Effect of Pretests on Children's Numerical Magnitude Representations

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Abstract

An accurate sense of numerical magnitude is crucial for children to develop an understanding of mathematics. As children begin learning about numbers, they develop a logarithmic sense of number magnitudes- one where the numbers they can't understand or count to are just considered "big" and are fairly indistinguishable- but as they learn more numbers, they shift to a linear, 1:1 representation of numbers. Some researchers suggest that in studies of the logarithmic to linear shift in numerical magnitude representation, strategy choice during the study's pretest could be influencing their data. In studies where children complete an unguided pretest, participants have to choose a particular strategy to utilize for each problem, a choice that is influenced by whether they have a more linear or logarithmic sense of the numbers in the pretest's range. If these strategies are incorrect or inappropriate, they prevent participants from acquiring or reinforcing new strategies learned during a feedback stage of the study because the "bad" strategy is forefront in their minds. Our study investigates this relation between pretest strategies and posttest performance. Participants were tested on a basic number line task for the pretest, feedback stage, and posttest, with the pretest differing between randomized groups; one group was tested on a number line ranging from 0-10, another was tested on 0-20, and the control group had no pretest. The results suggest that the children tested with a 0-10 number line on the pretest learned more from the feedback than children tested on a 0-20 number line. However, the randomized groups differed in their pre-experimental number magnitude knowledge, which may have affected our results.

Keywords: numerical magnitude, pretest, logarithmic to linear shift

Effect of Pretests on Children's Numerical Magnitude Representations

The ongoing exploration of how children learn mathematics is one that is becoming increasingly important in national and international communities of educators. While there are many factors influencing this learning, one of the most important factors is the development of numerical magnitude representations. An accurate sense of numerical magnitude is crucial for children to develop mastery in all domains of mathematics (Booth & Siegler, 2008). However, children do not begin with a sense of numerical magnitude that adults rely on in their everyday lives. Through age and experience with greater numerical magnitudes, they come to develop a sense of numerical magnitude necessary for all mathematical endeavors (Booth & Siegler, 2008).

Logarithmic to Linear Shift

As children begin learning about numbers, their first representation of numerical magnitudes is best fit by a logarithmic function (Booth & Siegler 2008). Children most often experience smaller numbers, usually 0 through 10; for example, they learn to count to 10 first before going higher, children's books focus heavily on smaller numbers, classroom math activities rarely use numbers above 20. Due to this biased exposure to small numbers, children greatly overestimate their magnitude. Numbers they have less exposure to or cannot count to are just considered "big" in their representations and are fairly indistinguishable (Siegler, 2009). This effect is seen in tasks where children are asked whether the answer to a problem such as 24 + 18 could equal 82. Since the child with a logarithmic representation considers 44 and 82 to be equally large, they would not consider 82 to be an unreasonable answer (Laski & Siegler, 2007).

Through experience with numerical magnitude, children develop a linear representation of numbers, a representation that matches the structure of society's number system and is therefore more accurate. The age where this shift occurs depends on the range of numbers. One effective way of tracking this shift is with a numerical estimation task, specifically a number line estimation task. Children of varying age groups are given an empty number line with clearly marked endpoints and asked to locate the position of a third number on the line. This task requires children to transfer their knowledge of numerical magnitude between the symbolic representation and its spatial location, giving experimenters an accurate measure of children's numerical representations.

When given an empty number line with endpoints of 0 and 100 and asked to estimate the location of numbers on that line, data from kindergarteners fits a logarithmic curve, whereas estimates from second graders produce a much more linear fit (Booth & Siegler, 2004). First graders are split between the two representations. However, on a number line from 0 to 1000, data from second graders produces a logarithmic representation, while fourth graders have mostly achieved the linear shift (Siegler & Booth, 2004).

While these examples show the logarithmic-to-linear shift on a long-term scale, it has also been shown on a short-term scale. Children can often show breakthroughs in representations in a single research session where they receive feedback on their answers (Siegler, Thompson, & Opfer, 2009). This abrupt shift is also seen on other numerical estimation tasks, such as measurement estimation (Booth & Siegler, 2006). One example of a measurement estimation task is the "zip" test. A number line is labeled as 1000 zips long, and the child is asked to draw a line that is *n* zips long (where *n* is smaller than 1000) (Booth & Siegler, 2006). The strength of this finding is directly applicable to teaching children in the classroom: Griffin, Case, and their colleagues (Griffin, 2004; Griffin et al., 1994) found that they could improve mathematical performance of low-income preschoolers by giving them games focused on activating linear representations.

