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Inefficiencies from Metropolitan Political and Fiscal Decentralization: Failures of Tiebout Competition*

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Abstract. We examine theoretically and quantitatively the welfare effects of decentralized (Tiebout) provision of local public goods as compared to uniform centralized provision. We show that inefficiencies associated with property taxation offset the potential welfare gains from matching provision to preferences under decentralized provision. We identify an externality in community choice as the major source of inefficiency: Poorer households crowd the suburbs while avoiding taxes by consuming little housing. Our quantitative findings are based on a variety of estimates including an estimated model of the Boston Metropolitan Area.

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1. Introduction.

The analogy between competition among firms in providing private goods and “Tiebout (1956) competition” among jurisdictions in providing local public goods is central to the economic study of local public finance. The basic idea is that household mobility will induce jurisdictions to provide efficient mixes of local public goods and taxes, or they will fail to attract residents. Beginning with the pioneering work of Oates (1969, 1972), a large literature examines when the analogy is sufficiently compelling so that inter-jurisdictional competition is efficient and the nature of departures from efficiency when these conditions are not fulfilled.1 For efficiency, essentially the tax system and housing-market prices must control any externalities in residential choice with also efficient governmental choice of the levels of the local public goods. While standard models frequently fail to meet the conditions for efficiency, economic intuition suggests that some Tiebout competition is better overall than none: The alternative of uniform centralized provision will do nothing to match heterogeneous preferences to provision of local public goods. This paper challenges this intuition, showing that, in practice, inefficiencies in Tiebout competition are large. Indeed, results from both calibrated and estimated models suggest that the inefficiencies arising from decentralization are of comparable magnitude to the preference-matching benefits of decentralization. Thus, rather than producing gains, decentralization may well result in aggregate welfare losses.

The model we consider is not contrived.2 A metropolitan area is made up of multiple jurisdictions with given boundaries. Households differ by income and a taste parameter with utility function over numeraire consumption, housing consumption, and the level of the local congested public good (e.g., per student educational expenditure). The local public good is financed by a property tax that is chosen by majority vote of residents of the jurisdiction. Households choose where to reside, and then vote in their jurisdiction and consume. 


2 We work with the “standard Tiebout model,” which, of course, abstracts from elements like commuting costs that are of interest to consider. We discuss various extensions to the analysis in the concluding section. More importantly, the model has been shown to fit well the data in a series of papers (Epple and Sieg, 1999, Epple, Romer, and Sieg, 2001, and Calabrese, Epple, Romer, and Sieg, 2006). We employ estimates from this research in Section 5, where the specification is reviewed.
findings regard cases when an income-stratified equilibrium exists, i.e., when a Tiebout-type equilibrium arises.³

We show computationally that welfare in aggregate, measured by aggregate compensating variation, is lower than in the analogous centralized equilibrium with the same political process for realistically specified parameters. We go on to show the same holds in an estimated model of the Boston metropolitan area. We know a priori that the Tiebout equilibrium will not be Pareto Efficient. First, majority choice of the tax level satisfies a median resident’s preference and will not generally satisfy the Samuelsonian condition for efficient provision of the local public good. Second, the property tax causes a distortion in the housing market, while a head tax would be non-distorting and efficient. Third, the latter distortions imply externalities in individual residential choice. With local head taxes chosen efficiently and equilibrium household choices of jurisdictions, the modified Tiebout allocation would generate substantial welfare gains. With the imperfect system, these potential welfare gains are not just lost, but are frequently reversed. We provide computational evidence that the most costly inefficiency is the externality in residential choices. Too many relatively poorer households move into richer jurisdictions. Efficient sorting would be more exclusive than arises in equilibrium. It is rather surprising that getting part way to an efficient Tiebout allocation is frequently less efficient than no sorting.

Section 2 provides a simple example that illustrates our key finding. Section 3 presents the theoretical model and associated positive and normative properties. A calibrated computational model is analyzed in Section 4. The estimated model and associated welfare calculations are presented in Section 5, along with discussion of some related analysis. Section 6 concludes. An appendix contains some proofs.

2. A Simple Example.

We begin with a brief summary of a simple example that is not intended to be realistic but illustrates our key finding. This example conforms to the theoretical model developed in detail in the next section so any elements that may be unclear here will be explained fully. We compare a centralized equilibrium where all households live in one jurisdiction and choose by majority vote a property tax to provide uniform provision of a congested public good to a Tiebout equilibrium where households choose between two jurisdictions each of which chooses

³ As described in detail below, such an equilibrium will arise for realistic parameter values when standard single-crossing conditions are satisfied.
by majority vote of residents a local property tax for local provision of the congested public
good.

In the simple example, 2/3 of the population of households are poor with income of
$25,000 and the remaining are rich with income of $100,000. Households have CES utility
function over housing services, the public good, and numeraire (composite private good)
consumption, with parameters in Table 1 presented later. In the centralized equilibrium, housing
services are competitively supplied with constant elasticity of supply equal to 3. Centralized
equilibrium has price per unit of housing services equal to $16.74, a tax rate per unit of housing
services equal to about 35%, and per household provision of the public good equal to $3,490.
Taxes collected from poor households are $1,745 and from rich households are $6,980, the
difference due, of course, to different levels of consumption of housing services.

In the Tiebout equilibrium, the two jurisdictions each have one-half the land area,
splitting the supply of housing services equally between the jurisdictions. The poor jurisdiction
contains all poor households consisting of 49% of the total population, and the remaining poor
and rich reside in the richer jurisdiction. A poor household is the pivotal voter in the former
jurisdiction and a rich household is the pivotal voter in the latter. Rounding, the property tax
rates remain 35% in each jurisdiction, but the per household public good expenditure equals
$1,758 in the poor jurisdiction and $5,217 in the richer jurisdiction. The net prices of housing
services ascend from $14.03 to $18.60. In the richer jurisdiction, taxes collected from poor
households are $1,762 while $7,047 from rich households.

Everyone is worse off in the Tiebout equilibrium, including land owners who are
absentee in the model. It is not surprising that the poor are worse off since they cannot so easily
free ride on the rich. The short answer as to why the rich are worse off is that the sorting
mechanism, housing prices, is very inefficient: The fiscal externality born by the rich from the
poor who move into the rich district is intensified by the housing scarcity in a jurisdiction smaller
than the local economy. The long answer is provided below, where we show that the finding
here (of an average welfare loss) is not at all pathological.

3. Theoretical Analysis.
a. Elements of the Model. Our intent is to examine an archetypical model of a metropolitan area
with property taxation. Households have a utility function over numeraire consumption x,
housing consumption h, and the level of the local public good g measured in dollars. Households
differ by endowed income y and a taste parameter $\alpha$, with the latter measuring taste for the local
public good as clarified below. The joint distribution on household type \((y,\alpha)\) is continuous and given by \(F(y,\alpha)\), with joint density function \(f(y,\alpha)\) assumed positive on its support \(S = [\alpha, \bar{\alpha}] \times [y, \bar{y}] \subseteq \mathbb{R}^2\). Let \(U = U(x,h,g; \alpha)\) denote the household utility function, strictly quasi-concave, increasing, and twice continuously differentiable in \((x,h,g)\). Further restrictions on \(U\) are discussed below.

We compare a Tiebout-type equilibrium having the metropolitan area divided into jurisdictions to the counterpart single-jurisdiction centralized equilibrium. Focusing first on the former case, the metropolitan area (MA) is divided into \(J\) jurisdictions, each with non-decreasing housing supply function \(H_j^*(p_j)\), where \(p_j^*\) denotes the net-of-tax or supplier price of housing, and \(j = 1,2,\ldots,J\) henceforth unless indicated otherwise. We assume absentee housing owners that supply housing competitively, but will account for their rents in our welfare calculations.\(^4\) We assume absentee housing owners simply because it is most standard.

Equilibrium is determined in three stages. First, households purchase a home in a jurisdiction. Second, they vote in their jurisdiction for a property tax that is used to finance the local public good. Last, the local public good is determined from local governmental budget balance, and households consume (although their housing consumption is determined in the first stage). Households have rational expectations, thus anticipate all continuation equilibrium values.

This specification conforms to the case sometimes called “myopic voting,” because households take as given residences, housing consumption, and the supplier price of housing when voting, which are all established in the first stage.\(^5\) We examine this case because it is historically the most studied case in the literature. We show in the robustness analysis in an appendix that the welfare loss we find from Tiebout sorting increases with other standard specifications of the timing of choices and thus voter beliefs that may be more appealing.

b. Positive Properties of Equilibrium. To provide a formal description of equilibrium, begin with the third stage. Let \(f_j(y,\alpha)\) denote the density of household types living in jurisdiction \(j\), \(t_j\) the property tax rate, and \(h_j(y,\alpha)\) housing consumption of household \((y,\alpha)\), all of which are given in

\(^4\) One interpretation is that the MA is divided into jurisdictions with fixed amounts of land, and land is combined with elastically supplied factors to produce units of housing. Then the “absentee housing owners” could just as well be absentee land owners. In Section 4, we provide a specific example of this.

\(^5\) The label “myopic voting” is potentially confusing since voters are fully rational given residence and housing consumption have been committed in the first stage. The “myopia” interpretation arises if households could move or otherwise adjust housing consumption after voting, but voters fail to recognize this. Equilibrium is the same with either interpretation because no such changes are made in equilibrium in either case.
the third stage. The gross housing price \( p_j \), local public good level \( g_j \), and household numeraire consumption are determined in the third stage, satisfying respectively:

\[
p_j = (1 + t_j)p_s^i; \tag{1}
\]

\[
g_j \int f_j(y, \alpha) dy d\alpha = t_j p_s^i H_j(p_s^i) \tag{2}
\]

and

\[
x = y - (1 + t_j)p_s^i h_j(y, \alpha); \tag{3}
\]

where \( p_s^i \) is also given, established in the first stage.\(^6\) The congestion assumption about the public good implicit in (2) is also fairly standard, as for public schooling, and avoids issues of economies in providing local public goods. Obviously, the third stage values exist and are unique for any input vector.

