of this event would decrease if we were to choose lower values for η and σ, but this would also decrease the probability of partnerships being dissolved at the intermediate date. We use a relatively low value for the idiosyncratic noise in order to have cross-sectional differences in funds that are mostly driven by differences in expected profitability. Notice, however, that with funds lasting ten years and investing in multiple projects, the notion that idiosyncratic noise has a relatively small effect on accumulated returns may not be completely unreasonable.

The parameter values were chosen through trial and error to approximate several unconditional moments of the cross section. We interpret the unconditional cross section as the pooled set of initial and follow-on funds.

When we average over the full cross section, the average fund size is $176.6M, close to the estimate of $172.2M by Kaplan and Schoar (2005). The probability of raising a follow-on fund in our cross section is 69 percent, close to the 70 percent Zarutskie (2008) reports for venture capital funds. The average partnership-wide return (i.e., gross-of-fee abnormal return) we generate is 1.40 percent annually, in-between the 1 percent reported by Hwang, Quigley and Woodward (2005) and the 3 percent reported by Phalippou and Gottschalg (2008).

In our simulated pooled cross section of funds, the average revenue a general partner collects from management fees and carried interest, over the lifetime of a fund, should, according to our model, represent 19.10 percent of assets under management. Using the same simulated partnership-wide returns, but applying a 2/20 rule instead of the model's predicted profit-sharing rule yields an average of 19.05 percent. Thus, the 2/20 contract widely used in practice produces in our cross section average revenues that are quantitatively similar to what our model predicts. These simulated averages are also close to the 18.9 percent that Metrick and Yasuda (2007) estimate by simulating alternative scenarios for real-life funds. The 2/20 rule in our cross section results in the average LP collecting an abnormal profit of \$6.0M over the two periods, which we argue is close to the zero-profit prediction of our model, given that this \$6.0M profit necessitates an average investment of \$176.6M over 10 or 20 years. In our simulations, the average LP receives annually an abnormal return, net of fees, of -0.49 percent according to our model and -0.51 percent according to the 2/20 rule. These numbers are slightly negative, similar to findings by Kaplan and Schoar (2005) and Phalippou and Gottschalg (2008). Finally, the abnormal return, net of fee, the average LP receives annually from investing in an initial fund is -1.13 percent according to both the model and the 2/20 rule. Since in this parameterization the 2/20 rule provides LPs investing in an initial fund with the same *expected* return as the optimal contract in our model, we use the *realized* return to a LP given by a 2/20 rule to specify the realized return a LP would receive in our model (remember: the functional form of our model's optimal contract is indeterminate).

We now turn to how the model behaves when we consider cross-sectional relationships between follow-on funds and past performance, the central concern of our paper.

Figure 5 plots the simulated relationship between returns earned on initial funds and the size of their follow-on funds. This relationship is positive, consistent with the empirical finding of Kaplan and Schoar (2005) and Phalippou and Gottschalg (2008) and with the predictions of other learning models such as Berk and Green (2004). In our simulated sample, regressing the logarithm of fund size for follow-on funds against abnormal returns to LPs from the initial fund and a constant yields a coefficient of 1.54, close to 5 times the estimates Kaplan and Schoar (2005) and Phalippou and Gottschalg (2008) report. Note that it is difficult to compare our regression results to theirs since for these regressions they use scaled NPV-like measures they respectively call *public market equivalent* and *profitability index*, rather than abnormal returns to LPs, to measure fund performance.[5]

In Figure 6 we plot the simulated relationship between the size of follow-on funds and the expected return to LPs investing in those funds. This relationship is positive and concave, consistent with the empirical finding of Kaplan and Schoar (2005), but contrasts with the predictions of Berk and Green (2004). In Berk and Green (2004), there is no predictability in returns net of fee due to competition between investors for the services of mutual fund managers. In our model, LPs reap financial benefits from a profitable sector at an intermediate date to ensure LPs do not turn to others and introduce competition between managers. In our simulated sample of initial and follow-on funds, regressing abnormal expected returns to LPs against the logarithm of the fund's size and a constant yields a coefficient of 0.06, consistent with estimates by Kaplan and Schoar (2005) and Phalippou and Gottschalg (2008).

The number $M(\phi_1)$ of GPs who, if solicited by an informed LP at the intermediate date, would be willing to invest in the sector increases with estimated profitability. This, in turn, increases the bargaining power of incumbent LPs. Figure 7 plots this relationship and the optimal number of partnerships $N^*(\phi_0)$ to invest in the sector at the initial date (in our simulation $N^*(\phi_0) = 4$). The figure can be divided in three regions: a region where initial returns are so low that all agents exit the sector, a region where returns are sufficiently high that uninformed GPs would to enter they became informed, and an intermediate region where only the GPs who have already paid their entry cost are willing to invest in the sector during the second period. In our model, the only incumbent LPs with bargaining power are those who invested in funds in the region with high returns. As a result, those LPs should be the only ones able to collect a share of their partnership's positive expected excess profits.

Figure 8 plots the relationship our model predicts between returns earned from initial funds and expected returns to be collected by incumbent LPs from follow-on funds. The figure also plots the returns each simulated LP would collect, on average, if it were facing a 2/20 compensation scheme instead (using, for each simulated initial scenario, 50,000 simulated follow-on scenarios). Similarly,

[5]For example, Kaplan and Schoar (2005) compute a fund's public market equivalent by discounting all of the fund's cash outflows to investors (i.e., net of fees) using the total return on the S&P 500 and then dividing the sum of these discounted cash outflows by the sum of the fund's discounted cash inflows that came from investors.

