3-2008

Wage Signaling: A Dynamic Model of Intrafirm Bargaining and Asymmetric Learning

Limor Golan
Carnegie Mellon University, limor@andrew.cmu.edu

Published In
International Economic Review, 50, 3, 831-854.

Wage Signaling: A Dynamic Model of Intrafirm Bargaining and Asymmetric Learning*

Limor Golan[1]

Abstract

The paper analyzes the effect of employer–worker bargaining on wage dynamics in the presence of asymmetric information between current and potential employers. A failure to reach an agreement leads to output loss. Because the disagreement points depend upon the worker's productivity, productive workers separate themselves from less-productive workers and signal their ability through wages. In existing models of asymmetric learning, wages are attached to publicly observable characteristics and wage growth occurs only when there is a change in observable characteristics. This model, in contrast, generates an increase in earnings dispersion in cohorts of workers with similar observable characteristics.

Keywords: Intrafirm Bargaining, Asymmetric Information, Wage Dynamics, Wage Dispersion. (JEL J3, J41, C78, D82, D83)

*First submission August 2005, Second submission March 12, 2008.

[1]I am grateful to John Kennan, Derek Neal and Peter Norman for guidance and encouragement. I thank Kevin Cotter, Dennis Epple, Igal Hendel, Barton Lipman, Hiromi Nosaka, Canice Prendergast, Larry Samuelson, Michael Waldman and two anonymous referees for useful comments and suggestions. I also thank the participants of the SITE 2002 Summer Workshop, and the seminar participants at Carnegie Mellon, Cornell, Emory, Georgetown, Rutgers, Tel-Aviv University, University of Illinois–Champaign, University of Pittsburgh, University of Wisconsin–Madison and Wayne state for discussions. All remaining errors are my own.

1 Introduction

When firms hire new workers, there is typically uncertainty regarding those workers' skills. In many jobs, such as white-collar and managerial jobs, actual productivity may vary substantially among workers with similar initial observable characteristics such as labor-market experience and education. Information about a worker's performance, however, is typically revealed privately to the worker and the current employer. It is therefore natural to think of the long-term employment relationship as one in which both the current employer and worker receive more precise information regarding the worker's ability than potential employers do.

In most existing literature on wage determination under asymmetric information between incumbent and potential employers, workers are treated as price takers (e.g., Greenwald 1986). If workers are price takers, the wage is the highest offer outside firms make. Therefore, wages do not reflect private information the current employer has, and all workers who look similar to outside firms earn the same wages.[2] Over the life cycle, earnings dispersion of a cohort of workers typically grows. One common explanation is that over time, there is learning on workers' productivity (see Gibbons and Farber 1996). Whereas some of the increase in variation in earnings is explained by changes in observable characteristics such as promotions and job assignments (see Waldman 1984 and Bernhardt 1995 for models of asymmetric information in which outside employers use privately observed signals to infer productivity), a substantial variation in earnings within jobs remains (see Baker, Gibbs and Holmstrom 1994a and b). The missing link is therefore, how the market learns about workers' productivity when information about a worker's productivity is private.

This paper departs from existing literature on asymmetric information (between current and potential employer) by explicitly considering negotiation processes in which neither the employer nor the worker is a price taker. This paper assumes workers cannot commit to long-

[2] Lazear (1986), Bernhardt and Scoones (1993), Pinkston (2005) analyze a model of asymmetric learning in an auction setting in which outside firms receive private information. This paper, however, focuses on a setting in which the current employer has an informational advantage over the outside firms.

term contracts and may renegotiate wages. When bargaining over compensation with workers, a failure or delay in reaching an agreement is costly as it leads to output loss. The insight of this paper is that since the cost of delay or failure to reach an agreement may depend upon worker's productivity, productive workers are able to separate themselves from less productive workers; thus wages may reflect actual differences in productivity.[3]

The main challenge in analyzing the equilibrium outcome is therefore the analysis of the bargaining outcome under this asymmetric information: If wages reflect productivity, outside firms can observe wages and infer worker's productivity. Because the employer loses his informational advantage, signaling productivity is costly for workers. In contrast to standard signaling models (e.g., Spence 1973), the cost of signaling is endogenous in this model and depends on the beliefs of outside employers. Thus, the derivation of the age–earnings profiles contributes to the understanding of the division of surplus between employers and workers in the presence of competitive labor markets and information asymmetries.

In the model, each worker is equally productive in all firms. Workers are heterogeneous with respect to their productivity and their publicly observable characteristics (such as age and education). When a worker is first hired, there is uncertainty regarding his skills. Early during the worker's career, the employer and worker learn about the worker's productivity.[4] Outside firms do not observe this information. Workers cannot commit to more than a one-period contract, and they can renegotiate wages. Potential employers observe wages and may make offers to employed workers. The result is a partially revealing equilibrium that sorts workers into two groups: a high-productivity group whose wages reveal the workers' actual productivity, and a low-productivity group whose wages depend only on publicly observable characteristics.

[3] In the literature on asymmetric information, wages only reflect information available to potential employer and not private information available to workers and current employers. Therefore, wages are not used to infer workers' ability.

[4] The information structure is similar to the one in Greenwald (1986). The uncertainty about the worker's productivity is resolved in the first period.

Workers from the low-productivity group earn wages equal to the offers outside firms make. Hence, these workers' abilities are not revealed to the market. These workers earn wages lower than their marginal productivity. Each worker chooses between earning a low revealing wage for a period followed by wage growth in later periods, and a larger wage in the current period followed by no wage growth in later periods. The bargaining outcome for the marginal worker who renegotiates the wage is strictly lower than the outside option. A high-productivity worker from the first group earns low wages early in his career because the outcome of the bargaining process reveals his productivity. Observing the wage, outside firms offer the worker his marginal revenue product. The current employer then matches the outside offers. This revealing wage compensates the employer for the loss of future informational rents, and hence is lower than the wage that would arise in bargaining under symmetric information. Moreover, the worker may choose to earn a wage below the initial market wage. The compensation to the employer (through a low wage payment) increases with the size of the future loss of informational rents. Informational rents are increasing in the worker's productivity and decreasing in the worker's initial market wage.

Wage innovations occur without increases in productivity (see Neal and Rosen 2000 for a survey of theories of income distribution). They occur because outside firms learn with a time lag about workers' abilities. If the equilibrium wage is a revealing wage, then in succeeding periods, outside firms offer the worker his productivity. In contrast, human capital theory predicts wages increase because workers become more productive as they accumulate experience. In models of learning and sorting, productivity increases because workers are sorted into jobs in which they are more productive. In particular, there are papers on sorting with asymmetric information between current and potential employers in which wages change with job assignment (see Waldman 1984 and Bernhardt 1995). In these models productivity changes when a worker is promoted. The fundamental difference between this paper and the papers above, however, is that firms have control over job assignments and they make take-it-or-leave-it offers to workers. Thus, workers are unable to signal their ability to the market.

In this model, wage dispersion of a cohort of workers increases as workers become more experienced. The first-period salary is the same for all workers with the same observable characteristics. In the second period, the wages of workers who negotiated salaries are different from those of workers who did not. The earnings dispersion in the third period is larger than the dispersion in the second period because the wages are lower in the second period when workers are investing in revealing information through wages. In the third period, workers who invested experience a wage increase and are paid their productivity whereas the wages of workers who did not negotiate in the second period remain constant. This feature is typical to models of investment (e.g., Ben-Porath 1967).

