INFLATION AND PROFESSIONAL FORECAST DYNAMICS:
an evaluation of stickiness, persistence, and volatility

Elmar Mertens 1  James M. Nason 2

1 Federal Reserve Board
2 North Carolina State University

The results presented here do not necessarily represent the views of the Federal Reserve System or the Federal Open Market Committee

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**Research Agenda**

**Research Question**

What is the relationship between survey forecasts and inflation?

**Inflation process is characterized by ...**

- drifting mean / trend component
- time-varying volatility in shocks to trend and gap
- time-varying persistence

**Evidence about survey forecasts says ...**

- surveys are good at forecasting inflation
- but there are also persistent forecast errors
- consistent with informational frictions in survey formation
QUESTIONS MOTIVATED BY INFORMATION FRICTIONS

① Does “stickiness” vary over time?

② How does “stickiness” interact with inflation?

③ Is “stickiness” related to monetary regimes?
1) **Stock-Watson-type UC model of inflation**

\[
\pi_t = \tau_t + \varepsilon_t \\
\tau_t = \tau_{t-1} + \varsigma \eta_{t-1} \\
\varepsilon_t = \varsigma \nu_{t-1} \\
\log \varsigma = \log \varsigma_{t-1} + \sigma \zeta_{l,t} \forall l = \eta, \nu \\
\lambda_t = \lambda_{t-1} + \sigma \lambda \zeta_{\lambda,t} \quad 0 \leq \lambda_t \leq 1 \\
\theta_t = \theta_{t-1} + \sigma \theta \zeta_{\theta,t} \mid \theta_t \mid \leq 1
\]

2) **Sticky/noisy information in survey forecasts**
1) **Stock-Watson-type UC model of inflation**

\[
\pi_t = \tau_t + \varepsilon_t \\
\tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \\
\varepsilon_t = \varsigma_{\nu,t-1} \nu_t \\
\log \varsigma^2_{l,t} = \log \varsigma^2_{l,t-1} + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \ \nu
\]

2) **Sticky/noisy information in survey forecasts**

\[
\lambda_t = \lambda_{t-1} + \sigma_{\lambda_t} \zeta_{\lambda,t} \\
\theta_t = \theta_{t-1} + \sigma_{\theta_t} \zeta_{\theta,t} \quad \theta_t \leq 1
\]
STOCK-WATSON SV ESTIMATES $\zeta_{t|T}$
Trend SV (black), Gap SV (red)
1) Filtered Trend is EWMA

\[ \tau_{t|t} = (1 - K_t)\tau_{t-1|t-1} + K_t\pi_t \]

where \( K_t \) is the Kalman gain for the trend
1) Filtered Trend is EWMA

\[ \tau_{t|t} = (1 - K_t) \tau_{t-1|t-1} + K_t \pi_t \]

where \( K_t \) is the Kalman gain for the trend

2) IMA representation

\[ \Delta \pi_t = (1 - \psi_t L) e_t \quad e_t = \pi_t - E(\pi_t|\pi^{t-1}) \]

\[ \frac{\partial \pi_{t+\infty}}{\partial e_t} = (1 - \psi_t) = K_t \]
STOCK-WATSON INFLATION PERSISTENCE

Long-run response $\frac{\partial \pi_{t+\infty}}{\partial e_t} = (1 - \psi_t)$
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + s_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = s_{\nu,t-1} \nu_t \]

\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \ \nu \]

2) Sticky/noisy information in survey forecasts
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = \varsigma_{\nu,t-1} \nu_t \]
\[ \log \varsigma^2_{l,t} = \log \varsigma^2_{l,t-1} + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]
\[ \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \]
\[ \varepsilon_t = \theta \varepsilon_{t-1} + \varsigma_{\nu,t-1} \nu_t \]

\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]
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\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall \ l = \eta, \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]
1) Stock-Watson-type UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \]

\[ \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1} \eta_t \]