Figure 9 compares the expected revenue per dollar of assets under management an incumbent GP would collect based on our model and on a 2/20 rule. These figures suggest that, under the current parameterization, our model's optimal contract generates more cross-sectional variation in the expected share of the profits going to the GPs than does a 2/20 rule. Since the GP and LP share the total expected payoff, this in turn dampens the variation in the returns going to the LPs. A way to quantify these cross-sectional variations is through the use of cross-sectional regressions. For example, in our simulated sample of partnerships, regressing a partnership's follow-on return to LPs from a 2/20 rule over its past returns to LPs and a constant yields a coefficient of 0.20, close to the 0.22 reported by Phalippou and Gottschalg (2008).[6] However, running the same regression, but using follow-on returns predicted by our model rather than by a 2/20 rule, yields a coefficient of 0.10, which represents only half of the persistence we get with the 2/20 rule. Note that in a different parameterization that resulted in more partnerships entering each sector at the initial date, we were able to reduce the difference in persistence predictions between our model and a 2/20 rule. But in that case, the simulated levels of performance persistence were significantly lower than the empirical estimate of 0.22 found in Phalippou and Gottschalg (2008).

It is however evident from Figure 8 that some follow-on funds raised in our model would not be raised if a 2/20 rule was implemented. Follow-on funds with relatively small, but positive ϕ would be profitable according to our model, but rational and informed LPs would refuse to finance such investments in a 2/20 environment. The fixed fee of 2 percent per year would end up eating more than the expected returns available in the sector, and LPs would expect to lose money on their investment. For this reason, we compare the persistence coefficients generated by our model and by a 2/20 rule when only considering follow-on funds that would be raised given both profit sharing schemes. For these funds, our model generates a level of performance persistence of 0.11 whereas the 2/20 rule generates a coefficient of 0.15. That is, considering funds that are simultaneously viable for LPs in a 2/20 rule and in our model reduces the difference in the cross-sectional implications between our model and a 2/20 rule. This, in turn, allows the calibrated model to more closely

[6] Phalippou and Gottschalg (2008) use a similar sample to that of Kaplan and Schoar (2005), but correct for potential sample-selection biases. The correction leads to an estimate of performance persistence (i.e., 0.22) that is smaller, yet significant, than what Kaplan and Schoar (2005) report (i.e., between 0.32 and 0.54, depending on the regression).

match observed outcomes of persistence without pushing the persistence under the 2/20 rule to implausible levels.

To summarize these findings, at parameters chosen to match the unconditional moments of the pooled cross section, such as average fund size and survival probability, we can ensure the right level of persistence for the 2/20 contract. The model, while producing significant levels of performance persistence, is unable to match the empirically observed level of persistence. The model closely matches the relationship between fund size and expected returns to LPs in follow on funds, but fund size is much too responsive to past performance. This latter behavior is closely related to the low level of persistence, as one would expect from the logic of Berk and Green (2004). Thus, relative to the benchmark of our model, performance is more persistent than for actual funds because, either flows in the data are less responsive than in the model or because the 2/20 contract is too rigid, or both.

There are two ways to increase in our model the persistence in the returns to LPs. First, by limiting the number of initial entrants, one can decrease the responsiveness of flows to performance. A monopoly, for example, would invest less aggressively in follow-on funds than a duopoly. Proceeding in this direction, however, also increases the persistence in follow-on returns using the 2/20 rule. So, while our model could match the observed point estimates, the 2/20 returns under such a parameterization would overstate them. Thus, the puzzle the model presents is why the sharing rule used in practice does not allow greater sensitivity in the GPs return to past performance. It may well be that our oversimplified implementation of the 2/20 rule, which ignores many of the variations in actual contract terms, overstates the rigidity in the GP's share across periods.

Alternatively, we can manipulate the parameters that control how the follow-on returns are split between the LP and GP, to give the LP more second-period bargaining power. The difficulty here is that increasing the LP's share of the follow-on funds returns, as it reduces the GP's expected return, must be offset with lower expected returns for the LP in the initial fund. This pushes the average return to the LP in the initial fund below the levels seen in the data.

8 Conclusion

In this paper, we present a simple model of private equity partnerships. Our model is based on evident differences in the institutional setting between mutual funds and private equity funds. In contrast to the model for mutual funds in Berk and Green (2004), the learning in our model pertains to profitability associated with an emerging sector or a new strategy, rather than just with ability specific to the manager. An optimal partnership contract in this setting leads to performance persistence because incumbent investors benefit, along with managers, from increases in the estimated profitability of a given investment following high realized returns. Sharing information rents with initial investors guarantees incumbent managers that their investors will not leave them at an intermediate date to form partnerships with non-incumbent managers, resulting in information spillovers and profit-reducing competition.

While the model clearly oversimplifies many features of the data, when calibrated to the empirical moments from past studies, it captures quantitatively many important aspects of the observed outcomes. However, flows are too responsive to past performance, relative to empirical results, and returns to LPs show less persistence. The source of this discrepancy appears to be the rigidity of the 2/20 contract used in practice, as the model predicts more variation in GP returns due to past performance than can be achieved through the 2/20 sharing rule.

APPENDIX

Proof of Lemma 1: First, consider $F(N, N+K)$ when K is even. We can rewrite $F(N, N+K)$ as $F(N, N+2\cdot T)$ where $T \equiv \frac{K}{2} \in \mathcal{N}^{++}$:

$$\begin{aligned} F(N, N+2\cdot T) &= \sum_{j=1}^{2T} (-1)^{j+1} f(N+j) \\ &= \sum_{j=1}^{T} \left[f(N+2j-1) - f(N+2j) \right]. \end{aligned} \tag{A-1}$$

For any $j \in \mathcal{N}^{++}$, the term $[f(N+2j-1) - f(N+2j)]$ is strictly positive because $f(N+2j-1) > f(N+2j)$. Hence, for any $T \in \mathcal{N}^{++}$ $F(N, N+2\cdot T)$ is a sum of strictly positive number and is also strictly positive.