This paper is related to several other bargaining papers. Wolinsky (2000) analyzes bargaining between worker and employer and derives implications for intrafirm bargaining when the information is symmetric, but bargaining affects earnings of all workers. In the solution, the firm and workers split the continuation value surplus (over the disagreement points). In the literature on bargaining under asymmetric information, the asymmetric information is typically between the parties that bargain (e.g., Kennan 2001). Two previous papers in asymmetric information between current employer and worker incorporate bargaining between worker and employer. Chang and Wang (1996) examine a two-period model of asymmetric information and the implications on investment in human capital. In their model, wages in the second period are determined by the Nash bargaining solution. Since the second period is the last period, there is no information externality and the bargaining solution is the solution that arises in a static model. The most closely related paper on asymmetric information between current and potential employers is Ricart I Costa (1988). That paper analyzes a two-period model of asymmetric information in which the worker and current employer observe output privately, and output is a noisy signal of worker's ability. In equilibrium, wages of some workers depend on productivity. This is because the market value of workers is increasing in their productivity since the market offers screening contracts in which wages are contingent on output. Ricart I Costa assumes workers are risk averse. If workers are risk neutral and contingent contracts are feasible, then wages are similar to wages in the symmetric-information case; workers earn the marginal productivity. Wages are not the expected productivity because workers are risk averse. My model is a wage-signaling model; workers can signal productivity via wages because they have bargaining power.[5]

The paper is organized as follows. Section 2 describes the model, section 3 contains equilibrium analysis, section 4 analyzes wage dynamics, section 5 discusses robustness, and section 6 concludes.

2 The Model

This section describes the economy and the model. The goal of the paper is to study the effect of bargaining outcomes on future earnings in a competitive environment. In order to do that, I need to include a specific bargaining game between the current employer and worker, and model competition between outside employers and the current employer. The bargaining problem is inherently dynamic, and I formulate a three-period model with the following structure. In the first period, employers compete over the worker. At the beginning of the first period, the information is symmetric and the contract offered determines prices that reflect no private information. During the first period, the current employer and the worker privately learn the productivity of the worker.[6] During the second period, a worker can choose to renegotiate the wage specified in his contract. The ability of workers to negotiate wages is central for the result that the market may learn about productivity, which is private information to the current employer and worker. The loss of output as a result of a disagreement, rather then outside opportunities, allows workers to earn wages that reflect their actual productivity because these

[5]In Ricart I Costa (1988), wages of some workers reflect productivity. The paper, however, does not analyze wage signaling since the model is a two-period model and wages can only reflect productivity in the second period.

[6]Employers can specify a wage rate for all future periods. This rate is determined at the beginning of the first period, when information is symmetric, and allows workers to choose between this rate and renegotiating.

costs depend on the actual productivity. The third period captures competition over workers after outside firms observe the second-period wage.

Next, I describe the model, the timing, the information structure of the game and the bargaining protocol.

2.1 The economy

There is one good in the economy, and its price is normalized to one. Each period, risk-neutral firms compete over risk-neutral workers. Firms and workers have a common discount factor δ. The output of each firm is normalized to be equal to the workers' labor inputs. Each worker is equally productive in all firms. Workers are heterogeneous with respect to their productivity, θ. A worker is characterized by an ability parameter. The productivity parameter is drawn from a known distribution with continuous density $f(\theta)$ on $[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} > 0$. Workers have no disutility from effort and have a reservation utility of zero; a worker's utility function is the discounted sum of wages. Workers work for three periods and then retire. If a firm hires an inexperienced worker, it takes the firm one period to train the worker and learn his productivity, and no output is produced during that period. The only cost of training a worker is the wage. I assume firms assign workers to jobs only if they know θ.[7] Output produced is not observed by outside firms, and is unverifiable by a court. Workers cannot commit to more than a one-period contract, but firms may commit to long-term contracts. Contracts cannot be contingent on output. If the worker renegotiates a contract, this contract no longer binds the employer. If a worker quits or is fired, he cannot return to work for the firm.

2.2 Timing and information structure of the game

I describe below the order in which workers and firms move, the information available to them in each stage, and the payoffs (see Figures 1–5).

[7] This is an assumption on offers made to employed workers. One possible interpretation is that job vacancies are created before the beginning of the period.

First period: At the beginning of the first period (Figure 2), neither firms nor workers observe the worker's productivity. Firms offer workers long-term contracts that specify wages for periods one, two and three. During this period, the worker and the current employer privately learn θ. At the end of the first period, the employer pays the worker the wage for the training period.

Second period: At the beginning of the second period (Figure 3), only the employer and worker know θ (outside firms observe past wages only). Outside firms make offers and the employer makes a counteroffer. The worker can stay with the current employer or accept an outside offer. If the worker joins an outside firm (Figure 4), he works for the wage he accepted.

Assumption 1 *There is no negotiation if the worker accepts an outside firm's offer.*

This assumption is made to avoid the analysis of bargaining between workers and firms under asymmetric information. If the worker rejects an outside offer, he cannot decide to join this firm later during this period. If a worker stays with the current employer (Figure 5), he can produce immediately or renegotiate the wage. In the case of renegotiation (a description of the renegotiation stage follows), if the employer and worker agree upon a wage, the worker produces. At the end of each period, the worker earns the wage agreed upon. If there was no production, the worker earns nothing.

Third period: The timing is similar to the timing in the second period (similar to the timing in Figures 3, 4, and 5), but the information set of outside firms is different.

Assumption 2 *At the beginning of the period, outside firms observe wages and turnover history. The date the wage contract was signed is verifiable.*

The purpose of this assumption is to provide outside firms with information on whether workers renegotiated the wage and in what stage agreement was reached.

3 Equilibrium Analysis

This section lays out the characterization of a particular perfect Bayesian equilibrium and its analysis in three steps. The first step characterizes the solution to a bargaining protocol between an employer and a worker in a static benchmark case, which is then incorporated in the dynamic game. The second step presents the characterization of the equilibrium wage in the second and third production periods. The third step derives the first-period equilibrium contract. This particular equilibrium is chosen because it is the simplest equilibrium that demonstrates the signaling and revelation of information to the market. Furthermore, the equilibrium contracts are ex-post incentive compatible, and many of the equilibrium's properties hold in other equilibria. I discuss the robustness of the equilibrium outcome and properties of other equilibria in section 5. All formal proofs are in the Appendix.

3.1 Bargaining protocol—static case

There are two rounds of offers, $t = 1, 2$, with no discounting between the two rounds. Both the employer and worker have reservation utility of zero. If agreement has not been reached at the first or the second bargaining round, nothing is produced and the payoffs are zero to both players. In each bargaining round, the employer and worker flip a coin to determine who makes the offer in that round only (i.e., each player makes an offer with probability $1/2$ in each round). For example, suppose that the worker makes the first offer in the first round (occurs with probability $1/2$). If the employer rejects it, then in the second bargaining round, the employer and worker flip a coin again and each player makes the last offer with probability $1/2$.

Lemma 1 *In every subgame perfect equilibrium, in round two the player who makes the offer offers the other player zero, and this offer is accepted. There exists an equilibrium in which agreement is always reached in the first round, and each player receives $\theta/2$.*

This game has multiple subgame perfect equilibria. In every equilibrium, the continuation value for both players, if an offer is turned down in the first round, is $\theta/2$. Therefore, the unique strategy of the player who receives an offer in round one is to turn down any offer below $\theta/2$, accept any offer above it, and accept an offer of $\theta/2$ with any probability in [0,1]. Therefore, there is a continuum of equilibria in which the probability of agreement in round one is any number between [0,1].[8]

3.2 Equilibrium outcome

Next, I incorporate the above bargaining protocol in the dynamic model.

Definition 1: A revealing wage is a wage that depends directly on the worker's productivity: The wage is an invertible function of θ.

Definition 2: A nonrevealing wage is a wage that depends only on publicly observable characteristics.

The following proposition describes the equilibrium outcome in the second and third periods.

Proposition 1 *There exists an equilibrium with the following properties.*

A) No turnover occurs in equilibrium.

B) There is a threshold ability level, θ^*. Workers with productivity above θ^* earn a revealing wage. Workers with ability below θ^* earn a nonrevealing wage.

$$\theta^* = 2\underline{\theta} \tag{1}$$

C) Revealing wages: Workers with ability above θ^* renegotiate the wage in the second period. Agreement is reached in the first bargaining round of the second period. The second-period Revealing wage is

$$w_2 = \frac{\theta}{2}(1 - \delta). \tag{2}$$

The third-period wage is the productivity:

$$w_3 = \theta. \tag{3}$$

[8] Neither one of the equilibria in which agreement may be reached in the second round is robust to arbitrarily small chance of breakdown or any other modifications that lead to discounting.