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\[ \log \varsigma_{l,t}^2 = \log \varsigma_{l,t-1}^2 + \sigma_l \zeta_{l,t} \quad \forall l = \eta, \nu \]

2) Sticky/noisy information in survey forecasts

\[ F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h} \]

... and add new time-varying parameters
1) Stock-Watson-type UC model of inflation

\[
\begin{align*}
\pi_t &= \tau_t + \varepsilon_t \\
\tau_t &= \tau_{t-1} + \zeta_{\eta,t-1} \eta_t \\
\varepsilon_t &= \theta_{t-1} \varepsilon_{t-1} + \zeta_{\nu,t-1} \nu_t \\
\log \varsigma^2_{l,t} &= \log \varsigma^2_{l,t-1} + \sigma_l \zeta_{l,t} \quad \forall \, l = \eta, \nu
\end{align*}
\]

2) Sticky/noisy information in survey forecasts

\[
F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h}
\]

...and add new time-varying parameters

\[
\begin{align*}
\lambda_t &= \lambda_{t-1} + \sigma_{\lambda} \zeta_{\lambda,t} \quad 0 \leq \lambda_t \leq 1 \\
\theta_t &= \theta_{t-1} + \sigma_{\theta} \zeta_{\theta,t} \quad |\theta_t| \leq 1
\end{align*}
\]
# RELATED LITERATURE

## Surveys and fundamentals
- Coibion & Gorodnichenko (2014), Nason & Smith (2014)
- Ang, Bekaert, & Wei (2007), Faust & Wright (2013)

## Inflation models
- Cogley & Sargent (2005), Cogley, Primiceri, & Sargent (2010)

## Particle filters
AGENDA

1. Sticky Information Model
2. Nonlinear State Space
3. Results
SI Law of Motion

\[
F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \\
= (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-j} \pi_{t+h}
\]
STICKY SURVEY FORECASTS
constant SI weight

**SI Law of Motion**

\[
F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \\
= (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-j} \pi_{t+h}
\]

**Implication: Persistent forecast errors**

\[
(E_t - F_t) \pi_{t+h} = \lambda (E_{t-1} - F_{t-1}) \pi_{t+h} + e_t
\]

Coibion & Gorodnichenko (2014, forth AER): "SI" law of motion consistent with...

- Sticky information (Mankiw & Reis, 2002)
- Noisy information/Rational inattention (Woodford, 2002; Sims, 2003; Mackowiak & Wiederholt, 2009)
STICKY SURVEY FORECASTS
constant SI weight

SI Law of Motion

\[ F_t \pi_{t+h} = (1 - \lambda) E_t \pi_{t+h} + \lambda F_{t-1} \pi_{t+h} \]
\[ = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j E_{t-j} \pi_{t+h} \]

Implication: Persistent forecast errors

\[ (E_t - F_t) \pi_{t+h} = \lambda (E_{t-1} - F_{t-1}) \pi_{t+h} + e_t \]

Coibion & Gorodnichenko (2014, forth AER):
“SI” law of motion consistent with . . .
- Sticky information (Mankiw & Reis, 2002)
- Noisy information/Rational inattention (Woodford, 2002; Sims, 2003; Mackowiak & Wiederholt, 2009)
STICKY SURVEY FORECASTS
NEW: time-varying SI weight

**SI Law of Motion**

\[ F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h} \]

\[ = \sum_{j=0}^{\infty} (1 - \lambda_{t-1-j}) \cdot \left( \prod_{l=0}^{j-1} \lambda_{t-1-l} \right) E_{t-j} \pi_{t+h} \]

**Implication: Persistent forecast errors**

\[ (E_t - F_t) \pi_{t+h} = \lambda_{t-1} (E_{t-1} - F_{t-1}) \pi_{t+h} + \epsilon_t \]

**Coibion & Gorodnichenko (2014, forth AER):**

“SI” law of motion consistent with . . .