It remains to consider $F(N, N+K)$ when K is odd. We can write:

$$F(N, N+K) = F(N, N+K-1) + (-1)^{K+1} f(N+K). \tag{A-2}$$

Since $K+1$ is even, the last term is positive, and the first term is positive by the above argument for K even. Hence, for any combination of $N \geq 1$ and $K \geq 1$, $F(N, N+K)$ will be strictly positive. $\blacksquare$

Proof of Proposition 1: For $N > M-2$ the result is self-evident from the expressions for the terminal values in equations (11)-(15). When $N \leq M-2$, we must separately consider the cases when $M-N$ is even and odd.

When $M-N$ is odd, and $M-N \geq 2$, we must establish the following inequalities hold:

$$V(\phi, N) = \frac{2\phi^2}{C} \sum_{j=1}^{M-N} \frac{(-1)^{j+1}}{(N+j)^2} > \frac{2\phi^2}{C} \sum_{j=1}^{M-N} \frac{(-1)^{j+1}}{(N+1+j)^2} = V(\phi, N+1), \tag{A-3}$$

and

$$W(\phi, N) = \frac{2\phi^2}{C} \sum_{j=1}^{M-N+1} \frac{(-1)^{j+1}}{(N+j)^2} > \frac{2\phi^2}{C} \sum_{j=1}^{M-N-1} \frac{(-1)^{j+1}}{(N+1+j)^2} = W(\phi, N+1). \tag{A-4}$$

We first show that the inequality in (A-4) is equivalent to (A-3). Subtracting the final term from the left-hand side of (A-4) we obtain:

$$W(\phi, N) - \frac{2\phi^2}{C}\frac{(-1)^{M-N+1}}{(N+(M-N+1))^2} = \frac{2\phi^2}{C}\sum_{j=1}^{M-N}\frac{(-1)^{j+1}}{(N+j)^2}, \tag{A-5}$$

which is identical to the left-hand side of (A-3). Subtracting the same expression from the right-hand side of (A-4) gives:

$$\begin{aligned} W(\phi, N+1) - \frac{2\phi^2}{C}\frac{(-1)^{M-N+1}}{(N+(M-N+1))^2} &= \frac{2\phi^2}{C}\sum_{j=1}^{M-N-1}\frac{(-1)^{j+1}}{(N+j+1)^2} + \frac{2\phi^2}{C}\frac{(-1)^{M-N}}{(N+(M-N+1))^2} \\ &= \frac{2\phi^2}{C}\sum_{j=1}^{M-N}\frac{(-1)^{j+1}}{(N+j+1)^2}, \end{aligned} \tag{A-6}$$

which is identical to the right-hand side of (A-3). Thus, if we can demonstrate that (A-3) holds, it will follow that (A-4) holds.

Subtracting $V(\phi, N+1)$ from $V(\phi, N)$, we see that, assuming $\phi > 0$, (A-3) will hold when:

$$\sum_{j=1}^{M-N}(-1)^{j+1}\left[\frac{1}{(N+j)^2} - \frac{1}{(N+1+j)^2}\right] > 0. \tag{A-7}$$

The term $\left[\frac{1}{(N+j)^2} - \frac{1}{(N+1+j)^2}\right]$ can be reduced to $\frac{2(N+j)+1}{(N+j)^2(N+j+1)^2}$ and is strictly decreasing in j, given that:

$$\begin{aligned} \frac{\partial}{\partial j}\left[\frac{2(N+j)+1}{(N+j)^2(N+j+1)^2}\right] &= \frac{2(N+j)(N+1+j) - [2(N+j)+1]\,[2(N+1+j)+2(N+j)]}{(N+j)^3(N+1+j)^3} \\ &= -\frac{[2(N+j)+1]\,[(N+1+j)+2(N+j)]}{(N+j)^3(N+1+j)^3} \\ &< 0. \end{aligned} \tag{A-8}$$

Consequently, Lemma 1 implies that the sum in equation (A-7) is strictly positive, making the inequality always satisfied.

Now consider the case where $M-N$ is even, and $M-N>2$. Then, we must establish:

$$V(\phi,N)=\frac{2\phi^2}{C}\sum_{j=1}^{M-N+1}\frac{(-1)^{j+1}}{(N+j)^2}>\frac{2\phi^2}{C}\sum_{j=1}^{M-N-1}\frac{(-1)^{j+1}}{(N+1+j)^2}=V(\phi,N+1), \tag{A-9}$$

and

$$W(\phi,N)=\frac{2\phi^2}{C}\sum_{j=1}^{M-N}\frac{(-1)^{j+1}}{(N+j)^2}>\frac{2\phi^2}{C}\sum_{j=1}^{M-N}\frac{(-1)^{j+1}}{(N+1+j)^2}=W(\phi,N+1). \tag{A-10}$$

Condition (A-9) is identical in form to inequality (A-4), and the same steps (subtracting the final term on the LHS from both sides) will show it holds if (A-10) holds. Inequality (A-10), however, is identical in form to condition (A-3), which we have already shown to hold as a consequence of (A-8).$\blacksquare$

Proof of Proposition 2: First, if $N\geq M-1$, then $W(\phi,N+1)$ and $\frac{W(\phi,N+1)}{Q*(\phi,N)}$ equal zero for any value of ϕ. If however $N<M-1$, the result is implied by Lemma 1 and the facts that $\frac{1}{(N+1+j)^2}$ and $\frac{N+1}{(N+1+j)^2}$ are both strictly positive and strictly decreasing in j. $\blacksquare$

Proof of Proposition 3: Total flows to GPs in the sector are $\overline{Q}=NQ^*(\phi,N)$, which by (3) can be written as:

$$\overline{Q}=\frac{2\phi}{C}\frac{N}{N+1}, \tag{A-11}$$

which is, evidently, increasing in ϕ.