Now consider the second, voting stage. Substitute (1) into (3), and then (3) into the utility function and write indirect utility of household \((y, \alpha)\) as a function of \((p_j, g_j)\):

\[
V(p_j, g_j; y, \alpha) = U(y - p_j h_j(y, \alpha), h_j(y, \alpha), g_j; \alpha). \tag{4}
\]

When voting on the property tax rate, households maximize \( V(\cdot) \) while correctly anticipating that \((p_j, g_j)\) will satisfy (1)-(2), taking as given \((f_j(y, \alpha), h_j(y, \alpha), p_s^i)\). Suppress the \( j \) indicator and compute the slope of an indifference curve of \( V = \) constant in the \((g, p)\) plane:

\[
\left. \frac{dp}{dg} \right|_{V = \text{const.}} = -\frac{V_g}{V_p} = \frac{U_g / U_y}{h_y(y, \alpha)}; \tag{5}
\]

where the arguments in the numerator of the right-hand side of (5) are the same as in the right-hand side of (4). We make the following “single-crossing assumptions” with respect to indifference curves in the \((g, p)\) plane.

\[
\frac{\partial}{\partial y} \left( \left. \frac{dp}{dg} \right|_{V = \text{const.}} \right) > 0; \quad \text{(SRI)}
\]

and

\[
\frac{\partial}{\partial \alpha} \left( \left. \frac{dp}{dg} \right|_{V = \text{const.}} \right) > 0. \quad \text{(SR\(\alpha\))}
\]

\(^6\) Because households will correctly anticipate all equilibrium values, a negative numeraire will never arise in equilibrium.
Assumption SRI, “slope rising in income,” means that the willingness to trade an increase in housing price for higher $g$ rises with income. Intuitively, from the right-hand side of (5), one can see that this corresponds to cases where the marginal value of $g$ rises faster with income than does housing demand. We provide examples of and evidence supporting this assumption below. The intended nature of the taste parameter is embodied in Assumption SR$\alpha$. For given income, higher-$$\alpha$$ households are also more willing to trade an increase in housing price for increased $g$.

Proposition 1 summarizes key properties of the voting stage, with properties illustrated in the panels of Figure 1.7

**Proposition 1:** Assume that $V(p_j,g_j;y,\alpha)$ is twice continuously differentiable and strictly quasi-concave in $(p_j,g_j)$ for $(p_j,g_j) > 0$. Assume also the Inada condition that $V_g \to \infty$ as $g \to 0$.

Then:

a. Majority voting equilibrium exists and is unique.

b. The equilibrium is the preferred choice of households $(y,\alpha)$ on the downward sloping locus $y_j^m(\alpha)$ satisfying:

$$\int \int f_j(y,\alpha) dy d\alpha = .5N_j; \quad (6)$$

$$N_j \equiv \int \int f_j(y,\alpha) dy d\alpha. \quad (7)$$

c. Households living in community $j$ with $(y,\alpha)$ to the “northeast” (“southwest”) of the $y_j^m(\alpha)$ locus in the $(\alpha,y)$ plane prefer a higher- (lower-) than equilibrium tax. (See Figure 1A.)

Proposition 1 is a generalization to taste variation of well known results in the literature and is a variation on Propositions 1 and 2 in Epple and Platt (1998). We provide a proof in the appendix for completeness.

Remarks on Proposition 1:

1. An example that satisfies the conditions for Proposition 1 is the CES utility function:

$$U = [\beta_x x^\rho + \beta_h h^\rho + \beta_g(\alpha) g^\rho]^\gamma/\rho,$$

with $\rho < 0$ and $\beta_g'(\alpha) > 0$. We examine a variant of the latter in detail in Section 4.

2. The analysis is much simpler without taste variation, in which case $V$ need not be quasi-concave to obtain the analogue of the results of Proposition 1.8 With taste variation, the quasi-

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7 The $y_j^b(\alpha)$ loci in Figure 1 partition the $(\alpha,y)$ plane into jurisdictions and are discussed below.
concavity condition need not necessarily be invoked. See Epple and Platt (1998) for an alternative condition.

Now consider the first-stage household choices and the implications for the full (three-stage) equilibrium. Households choose jurisdictions and housing consumption in this stage. Since households correctly anticipate all equilibrium values, their housing consumption satisfies ordinary demand, which we denote by $h_d$. Thus a household that chooses to live in jurisdiction $j$ consumes housing:

$$h = h_d(p_j, g_j, y, \alpha) \text{ for all } j \text{ and } (y, \alpha).$$

Given jurisdicational choices, housing market clearance in community $j$ determines the supplier price of housing:

$$\int_s h_d(p_j, g_j, y, \alpha) f_s(y, \alpha) dy d\alpha = H^*_j(p^*_j); \quad (9)$$

where $p_j$ satisfies (1) for correctly anticipated $t_j$.

To determine choice of jurisdiction, find indirect utility

$$\tilde{V}(p_j, g_j; y, \alpha) = U(y - p_j h_d(p_j, g_j, y, \alpha), h_d(p_j, g_j, y, \alpha), g_j; \alpha).$$

Households choose among the $J$ jurisdictions to maximize $\tilde{V}$, correctly anticipating equilibrium $\tilde{(p_j, g_j)}$, $j = 1, 2, \ldots, J$. Applying the Envelope Theorem, the slope of $\tilde{V} =$ constant in the $(g_j, p_j)$ plane is of the same form as the slope of $V =$ constant:

$$\left. \frac{dp}{dg} \right|_{\tilde{V}=\text{const.}} = -\frac{\tilde{V}_g}{\tilde{V}_p} = \frac{U_s / U_y}{h_d}; \quad (11)$$

but evaluated at the same argument values as is utility on the right-hand side of (10). We make the analogous single-crossing assumptions on $\tilde{V}$ as SRI and SR$\alpha$, which we reference as $\tilde{\text{SRI}}$ and $\tilde{\text{SR}}\alpha$. The two pairs of single-crossing assumptions are closely related, and, for example, are exactly the same in the CES example in Remark 1 to Proposition 1.

Summarizing, an equilibrium arises if the following conditions are satisfied: In each community $j$, $(p_j, g_j)$ satisfy (1) and (2). Household numeraire consumption satisfies (3). The tax rate in each community is the majority choice, where households maximize $V(\cdot)$ when voting. Housing consumption satisfies ordinary demand, (8), and the supplier price of housing in each community satisfies housing-market clearance (9). Residential choices maximize $\tilde{V}(\cdot)$.

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8 Existence of voting equilibrium then only requires SRI. And households with income higher (lower) than the median income in community $j$ prefer higher (lower) tax than the equilibrium tax.
There are two types of equilibria that can arise. Our interest is in Tiebout-type equilibria with differences among jurisdictions in levels of provision of the public good and with at least some households having strict preference for their choice of jurisdiction. Thus, assume for now that \( g_i \neq g_j \), for all jurisdictions \( i \neq j \). Proposition 2 summarizes key characteristics of such equilibria:

**Proposition 2:** Tiebout equilibria with jurisdictions numbered such that \( g_1 < g_2 < \ldots < g_J \):

a. Have ascending bundles: \( p_1 < p_2 < \ldots < p_J \).

b. Are stratified by income and the taste parameter: For given \( \alpha \), if household with income \( y_1 \) resides in higher-numbered jurisdiction than household with income \( y_2 \), then \( y_1 \geq y_2 \) with equality for at most one income level. For given \( y \), if household with taste parameter \( \alpha_1 \) resides in higher-numbered jurisdiction than household with taste parameter \( \alpha_2 \), then \( \alpha_1 \geq \alpha_2 \) with equality for at most one value of \( \alpha \).

c. Exhibit boundary indifference and strict preference for non-boundary households: Households that exist with income level \( y_i^B(\alpha) \), \( i > j = 1, 2, \ldots, J - 1 \), for whom:

\[
\tilde{V}(p_j, g_j; y, \alpha) = \tilde{V}(p_i, g_i; y, \alpha) = \max_{k=1,2,\ldots,J} \tilde{V}(p_k, g_k; y, \alpha)
\]  

form a boundary in the \((\alpha, y)\) plane that partitions residents between communities \( j \) and \( i \) (see Figure 1A). Households on a boundary are indifferent between their chosen residents while all other residents strictly prefer their residential choice.

Versions of these results are in the literature (see, e.g., Epple and Platt (1998)), and we just outline the logic here. Proposition 2a must hold to have anyone choose a lower numbered community. Proposition 2b follows from the single-crossing assumptions \( \text{SRI} \) and \( \text{SR}\alpha \).

Proposition 2c is essentially definitional. Typically, a boundary will be between communities \( j \) and \( j+1 \), but we cannot rule out that for some \( \alpha \) no types will choose a community (implying, e.g., a boundary might be between \( j \) and \( j+2 \) for some \( \alpha \)). Note that Proposition 2b implies that boundaries will be downward sloping.

Existence of Tiebout equilibrium in the three-stage model is not guaranteed, but is not unusual.\(^9\) We provide computed examples below. Multiplicity of Tiebout equilibria can arise if housing supplies differ across jurisdictions. For example, with two jurisdictions having different housing supplies, either might be the lower-g jurisdiction. A non-stratified equilibrium always

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\(^9\) Restrictions on preferences and technology sufficient for existence in the model with no taste variation are developed in Epple, Romer, and Filimon (1993).
exists in the model as well. Suppose, for example, that each jurisdiction has the same housing supply. Suppose, further, that households choose jurisdictions in the first stage such that \( f_j = f/J \) for all \( y \). Then the continuation equilibrium values are the same in each jurisdiction; the jurisdictions are clones. In turn, the initial residential choices are equilibrium ones since the households are indifferent to their community. These non-Tiebout equilibria do not require the same housing supplies; initial residence choices can be adjusted so that the same (p,g) values arise in each jurisdiction. There are also mixed equilibria generally where proper subsets of jurisdictions are clones, these acting like one jurisdiction in a fully stratified equilibrium. Such equilibria are unstable (see, e.g., Fernandez and Rogerson, 1996). We study here the (full) Tiebout equilibrium, obviously in cases where it exists.