Estimation ability is associated with many aspects of mathematics, being a high predictor for overall math achievement (Siegler & Booth, 2004), memory for numbers (Case & Sowder, 1990), and conceptual understanding of mathematics (LeFevre, Greenham, & Naheed, 1993). Estimation is also used consistently in our daily lives; estimating the number of people in a crowd, the length of time to eat a meal, or the distance to the mall are common problems where an exact number is not necessary. Ultimately, estimation is incredibly important in our daily lives as well as in all fields of mathematics. Because of its correlations with many aspects of mathematics, estimation tasks are often an accurate predictor of learning in math-based studies.

The Current Study

As seen in the age-dependent differences on the estimation task between the 0-100 and 0-1000 number lines, studies have shown that although children may show a linear representation on a particular number line, they revert to a logarithmic representation for number lines of greater range. This could, however, be due to the way in which researchers are testing this phenomenon. John E. Opfer and Clarissa A. Thompson (2008) suggest that the pretest incorporated into the majority of math-based studies could be influencing these data. In their study, the children showed learning by an increased use of linear representations when given feedback on a numerical estimation task. However, if given a pretest with numbers out of the child's linear range, the test activated the incorrect logarithmic representation. This activation stayed in the forefront of their minds throughout the rest of the study, preventing them from learning a more linear representation (Opfer & Thompson, 2008).

In studies where children complete an unguided pretest, one in which they solve similar problems to the task being tested without receiving feedback, they have to choose a particular strategy to utilize for each problem. Strategy choice is a crucial aspect to children's learning. For any given task, children may have multiple strategies that they could employ, but they choose the strategy that will help them complete the task with speed and accuracy. They may not know the best strategy for that particular task, but they choose from the ones they know (Siegler, 2005). According to the *overlapping waves* theory, the frequency of any given strategy fluctuates depending on their age, the task at hand, comfort with the use of that particular strategy, and other factors. Children often use multiple strategies in one experimental session (Siegler, 1995).

The overlapping waves theory is seen in number line estimation tasks when children have to decide whether to utilize a logarithmic or linear representation strategy. For a number line with numbers outside of their range of expertise, children choose a logarithmic strategy. While it may not be the correct strategy for solving the problem, it is the most effective for the child because they do not understand the magnitude differences for large numbers. If they choose a strategy that is incorrect or inappropriate for the task, that choice actually prevents the children from acquiring new strategies. The "bad" strategy is forefront in their minds, making the strategy harder to dislodge and a stronger competitor in their strategy choice for future problems. They then show less learning on the posttest because they have not been able to absorb the feedback from the learning phase of the study (Opfer & Thompson, 2008).

In the present study, we investigated whether pre-testing children around age five on a number line task impacts both performance and learning on more difficult number line tasks.

This question is relevant for the interpretation of data from past studies; the pretests incorporated into many studies could have affected the results. This question is also relevant for future math-based experiments, affecting study design and purpose. Even further, this question applies directly to the classroom in many ways, including the structure of homework and how new material is presented in class. By expanding the findings of Opfer and Thompson, we can generalize this phenomenon to a wider population and set of tasks. Also, including two extra pretests from their study, one that could potentially be beneficial to learning and one used as a control, will help us determine whether the "logarithmic" pretest is harmful, the "linear" pretest is beneficial, or both.

Most children around age five have a logarithmic representation of numbers greater than 10- all numbers greater than their limit are categorized as "big" and become fairly indistinguishable (Booth & Siegler, 2008). In our study, if the child chooses the logarithmic strategy on the pretest, which would be the case for a number range extending beyond 10, we predict that they will show less learning from feedback on the posttest. If the child chooses the correct linear strategy on the pretest, then they will be more likely to continue choosing that strategy during the feedback phase where it will be reinforced further, and therefore they will show more learning on the posttest. A pretest with a greater numerical range between endpoints should activate logarithmic representations of numbers on a number line, which would strengthen the possibility of the child choosing a logarithmic representation, whereas a pretest with smaller, better known numbers should activate a linear representation that would help on the posttest.

To test these ideas, we tested kindergarteners and preschoolers on a number line estimation task with three different pretests: one intended to activate a detrimental logarithmic representation, one intended to activate a beneficial linear representation, and one group with no pretest to be used as a control. After a short feedback session, the children then completed a posttest on the logarithmic number line to determine if they had learned from the feedback.