The second item follows if:

$$H(N)=\frac{N}{N+1} \tag{A-12}$$

is increasing in N. Differentiating,

$$H'(N)=\frac{(N+1)-N}{(N+1)^2}=\frac{1}{(N+1)^2}>0. \tag{A-13}$$

The third item follows if:

$$\frac{\partial \overline{Q}}{\partial \phi \partial N} > 0. \tag{A-14}$$

Since $\overline{Q}$ is linear in ϕ, however, this result will hold if $H'(N) > 0$, which was just established.∎

Proof of Proposition 4: From equations (6) and (7), evaluated at N rather than $N+1$, we have:

$$V(\phi, N) - W(\phi, N) = V(\phi, N+1) - W(\phi, N+1) \tag{A-15}$$

for all N. From (11) and (12):

$$V(\phi, M) - W(\phi, M) = \frac{2\phi^2}{C} \frac{1}{(M+1)^2}. \tag{A-16}$$

Finally, Proposition 1 shows $W(\phi, N)$ is decreasing in N, so that:

$$\begin{aligned} V(\phi, N) - W(\phi, N+1) &\geq V(\phi, N) - W(\phi, N) \\ &= \frac{2\phi^2}{C} \frac{1}{(M+1)^2} \\ &> 0 \end{aligned} \tag{A-17}$$

as long as M is finite.∎

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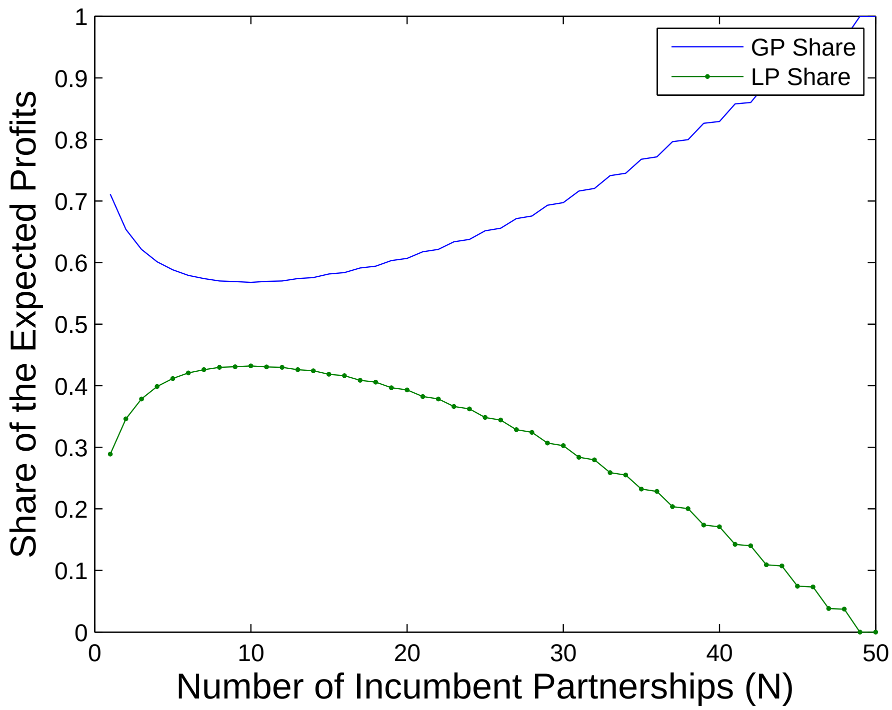

Figure 1: Fraction of total profits accruing to the GP and LP, as a function of N, the number of active partnerships. Parameter values are $M = 50$ (potential GPs), $\phi = 1$ (expected profitability), and $C = 2$ (coefficient in cost function).

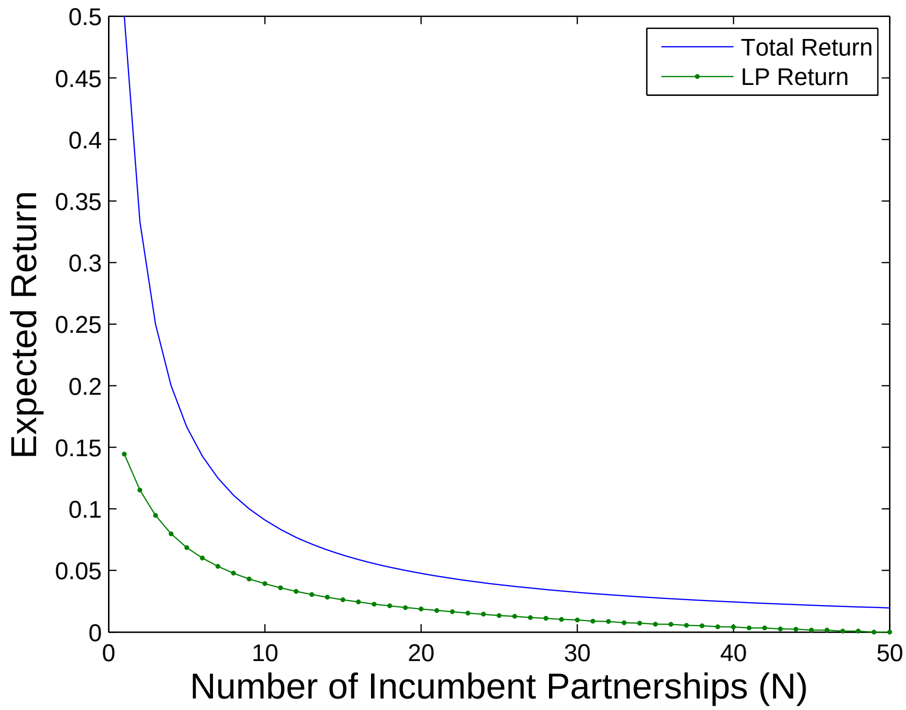

Figure 2: Expected return to the partnership and to the LP, as a function of N, the number of active partnerships. Parameter values are $M = 50$ (potential GPs), $\phi = 1$ (expected profitability), and $C = 2$ (coefficient in cost function).