- Sticky information (Mankiw & Reis, 2002)
- Noisy information/Rational inattention (Woodford, 2002; Sims, 2003; Mackowiak & Wiederholt, 2009)
SW-UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \quad \tau_t = \tau_{t-1} + s_{\eta,t-1} \eta_t \quad E_{t-1} \varepsilon_t = 0 \]

\[ E_t \pi_{t+h} = E(\pi_{t+1}|\tau^t, \varepsilon^t) = \tau_t \]
**SW-UC model of inflation**

\[
\pi_t = \tau_t + \varepsilon_t \quad \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1}\eta_t \quad E_{t-1}\varepsilon_t = 0
\]

\[
E_t\pi_{t+h} = E(\pi_{t+1}|\tau^t, \varepsilon^t) = \tau_t
\]

**Forecaster \( i \) receives noisy signal**

\[
s_t^i = \tau_t + e_t^i \quad e_t^i \sim N(0, \sigma_e^2) \quad F_t^i\pi_{t+h} = E(\tau_t|s_t^{i,t})
\]
### SW-UC model of inflation

\[ \pi_t = \tau_t + \varepsilon_t \quad \tau_t = \tau_{t-1} + \zeta_{\eta,t-1} \eta_t \quad E_{t-1}\varepsilon_t = 0 \]

\[ E_t \pi_{t+h} = E(\pi_{t+1}|\tau^t, \varepsilon^t) = \tau_t \]

### Forecaster \( i \) receives noisy signal

\[ s^i_t = \tau_t + e^i_t \quad e^i_t \sim N(0, \sigma^2_e) \quad F^i_t \pi_{t+h} = E(\tau_t|s^{i,t}) \]

\[ E(\tau_t|s^{i,t}) = E(\tau_t|s^{i,t-1}) + \kappa_{t-1} (s^i_t - E(s^i_t|s^{i,t-1})) \]
**SW-UC model of inflation**

\[ \pi_t = \tau_t + \varepsilon_t \quad \tau_t = \tau_{t-1} + \varsigma_{\eta,t-1}\eta_t \quad E_{t-1}\varepsilon_t = 0 \]

\[ E_t\pi_{t+h} = E (\pi_{t+1}|\tau^t, \varepsilon^t) = \tau_t \]

**Forecaster \( i \) receives noisy signal**

\[ s^i_t = \tau_t + e^i_t \quad e^i_t \sim N(0, \sigma^2_e) \quad F^i_t\pi_{t+h} = E(\tau_t|s^{i,t}) \]

\[ E(\tau_t|s^{i,t}) = E(\tau_t|s^{i,t-1}) + \kappa_{t-1} (s^i_t - E(s^i_t|s^{i,t-1})) \]

**Average Forecast**

\[ F_t\pi_{t+h} = \int_i F^i_t\pi_{t+h} \, di \]

\[ F_t\pi_{t+h} = \kappa_{t-1}E_t\pi_{t+h} + (1 - \kappa_{t-1})F_{t-1}\pi_{t+h} \]
NOISY INFORMATION EXAMPLE

**SW-UC model of inflation**

\[
\pi_t = \tau_t + \varepsilon_t \\
\tau_t = \tau_{t-1} + \zeta_{\eta,t-1} \eta_t \\
E_{t-1} \varepsilon_t = 0
\]

\[
E_t \pi_{t+h} = E (\pi_{t+1} | \tau^t, \varepsilon^t) = \tau_t
\]

**Forecaster \( i \) receives noisy signal**

\[
s_i^t = \tau_t + e_i^t \\
e_i^t \sim N(0, \sigma_{e_i}^2) \\
F_t^{i} \pi_{t+h} = E(\tau_t | s_i^{i,t})
\]

\[
E(\tau_t | s_i^{i,t}) = E(\tau_t | s_i^{i,t-1}) + \kappa_{t-1} (s_i^t - E(s_i^t | s_i^{i,t-1}))
\]

**Average Forecast**

\[
F_t \pi_{t+h} = \int_i F_t^{i} \pi_{t+h} \, di
\]

\[
F_t \pi_{t+h} = \kappa_{t-1} E_t \pi_{t+h} + (1 - \kappa_{t-1}) F_{t-1} \pi_{t+h}
\]