D) Nonrevealing wage: Let (w_2^c, w_3^c) *be the second- and third-period wages offered in the initial contract. Workers with ability below* θ^* *do not renegotiate with the employer and earn*

$$w_2^c = w_3^c = \underline{\theta}. \tag{4}$$

The derivation and proof of Proposition 1 are developed in the preceding subsections. Next, I provide an overview of the equilibrium outcome. No turnover occurs in equilibrium because all workers are equally productive in all firms.[9] The equilibrium is partially separating; there is an ability threshold that sorts the workers into two groups. More-able workers earn a wage in the second period that reveals θ, and after the second period earn a wage equal to their productivity. Workers with ability below the threshold level earn the wage specified in the initial contract, and their ability cannot be inferred directly from the wage; these workers earn this wage until retirement. Figure 6 describes the second-period wage as a function of θ, when $\underline{\theta} = 1$, and $\delta = 0.6$. The ability threshold level is $\theta^* = 2\underline{\theta}$. Below it, workers earn nonrevealing wages, and above it they earn revealing wages. Figure 7 shows the second- and third-period wages as a function of productivity.

To motivate the equilibrium, consider a similar game with different bargaining rules. In particular, consider a bargaining game in which workers always make the final offer. In this case, wages of workers are θ. Alternatively, if employers always make the final offer in all periods, wages of all workers are equal to the wage specified in the initial contract ($\underline{\theta}$). In this model, in which bargaining is a symmetric process, wages that are an outcome of the bargaining depend directly on ability, but the wage is lower than the wage that arises in symmetric-information bargaining.

Not all workers choose to renegotiate. It is costly for a worker to reveal ability because the employer loses future informational rents. The employer is compensated for that loss in the second period, and receives a one-period transfer. The employer's profit is an increasing

[9] One possible way to generate turnover is to introduce a firm–employee match component to the worker's productivity. The purpose of the paper, however, is to analyze wage signaling, and the main features of the equilibrium remain when workers' productivity varies across firms.

function of the worker's ability. The employer receives half of the current-period output plus a discounted value of future payoffs. The second-period revealing wage can be below $\underline{\theta}$ (see Figure 6). As a result, some workers do not reveal ability. These are workers for whom the additional present value of receiving θ for all subsequent periods does not exceed the cost required to reveal information. All these workers earn wages below their productivity.

3.3 Equilibrium strategies

I begin by describing the equilibrium strategies and beliefs of each player. The employing firm is denoted by f, outside firms are denoted by m, and workers by l. Let $k = \{l, m, f\}$ denote the player. Wage-offer strategies are denoted by W, and wage realization by w. I denote the wage-offer strategy in bargaining round r in production period t by $W^k_{t,r}$. Wage offers that are not part of the bargaining process (that is, offers made at the beginning of each production period) are denoted by W^k_t. With some abuse of notation w^m_t is the *highest* outside-firm offer. The wage history at the beginning of period $s+1$ is denoted by H_{s+1} and includes $w_1, ..., w_s$. Beliefs about the worker's productivity are denoted by $\hat{\theta}(H_s)$.

3.3.1 Third-period strategies

If a negotiation occurs in the third period, it is a one-period bargaining game with symmetric information (for any possible history). Therefore, the analysis in Lemma 1 applies. The expected payoff to each party is $\theta/2$. Let I^f_t be an indicator function that takes the value 1 if the worker remains with the employer and zero if he accepts an outside offer at the beginning of period t.

Lemma 2 *1. At the beginning of the period, the employer's strategy is to match an outside offer, $w^f_3 = w^m_3$ if $\theta \geq w^m_3$ and make no counteroffer otherwise.*

2. The worker's choice of employer is given by

$$I^f_t = \begin{cases} 1 & \text{if } \; w^m_3 \leq max\{\frac{\theta}{2}, w^f_3\}. \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

3. Workers renegotiate if the expected payoffs are greater than the wage offer $\theta/2 > w_3^f$.

4. At the beginning of period three, outside employers offer to workers who renegotiated in period two

$$W_3^m = \hat{\theta} \tag{6}$$

and they make no offers otherwise.

In the third period, for any history, all players' strategies maximize a one-period payoff. Offers are made only to workers who earn revealing wages, because it is not profitable to hire workers for one period when the wage does not convey information on productivity (by assumption).

3.3.2 Second-period strategies

Next, I describe the worker and employer second-period equilibrium bargaining strategies and the outside firms' beliefs after observing the bargaining outcome.

Lemma 3 *Outside firms' beliefs are*

$$\hat{\theta}(w_{2,r}^k) = \begin{cases} [\frac{2}{2-\delta}]w_2 & \text{if } \; w_{2,2}^k > 0 \\ [\frac{2}{1-\delta}]w_2 & \text{if } \; w_{2,1}^k > 0 \\ E[\theta|\theta^* \le \theta \le \overline{\theta}], & \text{if } \; w_{2,r}^k = 0 \\ E[\theta|\underline{\theta} \le \theta \le \theta^*], & \text{otherwise.} \end{cases} \tag{7}$$

The following strategies are optimal, given the third-period strategies described in Lemma 2 and the above beliefs,

1. Agreement is reached in the first round of the bargaining. The first-round offer made by either the employer or the worker is $W_{2,1}^k = \theta/2 - \delta\theta/2$.

2. If a second round is reached, the employer offers the worker $W_{2,2}^f = 0$, *and the worker accepts. If the worker makes an offer in the second round, he offers* $W_{2,2}^l = \theta - \delta\theta/2$, *and the employer accepts.*

Because the beliefs are monotonically increasing in the bargaining outcome, both on and off the equilibrium path, the worker's continuation payoff increases in the agreed-upon wage and the employer's continuation payoff decreases in that wage. As a result of this monotonicity, it is optimal for the worker to offer the highest wage that the employer will accept and for the employer to make the lowest wage offer that the worker will accept. The player who makes the offer, therefore, offers a wage that leaves the other player indifferent between rejecting and accepting the offer.

Consider the final bargaining round. The employer offers the worker zero, and the worker accepts. The worker's continuation value of rejecting an offer is the third-period payoff (as nothing is produced in the second period). This payoff is the same when agreement is reached and the wage is zero and when no agreement is reached. The worker's offer in the final round leaves the employer indifferent between rejecting and accepting the offer by construction. Rejection means that no information is revealed and the worker and employer split the third-period surplus. If an offer is accepted, however, it reveals information to the market, and the employer's third-period payoff is zero. Therefore, the worker's offer compensates the employer for loss of half of the surplus in the third period.

Since the probability of making the final offer is half and there is no discounting, the first-round offers leave both players indifferent between agreement in the first round and the continuation payoffs if no agreement is reached in that round. The continuation payoffs if no agreement is reached in the first round is the expected payoff from agreement in the second round. Because this continuation payoff is independent of who makes the first-round offer, these offers do not depend on the identity of the player who makes it.

When renegotiation occurs, outside firms infer the worker's productivity from the wage. Their beliefs in equation (7) are the inverse function of the equilibrium wage–offer strategies. On the equilibrium path, workers with ability below θ^* do not negotiate, therefore the beliefs $\hat{\theta} = E[\theta | \underline{\theta} \leq \theta \leq \theta^*]$ are consistent.

Next, I characterize the offers made at the beginning of the second period. Denote a contract offer made by outside firms for periods two and three by (w_2^m, w_3^m).

Lemma 4 *Outside firms offer at the beginning of period two a contract of* $W_2^m = 0, W_3^m = \underline{\theta}$.

At the beginning of the second period, outside firms have no new information about an employed worker's productivity. If a firm raids a worker in the second period with an offer of (w_2^m, w_3^m), it learns about the worker's productivity during that period and becomes informed in the third period. The employer does not match an offer if the expected wage payments weakly exceed the worker's expected productivity over the two periods. Therefore, outside firms suffer from a winner's curse and offering the lower bound of a worker's productivity over the two periods is optimal.[10] The lower bound is derived as follows. At the beginning of the third period, if a worker did not negotiate the wage, outside firms make no offers. By assumption, the first period is a learning period in which no output is produced (firms do not provide jobs to workers for whom ability is unknown). Any offer above this lower bound yields a weakly negative expected payoff.