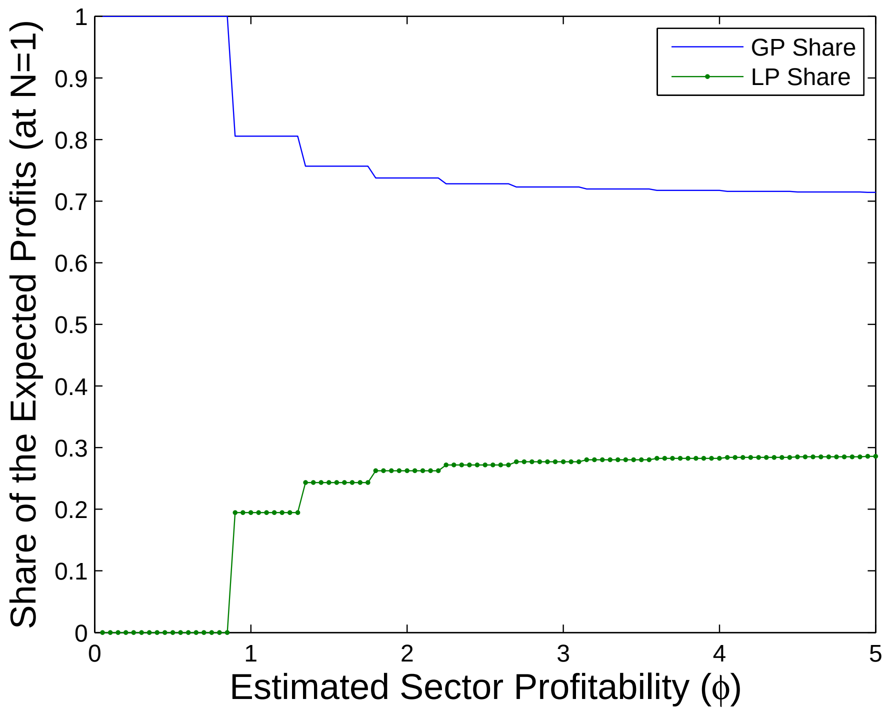

Figure 3: Share of profits accruing to the GP and the LP, as a function of ϕ, the expected profitability of the sector, with endogenous entry. Parameter values are $N = 1$ (incumbent GPs), $k = 0.02$ (fixed cost of entry), and $C = 2$ (coefficient in cost function).

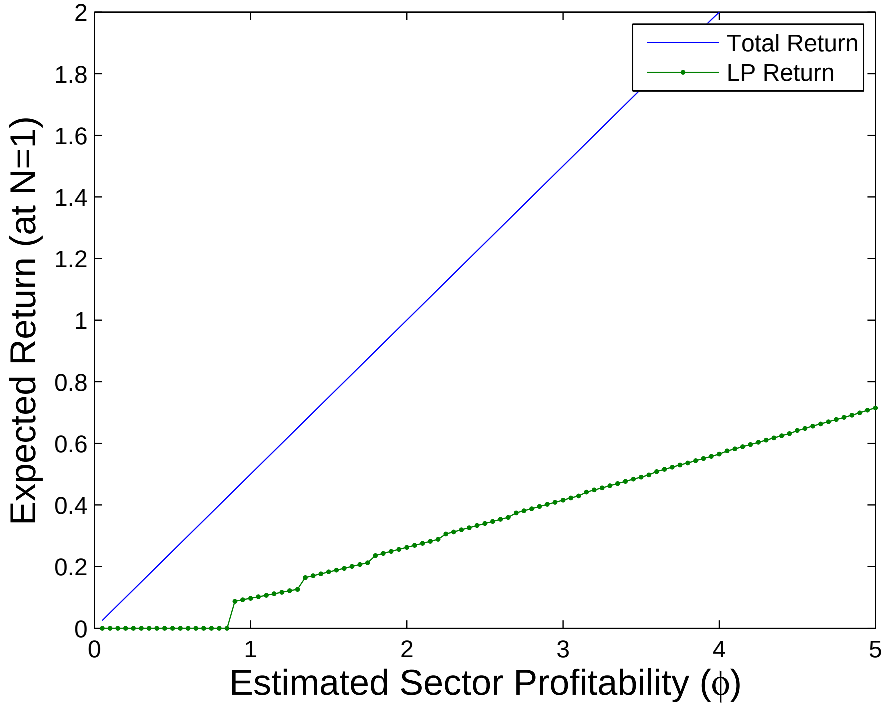

Figure 4: Expected return to the partnership and to the LP, as a function of ϕ, the expected profitability of the sector, with endogenous entry. Parameter values are $N = 1$ (incumbent GPs), $k = 0.02$ (fixed cost of entry), and $C = 2$ (coefficient in cost function).

