Inflation and Professional Forecast Dynamics: An Evaluation of Stickiness, Persistence, and Volatility

Elmar Mertens  
*Federal Reserve Board, em@elmarmertens.com*

James Nason  
*North Carolina State University, jmnason@ncsu.edu*

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INFLATION AND PROFESSIONAL FORECAST DYNAMICS: an evaluation of stickiness, persistence, and volatility

Elmar Mertens ¹  James M. Nason ²

¹Federal Reserve Board
²North Carolina State University

The results presented here do not necessarily represent the views of the Federal Reserve System or the Federal Open Market Committee

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What is the relationship between survey forecasts and inflation?

Inflation process is characterized by . . .
- drifting mean / trend component
- time-varying volatility in shocks to trend and gap
- time-varying persistence

Evidence about survey forecasts says . . .
- surveys are good at forecasting inflation
- but there are also persistent forecast errors
- consistent with informational frictions in survey formation
QUESTIONS MOTIVATED BY INFORMATION FRICTIONS

① Does “stickiness” vary over time?

② How does “stickiness” interact with inflation?

③ Is “stickiness” related to monetary regimes?
**1) Stock-Watson-type UC model of inflation**

\[
\pi_t = \tau_t + \epsilon_t \\
\tau_t = \tau_{t-1} + \varsigma \eta_t - \varsigma \\
\epsilon_t = \varsigma \nu_t - \varsigma \\
\log \varsigma_{2l,t} = \log \varsigma_{2l,t-1} + \sigma_l \zeta_{l,t}
\]

\( \forall l = \eta, \nu \)

**2) Sticky/noisy information in survey forecasts**

\[
F_t \pi_t + h = (1 - \lambda) E_t \pi_t + h + \lambda F_{t-1} \pi_t + h
\]

\( \lambda_t = \lambda_{t-1} + \sigma_\lambda \zeta_{\lambda,t} \)

\( 0 \leq \lambda_t \leq 1 \)

\( \theta_t = \theta_{t-1} + \sigma_\theta \zeta_{\theta,t} \mid \theta_t \mid \leq 1 \)
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = \varsigma_{\nu,t-1} \nu_t \]
\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \ \nu \]

2) Sticky/noisy information in survey forecasts
STOCK-WATSON SV ESTIMATES $\zeta_{t|T}$
Trend SV (black), Gap SV (red)
1) Filtered Trend is EWMA

\[ \tau_{t|t} = (1 - K_t)\tau_{t-1|t-1} + K_t\pi_t \]

where \( K_t \) is the Kalman gain for the trend
### PROPERTIES OF THE UCSV MODEL FOR INFLATION

1) **Filtered Trend is EWMA**

\[
\tau_{t|t} = (1 - K_t)\tau_{t-1|t-1} + K_t\pi_t
\]

where \(K_t\) is the Kalman gain for the trend

2) **IMA representation**

\[
\Delta\pi_t = (1 - \psi_t L)e_t \quad e_t = \pi_t - E(\pi_t|\pi_{t-1})
\]

\[
\frac{\partial\pi_t + \infty}{\partial e_t} = (1 - \psi_t) = K_t
\]
STOCK-WATSON INFLATION PERSISTENCE

Long-run response $\frac{\partial \pi_{t+\infty}}{\partial e_t} = (1 - \psi_t)$
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + s_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = s_{\nu,t-1} \nu_t \]
\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \ \nu \]

2) Sticky/noisy information in survey forecasts
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = \varsigma_{\nu,t-1} \nu_t \]

\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall l = \eta, \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + \zeta_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = \theta \varepsilon_{t-1} + \zeta_{\nu,t-1} \nu_t \]

\[ \log \sigma_{l,t}^2 = \log \sigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \ \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]
1) **Stock-Watson-type UC model of inflation**

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\pi_t = \tau_t + \varepsilon_t \\
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\log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \nu
\]

2) **Sticky/noisy information in survey forecasts**

\[
F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h}
\]
**1) Stock-Watson-type UC model of inflation**

\[
\begin{align*}
\pi_t &= \tau_t + \varepsilon_t \\
\tau_t &= \tau_{t-1} + s_{\eta,t-1} \eta_t \\
\varepsilon_t &= \theta_{t-1} \varepsilon_{t-1} + s_{\nu,t-1} \nu_t \\
\log s_{l,t}^2 &= \log s_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \nu
\end{align*}
\]

**2) Sticky/noisy information in survey forecasts**

\[
F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h}
\]

... and add new time-varying parameters
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = \theta_{t-1} \varepsilon_{t-1} + \varsigma_{\nu,t-1} \nu_t \]
\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall l = \eta, \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h} \]