**SI weight \( \lambda_t \) corresponds inversely to Kalman gain \( \kappa_t \)**
RECURSIVE SI LAW OF MOTION

Consider the case of a constant AR for the inflation gap . . .

\[ x_t = \begin{bmatrix} \tau_t \\ \varepsilon_t \end{bmatrix} \]

**UC model of inflation**

\[ \pi_t = \delta_x x_t \quad \Rightarrow \quad E_t \pi_{t+h} = \delta_x E_t x_{t+h} \]

\[ x_t = \Theta x_{t-1} + \Xi_{t-1} \omega_t \quad \Rightarrow \quad E_t x_{t+h} = \Theta^h x_t \]

**SI forecasts**

\[ F_t \pi_{t+h} = (1 - \lambda_{t-1}) E_t \pi_{t+h} + \lambda_{t-1} F_{t-1} \pi_{t+h} \]

\[ \Rightarrow \quad F_t \pi_{t+h} = \delta_x F_t x_{t+h} \]

\[ \Rightarrow \quad F_t x_{t+h} = \Theta^h F_t x_t \]

**Recursive SI representation**

\[ F_t x_t = (1 - \lambda_{t-1}) x_t + \lambda_{t-1} \Theta F_{t-1} x_{t-1} \]
AGENDA

1. Sticky Information Model
2. Nonlinear State Space
3. Results
“Linear” States $S_t$

\[
\begin{bmatrix}
    x_t \\
    F_t x_t
\end{bmatrix} = S_t = \begin{bmatrix}
    \Theta \\
    (1 - \lambda_{t-1})\Theta
\end{bmatrix} S_{t-1} + \begin{bmatrix}
    0 \\
    \lambda_{t-1}\Theta
\end{bmatrix} \begin{bmatrix}
    B_{t-1} \\
    (1 - \lambda_{t-1})B_{t-1}
\end{bmatrix} w_t
\]

“Non-Linear” States $V_t$

\[
V_t = \begin{bmatrix}
    \lambda_t \\
    \log \varsigma_{\eta,t}^2 \\
    \log \varsigma_{\nu,t}^2
\end{bmatrix} \sim p(V_t|V_{t-1})
\]
“Linear” States $S_t$

$$
\begin{bmatrix}
  x_t \\
  F_t x_t
\end{bmatrix} = S_t = \begin{bmatrix}
  \Theta_{t-1} \\
  (1 - \lambda_{t-1}) \Theta_{t-1}
\end{bmatrix} S_{t-1} + \begin{bmatrix}
  0 \\
  \lambda_{t-1} \Theta_{t-1}
\end{bmatrix} S_{t-1} + \begin{bmatrix}
  B_{t-1} \\
  (1 - \lambda_{t-1}) B_{t-1}
\end{bmatrix} \omega_t
$$

“Non-Linear” States $V_t$

$$
V_t = \begin{bmatrix}
  \lambda_t \\
  \log \kappa_{\eta,t}^2 \\
  \log \kappa_{\nu,t}^2 \\
  \theta_t
\end{bmatrix} \sim p \left(V_t | V_{t-1}\right)
$$
"Linear" States $S_t$

$$
\begin{bmatrix}
  x_t \\
  F_t x_t
\end{bmatrix} = S_t = \begin{bmatrix}
  \Theta_{t-1} & 0 \\
  (1 - \lambda_{t-1})\Theta_{t-1} & \lambda_{t-1}\Theta_{t-1}
\end{bmatrix} S_{t-1} + \begin{bmatrix}
  B_{t-1} \\
  (1 - \lambda_{t-1})B_{t-1}
\end{bmatrix} w_t
$$

Interaction between $\lambda_t$ and $(B_t, \Theta_t)$ and TVP-transition!