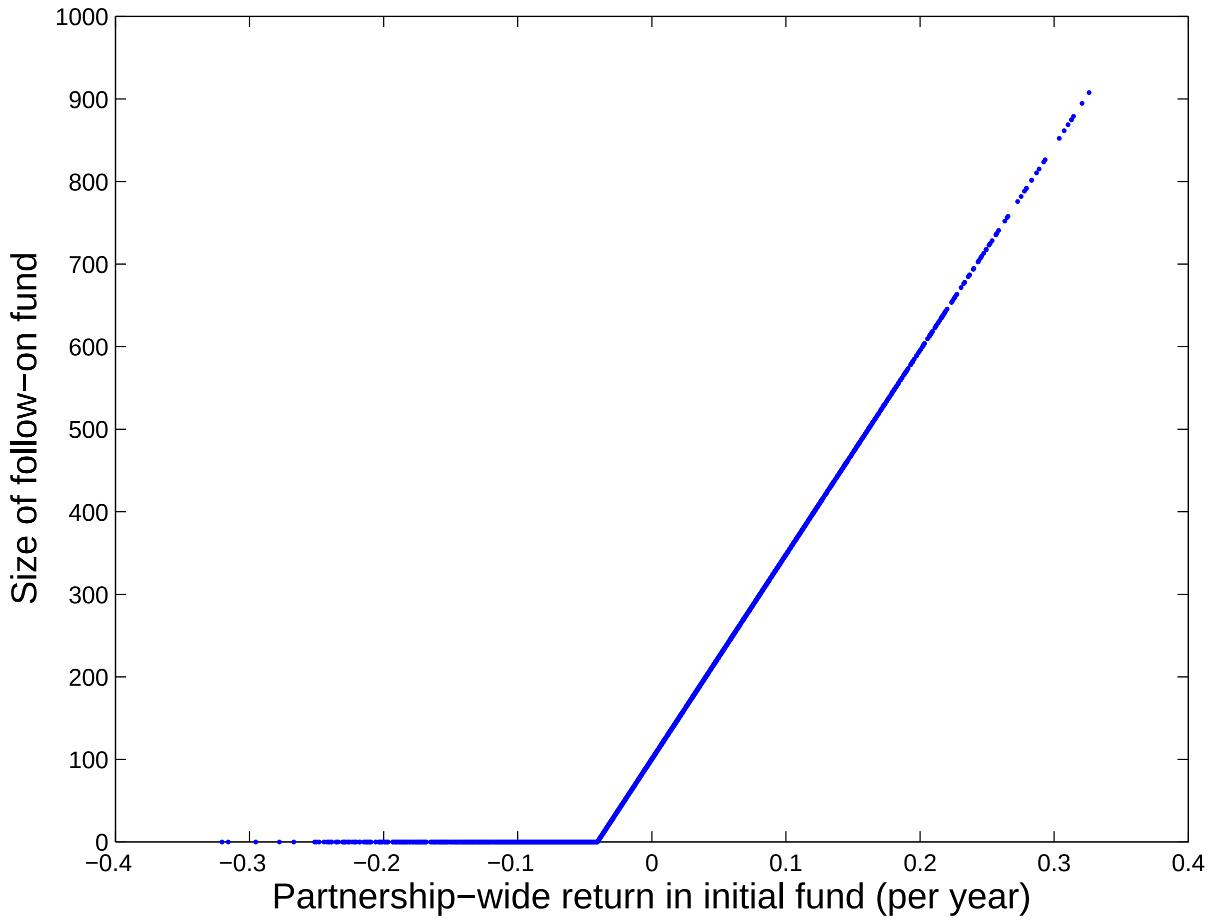

Figure 5: Size of follow-on fund as a function of the excess return earned on initial fund.

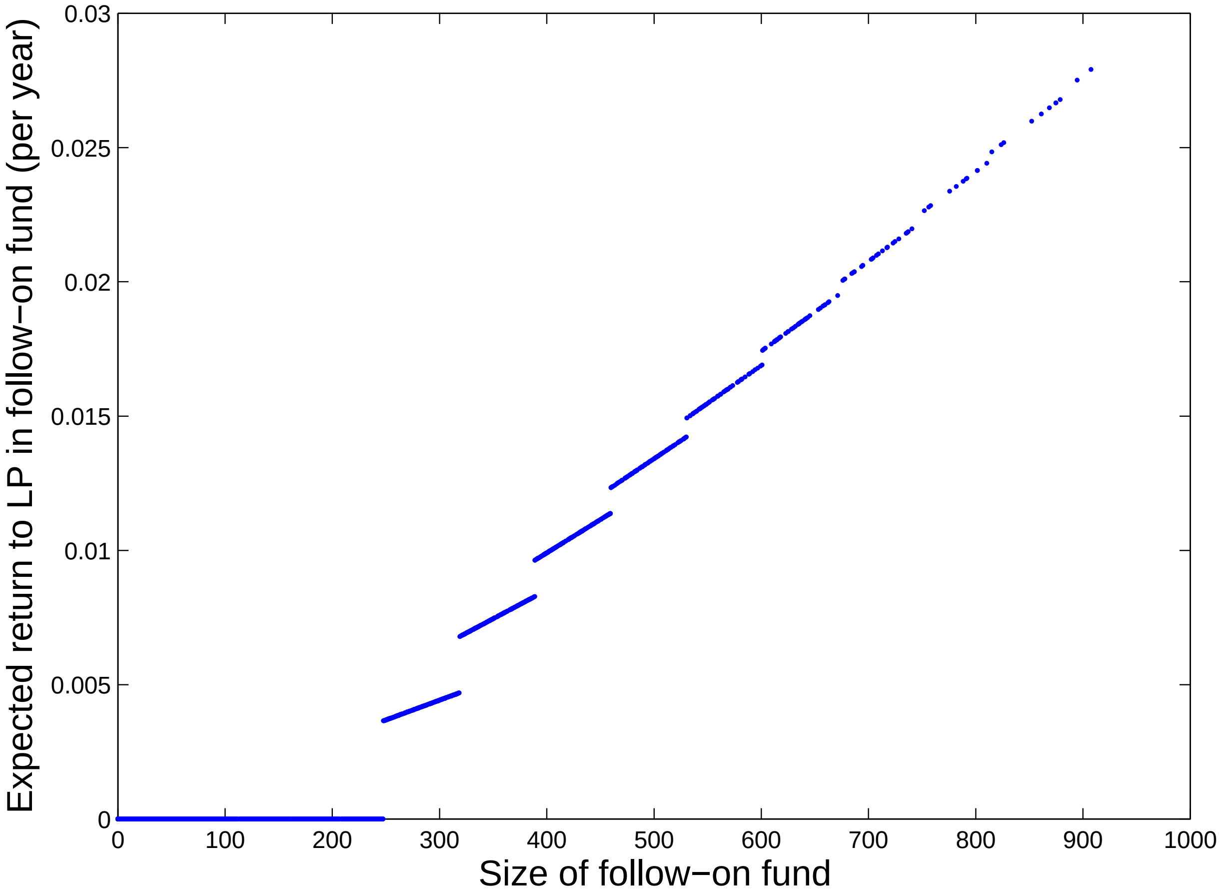

Figure 6: Expected return to LP in follow-on fund as a function of its size.