The comparison centralized equilibrium assumes the metropolitan area is one jurisdiction, with housing supply that is the usual aggregation of the jurisdictional housing supplies in the non-centralized case. Equilibrium is determined analogously to above, but with no alternative jurisdictions to choose from in the first stage and with one vote of the entire population for the tax rate, followed by consumption and provision of the public good. From above, it follows that centralized equilibrium exists and is unique. Obviously, no matching of preferences to public goods arises in the centralized case. Our interest is in the welfare comparison of the centralized equilibrium to the Tiebout equilibrium, when the latter exists. Note that the centralized equilibrium values correspond to those in the de-centralized non-stratified (clone) equilibrium discussed in the previous paragraph, so one can interpret the comparison this way as well.

c. Efficiency Considerations. The main finding in this paper is that potential efficiency gains from decentralization are, in practice, largely dissipated. In this sub-section, we first examine the social welfare problem to provide a theoretical perspective on the causes of the inefficiency we find. We then go on to clarify how we measure efficiency when we calculate welfare effects.

(i) The Planner’s Problem. We first characterize Pareto Efficient allocations. Let \( \omega(y,\alpha) > 0 \) denote the weight on household \((y,\alpha)\)’s utility in the social welfare function and \( \omega_R > 0 \) the same for the absentee initial housing owners.¹⁰ Let \( r(y,\alpha) \) denote the planner’s monetary transfer to household \((y,\alpha)\) and \( R \) the total transfer to the initial housing owners. The social planner is permitted to levy in community \( j \) both a head tax \( T_j \) and a property tax \( t_j \), the former necessary to obtain efficiency as we show. After solving this problem, we will then examine the

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¹⁰ We assume housing owners have quasi-linear utility functions and the social planner treats them all the same.
constrained efficiency problem that does not allow head taxation, as this will provide further insight into inefficiencies that arise in Tiebout (property-tax) equilibria. It is again convenient to work with an indirect utility function. Let:

\[ V^*(p_j, g_j, y + r(y, \alpha) - T_j, \alpha) \equiv \max_h U(y + r(y, \alpha) - T_j, h, g_j; \alpha); \]

where the solution to the maximization problem in (13) is given by \( h_d(p_j, y + r(y, \alpha) - T_j, g_j, \alpha), \) recalling that \( h_d(\cdot) \) denotes ordinary housing demand. Finally, let \( a_j(y, \alpha) \in [0, 1] \) denote the proportion of households \((y, \alpha)\) assigned by the planner to community \( j \).

The social planner’s problem is:

\[
\max_{r(y, \alpha), a_i(y, \alpha), T_i, p_i, h_i} \sum_{i=1}^{J} \left\{ \int_S \omega(y, \alpha) V^*(p_i, y + r(y, \alpha) - T_i, g_i, \alpha) a_i(y, \alpha) f(y, \alpha) dy d\alpha + \omega_R (R / J + \int_{n/(l+1)}^{n_i} H^i_s(z) dz) \right\} \\
\text{s.t.} \quad R + \int_S r(y, \alpha) f(y, \alpha) dy d\alpha = 0; \\
\int_S h_d(p_i, y + r(y, \alpha) - T_i, g_i, \alpha) a_i(y, \alpha) f(y, \alpha) dy d\alpha = H^i_s(p_i / (1 + t_i)), i = 1, 2, \ldots, J; \\
T_i \int_S a_i(y, \alpha) f(y, \alpha) dy d\alpha + \frac{t_i p_i}{1 + t_i} H^i_s(p_i / (1 + t_i)) = g_i \int_S a_i(y, \alpha) f(y, \alpha) dy d\alpha, i = 1, 2, \ldots, J; \\
a_i(y, \alpha) \in [0, 1] \text{ and } \sum_{i=1}^{J} a_i(y, \alpha) = 1 \forall (y, \alpha). \]

A solution to the problem is Pareto Efficient.\(^{11}\) Since the problem is written requiring competitive provision of housing and also requiring jurisdictional balanced budgets, it may appear we have imposed some second-best requirements on the “efficient” allocation. However, as discussed below, these impositions are consistent with first-best Pareto Efficiency (but see the previous footnote). As the social weights \((\omega(y, \alpha), \omega_R)\) are varied, alternative Pareto Efficient allocations are determined. If the utility possibilities set is convex, then all Pareto Efficient allocations are a solution to the problem for some set of weights.\(^{12}\) Note, too, that \( r(y, \alpha) = R = 0 \) will arise in the solution to the planner’s problem for some weights \((\omega(y, \alpha), \omega_R)\), which is the case most naturally compared to the market equilibrium allocation.

\(^{11}\) We treat the housing supplies to jurisdictions as a technological constraint. That is, we do not allow jurisdictional lines to be redrawn, which would effectively permit trading of housing between jurisdictions.

\(^{12}\) If the constrained utilities possibilities set is not convex, then one can still find all Pareto Efficient allocations as extrema of the planner’s problem. Some solutions would be local minima of the problem but would satisfy the same (first-order) conditions we derive below.
To solve the problem, write the Lagrangian function:

\[ L = \sum_{i=1}^{i} \left[ \int \omega V_i^c a_i \, d\alpha + \omega_i (R/J + \int_0^{1+i+1} H_i^d \, dz) \right] + \sum_{i=1}^{i} \lambda_i [(T_i - g_i) \int a_i \, d\alpha + \frac{t_i}{1+t_i} H_i^d] \right] + \sum_{i=1}^{i} \eta_i \left[ h_i a_i \, d\alpha - H_i^d \right] + \Omega [R + \int_0^{1} \, d\alpha]; \]

where \( \lambda_i, \eta_i, \) and \( \Omega \) are multipliers, we have suppressed arguments of functions, \( V_i^c \) is notation indicating that \( V_i^c \) has arguments corresponding to community \( i \), and constraint (18) is taken account of below. The first-order condition on \( (r(y,\alpha),R) \) can be written:

\[ -\Omega = \sum_{i=1}^{i} \omega U_i^a a_i + \sum_{i=1}^{i} \eta_i \frac{\partial h_i^j}{\partial y} a_i = \omega_R \forall (y,\alpha); \]

where \( U_i^\alpha \) is the partial derivative of \( U \) with respect to its first argument and the superscript indicates evaluation of the function at community \( i \) values. (We continue to use such notation below.) Let:

\[ \text{MSV}_i(y,\alpha) \equiv L_{a,\epsilon} = \omega V_i^c + \lambda_i [T_i - g_i] + \eta_i h_i^j \]

denote the marginal social value of assigning a measure \( a_i f(y,\alpha) \) of household type \( (y,\alpha) \) to community \( i \), which equals the first variation in the Lagrangian with respect to type \( (y,\alpha) \).\(^{13}\)

Now taking account of (18), the optimal household assignment criterion can be written\(^{14}\):

\[ a_i(y,\alpha) \in [0,1] \text{ as } \text{MSV}_i(y,\alpha) = \text{Max}_{j \neq i} \text{MSV}_j(y,\alpha) \forall (y,\alpha). \]

To write out the remaining first-order conditions, let:

\[ N_i = \int a_i(y,\alpha)f(y,\alpha)\,d\alpha \text{ and } \epsilon_i^j = \frac{H_i^j}{H_i^i} \frac{p_i}{(1+t_i)} \]

denote respectively the number of residents of community \( i \) and the elasticity of housing supply. We have:

\[ L_{t_i} = 0 \rightarrow -\omega_R + \lambda_i (1-t_i \epsilon_i^j) + \frac{1+t_i}{p_i} \eta_i \epsilon_i^j = 0; \]

\[ L_{t_i} = 0 \rightarrow -\int a_i f(y,\alpha)\,d\alpha + \lambda_i N_i - \eta_i \int \frac{\partial h_i^j}{\partial y} a_i \, d\alpha = 0; \]

\(^{13}\) This is scaled by \( f(y,\alpha) \) just to be comparable across types.

\(^{14}\) If the middle line of (22) characterizes the solution for a household \( y \), then the summation constraint in (18) comes into play. However, we will focus on cases where this does not characterize the optimum as discussed below.
\[ L_{g_i} = 0 \rightarrow \int_s \omega U^i \alpha_f dyd\alpha + \eta \int_s \frac{\partial h^i}{\partial g_i} a_f dyd\alpha - \lambda_i N_i = 0; \]  
\[ \text{and} \]
\[ L_{p_i} = 0 \rightarrow \frac{1 + t_i}{H_i} \left[ \eta \int_y \frac{\partial h^i}{\partial p} a_f dyd\alpha - \int_y \omega U^i h^i a_f dyd\alpha \right] + t_i \lambda_i (1 + \varepsilon^i) - \frac{\eta_i (1 + t_i) \varepsilon^i}{p_i} + \omega_n = 0. \]  

We restrict attention to cases where it is efficient to have *differentiated communities* as in Tiebout allocations. This conforms to cases such that \( a_i(y, \alpha) = 1 \) for some community \( i \) for a.e. household (see (21) and (22)). The alternative has homogeneous communities. Whether differentiation is optimal depends on the utility weights in the social welfare function. Essentially we want to examine when equilibrium allocations with differentiation are associated with externalities in community choice.

First we confirm what is very intuitive: The social optimum will have no property taxation, just head taxes. More to our purposes, unilateral household choice of residence with an efficiently chosen head tax would be consistent with the efficient allocation. We will then go on to examine the second-best problem assuming only property taxes are allowed.

**Proposition 3:** In an efficient differentiated allocation: (a) \( t_i = \eta_i = 0 \) and \( T_i = g_i \); (b) \( g_i \) satisfies the Samuelsonian condition,\(^{15}\) and (c) households are assigned to the community where \( V_i^e \) is at a maximum.

The proof of Proposition 3 is in the appendix.

**Remarks:**

1. It is straightforward to confirm that the same results obtain if the planner also assigns housing consumption to each household and if the government budget constraint is economy-wide, rather than local. Regarding the former, households would, of course, be assigned the level of housing they demand. Regarding the latter, direct income transfers permit the government to accomplish the same set of utility levels as would also allowing transfers across jurisdictions. The reason we have specified the problem imposing competitive housing consumption and jurisdictional budget balance is because we want to impose these requirements in the second-best analysis that follows.

