The Joint Services of Money and Credit

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Credit card transactions $\subset$ monetary supply components?
How do such theories work?

- Credit cards balances $\not\subset$ monetary assets.
- Credit cards balances $\subset$ liabilities.
- Credit cards balances used for current period transactions provide monetary services.
- Aggregation and index number theory measure service flows.
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**Intertemporal Allocation**

\[
\text{max } u_t = u_t(m_t, ..., m_{t+T}; c_t, ..., c_{t+T}; x_t, ..., x_{t+T}; A_{t+T})
\]

\[
= U_t(v(m_t, c_t), v_{t+1}(m_{t+1}, c_{t+1}), ..., v_{t+T}(m_{t+T}, c_{t+T}));
\]

\[
V(x_t), V_{t+1}(x_{t+1}); A_{t+T})
\]

subject to

\[
p'_s x_s = \omega_s L_s + \sum_{i=1}^{n} [(1 + r_{i,s-1}) p^*_{s-1} m_{i,s-1} - p^*_s m_{is}]
\]

\[
+ \sum_{j=1}^{k} [p^*_s c_{j,s} - (1 + e_{j,s-1}) p^*_{s-1} c_{j,s-1}]
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\[
+ [(1 + R_{s-1}) p^*_s A_{s-1} - p^*_s A_{s}].
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= U_t(\nu(m_t, c_t), \nu_{t+1}(m_{t+1}, c_{t+1}), \ldots, \nu_{t+T}(m_{t+T}, c_{t+T}));
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\]

\[
+ [(1 + R_{s-1})p_{s-1}^* A_{s-1} - p_s^* A_s].
\]
Let

\[ \rho = \begin{cases} 
1, & \text{if } s = t, \\
\prod_{u=t}^{s-1}(1 + R_u), & \text{if } t + 1 \leq s \leq t + T.
\end{cases} \]
Then

\[
\begin{align*}
&\sum_{s=t}^{t+T} \left( \frac{p'_s}{\rho_s} \right) x_s + \sum_{s=t}^{t+T} \sum_{i=1}^{n} \left[ \frac{p^*_s}{\rho_s} - \frac{p^*_s(1+r_{i,s})}{\rho_{s+1}} \right] m_{i,s} + \\
&\sum_{i=1}^{n} \frac{p^*_{t+T}(1+r_{i,t+T})}{\rho_{t+T+1}} m_{i,t+T} + \frac{p^*_{t+T}}{\rho_{t+T}} A_{t+T} + \\
&\sum_{s=t}^{t+T} \sum_{j=1}^{k} \left[ \frac{p^*_s(1+e_{j,s})}{\rho_{s+1}} - \frac{p^*_s}{\rho_s} \right] c_{j,s} \\
&= \sum_{s=t}^{t+T} \left( \frac{\omega_s}{\rho_s} \right) L_s + \sum_{i=1}^{n} (1 + r_{i,t-1}) p^*_{t-1} m_{i,t-1} + (1 + R_{t-1}) A_{t-1} p^*_{t-1} + \\
&\sum_{j=1}^{k} \frac{p^*_{t+T}(1+e_{j,t+T})}{\rho_{t+T+1}} c_{j,t+T} + \\
&\sum_{j=1}^{k} (1 + e_{j,t-1}) p^*_{t-1} c_{j,t-1}.
\end{align*}
\]
The nominal user cost of monetary asset holding $m_{is}$ is

\[ \pi^*_{is} = \frac{p^*_s}{\rho_s} - \frac{p^*_s(1 + r_{is})}{\rho_s + 1} = \frac{p^*_t}{1 + R_t}. \]
Likewise, the nominal user cost of credit card service \( c_{js} \) is

\[
\tilde{\pi}_j^* = \frac{p_s^*(1 + e_{js})}{\rho_{s+1}} - \frac{p_s^*}{\rho_s} = p_t^* \frac{e_{jt} - R_t}{1 + R_t}.
\]
Extension to Revolving Credit

Method 1: Define $c_{js}$ to be total debt balances in the credit card account.

Method 2: Define $c_{js}$ to be credit card balances used for purchases during period $s$. 
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The budget constraint

$$p'_s x_s = \omega_s L_s + \sum_{i=1}^{n} \left[ (1 + r_{i,s-1}) p_{s-1}^* m_{i,s-1} - p_s^* m_{is} \right]$$

$$+ \sum_{j=1}^{k} \left[ p_s^* y_{j,s} - (1 + e_{j,s-1}) p_{s-1}^* y_{j,s-1} \right]$$

$$+ \left[ (1 + R_{s-1}) p_{s-1}^* A_{s-1} - p_s^* A_s \right],$$

where $y_{js} = c_{js} + z_{js}$. 