3.3.3 Characterization of the threshold ability

As discussed above, the negotiation outcome is *monotonically increasing* in productivity: High-productivity workers renegotiate and earn a wage above the wage earned by workers who do not renegotiate. The following corollary characterizes the optimal decision to renegotiate.

Corollary 1 *The ability threshold for negotiation is* $\theta^* = 2\underline{\theta}$. *All workers who earn a revealing wage with productivity in the range of*

$$\theta < \frac{2\underline{\theta}}{1-\delta} \tag{8}$$

earn wages below the outside option (that is, $w_2 < \underline{\theta}$*).*

The ability threshold for negotiation is the level that equates the continuation value of negotiation to the payoff of earning the wages specified in the initial contract:

$$\frac{\theta^*}{2}(1-\delta) + \delta\theta^* = w_2^c + \delta w_3^c = \underline{\theta} + \delta\underline{\theta}. \tag{9}$$

[10] A solution to the game with simultaneous offers exhibits the same features.

Marginal workers with productivity above the threshold ability level earn a wage below the wage earned by workers who do not renegotiate. These workers invest in changing potential employers' perceptions of their productivity, and thus earn a high wage in the third period. The cost of revealing ability to low-ability workers (earning a wage below θ in the second period) is higher than the benefit (the wage increase in the third period is too small for these workers). A proof that these workers cannot profitably deviate if they renegotiate is in the appendix. Notice that the second-period wages are not monotonic in ability. Such equilibrium can be sustained because of the assumption that outside firms can verify the date on which the contract was signed, and therefore know which workers renegotiated and which did not.

3.3.4 First-period equilibrium wages

This section derives the first-period equilibrium contract.

Proposition 2 (First-period wage) *The first-period contract, in which $w_2^c = w_3^c = \underline{\theta}$ and*

$$w_1(\underline{\theta}) = \delta \left\{ (1+\delta) \int_{\underline{\theta}}^{\theta^*} (\theta - \underline{\theta}) f(\theta) \partial\theta + \int_{\theta^*}^{\overline{\theta}} (\theta - w_2(\theta)) f(\theta) \partial\theta \right\}$$

is optimal.

At the beginning of the first period, the information is symmetric and the market is ex-ante competitive. The equilibrium wage clears the market, and a firm's expected profit from hiring a worker is zero. The first-period wage is therefore the employer's expected profit. The probability that a worker will earn a nonrevealing wage in the second period is the probability that $\theta \leq \theta^*$. The profit is $\theta - w_2(\theta)$ if the worker's skill level is above θ^*, where w_2 is given by equation (2) weighed by the probability that the worker's ability is above θ^*, and it is $(\theta - \underline{\theta})(1+\delta)$ if the worker's ability is below θ^*. Each worker chooses a contract that maximizes the expected lifetime earnings:

$$EU = w_1 + \delta E_\theta[w_2(\theta, w_2^c, w_3^c)] + \delta^2 E_\theta[w_3(\theta, w_2^c, w_3^c)]. \tag{10}$$

Substituting the first-period wage in equation (10) shows that the worker's expected lifetime earnings is the discounted present value of the expected productivity in all periods, $\delta E_\theta(\theta) +$

$\delta^2 E_\theta(\theta)$.

4 Discussion of the Equilibrium Properties

Workers earn a wage larger than their productivity in the first production period. This is a typical feature of competitive asymmetric learning models. Because employers earn informational rents in the second period, the first-period wage includes the future expected information rent, and employers earn zero expected profits. One possible interpretation of this result is that the high first-period salary includes a one-time signing bonus.

The mechanism that allows wage growth is different from the ones proposed in the literature. In models of investment in human capital, wage growth occurs because workers become more productive as they accumulate experience. This is either due to investment in skills or on-the-job training. In models of learning and sorting (e.g., Gibbons and Waldman 1999), workers become more productive as new information about their productivity arrives because they are sorted into jobs for which they are better suited. Wage growth in this model occurs without changes in productivity. Information revealed in the first period is signaled to the market in the second period and occurs in one stage. High-productivity workers invest in changing outside employers' perceptions of their skills.[11]

This model demonstrates that earnings dispersion of a cohort of workers can increase without changes in publicly observable characteristics or productivity growth, even when employers privately learn workers' productivity.[12] Workers who initially look similar and earn the same initial wage may have different wage patterns as they become experienced. In particular, high-

[11]Notice that wage levels of workers who invest may be lower than wages of those who do not invest in period two. It is possible to extend the model to allow instead for workers to work a different amounts of time; instead of a nonmonotonic wage schedule, the average hourly wage is nonmonotonic in such a model. In that case, working longer hours may be used as a signal as opposed to the actual total wage.

[12]Baker, Gibbs and Holmstrom (1994) find that there is a substantial variation in wage levels and wage growth within job levels.

productivity workers who invest in revealing productivity to the market earn their marginal product late in their careers. Low-productivity workers never earn their marginal product. Thus, differences in productivity cause wage dispersion to increase with experience. This component of investment in the model implies that workers who earn "low" revealing wages will earn higher wages in the future compared to workers who earned nonrevealing wages. This feature is similar to predictions of models in which workers invest in human capital (e.g., Ben-Porath 1967).

Wage growth in this paper depends on initially observable characteristics and on the worker's actual productivity.[13] Thus, workers who initially look similar and earn the same initial wage will differ in their wage patterns due to differences in productivity. There are several reasons in the literature for equally productive workers to earn different wages. One of them is luck, affecting output realizations in learning models and offer arrivals in search models. In this model, allowing for variation in publicly observable characteristics (so $\underline{\theta}$ varies across worker groups), workers with high ability indicators (large $\underline{\theta}$) are more likely to receive nonrevealing wages and experience no wage increase compared to equally productive workers with lower outside options. If the difference between the worker's actual productivity and the outside option is small, the worker may choose a nonrevealing wage. In this case, the worker's wage level can be high, but his wage does not increase. This model, therefore, predicts that given two equally productive workers, the worker with lower wages may earn larger raises. Holmstrom (1994): Controlling for performance rating, workers with the same tenure in the same job level in higher salary quartiles earn lower proportional salary increases.[14]

[13] While differences in observable characteristics are not modeled explicitly, it is straightforward to include them so they affect $\underline{\theta}$.

[14] The study suggests this may be a result of a firm policy to reduce wage dispersion within job levels.

5 Robustness

This section analyzes the properties of other equilibria. In particular, it shows that a common feature of all the equilibria in this game is that the second-period wages reveal some information on workers' productivity for sufficiently large dispersion of the productivity distribution.

Corollary 2 *There exists a continuum of equilibria with a first-period contract of w_1, w_2^c, w_3^c, in which the second- and third-period nonrevealing wages are as described in Proposition 1. The threshold ability is $\theta^* = (w_2^c + \delta w_3^c)2/(1+\delta)$. The first-period wage is*

$$w_1(\underline{\theta}, w_2^c, w_3^c) = \delta \int_{\underline{\theta}}^{\theta^*} (\theta - w_2^c) f(\theta) \partial\theta + \delta^2 \int_{\underline{\theta}}^{\theta^*} (\theta - w_3^c) f(\theta) \partial\theta + \delta \int_{\theta^*}^{\overline{\theta}} (\theta - w_2(\theta)) f(\theta) \partial\theta$$

The initial contract is not unique. The firms and workers are risk neutral and there are no frictions in the capital markets. As a result, players only maximize expected discounted lifetime earnings. Therefore, contracts with different second- and third-period wages may be offered, and the threshold ability and first-period wage change accordingly. The first period wage is derived from the zero-expected-profit condition. Other equilibria with second- and third-period wages below $\underline{\theta}$ will have a lower threshold level for renegotiation and higher first-period wages. When $w_2^c = w_3^c = 0$, the contract is equivalent to a spot contract. Notice, in this case, there is an equilibrium in which all workers negotiate and information on all workers is revealed to the market. If workers were risk-averse, an optimal contract should always include long-term commitment to wages in the second and third periods.