"Non-Linear" States $V_t$

$$
V_t = \begin{bmatrix}
  \lambda_t \\
  \log \zeta_{\eta,t}^2 \\
  \log \zeta_{\nu,t}^2 \\
  \theta_t
\end{bmatrix} \sim p(V_t|V_{t-1})
$$
**Data and Measurement Vector**

**Measurement Vector**

\[ y_t = \begin{bmatrix} \pi_t^* \\ \pi_{t,t+1}^{SPF} \\ \vdots \\ \pi_{t,t+5}^{SPF} \end{bmatrix} = \begin{bmatrix} \pi_t \\ F_t \pi_{t+1} \\ \vdots \\ F_t \pi_{t+5} \end{bmatrix} + \begin{bmatrix} \xi_{t,\pi} \\ \xi_{t,t+1} \\ \vdots \\ \xi_{t,t+5} \end{bmatrix} = C_t S_t + \xi_t \]

**Data**

- Real-time measure of realized inflation \( \pi_t^* \)
- SPF surveys for GDP/GNP deflator 1968:Q4 – 2015:Q2
- Forecast horizons up to one year out
- Surveys are collected mid-quarter \( t \), treated as \( F_{t-1}(\cdot) \)
Estimation Strategy

Nonlinear state space with conditional linearity

Data: \( Y_t \sim p(Y_t|S_t, V_t; \Psi) \)

States: \( S_t \sim p(S_t|S_{t-1}, V_{t-1}; \Psi) \)
\( V_t \sim p(V_t|V_{t-1}; \Psi) \)

\( S_t|(Y^t, V^t; \Psi) \sim N(S_{t|t}, \Sigma_{t|t}) \)

Previous draft of the paper:

Particle filtering and smoothing conditional on calibrated \( \Psi \)

Revised draft: “Particle Learning”

Online estimation of \( \Psi \)
embedded in particle filter and smoother
(see Storvik, 2002; Carvalho et al, 2010)
AGENDA

1. Sticky Information Model
2. Nonlinear State Space
3. Results
• Joint UC-SI state space

• TVP-AR(1) in inflation gap

• GDP/GNP deflator, real time 1968:Q3 – 2015:Q1

• SPF for $h = 1, \ldots, 5$

• Estimated with particle learning
$F_t \pi_t = (1 - \lambda_{t-1}) \pi_t + \lambda_{t-1} F_{t-1} \pi_t$
INFLATION GAP
RE (black), SI (red), filtered estimates
SPF AND TREND INFLATION
One-step ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Two-steps ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Three-steps ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Four-steps ahead forecast (red), inflation (blue), SI trend (black)
SPF AND TREND INFLATION
Five-steps ahead forecast (red), inflation (blue), SI trend (black)
STOCHASTIC VOLATILITY IN TREND SHOCKS

top: filtered, bottom: smoothed
STOCHASTIC VOLATILITY IN GAP SHOCKS

top: filtered, bottom: smoothed
GAP AR COEFFICIENT $\theta_t$

top: filtered, bottom: smoothed
SI WEIGHT $\lambda_t$

top: filtered, bottom: smoothed

[Graph with two lines showing changes over time from 1970 to 2015, one for filtered and one for smoothed data.]
SI WEIGHT AND MODEL SPECIFICATION

$\lambda_t$: TVP-AR(1) in red
SI WEIGHT AND MODEL SPECIFICATION

$\lambda_t$: TVP-AR(1) in red, Const-AR with $\theta = 0$ in black
SI WEIGHT AND (ONE MINUS) INFLATION PERSISTENCE

Blue: IMA coefficient $\psi_t$ from $\Delta \pi_t = (1 - \psi_t L) e_t$
1 Does “stickiness” vary over time?
   Yes! Surveys have been quite sticky over the last couple of decades, but they were much less sticky before the mid-1980s.

2 How does “stickiness” interact with inflation?
   Stickiness seems to rise with falling inflation persistence and decreasing trend volatility.

3 Is “stickiness” related to monetary regimes?
   For future research: Stickiness seems to coincide with “well anchored” inflation expectations.