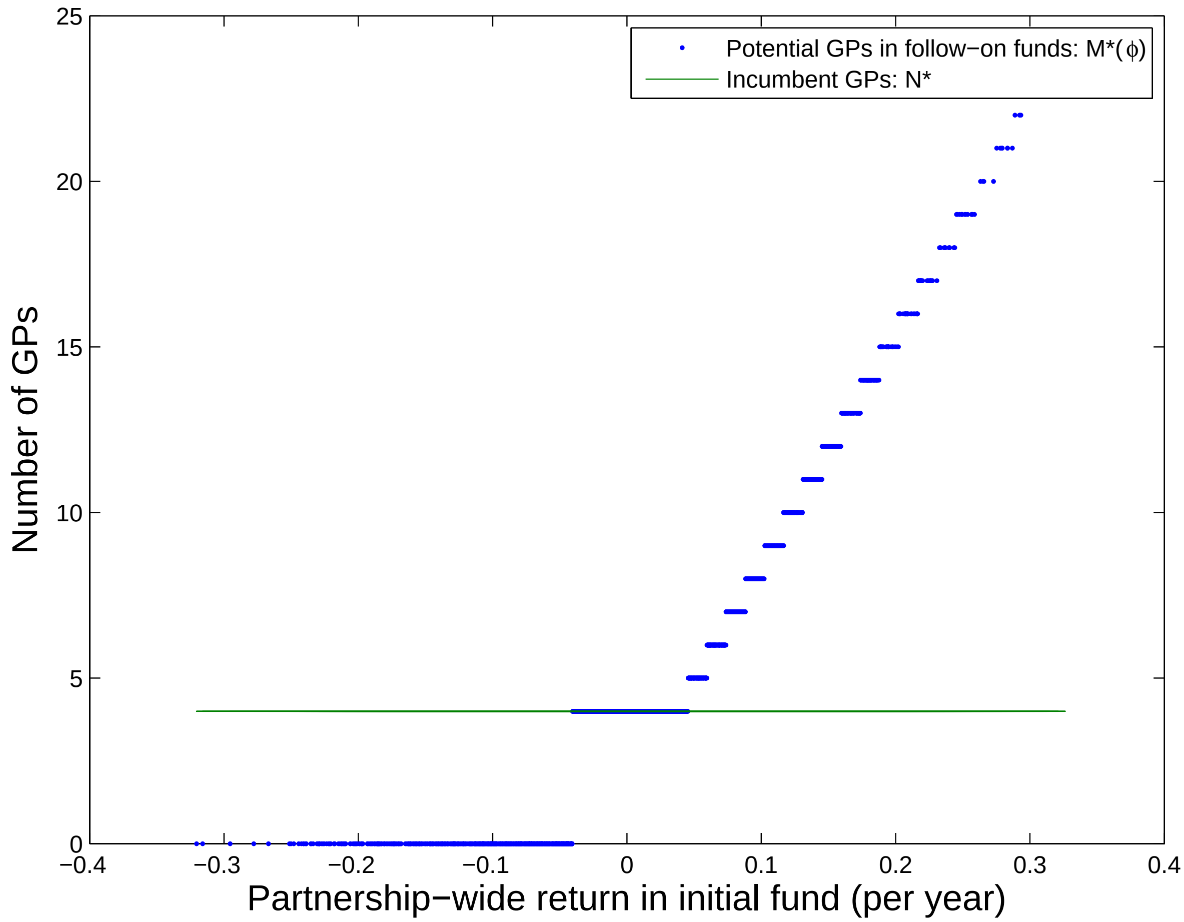

Figure 7: Number of potential GPs and incumbent GPs as a function of initial fund's partnership-wide excess return.