Methods

Participants

Kindergarteners and preschoolers from the Children's School at Carnegie Mellon University participated in the study. They were ages 4 and 5, with an average age of 5.22 years. Out of the total 32 participants, 14 were from preschool classrooms and 18 were in the kindergarten. Just two weeks earlier, the kindergarteners had started doing class activities using a large in-class number line from 0-20. This may have contributed to the kindergarteners being better at the task overall, the preschoolers seemed to have enough experience with number lines understand the concept of a number line and the presented task. All of the participants were run within two weeks, limiting the amount of learning that could have been acquired in the classroom while the study was being conducted.

Materials and Procedure

The study was conducted on a laptop with a plugged-in mouse, facilitating easier motor movements for the participants than the trackpad. The experimenter only assisted the participant with computer skills where necessary. The Java program for the study recorded data from the location on the monitor where the participant clicked.

The study consisted of a pretest, a feedback phase, and a posttest, as well as one example at the beginning ("0" on a 0-20 number line) for the experimenter to demonstrate the task. The pretest and posttest consisted of 15 numbers each, and the feedback phase consisted of 6 problems. The feedback trials consisted of three numbers that were tested on the posttest and three that were not. The order of numbers within each phase was random. There were three experimental conditions, and participants were randomized assigned to them. The number of preschooler and kindergarten participants were about equal across conditions.

In each example presented to the participants, a number was displayed above a number line with clearly marked endpoints. The participant was directed to click the place on the line where that number belonged. A green line would appear where participants clicked, and then they were instructed to press the "Next" button at the top of the screen. This movement to press the button, along with instructions from the experimenter, prevented them from clicking in the same location repeatedly. In the feedback phase, there were green stars along the top of the screen to indicate that it was a different section. When the participant clicked on the number line, a red line would indicate where they clicked, and a separate line would indicate the location of the correct answer. If their guess was within 10% of the correct answer, the line indicating the correct answer was red, and the monitor would display the message "Great!". If the participant's guess was outside of that range, then a blue line indicated the correct answer and the message displayed was "Not Quite." Each message was verbally reinforced by the experimenter, while they urged the child to try the next one. This section particularly excited the participants, possibly because they felt rewarded when they were close to the right answer.

The three variations differed only in their pretest. One variation (the large number pretest) was a number line from 0-20, the second variation (the small number pretest) was a number line from 0-10, and the third variation (the control) had no pretest at all. The numbers chosen for the large number pretest were the same as those on the posttest, and the numbers chosen for the small number pretest included only two numbers that were not on the posttest. The 0-10 number line was chosen based on previous studies of a similar nature, which found that 4- and 5-year-olds are more linear than logarithmic on numbers in that range, whereas they are more logarithmic than linear on 0-20 number lines. Using the 0-10 number line on the pretest would ideally activate the participants' linear representation strategies, which would aid in strategy choice during the feedback, helping participants learn more. Most children around age 5 do not have a linear representation of a 0-20 scale, so activating this logarithmic representation should hinder their ability to learn from the feedback and make it possible for us to assess learning within this range. The no-pretest condition was used as a standard for comparison to help determine whether the 0-10 pretest had a positive effect on learning, whether the 0-20 pretest was detrimental, or both.

Results

Accuracy

First, we examined the accuracy of participants' estimates using absolute error (PAE). PAE is defined as the magnitude of the difference between the exact value and the approximation, which is the participant's estimation in our study. The formula used to determine PAE is |(given number – participant's estimate) / given number| * 100. For example, if the participant is asked to guess the location of 8 on a 0-10 number line and their estimate is at the location of 6, then their PAE is |(8-6)/8|*100 = 25%. However, if they guess at 7, then their PAE is 12.5%. A participant with a lower PAE is better at the task because their guesses are more accurate.

As shown in Table 1, the PAE analysis of individual data shows that PAE's of the small number pretest condition decreased from pretest to posttest, indicating a significant improvement ($t(10) = 5.72, p<.05$). The analysis also shows that the large number pretest condition had no significant improvement ($t < 1$).

Linearity

We then computed the best fitting linear and logarithmic functions relating the actual number to the participant's estimate. Whichever R^2 value is higher between the two functions indicates which function fits the data better. At pretest, the small number pretest condition was significantly more linear than logarithmic ($t(10) = 1.3, p=.12$), as predicted. There was no significant difference between the linear and logarithmic functions at pretest for the large number pretest condition ($t<1$), showing that each function fit the data equally well. At posttest, both functions fit the data equally well for all conditions ($t<1$ for all three conditions).