2. A key implication of Proposition 3 is that if a community were to use head taxation to provide the local public good optimally, then household choice of communities would be socially optimal. Unilateral choice of community would lead households to choose the community where

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\(^{15}\) Equation (A13) in the appendix states the Samuelsonian condition.
\( V^c \) is at a maximum, which, by Proposition 3c, is efficient. Likewise, competitive provision of housing is efficient. The non-distorted price of housing and the head tax efficiently price access to communities. There are no externalities in community choice in this case.

3. This proposition can be viewed as a generalization of the celebrated decentralization theorem of Oates (1972). Our framework follows Oates in assuming no spillovers, costs of provision the same for the centralized as for the decentralized case, and centralization entails uniform provision.\(^{16}\) Our result extends Oates’s theorem by permitting households to be mobile and by establishing that optimally chosen head taxes achieve the efficient decentralized allocation when households are mobile.

To determine the character of jurisdictional choice externalities in the property tax equilibrium, we now examine the planner’s problem assuming head taxation is not allowed. Set \( T_i = 0 \) everywhere above and drop the first-order condition describing the efficient choice of \( T_i \), i.e., (25). With \( T_i = 0 \), the other first-order conditions remain valid.\(^{17}\) Of course, \( t_i \) will be positive here and is optimally chosen by the planner, but we will also discuss later the alternative where \( t_i \) is suboptimal. Household choice of a jurisdiction would now be associated with an externality, and its character is the focus. With reference to (21)-(22), the value of what we call the “jurisdictional choice externality (JCE)” of household \((y, \alpha)\) in jurisdiction \( i \) is given by:

\[
\text{JCE}_i(y, \alpha) \equiv -\lambda_i, g_i + \eta_i, h_i(p, y + r(y, \alpha), g_i, \alpha).
\]  

(28)

\( \text{JCE}_i(y, \alpha) \) equals the social value of choice of community \( i \) by household \((y, \alpha)\) in excess of the household’s own (weighted) utility. Thus \( \text{JCE}_i(y, \alpha) \) measures the social benefit or cost imposed on others when household \((y, \alpha)\) chooses to locate in jurisdiction \( i \). We assume now to simplify the analysis that housing demand is independent of \( g_i \), as arises in the cases we analyze below.\(^{18}\)

To convey the main results here, we introduce a bit more notation. Let \( h_c(\cdot) \) denote a household’s compensated demand function for housing. Let:

\[
\tau_i(y, \alpha) \equiv \frac{t_i, p, h_d(p, y + r(y, \alpha), \alpha)}{(1 + t_i)}; \tag{29}
\]

and

\(^{16}\) See Oates (1999, 2006) for a detailed discussion of the assumptions underlying the theorem and the discussion in our concluding section.

\(^{17}\) We continue to study cases with differentiated allocations.

\(^{18}\) We will also indicate what changes if housing demand does depend on \( g_i \).
\[ \theta_i \equiv \frac{(1 + t_i)\varepsilon_s^i}{(1 + t_i)\varepsilon_s - \int S \frac{\partial h_s^i}{\partial p_i} p_i a_i f d y d \alpha}. \] (30)

Observe that \( \tau_i \) is household \((y, \alpha)\)’s tax payment in jurisdiction \(i\), and \( \theta_i \in [0, 1] \) where the integral term in the denominator of \( \theta_i \) is elasticity of the compensated demand. We have:

**Proposition 4:** (a) The jurisdictional choice externality in the planner’s solution satisfies:

\[ \text{JCE}_i(y, \alpha) = -\lambda_i [g_i - \tau_i(y, \alpha)\theta_i]; \] (31)

with

\[ \lambda_i = \int_s oU'_i a_i f d y d \alpha \frac{N_i}{N} > 0, \] (32)

(where \( U'_i \) is the derivative of utility with respect to \( g_i \)).

(b) \( \text{JCE}_i(y, \alpha) \to -\lambda_i g_i \) as \( \varepsilon_s^i \to 0; \) \( \text{JCE}_i(y, \alpha) \to -\lambda_i (g_i - \tau_i(y, \alpha)) \) as \( \varepsilon_s^i \to \infty. \)

(c) \( \text{JCE}_i(y, \alpha) \) is negative for all households in community \(i\) with housing demand below the mean.

**Proof of Proposition 4:** (a) Substitute from (24) and (A11) from the appendix into (27) to obtain:

\[ \eta_i = -\lambda_i \frac{t_i p_i \varepsilon_s^i}{p_i \int S \frac{\partial h_s^i}{\partial p_i} a_i f d y d \alpha - (1 + t_i)\varepsilon_s^i}. \] (33)

Substituting (29), (30), and (33) into (28), yields (31). Expression (32) follows from (26) using our assumption that housing demand is independent of \( g_i \), and the value of \( \lambda_i \) is obviously positive.\(^{19}\)

(b) These results follow trivially from (31) and the definition of \( \theta_i \) (i.e., (30)).

(c) This follows from (31) since \( \theta_i \in [0, 1] \) and \( g_i \) equals the tax payment of the household in community \(i\) with average housing consumption. \[\Box\]

**Remarks:**

1. The main implication is that an equilibrium allocation with efficient property tax will have too many households choosing jurisdictions with high \( g \)’s, especially poorer households (assuming housing demand is normal). Intuition suggests, and (31) confirms, that the JCE will be proportional to the difference between the value of the service the household consumes \((g)\)

\(^{19}\) If housing demand depends on \( g_i \), then a sufficient condition for \( \lambda_i \) to be positive is that housing demand is non-increasing in \( g_i \). The remaining results in Proposition 4 are as stated.
and the tax paid by that household. The “tax paid” by a household equals the tax shifted forward, τθ. In the limit with perfectly elastic housing supply, a household’s consumption of housing corresponds to new production, and taxes are effectively collected from the household in proportion to housing consumption. In this case, households with income above (below) the community average and thus housing consumption above (below) the average, exert a positive (negative) externality. At the opposite extreme with housing supply elasticity equal to 0, every household’s consumption of housing displaces that of other households in the community, and taxes to cover consumption of g are fully absorbed by the absentee housing owners. Here every household exerts a negative externality proportional to g in the community.

2. In light of the preceding, it is tempting to conclude that the aggregate loss from the JCE is highest when the elasticity of housing supply is zero. However, the aggregate loss associated with the JCE depends not only on the externality created by a household’s location choice, but also on the number of households who make location choices that depart from the efficient allocation. That number will tend to be smaller when the housing supply elasticity is low than when it is high, for the following reason. Consider two communities. As households move from the poor to the wealthy community, the housing price in the wealthy community rises more rapidly when the housing supply elasticity is low. Similarly the price in the poor community falls more rapidly when the supply elasticity is low. These price effects limit the extent to which location choices depart from the optimum. Hence, when the housing price elasticity is low (high), the externality from a poor household that chooses the wealthy community is high (low), but the number of households whose location deviates from the optimum is low (high). The net effect of a change in the housing price elasticity is then ambiguous. We find computationally that welfare rises as the housing supply elasticity falls for low and intermediate elasticity values (see the appendix on robustness).

In any case, poorer households that consume less housing have an incentive to crowd richer jurisdictions. The equilibrium does not exhibit efficient choice of property tax due to majority choice, but this distortion is typically minor. *We show below that the theoretical*

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20 Note that there is no JCE if the local public good is not congested. In this case, absent other distortions, housing prices alone would ration efficiently community access and housing consumption. Assuming property taxation, full efficiency would arise with inelastic housing supplies and with taxes set so that the local Samuelsonian conditions are satisfied.
distortion identified here – poorer types crowding richer jurisdictions – is key to the welfare losses from Tiebout sorting that arise with property taxation.\(^{21}\)

3. Note from (33) that the multiplier (\(\eta\)) on the housing-market clearance condition is positive except when the housing supply elasticity is 0. This is because the gross housing price inefficiently deters housing consumption and is not enough to deter poor households from moving into high-g communities. Requiring housing consumption in excess of demand could then improve efficiency.\(^{22}\) If the tax \(t_i\) is inefficient (i.e., is not chosen by the planner), one finds that\(^{23}\):

\[
\eta_i = \left[ \int_s \omega U^i_t h^i_{a} f_{dyd\alpha} - (1 + \varepsilon^i) \frac{H^i}{N} \int_s \omega U^i_t a_{fyd\alpha} \right] \\
\left[ \int_s \frac{\partial h^i_{a}}{\partial P_i} f_{dyd\alpha} + t_i \frac{\partial h^i_{a}}{\partial P_i} a_{fyd\alpha} - \frac{(1 + t_i)\varepsilon^i H^i_t}{p_i} \right]^{-1}.
\]

(34)

Now \(\eta_i\) can be positive or negative. This is because \(g_i\) might be over-provided (conditional on using property taxation) and limiting housing consumption would reduce this distortion.

(ii) Measuring Welfare. We treat the centralized equilibrium as the status quo and use (the negative of) aggregate compensating variation associated with the Tiebout equilibrium as our welfare measure. Let \(U^c(y,\alpha)\) denote utility of household \((y,\alpha)\) in the centralized equilibrium and \(U^T(y,\alpha)\) utility in Tiebout equilibrium. Let \(v(y,\alpha)\) denote compensating variation, defined in \(U^c(y,\alpha) = U^T(y+v,\alpha)\). Let \(CV = \int y v(y,\alpha)f(y,\alpha)dyd\alpha\) denote aggregate household compensating variation. Let \(R^c = \sum_{j=1}^{J} \int_0^{p^*_j} H^c_j(p)dp\) denote housing rents in the centralized equilibrium, where \(p^*_j\) denotes the net housing price. Let \(R^T = \sum_{j=1}^{J} \int_0^{p^*_j} H^T_j(p)dp\) denote housing rents in the Tiebout equilibrium. Compensating variation of the absentee landlords is given by: \(R^c - R^T\). We report \(W^T = -[CV + R^c - R^T]\) as our welfare measure, while also reporting aggregate consumer welfare (-CV). The negative of compensating variation is reported, so a positive value indicates a gain from Tiebout sorting.