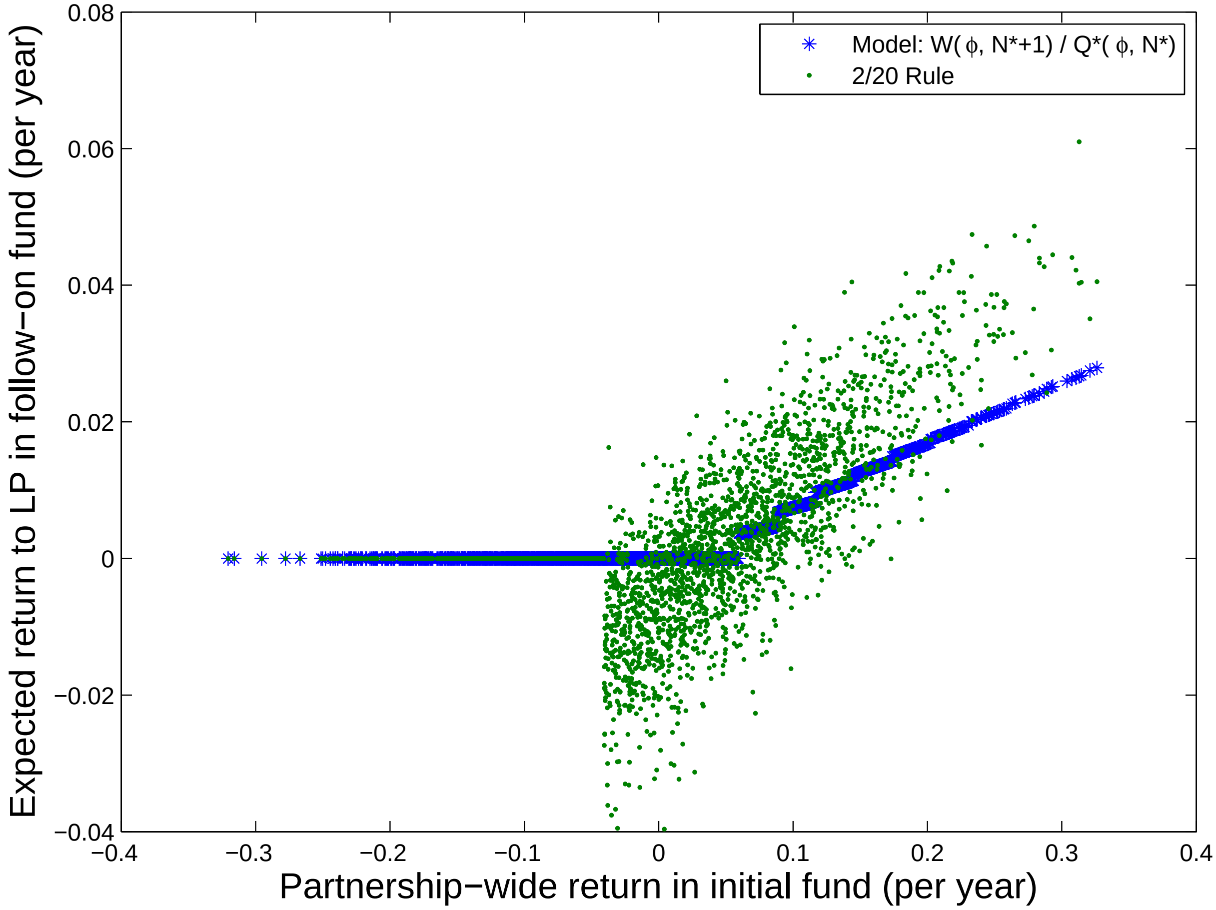

Figure 8: Expected returns earned by incumbent LPs in follow on funds, from the model and using a 2/20 rule, as a function of of initial fund's partnership-wide excess return.

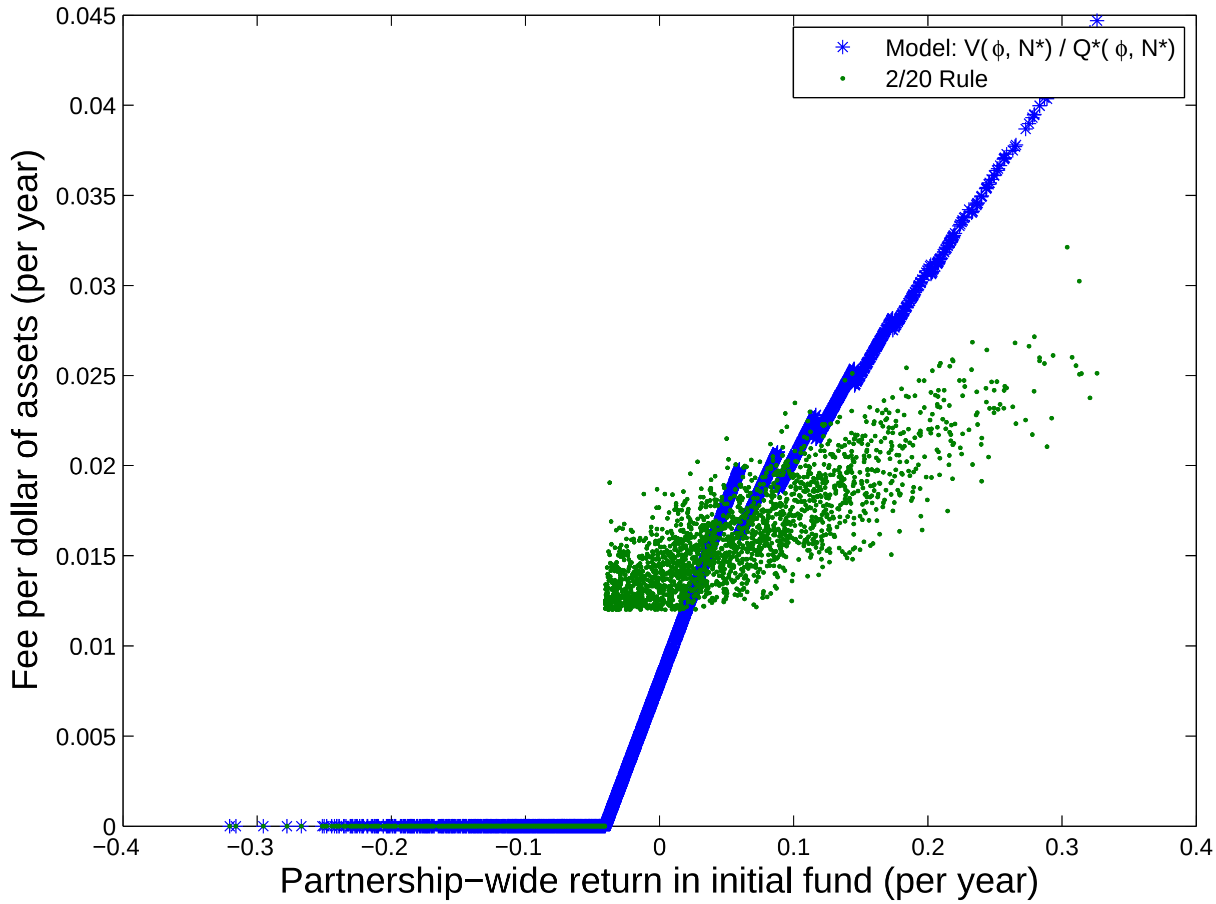

Figure 9: Expected fees earned by incumbent GPs in follow on funds, from the model and using a 2/20 rule, as a function of of initial fund's partnership-wide excess return.