... and add new time-varying parameters

\[ \lambda_t = \lambda_{t-1} + \sigma_\lambda \zeta_{\lambda,t} \quad 0 \leq \lambda_t \leq 1 \]
\[ \theta_t = \theta_{t-1} + \sigma_\theta \zeta_{\theta,t} \quad |\theta_t| \leq 1 \]
### RELATED LITERATURE

#### Surveys and fundamentals
- Coibion & Gorodnichenko (2014), Nason & Smith (2014)
- Ang, Bekaert, & Wei (2007), Faust & Wright (2013)

#### Inflation models
- Cogley & Sargent (2005), Cogley, Primiceri, & Sargent (2010)

#### Particle filters
AGENDA

1 Sticky Information Model
2 Nonlinear State Space
3 Results
SI Law of Motion

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]

\[ = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-j} \pi_{t+h} \]
STICKY SURVEY FORECASTS
constant SI weight

**SI Law of Motion**

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]

\[ = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-j} \pi_{t+h} \]

**Implication: Persistent forecast errors**

\[ (E_t - F_t) \pi_{t+h} = \lambda (E_{t-1} - F_{t-1}) \pi_{t+h} + e_t \]

Coibion & Gorodnichenko (2014, forth AER): "SI" law of motion consistent with...

- Sticky information (Mankiw & Reis, 2002)
- Noisy information/Rational inattention (Woodford, 2002; Sims, 2003; Mackowiak & Wiederholt, 2009)
STICKY SURVEY FORECASTS
constant SI weight

**SI Law of Motion**

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\[ (E_t - F_t) \pi_{t+h} = \lambda (E_{t-1} - F_{t-1}) \pi_{t+h} + e_t \]

**Coibion & Gorodnichenko (2014, forth AER):**

“SI” law of motion consistent with . . .

- Sticky information (Mankiw & Reis, 2002)
- Noisy information/Rational inattention (Woodford, 2002; Sims, 2003; Mackowiak & Wiederholt, 2009)
STICKY SURVEY FORECASTS
NEW: time-varying SI weight

SI Law of Motion

\[ F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h} \]

\[ = \sum_{j=0}^{\infty} (1 - \lambda_{t-1-j}) \cdot \left( \prod_{l=0}^{j-1} \lambda_{t-1-l} \right) E_{t-j} \pi_{t+h} \]

Implication: Persistent forecast errors

\[ (E_t - F_t) \pi_{t+h} = \lambda_{t-1} (E_{t-1} - F_{t-1}) \pi_{t+h} + e_t \]

Coibion & Gorodnichenko (2014, forth AER):

“SI” law of motion consistent with . . .

• Sticky information (Mankiw & Reis, 2002)
• Noisy information/Rational inattention (Woodford, 2002; Sims, 2003; Mackowiak & Wiederholt, 2009)
SW-UC model of inflation

\[
\pi_t = \tau_t + \epsilon_t \quad \tau_t = \tau_{t-1} + \zeta_{\eta,t-1}\eta_t \quad E_{t-1}\epsilon_t = 0
\]

\[
E_t \pi_{t+h} = E(\pi_{t+1}|\tau^t, \epsilon^t) = \tau_t
\]
## SW-UC model of inflation

\[
\pi_t = \tau_t + \varepsilon_t \quad \tau_t = \tau_{t-1} + s_{\eta,t-1} \eta_t \quad E_{t-1} \varepsilon_t = 0
\]

\[
E_t \pi_{t+h} = E (\pi_{t+1} | \tau^t, \varepsilon^t) = \tau_t
\]

## Forecaster \(i\) receives noisy signal

\[
s_t^i = \tau_t + e_t^i \quad e_t^i \sim N(0, \sigma_e^2) \quad F_t^i \pi_{t+h} = E(\tau_t | s_t^{i,t})
\]
<table>
<thead>
<tr>
<th>SW-UC model of inflation</th>
</tr>
</thead>
</table>
| $\pi_t = \tau_t + \varepsilon_t$  
$\tau_t = \tau_{t-1} + \zeta_{\eta,t-1} \eta_t$  
$E_{t-1} \varepsilon_t = 0$  
$E_t \pi_{t+h} = E(\pi_{t+1}|\tau^t, \varepsilon^t) = \tau_t$ |

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| $s^i_t = \tau_t + e^i_t$  
$e^i_t \sim N(0, \sigma_e^2)$  
$F^i_t \pi_{t+h} = E(\tau_t|s^{i,t})$  
$E(\tau_t|s^{i,t}) = E(\tau_t|s^{i,t-1}) + \kappa_{t-1} (s^i_t - E(s^i_t|s^{i,t-1}))$ |
## NOISY INFORMATION EXAMPLE

### SW-UC model of inflation

\[
\pi_t = \tau_t + \epsilon_t \quad \tau_t = \tau_{t-1} + \varsigma_{\eta, t-1} \eta_t \quad E_{t-1} \epsilon_t = 0
\]

\[
E_t \pi_{t+h} = E \left( \pi_{t+1} \mid \tau^t, \epsilon^t \right) = \tau_t
\]