We know a priori that the Tiebout equilibrium is not Pareto Efficient. If households choose residences and housing followed by efficient public good provision satisfying the

\(^{21}\) The idea that the “poor chasing the rich” can be associated with a departure from efficiency in Tiebout economies is well known. See Rubinfeld (1987) for a discussion and early references. Our contribution is mainly to quantify this inefficiency and provide evidence that it may be extreme.

\(^{22}\) See Calabrese, Epple, and Romano (2007) on residential zoning that improves efficiency.

\(^{23}\) This is found by solving the planner’s problem with \(t_i\) exogenous, hence suppressing condition (24). We continue to assume that \(T_i\) must be 0.
Samuelsonian condition financed by a head tax, then equilibrium would be efficient (Proposition 3). Such an allocation would maximize our welfare measure (i.e., the negative of aggregate compensating variation).\(^{24}\) In the Tiebout equilibrium we study, the housing market distortion from use of a property tax to finance public provision is generally inefficient. Likewise, majority choice of the level of provision of the local public good is generally inefficient. These inefficiencies further imply that externalities arise in the individual choice of residences as we have shown. We know, then, that if we calculate welfare analogously in going from the centralized equilibrium to the efficient head-tax equilibrium, denoted by \(W^H\), that \(W^H > 0\) and \(W^T > W^T\). In spite of the latter inequality, we perceive a strong belief among economists that \(W^T > 0\) is to be expected: Some aggregate welfare gains will arise from the equilibrium matching of households to relatively desired public good levels.\(^{25}\) In fact, we will see that this belief appears to be overly optimistic. We show below that it is frequently the case that \(W^T < 0\) when \(W^H\) is substantial.

4. Computational Analysis
We first examine a calibrated computational model that demonstrates the tendency for decentralization to be inefficient. This computational model also permits us to delineate the magnitudes of the various sources of inefficiency. We then employ a more general model estimated using data from the Boston Metropolitan Area (MA) in the next section, and, again, find a welfare loss.

a. Calibration of the Model. The calibrated model abstracts from taste differences, with then households differing only by income. Household utility is assumed to be CES:

\[
U = \left[ \beta_x x^\rho + \beta_h h^\rho + \beta_g g^\rho \right]^{1/\rho}.
\]

We must calibrate the MA income distribution, the number of jurisdictions, and the parameters of the utility function and housing supply functions. The distribution of MA income is calibrated using data from the 1999 American Housing Survey (AHS).\(^{26}\) Median income reported by the AHS is $36,942. Using data for the 14 income classes reported by the AHS, we estimate mean household income to be $54,710. These values and our assumption that the income distribution is lognormal imply \(\ln y \sim N(10.52,.785)\).

\(^{24}\) This is proved in an appendix available on request.
\(^{25}\) Our perception of the consensus belief is that \(v(y, \alpha)\) will be positive for those that choose poorer (low \(g\)) communities in Tiebout equilibrium (i.e., there will be a welfare loss for them), but \(v(y, \alpha)\) will be negative and offsetting for those that choose richer communities. We find that the offset does not typically occur.
\(^{26}\) http://www.census.gov/hhes/www/housing/ahs/99dtchrt/tab2-12.html
We assume constant elasticity housing supply function in each jurisdiction. Such a housing supply function arises if units of housing are produced competitively by combining a jurisdiction’s inelastically supplied land $L_j$ with an elastically supplied factor $q$ according to constant-returns production function: $h = L^\gamma q^{1-\gamma}$, $\gamma \in (0,1)$. Specifically, then

$$H_j^t = L_j \left( p_s^t \right)^{1-\gamma} \left( \frac{1-\gamma}{w} \right)^{1-\gamma},$$

(36)

where $w$ is the given price of input $q$. The quantity of housing available at given housing price then varies across jurisdictions proportionately to their land endowment. In our baseline calibration, we assume five local jurisdictions in the MA – a large city and four smaller suburbs that have equal area. The total land supply in the MA is normalized to 1. The city is assumed to have 40% of the total land area and each of the suburbs 15%. We assume that the city is the poorest jurisdiction. The jurisdictions are numbered from poorest to richest: Hence, $L_1 = .4$, and $L_2 = L_3 = L_4 = L_5 = .15$, where $L_j$ equals community $j$’s land share. The parameter $\gamma$ equals the share of land inputs in housing in our model. Based on the empirical evidence (see the discussion in Epple and Romer, 1991), we set $\gamma = \frac{1}{4}$. Note from (36) that this implies a housing supply elasticity equal to 3.27

The remaining parameter values are $\rho, \beta_x, \beta_h,$ and $\beta_g$ from the utility function (35), and $w$ from the housing supply function (36). The calibrated parameters are summarized in Table 1. The remaining calibration is based on the single jurisdictional equilibrium for simplicity. First, we set $\beta_x = 1$, a normalization. While less obvious, $w$ is also a “free parameter,” which we also then set equal to 1. To see this, note from (36) that the housing supply function for the MA is:

$$H_s = \left( w \right)^{\frac{\gamma-1}{\gamma}} \left( p_s \right)^{1-\gamma} \left( 1-\gamma \right)^{1-\gamma},$$

and this is the only place that $w$ appears in the model. For any $\gamma$, changing $w$ is equivalent to changing the units of measurement of housing. No equilibrium values relevant to utilities then vary with $w$.

The values of $\beta_g, \beta_h,$ and $\rho$ are set so that in the single jurisdictional equilibrium the median voter chooses $t = .35$, the net-of-tax expenditure share on housing equals .20, and the price elasticity of housing is very close to -1. A $t = .35$ implies a tax rate on property value that

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27 This housing supply elasticity is within the range of estimates for new housing, though estimates vary substantially. See Dipasquale (1999), Blackley (1999), and Somerville (1999). Dipasquale and Wheaton (1992) estimate the long run rental housing supply elasticity to be 6.8. Other estimates also find a higher elasticity than 3 (see Mayer and Sommerville, 2000, and Epple, Gordon, and Sieg, forthcoming). In an appendix, we show that increasing the housing supply elasticity results in a higher welfare loss from Tiebout sorting than in our baseline calculation until housing supply becomes very elastic (over 10).
is realistic, on the order of 2.5% to 3.0%. The expenditure share on housing of .20 is in the range of values estimated in the literature (see Hanushek and Quigley (1980). Likewise, the housing market literature indicates a price elasticity close to -1. The implied values of $\beta_g$ and $\beta_h$ are, respectively, 0.094 and .356. We set $\rho = -.01$, which implies a price elasticity of housing demand equal to -.993, while also implying SRI and existence of a Tiebout equilibrium when there are multiple jurisdictions.

<table>
<thead>
<tr>
<th>Table 1: Parameter Values Baseline Model</th>
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<tbody>
<tr>
<td>$\beta_x$</td>
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<tr>
<td>1.00</td>
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</tbody>
</table>

b. Findings. Table 2 summarizes the findings in this baseline specification, with positive results in the upper panel and normative results in the lower panel. Recall that we report the negative of compensating variation values so that gains from Tiebout sorting correspond to positive values. Column 2 of the upper panel shows key values in the Tiebout equilibrium and column 1 corresponding values in the centralized equilibrium where the MA is one jurisdiction. Ignore the other columns for the moment. The Tiebout equilibrium is income stratified, supported by ascending housing prices, although the property tax rates vary little and are very close to that in the centralized equilibrium. Because these tax rates apply to substantially different housing expenditures, the public good levels vary substantially.

The lower panel shows the welfare effects. Only the poor and very rich are better off in the Tiebout equilibrium, with 95% worse off. Figure 2A graphs the negative of compensating variation against household income. On average consumers are worse off, with an average (minus) compensating variation of $41. The absentee land owners experience a negligible welfare loss. Column 5 reports values in the efficient allocation discussed above, and Figure 2B graphs welfare gains against income. Although 74% of households are worse off in this allocation, households experience an average welfare gain of $726 and land owners an average

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28 Observed property tax rates are expressed as a percent of property value. In our model, rates are expressed as a percentage of annual implicit rent. Employing the approach of Poterba (1992), Calabrese and Epple (2006) conclude that tax rates on annualized implicit rents can be converted to rates on property values using a conversion rate on the order of 7% to 9%. Thus, our annualized rate of .35 translates to a tax rate on property value on the order of 2.5% to 3%, which is the order of magnitude of observed property tax rates.


30 A $\rho = 0$ implies a Cobb-Douglas utility function and a price elasticity of demand for housing exactly equal to -1. In this case, SRI fails and an equilibrium with Tiebout sorting does not arise.
gain of $711. Hence, the environment is one where Tiebout sorting could lead to substantial welfare gains on average, yet these not only fail to be realized in property tax equilibrium but are reversed.

To delineate the sources of the welfare losses that arise in Tiebout equilibrium, we calculate two other allocations. As discussed above, three inefficiencies arise in Tiebout equilibrium: First, property taxation distorts housing consumption with the usual deadweight loss. Second, majority choice of the tax rate reflects the preference of the median-income household in a jurisdiction, which generally differs from the choice that would maximize average welfare. Third, externalities arise in household choice of jurisdiction, which we show is the primary source of welfare loss.

The second, majority voting inefficiency, is generally believed to be small in these models. To verify this here, we compute multi-jurisdictional equilibrium with majority choice of a head tax. Equilibrium is determined precisely as in the property-tax model, but voting is over a local head tax that fully finances the local public good. Versions of Propositions 1 and 2 apply to this variation of the Tiebout sorting model. Values for this equilibrium are shown in column 3 of Table 2. The head taxes are, of course, equal to the levels of public good provision. Comparing column 3 to column 5, one sees that the allocation is very close to the efficient allocation. The welfare gain relative to the single-jurisdictional equilibrium is 99.8% of the potential welfare gain from household sorting.

The welfare loss in the Tiebout equilibrium is then largely attributable to property taxation and household jurisdictional choice externalities rather than voting bias. To delineate these effects, we assign households to jurisdictions as arises in the efficient allocation, but then they vote for a local property tax to finance the public good. Hence, this allocation essentially removes externalities from household choice of jurisdiction, while retaining the property tax distortion (as well as the small voting bias). This is not an equilibrium allocation because some households would prefer to move. The associated values are reported in column 4 of Table 2.