In addition, we analyzed the difference in R^2 values from pretest to posttest to determine whether participants became more linear between pretest and posttest. The analysis of R^2 values of individuals showed a nearly significant increase in linearity in the small number pretest group ($t(10) = 2.2, p=.06$), as predicted, whereas those in the large number pretest group stayed about the same ($t<1$) (Table 2). This finding is important considering that the small number pretest condition was given a pretest where they were expected to be linear.

Within each condition, we took the medians of all of the participants' estimates on each trial. At posttest, the small number pretest condition showed nearly significant linearity ($t(14) = 1.9, p=.08$), whereas the control condition was not significantly linear ($t<1$). This finding suggests that the small number pretest was better than no pretest at all.

The number of participants who were significantly linear turned out to be a poor predictor of the tasks, as the number of those who were significantly linear was small at pretest and decreased at posttest in every group.

Discussion

Overall, the data supported our predictions about the effect of strategy choice at pretest. While the linearity of the data was similar across groups at posttest, the small number pretest group showed far more improvement from their pretest linearity. The small number pretest group also displayed more accurate results, as seen by their PAE scores. Their estimates were almost 10% more accurate than at pretest, whereas the large number pretest group did not show almost any improvement.

Participants in the small number pretest condition improved after the feedback stage, whereas participants in the large number pretest condition did not show improvement. A possible explanation for this finding comes from the fact that the large number pretest participants were already close to ceiling at the pretest. Those participants chose a linear strategy more often than the small number pretest group on the pretest, so while it does not appear that they learned anything from the feedback stage, there was less room for them to show improvement. Another possibility is that since the kindergarteners had been using a 0-20 number line in their classroom, it took the small number pretest group a little longer to acclimate to the task. However, since the pattern of results was similar across the grades, we can discount a strong effect of curriculum.

The fact that the small number pretest group showed more improvement than the other two groups at posttest gives some support to our hypothesis that a pretest activating a linear representation for the rest of the study is ultimately beneficial. While fewer students in the small number pretest group started out as having a linear representation than in the large number pretest group, that strategy was still forefront in their minds, resulting in improvement by posttest. Also, there may have been other factors contributing to the fewer linear students in the small number pretest group, reflected in the fact that the large number pretest group was already at ceiling.

Giving a pretest where children chose a logarithmic strategy was not detrimental to their learning, but it was more helpful for them to have the linear strategy in the forefront of their minds. These findings can apply directly to learning and classroom planning. Many teachers do a short review with their students before building on a previously learned topic, which is similar to doing a pretest. By choosing a review activity where children are more likely to choose a correct strategy, that strategy is primed for the rest of the activities and allows for greater learning in the classroom.

Further research is necessary to fully understand this phenomenon, particularly because of the skewed data in the large number pretest group's pretest. A larger population of children from more diverse schools and backgrounds would give a more accurate depiction of this problem. Also, the concept of pretest effects could be applied to many other domains, both math and non-math related tasks. It would be interesting to investigate this effect in settings such as reading and the sciences. For example, if a teacher is explaining to her class how to put physics equations into real applications, it may help to have examples at the beginning of the lesson reminding students of correct use of basic physics equations and concepts. Having a short reminder section where the students are more likely to choose the correct strategy may benefit them for the rest of the more complicated problems. The correct strategies for equation uses will be at the forefront of their minds, helping them learn new applications utilizing those principles. This principle could be expanded to many areas of education.

Figures

Table 1. PAE averages of individual data

Overall PAE	Pretest	Posttest	Improvement
Large number pretest	.188	.181	.008
Small number pretest	.279	.144	.135
Control		.232	

Table 2. Averages of individual R^2 values of linear and logarithmic regressions

R^2	Pretest Lin	Pretest Log	Posttest Lin	Posttest Log
Large number pretest	.56	.53	.56	.56
Small number pretest	.48	.38	.68	.64
Control			.42	.38

Table 3. PAE analysis of preschoolers

Preschoolers	Pretest	Posttest	Improvement
Large number pretest	.211	.226	-.015
Small number pretest	.335	.182	.153
Control		.302	

Table 4. PAE analysis of kindergarteners

Kindergarteners	Pretest	Posttest	Improvement
Large number pretest	.161	.129	.032
Small number pretest	.246	.123	.123
Control		.186	

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