### Forecaster \( i \) receives noisy signal

\[
s^i_t = \tau_t + e^i_t \quad e^i_t \sim N(0, \sigma^2_e) \quad F_t^i \pi_{t+h} = E(\tau_t \mid s^{i,t})
\]

\[
E(\tau_t \mid s^{i,t}) = E(\tau_t \mid s^{i,t-1}) + \kappa_{t-1} \left( s^i_t - E(s^i_t \mid s^{i,t-1}) \right)
\]

### Average Forecast

\[
F_t \pi_{t+h} = \int_i F_t^i \pi_{t+h} \, di
\]

\[
F_t \pi_{t+h} = \kappa_{t-1} E_t \pi_{t+h} + (1 - \kappa_{t-1}) F_{t-1} \pi_{t+h}
\]
### SW-UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \quad \tau_t = \tau_{t-1} + s_{\eta,t-1} \eta_t \quad E_{t-1} \varepsilon_t = 0 \]

\[ E_t \pi_{t+h} = E (\pi_{t+1} | \tau^t, \varepsilon^t) = \tau_t \]

---

### Forecaster \( i \) receives noisy signal

\[ s^i_t = \tau_t + e^i_t \quad e^i_t \sim N(0, \sigma^2_e) \quad F_t^i \pi_{t+h} = E(\tau_t | s^{i,t}) \]

\[ E(\tau_t | s^{i,t}) = E(\tau_t | s^{i,t-1}) + \kappa_{t-1} (s^i_t - E(s^i_t | s^{i,t-1})) \]

---

### Average Forecast \( F_t \pi_{t+h} = \int_i F_t^i \pi_{t+h} \, di \)

\[ F_t \pi_{t+h} = \kappa_{t-1} E_t \pi_{t+h} + (1 - \kappa_{t-1}) F_{t-1} \pi_{t+h} \]

---

SI weight \( \lambda_t \) corresponds inversely to Kalman gain \( \kappa_t \)
RECURSIVE SI LAW OF MOTION
consider the case of a constant AR for the inflation gap . . .

**UC model of inflation**

\[ x_t = \begin{bmatrix} \tau_t \\ \varepsilon_t \end{bmatrix} \]

\[ \pi_t = \delta_x x_t \quad \Rightarrow \quad E_t \pi_{t+h} = \delta_x E_t x_{t+h} \]

\[ x_t = \Theta x_{t-1} + \Xi_{t-1} \omega_t \quad \Rightarrow \quad E_t x_{t+h} = \Theta^h x_t \]

**SI forecasts**

\[ F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h} \]

\[ \Rightarrow \quad F_t \pi_{t+h} = \delta_x F_t x_{t+h} \]

\[ \Rightarrow \quad F_t x_{t+h} = \Theta^h F_t x_t \]

**Recursive SI representation**

\[ F_t x_t = (1 - \lambda_{t-1}) x_t + \lambda_{t-1} \Theta F_{t-1} x_{t-1} \]
AGENDA

1. Sticky Information Model
2. Nonlinear State Space
3. Results
“Linear” States $S_t$

\[
\begin{bmatrix}
    x_t \\
    F_t x_t
\end{bmatrix} = S_t = 
\begin{bmatrix}
    \Theta \\
    (1 - \lambda_{t-1}) \Theta \\
    0 \\
    \lambda_{t-1} \Theta
\end{bmatrix}
\begin{bmatrix}
    S_{t-1} \\
    \lambda_{t-1} \Theta
\end{bmatrix} + 
\begin{bmatrix}
    B_{t-1} \\
    (1 - \lambda_{t-1}) \Theta
\end{bmatrix}
\begin{bmatrix}
    S_{t-1} \\
    \lambda_{t-1} \Theta
\end{bmatrix} \cdot w_t
\]

“Non-Linear” States $V_t$

\[
V_t = \begin{bmatrix}
    \lambda_t \\
    \log \zeta_{\eta, t}^2 \\
    \log \zeta_{\nu, t}^2
\end{bmatrix} \sim p (V_t | V_{t-1})
\]
“Linear” States $S_t$

\[
\begin{bmatrix}
  x_t \\
  F_t x_t 
\end{bmatrix} = S_t = \begin{bmatrix}
  \Theta_{t-1} \\
  (1 - \lambda_{t-1}) \Theta_{t-1} \lambda_{t-1} \Theta_{t-1}
\end{bmatrix}
\begin{bmatrix}
  \Theta_{t-1} \\
  \lambda_{t-1} \Theta_{t-1}
\end{bmatrix} S_{t-1} + \begin{bmatrix}
  B_{t-1} \\
  (1 - \lambda_{t-1}) B_{t-1}
\end{bmatrix} w_t
\]