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31 If median income were equal to mean income and if the (indirect) utility function were linear, then the preference of the median-income household would maximize average welfare. But neither of these conditions is satisfied. These biases are well known.
32 The ascending bundles property trivially regards the head tax, not the housing price. Since the head tax is a deterrent to moving into a jurisdiction, it is theoretically possible that housing prices could decline with the level of the public good.
33 Because the calibration of income begins at 0, some households in the poorest jurisdiction cannot afford to pay the $1691 head tax. The proportion of the population is only .000251, so we simply ignore this.
We see that most of the potential welfare gain from efficient sorting arises in this allocation; about 80%.

We conclude that the jurisdictional choice externality is the main cause of the welfare loss we find. As already discussed, relatively poor households crowd into richer jurisdictions to consume high levels of the public good, while free riding on richer households that pay more in taxes and on the absentee land owners. As one way to illustrate the free riding in the property-tax Tiebout equilibrium, we compute the ratio of the tax payment (or housing consumption) of the poorest resident to the mean-income resident in the four suburbs. Moving up the wealth hierarchy of suburbs, these ratios equal .84, .84, .80, and .57. From another perspective, referring to Table 2, we see that the equilibrium populations of the richer jurisdictions are substantially higher and the income levels substantially lower than in the efficient allocation. The fundamental explanation for the welfare loss in Tiebout property tax equilibrium is that the resulting sorting of households is inefficient; it’s not stratified enough! While we are in a second-best economy so that “anything can happen,” we, nevertheless, find this counterintuitive. Because the poor free ride on the rich in the centralized equilibrium, it would seem that some degree of sorting would lessen this externality. But carving the MA into jurisdictions entails limiting within jurisdiction housing supplies, which intensifies the externality losses as the relatively poor crowd relatively rich jurisdictions. Given property taxation, the model indicates that the degree of Tiebout sorting is crucial for welfare gains to be realized.34 While it is well known that Tiebout sorting is not good for poorer types, that it will sometimes be also bad on average makes it difficult to support such decentralization.

The welfare loss we find is not contrived. To examine robustness, we vary the equilibrium concept with respect to the nature of assumed voter beliefs and the parameters of the model. The analysis is an appendix available on request (or at www.xxxxxxx), and we very briefly summarize here. Two alternative specifications of timing of choices and voter beliefs are analyzed. An alternative where voters anticipate changes in housing consumption, but not in jurisdictional choice, has negligible effects on our findings. An alternative where voters anticipate both changes in housing consumption and jurisdictional choice leads to substantially higher welfare losses from Tiebout sorting in property tax equilibrium.

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34 One might argue that the explanation points to a misallocation of housing to jurisdictions. But this is essentially the same as saying that a misallocation of households to jurisdictions arises, which is the better perspective for given jurisdictional housing supplies. More importantly, the problem does not go away by redrawing jurisdictional boundaries (see our robustness analysis). Access must be limited beyond that caused by housing prices.
Increasing or decreasing the number of jurisdictions by one has very minor effects. Reducing $\rho$, hence the elasticity of substitution in the CES utility function, leads to welfare gains from Tiebout sorting under property taxation. Reducing $\rho$, hence the elasticity of substitution in the CES utility function, leads to welfare gains from Tiebout sorting under property taxation.35 Households consume more housing and are more reluctant to reduce their housing consumption. However, even with $\rho$ reduced to $.5$, thus doubling the elasticity of substitution relative to our benchmark calibration, less than one third of the potential gains from decentralization are realized.

Reducing $\gamma$ increases housing supply elasticities and welfare losses from Tiebout sorting with property taxation increase rapidly until the housing supply elasticity is very high (about 10). Housing prices rise more slowly as poor households move into richer jurisdictions, worsening the effect of the jurisdictional choice externality as more such movement takes place. Increasing $\gamma$ has the reverse effects.

Increases in $\beta_g$, the weight on the public good in the utility function, increase demand for the local public good and exacerbates the inefficiencies and welfare losses in Tiebout property tax equilibrium. Increasing $\beta_h$, the weight on housing in the utility function, increases demand for housing. Property tax rates fall and households find it more difficult to substitute away from housing. As a consequence, inefficiencies in Tiebout property tax equilibrium are reduced.

While we do not always find losses from Tiebout sorting under property taxation, we find a loss in our baseline calibration and we find it is fairly persistent over a variety of parameter variations. Moreover, when gains arise, they are typically a small percentage of potential gains from efficient sorting. These computational findings provide motivation for further pursuit of the efficiency question. The next section develops our main quantitative findings that are based on an estimated model.

5. Econometric Model and Findings

a. The Econometric Model and Estimated Parameters. The framework used in the econometric analysis is set forth in Epple and Sieg (1999), Epple, Romer, and Sieg (2001), and Calabrese, Epple, Romer, and Sieg (2006). Here we briefly summarize the econometric model and estimates. Households differ in income and taste for the local public good. The specification of the local public good is generalized relative to the above model by inclusion of a jurisdictional peer effect to better fit the data. Suppressing the jurisdictional subscripts and letting $q$ denote the quality of the local public good, the following indirect utility function is used:

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35 We cannot consider higher values of $\rho$ since any significant increases would violate SRI and preclude sorting of types in property tax equilibrium.
\[ \tilde{V}(q,p;y,\alpha) = \{\alpha q^1 + [e^{\frac{1-v}{1-\nu}} e^{\frac{\mu p^{1-v}}{1-\nu}}]^{1/\nu} \}; \] (37)

where

\[ q = g \cdot \bar{y}^\eta; \] (38)

and \( \bar{y} \) is the mean income in the jurisdiction. Peer effects might operate through educational spillovers in the classroom, through parental monitoring of teachers and school administrators, or through other channels.\(^{36}\) Note that the indirect utility function in (37) is the standard one that allows housing consumption to vary with prices, as specified in (10) above. This specification has the following useful properties.\(^{37}\) It is separable in public- and private-good components, substantially simplifying estimation. The implied elasticity of substitution between the public good component and private good component is constant and equal to \(1/(1-\tau)\). Using Roy’s identity, the implied housing demand function is \( h_B = B^\nu p^\eta \). Thus housing demand has constant price and income elasticities, as is common in empirical analysis. The single-crossing conditions for stratified community choice, i.e., \( \sim SRI \) and \( \sim SR\alpha \), are satisfied if \( \tau < 0 \).\(^{38}\) This condition is tested empirically, allowing \( \tau \) to be any value, and \( \tau \) is found to be negative and statistically very significant (Epple and Sieg, 1999). Hence, the central sorting conditions in the theoretical model are supported.

The metropolitan population density function, \( f(y,\alpha) \), is taken to be bi-variate lognormal:

\[
\begin{pmatrix}
\ln y \\
\ln \alpha
\end{pmatrix} \sim N
\begin{pmatrix}
\mu_{lny} \\
\mu_{ln\alpha}
\end{pmatrix}
\begin{bmatrix}
\sigma_{lny}^2 & \lambda \sigma_{lny} \sigma_{ln\alpha} \\
\lambda \sigma_{lny} \sigma_{ln\alpha} & \sigma_{ln\alpha}^2
\end{bmatrix}
\]
(39)

The model then has ten parameters to be estimated: \( \tau,\nu,\eta,B,\phi,\mu_{lny},\mu_{ln\alpha},\sigma_{lny},\sigma_{ln\alpha}, \) and \( \lambda \).

Estimation is based on data for the 92 municipalities in the Boston Metropolitan Area. Data are used for 1980, which precedes the state imposition of property tax limits (Proposition 2½). The Boston metropolitan area is particularly well suited to estimation of the model. In Massachusetts school districts and municipalities are coterminous. Property taxes were the


\(^{37}\) There is not a closed form for the associated (direct) utility function. An appendix, available from the authors, provides detail on this specification, including demonstration that it satisfies all the standard properties (e.g., quasi-concavity in prices) of an indirect utility function.

\(^{38}\) This condition also ensures that voting equilibrium exists in the second stage. This is shown in the appendix discussed in the previous footnote.
primary source of local revenues during this period, and residential property tax revenue tracks well educational expenditure per student in the 92 municipalities with a correlation coefficient of 0.73. Per student educational expenditure is then used for g in the estimation.

Estimation proceeds in two stages. Table 3 reports the estimates. In stage one (Epple and Sieg, 1999), the mean and standard deviation of ln(y) are estimated first, using the metropolitan income distribution. Three additional parameters are estimated by utilizing the stratification and boundary-indifference conditions. The boundary-indifference conditions and the indirect utility function imply the following expression for the boundary loci between jurisdictions j and j+1:

$$\ln(\alpha) + \frac{(y^{1-\nu} - 1)}{1-\nu} = \ln \left( \frac{Q_{j+1} - Q_j}{q_j - q_{j+1}} \right) \equiv K_j;$$

(40)

where:

$$Q_j = e^{-\frac{\ln y_{j}^{\nu} - 1}{\eta}}. \quad (41)$$

There are 91 such loci partitioning the metropolitan population into 92 municipalities. A minimum-distance estimator is then used to match quartiles of the 92 income distributions implied by the model to quartiles of the 92 income distributions that are estimated by the US Census. The additional parameters identified here are: $\tau/\sigma_{\ln \alpha}, \nu, \text{ and } \lambda$. In addition to the 92 income distributions, the populations of the 92 municipalities are used in this stage. The $K_j$ are solved out at each point in the parameter search. Hence, housing values and public good qualities are not needed at this stage. The asymptotics are with respect to the sample size taken by the Census so the parameters are estimated with a high degree of precision. The ratio, $\tau/\sigma_{\ln \alpha}$, is negative and statistically significant, supporting the single-crossing assumptions. Note that this first-stage estimator invokes only necessary conditions for equilibrium, so uniqueness of equilibrium is not a concern.