It is also possible that the second- and third-period wage in the initial contract are sufficiently high for negotiation to be unprofitable. For example, consider a contract $w_2^c = w_3^c = \overline{\theta}/2$, it is possible to find beliefs such that no one renegotiates. The first-period wage will adjust so firms make, on average zero, profits: $w_1^c = (\delta + \delta^2)[E(\theta) - \overline{\theta}/2]$.[15] Notice, however, that contracts that guarantee more than $\underline{\theta}$ may be less stable in the sense that retaining employees is not incentive compatible. All contracts in which the guaranteed wages do not exceed

[15]For distributions in which $E(\theta) - \overline{\theta}/2 < 0$, sustaining such equilibrium requires negative transfer.

$\underline{\theta}$ require only firms' commitment to wages, not employment. All the equilibria characterized above feature signaling, but a high initial contract lowers the amount of information revealed to the market and decreases the earnings dispersion.[16] Lastly, in all the equilibria characterized above, agreement may be reached in the second round (with any probability in [0,1]) because both players have equal continuation values of turning down and accepting the first-round offer. These equilibria are not robust to a small chance of breakdown or any other modification of the game that leads to discounting between the bargaining rounds.

In this game, the beliefs and outside firms' offers affect the offers made and accepted in bargaining. The costs of signaling in terms of second-period wages—as well as the returns, the third period wages—are determined endogenously. These beliefs may generate equilibria with properties different from the properties of the equilibria described above. The following proposition characterizes equilibrium outcomes that are common to all the perfect Bayesian equilibria of this game.

Proposition 3 *The following holds in any equilibrium of this game.*

1. Suppose $w_2^c + \delta w_3^c > \underline{\theta}(1+\delta)/2$*. Then there exists no equilibrium in which all workers renegotiate in period two.*

2. For $w_2^c + \delta w_3^c < \overline{\theta}[\delta/4 + 1/2]$*, there exists no equilibrium in which no worker renegotiates in period two.*

3. Suppose $4(w_2^c + \delta w_3^c)/[(1+\delta)(2-\delta)] < \overline{\theta}$*. Then there exists no equilibrium in which all workers who renegotiate earn the same wage in the second period.*

The first part of the proposition states that as long as the wages in the initial contract are not too low, some workers do not renegotiate. The intuition to this result is that the continuation payoff of renegotiation cannot exceed half of the lifetime surplus produced. The low-productivity workers earn more than half of the surplus (for example, the lowest type, $\underline{\theta}$, receives the entire surplus if the contract is $w_2^c = w_3^c = \underline{\theta}$) and therefore do not negotiate.

[16] If workers are risk averse, the initial contract is unique.

The second part provides a sufficient condition for renegotiation to occur in equilibrium.[17] Under this condition, in every equilibrium of the game there is a threshold ability above which workers renegotiate. The intuition to the result is that there is a lower bound on the worker's share of the surplus in any possible equilibrium (the appendix shows that it is $\theta/2 + \delta\theta/4$). Therefore, if the contract wages are low relative to the surplus produced by high-productivity workers, these workers always negotiate.

The proposition above does not rule out the existence of equilibria in which workers who renegotiate in the second period earn the same wages. The third part states that some of the earnings dispersion in the second period is caused by variation in earnings of workers who renegotiate. It rules out the possibility of equilibria in which large segments of types negotiate and earn the same second-period wage.[18]

6 Conclusions

This paper develops a model of intrafirm bargaining in which the worker and current employer learn privately about the worker's productivity. The symmetry of the bargaining leads to an equal division of the surplus between the employer and workers who negotiate. This is similar to the division of surplus in a bargaining game in which there are no outside employers competing over workers, and each player receives half of the surplus in each period. In the equilibrium analyzed, however, the worker receives a smaller portion of the share in the second period, to compensate the employer for loss of the third-period surplus which results from the signaling; he earns a larger share in the third period.

There may be other signaling mechanisms such as job assignment. If job assignment is observed and conveys information about workers' skills, then negotiation over job assignment may arise. See MacLeod and Malcomson (1988) for a reputation model of moral hazard and

[17] If the initial contract, for example, is the one in Proposition 1, the condition is that $\overline{\theta} > \underline{\theta}4(1+\delta)/(2+\delta)$.

[18] If the initial contract, for example, is the one in Proposition 1, the condition is that $\overline{\theta} > \underline{\theta}4/(2-\delta)$.

adverse selection. Hierarchy in their paper arises endogenously. Negotiation over job assignment will occur when workers' and firms' incentives are not aligned. In this framework, however, conflict may not arise when job assignment is incorporated, if all the relevant information can be conveyed through wage signaling.[19]

Limor Golan, Tepper School of Business, Carnegie Mellon University, Pittsburgh, Pennsylvania, USA.

Appendix

Proof of Lemma 1: Solving backwards from the final round, offering 0 and accepting are the unique equilibrium strategies. This is a standard result of equilibrium in an alternating-offers bargaining game. In round 1, the continuation value of rejecting an offer is $\theta/2$. Thus, offering $\theta/2$, and accepting any offer x, when $x \geq \theta/2$ are optimal strategies. The equilibrium outcome is that agreement is reached in the first round and each player receives $\theta/2$. Q.E.D.

Proof of Lemma 2. 3. The expected payoff from negotiation is $\theta/2$ as established in Lemma 1. The worker renegotiates if the expected payoff exceeds the offer w_3^f. This strategy maximizes the third-period wage. 2. Solving backwards, the worker remains in the firm if the expected payoff in the firm—$\max\left\{w_3^f, \theta/2\right\}$—exceeds the outside offer w_3^m. 1. The employer matches an outside offer if $\theta \geq w_3^m$. The worker accepts the employer's offer if there is a tie. Therefore, matching the market offer is the lowest offer accepted. Making an offer that is accepted by the worker if $\theta < w_3^m$ yields a negative payoff. Thus, making no offer is an optima strategy. 4. The offer in equation (6) is the optimal competitive offer (the expected productivity of the worker) when there is a Bertrand competition between equally informed firms. Making no offers if there was no renegotiation is optimal by assumption. Q.E.D.

Proof of Lemma 3: Given the optimal strategies in period three, and beliefs in equation

[19]See an earlier version of this paper, Golan (2002), for formal analysis. Golan (2005) develops a model with promotions demonstrating that the asymmetric information may not affect job assignment.

(7), I compute the continuation payoffs for the employer at the renegotiation stage in period two. I first show that the worker makes the highest offer the employer accepts, and that if the employer makes the offer, he offers the lowest wage accepted by the worker. Define ξ_t to be an indicator function that takes the value 0 if negotiation occurred in period t, and the agreed-upon wage is positive. It takes the value 1 otherwise (no negotiation, negotiation and failure to reach agreement, or a wage zero). Consider an offer made by player k in round r: The worker's continuation payoff if the offer is accepted is

$$U(w_{2,r}^k, \theta) = w_{2,r}^k + \delta[\xi_3(w_{2,r}^k)\frac{\theta}{2} + (1 - \xi_3(w_{2,r}^k))W_3^m(w_{2,r}^k)], \tag{11}$$

where the first element is the current-period payoff and the other elements are the expected payoff in period three. Notice that $\xi_3(w_{2,r}^k)$ is (weakly) decreasing in the current bargaining outcome. That is, it takes the value one if the market's offers are below $\theta/2$ and zero if above; however, the offers as described in equation (6) are strictly increasing in the bargaining outcome. Thus, the third-period payoff, $\xi_3(w_{2,r}^k)\theta/2 + (1 - \xi_3(w_{2,r}^k))W_3^m(w_{2,r}^k)$, weakly increases in $w_{2,r}^k$. The employer's continuation payoff, $\pi(w_{2,r}^k)$, is

$$\pi(w_{2,r}^k) = (1 + \delta)\theta - U(w_{2,r}^k, \theta). \tag{12}$$

Since $U(w_{2,r}^k, \theta)$ increases in $w_{2,r}^k$, $\pi(w_{2,r}^k)$ decreases in $w_{2,r}^k$.