“Non-Linear” States $\mathcal{V}_t$

\[
\mathcal{V}_t = \begin{bmatrix}
  \lambda_t \\
  \log \varsigma_{\eta,t}^2 \\
  \log \varsigma_{\nu,t}^2 \\
  \theta_t
\end{bmatrix} \sim p (\mathcal{V}_t | \mathcal{V}_{t-1})
\]
"Linear" States $S_t$

\[
\begin{bmatrix}
  x_t \\
  F_t x_t
\end{bmatrix} = S_t = \begin{bmatrix}
  \Theta_{t-1} & 0 \\
  (1 - \lambda_{t-1}) \Theta_{t-1} & \lambda_{t-1} \Theta_{t-1}
\end{bmatrix} S_{t-1} + \begin{bmatrix}
  B_{t-1} \\
  (1 - \lambda_{t-1}) B_{t-1}
\end{bmatrix} \omega_t
\]

Interaction between $\lambda_t$ and $(B_t, \Theta_t)$ and TVP-transition!

"Non-Linear" States $\mathcal{V}_t$

\[
\mathcal{V}_t = \begin{bmatrix}
  \lambda_t \\
  \log \varsigma_{\eta,t}^2 \\
  \log \varsigma_{\nu,t}^2 \\
  \theta_t
\end{bmatrix} \sim p(\mathcal{V}_t|\mathcal{V}_{t-1})
\]
### Measurement Vector

\[
y_t = \begin{bmatrix}
\pi_t^* \\
\pi_{t,t+1}^{SPF} \\
\vdots \\
\pi_{t,t+5}^{SPF}
\end{bmatrix} = \begin{bmatrix}
\pi_t \\
F_t \pi_{t+1} \\
\vdots \\
F_t \pi_{t+5}
\end{bmatrix} + \begin{bmatrix}
\xi_{t,\pi} \\
\xi_{t,t+1} \\
\vdots \\
\xi_{t,t+5}
\end{bmatrix} = C_t S_t + \xi_t
\]

### Data

- Real-time measure of realized inflation \( \pi_t^* \)
- SPF surveys for GDP/GNP deflator 1968:Q4 – 2015:Q2
- Forecast horizons up to one year out
- Surveys are collected mid-quarter \( t \), treated as \( F_{t-1}(\cdot) \)
Nonlinear state space with conditional linearity

Data: \( Y_t \sim p(Y_t|S_t, V_t; \Psi) \)

States: \( S_t \sim p(S_t|S_{t-1}, V_{t-1}; \Psi) \)
\( V_t \sim p(V_t|V_{t-1}; \Psi) \)
\( S_t|(Y^t, V^t; \Psi) \sim N(S_{t|t}, \Sigma_{t|t}) \)

Previous draft of the paper:
Particle filtering and smoothing conditional on calibrated \( \Psi \)

Revised draft: “Particle Learning”
Online estimation of \( \Psi \)
embedded in particle filter and smoother
(see Storvik, 2002; Carvalho et al, 2010)
AGENDA

1. Sticky Information Model
2. Nonlinear State Space
3. Results
• Joint UC-SI state space

• TVP-AR(1) in inflation gap

• GDP/GNP deflator, real time 1968:Q3 – 2015:Q1

• SPF for $h = 1, \ldots, 5$

• Estimated with particle learning
\[ F_t \pi_t = (1 - \lambda_{t-1}) \pi_t + \lambda_{t-1} F_{t-1} \pi_t \]
INFLATION GAP
RE (black), SI (red), filtered estimates
SPF AND TREND INFLATION
One-step ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Two-steps ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Three-steps ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Four-steps ahead forecast (red), inflation (blue), SI trend (black)
STOCHASTIC VOLATILITY IN TREND SHOCKS

Top: filtered, bottom: smoothed
STOCHASTIC VOLATILITY IN GAP SHOCKS

Top: filtered, bottom: smoothed
SI WEIGHT $\lambda_t$

*top: filtered, bottom: smoothed*
SI WEIGHT AND MODEL SPECIFICATION

$\lambda_t$: TVP-AR(1) in red
SI WEIGHT AND MODEL SPECIFICATION

$\lambda_t$: TVP-AR(1) in red, Const-AR with $\theta = 0$ in black
SI WEIGHT AND (ONE MINUS) INFLATION PERSISTENCE

Blue: IMA coefficient $\psi_t$ from $\Delta \pi_t = (1 - \psi_t L) e_t$
1. Does “stickiness” vary over time?
   Yes! Surveys have been quite sticky over the last couple of decades, but they were much less sticky before the mid-1980s.

2. How does “stickiness” interact with inflation?
   Stickiness seems to rise with falling inflation persistence and decreasing trend volatility.

3. Is “stickiness” related to monetary regimes?
   For future research: Stickiness seems to coincide with “well anchored” inflation expectations.