Table 3: Parameter Estimates

(Standard errors are in parentheses)

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<tr>
<th>$\mu_{\ln y}$</th>
<th>$\sigma_{\ln y}$</th>
<th>$\lambda$</th>
<th>$\tau/\sigma_{\ln \alpha}$</th>
<th>$\nu$</th>
<th>$B$</th>
<th>$\phi$</th>
<th>$\mu_{\ln \alpha}$</th>
<th>$\sigma_{\ln \alpha}$</th>
<th>$\eta$</th>
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<td>9.790 (0.002)</td>
<td>.755 (0.004)</td>
<td>-.019</td>
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</table>

$^a$Set at minimum in the program. $^b$Fixed since weakly identified.
In the second stage (Calabrese, et al., 2006), the remaining parameters are estimated using maximum likelihood. Parameters identified in the first stage are fixed at the values estimated in that stage. In the second stage, at each step in the parameter search, the theoretical model with household sorting and myopic voting over property tax rates is solved for equilibrium values of property tax rates, expenditures on \( g \), and aggregate housing values in each of the 92 communities. The observed values of these variables in the 92 communities are then presumed to be equal to the values implied by the model plus measurement error. Since estimation at this stage involves matching house values of the model to those in the data, the price elasticity of housing demand is only tenuously identified.\(^{39}\) Hence, that parameter is fixed at \( \eta = -0.3 \), and remaining parameters are estimated. We vary \( \eta \) below. Calabrese, et al. (2006) also fixed \( \ln \sigma_{\text{nu}} \) at 0.1 producing a good fit to the data.\(^{40}\) We have since confirmed that this closely corresponds to the estimated value (equal to 0.0998). Calabrese, et. al. (2006) prove that, conditional on the first stage results, the equilibrium in the second stage is unique. Thus, the estimation procedure is not vulnerable to concerns about multiple equilibria.

Figure 3 relates observed jurisdictional values to those predicted by the estimated model. Jurisdictions are ordered by increasing median household income. Following Poterba (1992), the observed property tax rate is converted from that on home value to that on the implied net rental rate per unit of housing to correspond to our model of housing services and prices. Note that the dollar values for educational expenditure per household and housing value are in 1980 dollars in Figure 3. The figure at once illustrates the substantial Tiebout sorting and the predictive power of the estimated model.

b. Welfare Effects of Tiebout Sorting. The econometric analysis summarized above does not generate an estimate of the elasticity of housing supply, which is needed to perform our welfare computations. We assume constant elasticity housing supply and use the same housing supply elasticity as in the baseline computational model above (i.e., equal to 3). The equilibrium with 92 communities is computed as part of the estimation. To calculate the counter-factual

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39 Data on housing values is only available at the jurisdictional level, which permits identification of key parameters, but does not provide good data for estimating the price elasticity.

40 \( \sigma_{\text{nu}} = 0.1 \) may give the impression that taste variation is then negligible, but that is incorrect because the magnitude of \( \sigma_{\text{nu}} \) depends on choice of scaling. For example, \( \sigma_{\text{nu}} \) can be rescaled by expressing the first argument in the indirect utility function (50) not as \( \alpha \) but instead as \( \alpha \). The important point here is that second-stage estimates preserve the decomposition of the metropolitan distribution of income within and across communities that is captured in the first-stage estimates. As reported in Epple and Sieg (1999), 89% of the total variance in metropolitan income is within-community variance and 11% is across communities.
equilibrium with a single metropolitan government, land area for housing must be obtained. Given the housing supply elasticity, using (36), the implied land area used for housing in each municipality can be calculated. These values are aggregated to obtain land area for housing with a single metropolitan government. This then permits calculation of the equilibrium with a single metropolitan government and no sorting of households.

In the computed single jurisdictional equilibrium, we obtain $g = 1100.42$, $t = 0.43$, and $p = 2.07$. By way of comparison, the population weighted means in the 92-community equilibrium are: $\bar{g} = 1127; \bar{t} = 0.41; \bar{p} = 2.35$. We then compute compensating variation for households and initial land owners as above. The household average compensating variation in going to the multi-jurisdictional equilibrium is $478 and the per household CV for land owners equals -$162. Hence, the Tiebout equilibrium implies a welfare loss equal to $316 per household. This equals 1.3% of 1980 per household income or, if expressed using year 2000 values to be comparable to the calibrated model in Section 3, the loss equals $711.

c. Sensitivity Analysis. The Calabrese, et al. (2006) model differs theoretically from the model developed earlier in this paper in its inclusion of a peer effect. Also, the assumed value of the price elasticity of housing demand ($\eta$) is more inelastic than that implied by our earlier calibration. Here we briefly examine consequences of better aligning the two models.

The welfare calculation in the preceding section assumes that centralization equalizes both expenditure per student and peer quality across jurisdictions. Of course, equalizing expenditure will not equalize peer quality without policy intervention that directly addresses peer sorting. Hence, we performed an alternative calculation in which expenditure per student is equalized while peer sorting is preserved. The household average compensating variation from expenditure equalization is $403, and the per household CV for land owners equals -$109. Hence, the Tiebout equilibrium implies a welfare loss equal to $294 per household or 1.2% of 1980 per household income. Thus, expenditure equalization achieves most of the potential welfare gain from centralization. This is of considerable practical importance, since existing school finance equalizations focus on expenditure equalization.

Turning to the price-elasticity of demand for housing, we re-estimated the model with $\eta$ equal to -.5 while continuing to include the peer effect. The welfare loss per household from Tiebout sorting rises to $811. The welfare loss continues to rise as $\eta$ is further reduced. However, the implied variation of housing prices across communities becomes to extreme to be
plausible as the housing price elasticity declines below -.5.\textsuperscript{41} But the price inelastic specification of housing demand in Calabrese, et al. (2006) model does \textit{not} underlie the welfare loss from Tiebout sorting we find here.

We also calculate the welfare effects of Tiebout sorting shutting down the peer effect while retaining the remaining parameter values.\textsuperscript{42} With $\eta = -.3$, a slight average welfare gain equal to $32$ per household arises from Tiebout sorting. With $\eta = -.5$, a slight average welfare loss arises equal to $31$ per household.

The results from the estimated model indicate the inefficiencies from Tiebout sorting under property taxation offset the potential gains, consistent with the findings of the calibrated model. Thus, findings from the estimated model reinforce this paper’s key finding.

\textbf{d. Relationship to Prior Work.} A very influential estimate of welfare \textit{gains} from decentralization is provided by Bradford and Oates (1974). They, as we, focus on education as the key service provided by local governments, and they compare decentralized to uniform provision. \textit{Assuming efficient decentralized provision}, they estimate a welfare loss from centralization equal to 50\% of the population-weighted sum of the absolute changes in expenditures that would arise from equalization of expenditures. For comparison, we calculate the welfare loss (using compensating variation) of moving from the \textit{efficient} decentralized equilibrium in our model (column 5) to the efficient centralized equilibrium (values not shown), obtaining $1,212$. This is remarkably close to the estimated welfare loss of centralization obtained by assuming that the percentage welfare loss from the Bradford-Oates calculation would be realized in our model. These results highlight that the differing conclusions that we obtain in this paper about the welfare effect of decentralization are not due to differences with respect to the potential gains from decentralization. Rather, we find that the inefficiencies associated with the current institutional structure dissipate those potential gains.

Another related paper is Fernandez and Rogerson (1998), who investigate the differences in human capital accumulation between a decentralized and a centralized economy. They find that centralization is beneficial from the perspective of human capital accumulation in a steady state. Using their household utility function and calibration of it, we find the static welfare

\textsuperscript{41} With $\eta = -.5$, the predicted housing price rises by a factor of 8.87 from the poorest to the richest jurisdiction. Letting $\eta$ drop to -.9, the factor rises to 134. We believe that the model would need to introduce preference heterogeneity in demand for housing to accommodate more price elastic housing demand while having realistic predictions.

\textsuperscript{42} The fit of the model is substantially reduced by shutting down the peer effect. However, we include this additional calculation to illustrate the robustness of our finding.
difference between the centralized and decentralized allocations to be approximately zero.\textsuperscript{43} Thus, we find the same gross inefficiencies in decentralized equilibrium using an independently specified model.\textsuperscript{44}

6. Concluding Remarks

Efforts to reduce inequalities in the local public finance of schooling have led to major changes in education policy in much of the U.S.\textsuperscript{45} Few economists would challenge the notion that those inequalities, arising from Tiebout sorting, have lowered the welfare of many, especially the poor. However, distributional issues aside, we, and we believe most other economists, had believed the Tiebout process to be efficiency enhancing. While the presence of inefficiencies in local property tax equilibria is understood, we know of no research that quantifies the net effects of such allocations when explicit account is taken of the effects of mobility.\textsuperscript{46} In pursuing such an analysis here, we have found that these inefficiencies are substantial and overturn potential average welfare gains in both a standard calibrated model and an estimated model. It is surprising to find that the welfare effects run counter to basic intuition concerning the Tiebout process. From a policy perspective, however, the findings are encouraging in suggesting that equity-motivated efforts to reduced differences in educational expenditures per student may come at little if any cost in allocative efficiency.

The finding that decentralization as manifest in practice results in average welfare losses has led us to investigate the main source of the inefficiency. We find that the externality in choice of residence is the primary source of loss. Thus, ironically, the mobility that Tiebout emphasized as essential to the realization of potential efficiency gains of decentralization is also the culprit in preventing those gains from being realized.

\textsuperscript{43} In our appendix on robustness (www.xxx), more detail on the Fernandez and Rogerson (1998) model is provided along with our related calculations.

\textsuperscript{44} Another, mainly empirical, literature examines the consequences of centralization versus decentralization for growth. See Brueckner (2006) for a theoretical analysis and for references.

\textsuperscript{45} See, for example, Evans, Murray, and Schwab (1998).