Given the above properties, I begin solving for the offer strategies in the final bargaining round. If the employer makes an offer, he offers the worker $W_{2,2}^f = 0$. Given the firm's beliefs and offers in equation (6), outside firms make no offers and the worker renegotiates. It is optimal for the worker to accept the offer because the continuation payoff in both cases is: $0 + \delta\theta/2$.

Next consider the worker's offer in the second round. If the employer rejects the last offer, his continuation payoff is $0 + \delta\theta/2$. The worker's offer maximizes his continuation payoff in equation (11). The worker makes the highest wage offer accepted by employer: $\pi(w_{2,2}^l) = \delta\theta/2$. Any offer below it will be accepted but will yield lower payoff for the worker (in both the second and third periods) and therefore is not optimal. Given the beliefs and third-period offers, for any offer above $\theta-\delta\theta/2$, the employer's profit if he accepts is zero in the third period and the second-period payoff is lower than the payoff if the employer rejects the offer (by construction). Thus, the worker offers $W^l_{2,2} = \theta - \delta\theta/2$ and the employer accepts the offer. The employer's payoff is the same as the continuation payoff of rejecting the offer $\pi(w^l_{2,2}) = \theta - (\theta - \delta\theta/2) + 0 = \delta\theta/2$. This proves part 2 of the Lemma.

In the first bargaining round, any offer made leaves the player who receives it indifferent between accepting and rejecting. The continuation value for the worker, if the first-round offer is rejected is the expected payoff from the offers in round two derived above, given that the probability of making an offer is 1/2. The worker's continuation payoff is

$$\frac{1}{2}(\theta - \delta\frac{\theta}{2} + \delta\theta) + \frac{1}{2}\delta(\frac{\theta}{2}) = (1+\delta)\frac{\theta}{2}. \tag{13}$$

The employer's continuation value is, therefore, $(1+\delta)\theta/2$ as well.

Therefore, the lowest wage offer made by the employer and accepted by the worker satisfies

$$W^f_{2,1} + \delta W_3(w^f_{2,1}) = (1+\delta)\frac{\theta}{2}. \tag{14}$$

Consider the offer in part 1 of the Lemma, $\theta/2-\delta\theta/2$. The beliefs in equation (7) and strategies in equation (6) imply that the third-period offer is θ. Plugging these wages into equation (14) above shows that this offer is accepted and satisfies the equality. The employer's surplus if this offer is accepted is $(1+\delta)\theta/2$, therefore, this offer maximizes the employer's payoff (higher wage offer decreases payoff, lower offer is rejected, resulting in the same expected payoff).

Similarly, the worker's first-round offer satisfies

$$\theta - W^l_{2,1} + \delta(\theta - W_3(w^l_{2,1})) = (1+\delta)\frac{\theta}{2}. \tag{15}$$

The wage $\theta/2 - \delta\theta/2$, implies a third-period offer of θ. Plugging these wages into equation (15) above shows that this offer is accepted and satisfies the equality. The worker's surplus if this offer is accepted is $(1+\delta)\theta/2$ and this offer is optimal. Thus, the first-round offers in part 1 of the Lemma are optimal.

Next, I show that given the equilibrium bargaining strategies, and given a threshold ability θ^*, the beliefs satisfy the consistency requirement on the equilibrium path.

a. On the equilibrium path, agreement is always reached in round one, the wage offered is $w_{2,1}^k = \theta/2 - \delta\theta/2$. The beliefs, by construction, are the inverse of the wage offer function $\hat{\theta} = (W_{2,r}^k)^{-1} = \theta$, and therefore, are consistent. Since workers' strategy is to negotiate iff $\theta \geq \theta^*$, the beliefs if no renegotiation occurs, that $E[\theta|\underline{\theta} \leq \theta \leq \theta^*]$, are consistent. Q.E.D.

Proof of Lemma 4: Consider an offer at the beginning of period two: $w_2^m = x, w_3^m = y$, such that $x \geq 0, y > \underline{\theta}$. Given that $w_2^c = w_3^c = \underline{\theta}$, the employer matches any outside offer in which

$$\theta + \delta\theta \geq x + \delta y.$$

Therefore, outside firms can only raid an employed worker if

$$\theta + \delta\theta < x + \delta y. \tag{16}$$

Consider the profits from hiring a worker in that case. If a worker accepts an outside offer, he decides to renegotiate in the third period if $\theta/2 > y$. Equation (16) implies that in this case $x > \theta + \delta\theta/2$, and the outside firm's profit over the two periods is negative. If $y > \theta/2$, the worker will not renegotiate and will earn y. The condition in 16 implies that the outside firm's profit is negative. Any lower offer will not increase the outside firm's payoffs as workers will not accept these offers. Q.E.D.

Proof of Corollary 1: Equation (9) derives the threshold ability by comparing the value of receiving a revealing wage in the first period and the productivity the following period to the present value of receiving the wages specified in the initial contract. Next, I show that workers below the threshold cannot profitably deviate. Suppose a worker with productivity below θ^* renegotiates. The negotiation strategies and beliefs are derived on and off the equilibrium path; according to strategies in Lemma 3, the agreement is reached in round one, the wages are $\theta/2 - \delta\theta/2$, and the beliefs in equation (7) are $2/(1-\delta)[\theta/2 - \delta\theta/2] = \theta$. Therefore the third-period outside wage offer is θ. The employer matches the offer, and the third-period wage is θ.

The expected payoff from deviation is therefore $\theta/2 - \delta\theta/2 + \delta\theta = (1+\delta)\theta/2$. From equation (9), $\theta < \theta^* = 2\underline{\theta}$. Therefore, the payoff from deviation is $\theta/2 + \delta\theta/2 < 2\underline{\theta}(1+\delta)/2 = (1+\delta)\underline{\theta}$. Q.E.D.

Proof of Proposition 2: The first-period wage in equation (17) is obtained by imposing a zero-profit condition. The expected profit is the sum of the second- and third-period expected profit from a worker with ability below the threshold plus the expected profit from a worker with ability above it weighted by the probabilities:

$$\pi(\underline{\theta}) = \delta\left\{(1+\delta)\int_{\underline{\theta}}^{\theta^*}(\theta - \underline{\theta})f(\theta)\partial\theta + \int_{\theta^*}^{\overline{\theta}}(\theta - w_2(\theta))f(\theta)\partial\theta\right\} - w_1.$$

Equating the profit to zero gives the first-period wage in the proposition. Because of competition, given w_2^c, w_3^c, no firm can offer a different first-period wage to increase profit. Next, I show that there is no alternative contract in which the second- and third-period wage are not $\underline{\theta}$, that can increase the payoffs of the deviating firm. The worker's expected payoff from the above contract is,

$$U(\underline{\theta}) = \delta\left\{(1+\delta)\int_{\underline{\theta}}^{\theta^*}(\theta - \underline{\theta})f(\theta)\partial\theta + \int_{\theta^*}^{\overline{\theta}}(\theta - w_2(\theta))f(\theta)\partial\theta\right\} \tag{17}$$

$$+\,\delta(1+\delta)\int_{\underline{\theta}}^{\theta^*}\underline{\theta}f(\theta)\partial\theta + \delta\int_{\theta^*}^{\overline{\theta}}w_2(\theta)f(\theta)\partial\theta + \delta^2\int_{\theta^*}^{\overline{\theta}}\theta f(\theta)\partial\theta = \delta(1+\delta)\int_{\underline{\theta}}^{\overline{\theta}}\theta f(\theta)\partial\theta$$

Suppose a firm offers a contract with lower future wages. That is, either $w_2 = x$ or $w_3 = y$ or both are below $\underline{\theta}$. Given the second- and third-period strategies, the threshold level of renegotiation will be lower: $w_2(\theta^*) + \delta w_3(\theta^*) = x + \delta y$. The equilibrium strategies are specified for any wages, w_2, w_3 (see Lemma 3 part 1). Thus, agreement is reached in the first round. The expected payoff for the worker in periods two and three are

$$\delta(x+\delta y)F[\theta^*(x,y)] + \delta\int_{\theta^*(x,y)}^{\overline{\theta}}w_2(\theta)f(\theta)\partial\theta + \delta^2\int_{\theta^*(x,y)}^{\overline{\theta}}\theta f(\theta)\partial\theta.$$

Because the threshold ability for renegotiation decreases, these payoffs are smaller. The employer can only hire a worker if the first-period wage is higher so the worker's expected payoff is $\delta(1+\delta)\theta$. Offering such a contract does not increase the employer's profit. Suppose a firm offers a contract with higher future wages. Either $x > \underline{\theta}$ or $y > \underline{\theta}$ or both. As argued above, any worker who renegotiates will receive third- and second-period wages as described in equations (2) and (3). Since the first-period wage is computed so the worker earns the expected lifetime productivity in the firm and since this productivity is maximized when the worker remains in the firm, offering a higher wage clearly does not increase the firm's profit. Q.E.D.