\textsuperscript{46} Brueckner (2004) compares a Tiebout equilibrium with taxation of mobile capital to the centralized alternative and shows by simulation that welfare can be higher or lower in the Tiebout equilibrium. Brueckner’s focus is on the trade off from inefficient tax competition for mobile capital, the focus of the tax competition literature, and the gains from matching levels of public goods to diverse preferences. Our research differs in several important ways. The tax we investigate is on (immobile) housing, so the fundamental inefficiency is Brueckner’s analysis is not present here. Brueckner’s Tiebout sorting is efficient in that a community forms for every preference type, while we treat community boundaries as given with an infinite number of different types and households then select their preferred community. We offer a quantitative assessment of the likely magnitude of welfare effects.
We think that our findings are important, but realize that our analysis abstracts from potentially important elements. Our perspective is that the efficiency analysis of empirical Tiebout equilibrium calls for more research. We very briefly discuss a few issues that might be relevant. In a very influential paper, Hamilton (1975) argued that zoning can overcome the inefficiencies associated with property taxation. In Calabrese, Epple, and Romano (2007), we pursue a theoretical and quantitative analysis of equilibrium residential zoning that supports Hamilton’s argument that zoning can serve as a substitute for head taxation. We show that local public choice of a zoning restriction on housing quality combined with a property tax closely mimics head taxation, and almost all potential Tiebout welfare gains are realized. That analysis is in the context of a model in which households differ only with respect to income. Whether such results carry over to an environment with preference heterogeneity is an important open question.\textsuperscript{47} More importantly, the empirical relevance of residential zoning is unclear. Evidence on the extent of intra-community household heterogeneity (Epple and Sieg, 1999; Hardman and Ioannides, 2004; Pack and Pack (1977)) and lot size heterogeneity (Epple, 2006) is not supportive of the argument that zoning is a good substitute for head taxes.

Our analysis abstracts from commuting costs.\textsuperscript{48} A conjecture is that commuting costs will reduce gains from Tiebout sorting simply because the implied geographical sorting is unlikely to be consistent with otherwise optimal residential choice that would arise in a centralized regime.

Jurisdictional competition in the provision of local public goods introduces a potential productivity gain from decentralization. This has been intensively studied in the provision of public schooling, but there is anything but agreement on its effects (see e.g., Hoxby, 2000 and Rothstein, 2007). On the other hand, inter-jurisdictional spillovers from local public goods and scale economies support centralization.

We have emphasized schooling as the primary local public good. The existence of private schooling raises the question as to how the relative efficiency of centralized versus decentralized public provision is affected by private market alternatives. Nechyba (2003) finds that relatively wealthy households that choose private schools will live in poorer school districts in Tiebout equilibrium, implying more limited household sorting. He finds as well that Tiebout

\textsuperscript{47}The modeling challenge for such a generalization is the difficulty in characterizing voting equilibrium when there are multiple sources of voter heterogeneity and multiple policy instruments.

\textsuperscript{48} See deBartolome and Ross (2003,2004) for analysis of Tiebout metropolitan equilibria with commuting costs.
equilibrium increases choice of private schools relative to centralized provision of public schooling. The net effect on welfare of these forces is unclear and of interest to investigate.

Finally, communities provide multiple local public goods. While this increases the potential gains from decentralization, it might heighten the residential choice externality that we find to be so damaging. A major research challenge in examining this is, of course, finding a satisfactory characterization of political equilibrium when multiple public goods are provided.

This paper does not, of course, refute Tiebout’s argument. Rather, it tells a cautionary tale about applying first-best arguments in a second-best environment. We think that it is of much interest to explore further the quantification of the welfare effects of local public goods equilibria.
References


### Table 2

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<th>Positive Properties</th>
<th>Efficient Allocation</th>
<th>Property Tax / Fixed Boundaries</th>
<th>Head Tax</th>
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**Distributional and Welfare Results**

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*The P’s are net housing prices; the Y’s minimum incomes in the jurisdictions; the N’s are the percentage populations; the t’s are the property tax rates; and the g’s are public good expenditures.
Figure 1A

Figure 1B

Indifference curve of a medium-preference voter

\[ p_j = p_i^j + \frac{N}{H_s} g_j \]
Figure 2A

Minus Compensating Variation by Income

Figure 2B

Minus Compensating Variation in Efficient Allocation
Actual and Predicted Values for Tax Rates, Government Spending, and House Values in Boston Metropolitan Area

Figure 3
(Solid line is predicted value.)
Appendix. Proof of Proposition 1: We suppress the community $j$ indicator in the proof.

a. Substituting (1) into (2), a voter’s preferred choice of $t$ corresponds to the choice of $(p,g)$ that solves:

$$\max_{p,g} V(p,g; y,\alpha)$$

subject to:

$$gN = (p-p_s)H_s(p);$$

where, recall, $p_s$ and $H_s$ are fixed in this stage, as well as $N$. Since $V$ is strictly quasi-concave and the constraint (A2) is linear, voter preferences are single peaked. Thus majority voting equilibrium exists and is the preference of a median-preference voter. Strict quasi-concavity of $V$ and linearity of the constraint (along with Inada condition) imply every voter’s preferred choice is unique and interior; thus equilibrium is unique.

b. and c. The first-order conditions for a voter’s preferred choice are:

$$-\frac{V_g}{V_p} = \frac{N}{H_s}$$

and (A2). Let $g^*(y,\alpha)$ denote the preferred choice of $g$ by voter $(y,\alpha)$. Differentiating (A2) and (A3) one obtains:

$$\frac{\partial g^*}{\partial \alpha} = -\frac{\partial (-V_g/V_p)}{\partial p} \left( -\frac{V_g}{V_p} \right) + \frac{\partial (-V_g/V_p)}{\partial g} > 0$$

and

$$\frac{\partial g^*}{\partial y} = -\frac{\partial (-V_g/V_p)}{\partial p} \left( -\frac{V_g}{V_p} \right) + \frac{\partial (-V_g/V_p)}{\partial g} > 0.$$

The denominators in (A4) and (A5) are (with sign) positive by strict quasi-concavity of $V$. The numerators are positive by SRa and SRI (see (5)), implying the inequalities. Let $g^e$ denote equilibrium $g$, which satisfies $g^e = g^*(y,\alpha)$ for median preference voters. Let $y^m(\alpha)$ satisfy the latter equation, which is continuous and unique by (A4) and (A5). Differentiating $g^e = g^*(y,\alpha)$ one obtains:

$$\frac{dy^m}{d\alpha} = -\frac{\partial g^*}{\partial \alpha} / \frac{\partial g^*}{\partial y} < 0;$$

(A6)
the inequality by (A4) and (A5). Thus the locus of median-preference voters is downward sloping as illustrated in Figure 1A.49

Any voter in community j with \((y, \alpha)\) to the southwest of the \(y^m(\alpha)\) locus has flatter indifference curve through \((g^e, p^e)\) in Figure 1B than any median preference voter by the single-crossing conditions. (Any median preference voter has indifference curve with the same slope through \((g^e, p^e)\).) Such voters: (i) prefer lower \(g\) and \(p\) than \((g^e, p^e)\); and (ii) would vote against any tax leading to higher \(g\) and \(p\). The reverse is true for any voters with \((y, \alpha)\) to the northeast of the \(y^m(\alpha)\) locus. Voting equilibrium then requires (6), which completes the proof of Part b. By (1) and (2), preference for higher (lower) \(g\) and \(p\) corresponds to preference for higher (lower) \(t\), implying Part c.

Proof of Proposition 3: (a) First we show that \(t_i = \eta_i = 0\). From (20) and that the allocation is differentiated:

\[\omega U_i^j + \eta_i \left( \frac{\partial \tilde{h}_d^i}{\partial y} \right) = \omega_r \quad \text{for all households} \ (y, \alpha) \ \text{assigned to community} \ i. \quad \text{(A7)}\]

Multiply through (A7) by \(a\) and integrate to obtain:

\[\int_s \omega U_i^j a_i f dyd\alpha + \eta_i \int_s \frac{\partial \tilde{h}_d^i}{\partial y} a_i f dyd\alpha = N_i \omega_r. \quad \text{(A8)}\]

Then (A8) and (25) imply:

\[\lambda_i = \omega_r. \quad \text{(A9)}\]

Also (A9) and (24) imply:

\[t_i \omega_r = \frac{\eta_i (1 + t_i)}{p_i}. \quad \text{(A10)}\]

Since \(\omega_r > 0\), if \(t_i = 0\), then \(\eta_i = 0\) and the reverse. Now we show that \(t_i \neq 0\) implies a contradiction. Multiply through (A7) by \(h_d^i a_i f\) and integrate to obtain:

\[\int_s \omega U_i^j h_d^i a_i f dyd\alpha = \omega_r H_i^s - \eta_i \int_s \frac{\partial \tilde{h}_d^i}{\partial y} h_d^i a_i f dyd\alpha; \quad \text{(A11)}\]

where we have substituted the housing market clearance condition ((16)). Now substitute from (A9), (A10), and (A11) into (27) to get:

\[\eta_i \left\{ \frac{1 + t_i}{H_i^s} \left( \int_s \frac{\partial \tilde{h}_d^i}{\partial p_i} a_i f dyd\alpha + \int_s \frac{\partial \tilde{h}_d^i}{\partial y} h_d^i a_i f dyd\alpha - \frac{1 + t_i}{t_i p_i} H_i^s \right) + \frac{1 + t_i}{p_i} \left( 1 + \epsilon_i^s \right) - \frac{1 + t_i}{t_i p_i} \epsilon_i^s + \frac{1 + t_i}{t_i p_i} \right\} = 0.\]

This simplifies to:

\[49\] The proof does not require that \(y^m(\alpha)\) is everywhere interior to the set of residents as in the example in Figure 1A.
The term in parentheses in the integrand in (A12) is the slope of the compensated demand for housing and is then negative. Hence, the integral term is negative, implying \( \eta_i = 0 \). This contradicts (A10), so it must be that \( t_i = \eta_i = 0 \).

Since \( t_i = 0 \), \( T_i = g_i \) by local budget balance (i.e., (17)).

(b) Using \( \eta_i = 0 \), substitute from (25) into (26). Then use that \( \omega U^i_1 \) equals a constant from (A7) to obtain the Samuelsonian condition for a congested public good:

\[
\int_s \frac{U^i_1}{U^i_1} a_i fdyd\alpha = N_i. \tag{A13}
\]

(c) Using the results in part (a), (21) and (22) imply that a household is optimally assigned to the community where \( V^e_i \) is maximized.

\[\blacksquare\]