Proof of Corollary 2. All negotiation strategies for periods three and two remain the same and characterize any possible initial contract. The threshold ability, θ^*, is derived in equation (9) for a general contract. The proof of no deviation for workers below θ^* in Proposition 1 does not depend on the value of the initial contract. The first-period contract is derived to satisfy the zero-profit condition and the proof of no profitable deviation from the initial contract is the same as the proof in Proposition 2. Q.E.D.

Proof of Proposition 3: 1. Consider type $\underline{\theta}$. In order for this type to renegotiate, payoffs should be weakly greater than $w_2^c + \delta w_3^c$. Suppose there exists an equilibrium in which all types renegotiate:

a. In any such equilibrium, $w_3^m(\underline{\theta}) \leq \underline{\theta}$. Otherwise, expected profits of outside firms are negative.

b. Consider a worker of type $\underline{\theta}$ negotiating and consider the employer's offer in round two. The worker's minimal payment in period three is $1/2\underline{\theta}$. Therefore, any offer made leaving the worker with a continuation value of $\delta 1/2\underline{\theta}$ is accepted by the worker. Thus, the employer always receives $\underline{\theta}(1 + \delta/2)$ if he makes the offer. If the worker makes the offer, he can have a continuation value of at most $\underline{\theta} + \underline{\theta}\delta/2$.

Because each player makes an offer with probability 1/2, the maximum surplus a worker can earn is $1/2(\underline{\theta} + \underline{\theta}\delta/2) + 1/2 \times \delta/2 \times \underline{\theta} = 1/2(1 + \delta)\underline{\theta}$. This is lower than $w_2^c + \delta w_3^c$.

2. Suppose such an equilibrium exists. Then no type deviates and renegotiates. The third-period wage is

$$w_3(\theta) = \begin{cases} \frac{\theta}{2}, & \text{if } \theta \geq w_3^c, \\ w_3^c, & \text{otherwise.} \end{cases} \tag{18}$$

Consider the second-period bargaining and assume no one renegotiates. In that case, the worker's payoff is $w_2^c + \delta max\{\theta/2, w_3^c\}$. Suppose that type $\overline{\theta}$ deviates and renegotiates. In round two of the bargaining, he makes an offer with probability half. For any beliefs and third-period market offers that follow negotiation, the current employer makes at most $\delta\theta/2$ if he rejects an offer. Therefore, any wage such that $\overline{\theta} - w_{2,r}^k \geq \delta\overline{\theta}/2$ is accepted. Therefore, the lower bound on the worker's continuation payoff if he makes offers in round two is $\overline{\theta}[1-\delta/2] + \delta\overline{\theta}/2$. With probability 1/2, the employer makes the offer. The worst outcome for the worker in the second period is 0. Regardless of the outcome of period two, the lowest expected third-period wage is $\overline{\theta}/2$. Therefore, the worker can *always* profitably deviate if $1/2\times\overline{\theta}[1-\delta/2+\delta/2]+1/2\times\delta/2\times\overline{\theta} > w_2^c + \delta w_3^c$. Therefore, if $\overline{\theta}/2 + \overline{\theta}\delta/4 > w_2^c + \delta w_3^c$ the high type will always renegotiate.

3. Part 1 of the proof establishes that the continuation value of second-round negotiation is at most $(1+\delta)\theta/2$, and part 2 of the proof establishes that its lower bound is $\theta/2 + \delta\theta/4$. Suppose there exists an equilibrium in which agreement is reached is round one and $w_{2,1}^k(\theta^*) = w_{2,1}^k(\overline{\theta})$. The following conditions are necessary for such an equilibrium to exist. First, the wage is accepted by $\overline{\theta}$. Therefore, $w_{2,1}^k + \delta\overline{\theta}/2 \geq \overline{\theta}/2 + \delta\overline{\theta}/4$. The lowest accepted wage is therefore $w_{2,1}^k = \overline{\theta}/2 - \delta\overline{\theta}/4$. Second, the employer agrees to the offer if made by type θ^*, $\theta^* - \overline{\theta}/2 - \delta\overline{\theta}/4 + \delta\theta^*/2 \geq (1+\delta)\theta^*/2$. The highest type that renegotiates in any possible equilibrium is $(1+\delta)\theta^*/2 = w_2^c + \delta w_3^c$. Therefore if $2(w_2^c + \delta w_3^c)/(1+\delta) < \overline{\theta}(1-\delta/2)$, the necessary conditions fail. Therefore, no equilibrium in which agreement is reached in round one and $w_{2,1}^k(\theta^*) = w_{2,1}^k(\overline{\theta})$ exists.

Next I show that if $\theta^* < \overline{\theta}$, there exists no equilibrium in which agreement is reached in round two and $w_{2,2}^k(\theta^*) = w_{2,2}^k(\overline{\theta})$. Suppose $w_{2,2}^l(\theta^*) = w_{2,2}^l(\overline{\theta})$. As established above, the lowest wage offer made by type $\overline{\theta}$ is $w_{2,2}^l \geq \overline{\theta} - \delta\overline{\theta}/2$. A sufficient condition for such an offer to be rejected if made by type $\theta^* < \overline{\theta} - \delta\overline{\theta}/2$. Since the condition in the proposition is that $\theta^* < \overline{\theta} - (2-\delta)/2$, the condition is satisfied. Q.E.D.

References

[1] Baker, G., M. Gibbs and B. Holmstrom, "The Wage Policy of a Firm," *Quarterly Journal of Economics* 109 (1994a), 881–919.

[2] Baker, G., M. Gibbs and B. Holmstrom, "The Internal Economics of the Firm: Evidence from Personnel Data," *Quarterly Journal of Economics* 109 (1994b), 921–955.

[3] Ben-Porath, Y., "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy* 75(4) (1967), 352–365.

[4] Bernhardt D. and D. Scoones, "Promotion, Turnover, and Preemptive Wage Offers," *American Economic Review* 83 (1993), 771–791.

[5] Bernhardt, D., "Strategic Promotion and Compensation," *Review of Economic Studies* 62 (1995), 315–339.

[6] Chang, C. and Y. Wang, "Human Capital Investment under Asymmetric Information: The Pigovian Conjecture Revisited," *Journal of Labor Economics* 14 (July 1996), 505–519.

[7] Farber, H. and R. Gibbons, "Learning and Wage Dynamics," *Quarterly Journal of Economics* 111 (1996), 1007–1047.

[8] Gibbons, R. and M. Waldman, "A Theory of Wage and Promotion Dynamics in Internal Labor Markets," *Quarterly Journal of Economics*, 114 (November 1999), 1321–1358.

[9] Golan, L., "Internal Labor Markets, Asymmetric Information and Wage Dynamics," Ph.D. Dissertation, University of Wisconsin–Madison, 2002.

[10] Golan, L., "Counteroffers and Efficiency in Competitive Markets with Asymmetric Information," *Journal of Labor Economics*, 23(2) (2005), 373–392.

[11] Greenwald, B., "Adverse Selection in the Labour Market," *Review of Economics Studies* 52 (1986), 325–347.

[12] Kennan, J., “Repeated Bargaining with Persistent Private Information,” *Review of Economics Studies*, 68, (October 2001), 719–755.

[13] Lazear, E., “Raids and Offer Matching,” in Ronald G. Ehrenberg, ed., *Research in Labor Economics*, Vol. 8, Part A (Greenwich, CT: JAI Press, 1986), 141–165.

[14] MacLeod, B. and J. Malcomson, “Reputation and Hierarchy in Dynamic Models of Employment,” *Journal of Political Economy*, 96(4) (August 1988), 832–854.

[15] Neal, D. and S. Rosen, “Theories of Labor Earnings,” in A. B. Atkinson and F. Bourguignon, ed., *Handbook of Income Distribution* (Amsterdam: North Holland, 2000).

[16] Pinkston, J. “A Model of Asymmetric Employer Learning with Testable Implications,” BLS Working Paper number 390, 2005.

[17] Ricart I Costa, J., “Managerial Task Assignment and Promotion,” *Econometrica* 56 (1988), 449–466.

[18] Spence, M., “Job Market Signaling,” *The Quarterly Journal of Economics* 87(3) (August 1973),

[19] Waldman, M., “Job Assignments, Signalling and Efficiency,” *Rand Journal of Economics*, 15(2), (Summer 1984), 255–267.

[20] Wolinsky A., “A Theory of the Firm with Non-Binding Employment Contracts,” *Econometrica* 68(4) (July 2000), 875–910.

Figure 1: Timeline

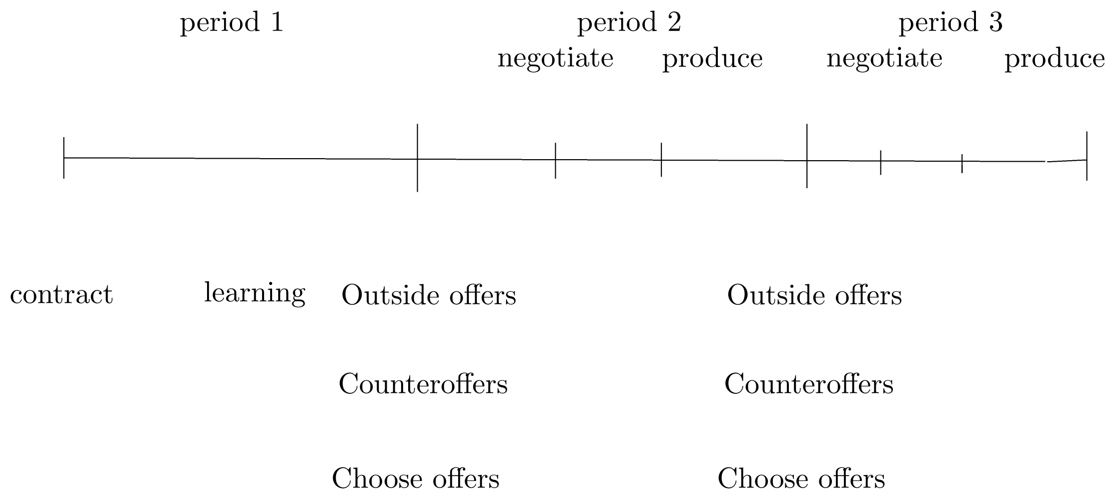

Figure 2: Period one—Learning

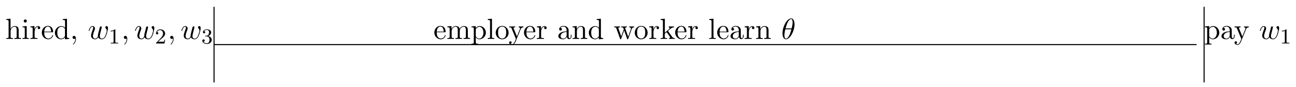

Figure 3: End of period 1 / Beginning of period 2

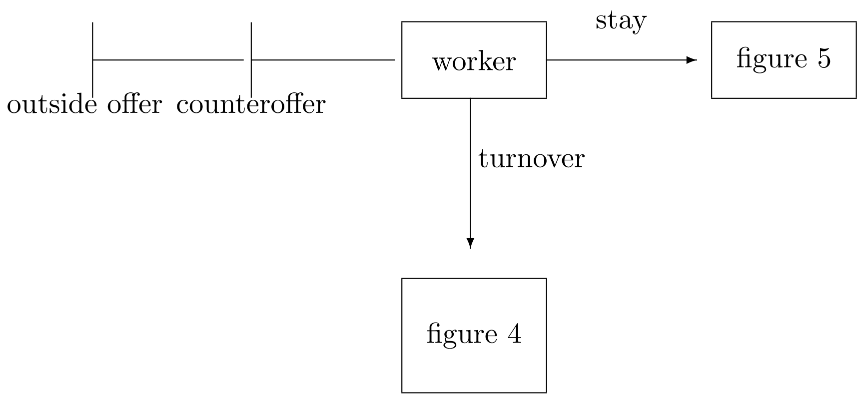

Figure 4: Period 2—Turnover

Figure 5: Period 2—Work for first-period employer

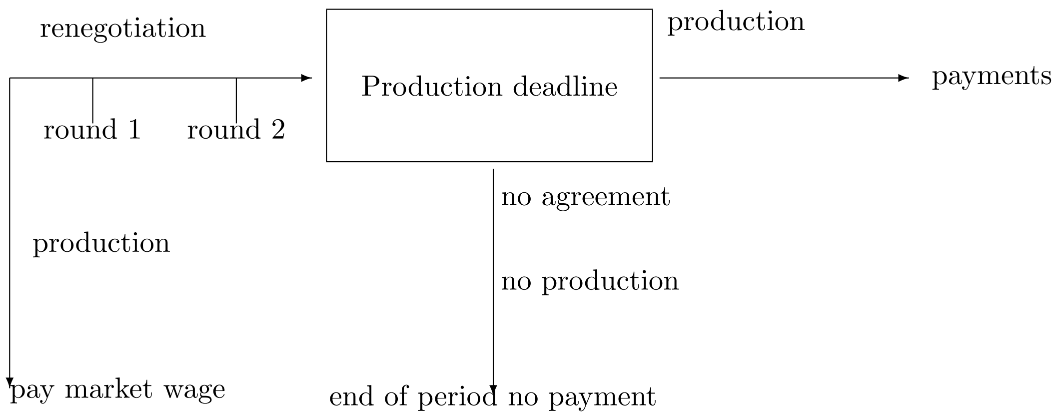

Figure 6: Second-period wage as a function of ability

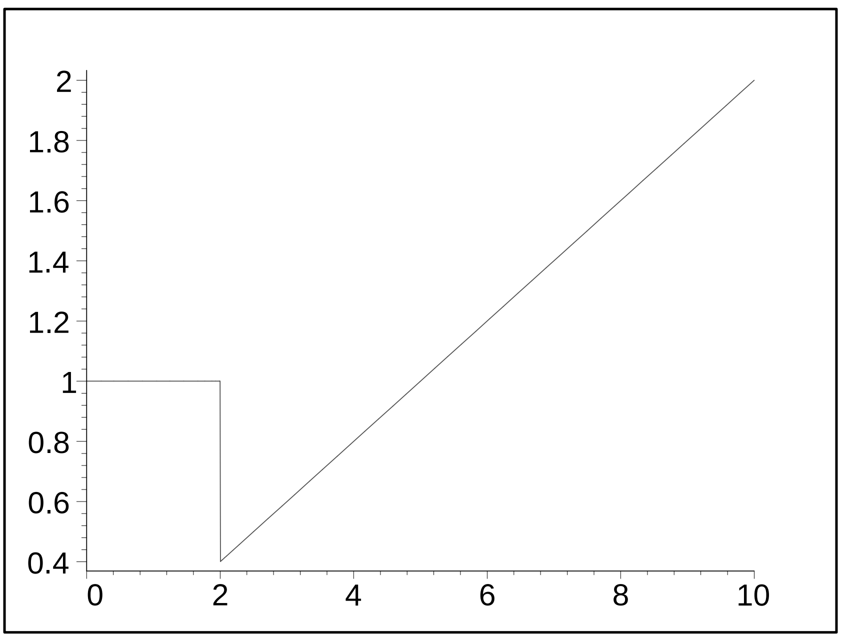

Figure 7: The second- and third-period wages as a function of productivity.