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Thus,

\[ p'_s x_s = \omega_s L_s + \sum_{i=1}^{n} [(1 + r_{i,s-1})p^*_s m_{i,s-1} - p^*_s m_{is}] + \sum_{j=1}^{k} [p^*_s c_{j,s} - (1 + e_{j,s-1})p^*_{s-1} c_{j,s-1}] + \sum_{j=1}^{k} [p^*_s z_{j,s} - (1 + e_{j,s-1})p^*_{s-1} z_{j,s-1}] + [(1 + R_{s-1})p^*_{s-1} A_{s-1} - p^*_s A_s]. \]
Comparison of The Two Methods:

1. Method 1: If $c_{js}$ is defined to be total debt balances in the credit card account:
   - All of the theory in this paper would be unchanged.
   - But the interpretation of inclusion of credit card debt in the utility function would be altered in a somewhat disturbing manner.

2. Method 2: If $y_{js}$ is defined to be total debt balances in the credit card account, where $y_{js} = c_{js} + z_{js}$:
   - Only current period credit card purchases $c_{js}$ provide transactions services included in the aggregate.
   - Theoretically preferable to Method 1, but has heavier data requirements.

Note: Empirical test needed to determine whether the aggregates are robust to Method 1 versus Method 2. If not robust, then Method 1 not justified.
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The assumptions on homogeneous blockwise weak separability of the intertemporal utility function are sufficient for consistent two-stage budgeting. See Green (1964, theorem 4).

1. Stage 1: The consumer selects real expenditure on augmented monetary services, $I_t^*$, and on aggregate consumer goods for each period within the planning horizon, along with terminal benchmark asset holdings, $A_{t+T}$.

2. Stage 2: Augmented monetary services $I_t^*$ are allocated over demands for the current period services of monetary assets and credit cards. That decision is to select $m_t$ and $c_t$ to

$$\max \nu(m_t, c_t),$$

subject to

$$\pi_t^* m_t + \tilde{\pi}_t^* c_t = I_t^*.$$
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\]
The exact quantity aggregate is the level of the indirect utility produced by the utility maximization problem:

\[
\mathcal{M} = \max \{ \nu(m_t, c_t) : \pi'_t m_t + \tilde{\pi}'_t c_t = I_t \} \\
= \max \{ \nu(m_t, c_t) : \pi'^*_t m_t + \tilde{\pi}'_t c_t = I^*_t \} \\
= \nu(m_t, c_t) \\
= \mathcal{M}(m_t, c_t) \\
= \text{augmented monetary aggregate.}
\]
An exact dual pair of price and quantity aggregates satisfies Fisher’s factor reversal test:

\[ \Pi(\pi_t, \tilde{\pi}_t) = \frac{I_t}{M_t} \]
Since $\nu$ is linear homogeneous, it follows from Barnett (1987) that,

$$\Pi(\pi_t, \tilde{\pi}_t) = \left[ \max_{\{m_t, c_t\}} \left\{ \nu(m_t, c_t) : \pi'_t m_t + \tilde{\pi}'_t c_t = 1 \right\} \right]^{-1}.$$

Define the cost function

$$E(\nu_0, \pi_t, \tilde{\pi}_t) = \min_{\{m_t, c_t\}} \left\{ \pi'_t m_t + \tilde{\pi}'_t c_t : \nu(m_t, c_t) = \nu_0 \right\}.$$
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$$
It can be proved that

\[ \Pi(\pi_t, \tilde{\pi}_t) = E(1, \pi_t, \tilde{\pi}_t) = \min_{\{m_t, c_t\}} \{ \pi'_t m_t + \tilde{\pi}'_t c_t : \nu(m_t, c_t) = 1 \}. \]

So

\[ \Pi(\pi_t, \tilde{\pi}_t) = \frac{I_t}{M_t} = E(1, \pi_t, \tilde{\pi}_t). \]
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So

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In summary, we have

\[ M_t = \max \{ g_1(m_t) : \pi_t^* m_t = \Pi^*_m M_t \} \]

and

\[ C_t = \max \{ g_2(c_t) : \tilde{\pi}_t^* c_t = \Pi^*_c C_t \} . \]

Thus, the optimal values of the monetary and credit card quantity aggregates are related to the joint aggregate in the following manner:

\[ \mathcal{M}_t = \tilde{\nu}(M_t, C_t) . \]
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Discrete Time Approximations to the Divisia Index: the Törnqvist-Theil Approximation:

\[
\log M(t) - \log M(t-1) \\
= \sum_{i=1}^{n} \bar{\omega}_{it} (\log m_{it} - \log m_{i,t-1}) + \sum_{i=1}^{k} \bar{\tilde{\omega}}_{it} (\log c_{it} - \log c_{i,t-1}),
\]

where \( \bar{\omega}_{it} = (\omega_{it} + \omega_{i,t-1})/2 \) and \( \bar{\tilde{\omega}}_{it} = (\tilde{\omega}_{it} + \tilde{\omega}_{i,t-1})/2 \).
Risk Adjustment

Choose the deterministic point \((m_t, c_t, x_t, A_t)\), and the stochastic process \((m_s, c_s, x_s, A_s)\), \(s = t + 1, \ldots, \infty\), to maximize

\[
u(m_t, c_t, x_t) + E_t[\sum_{s=t+1}^{\infty} \frac{1}{1+\xi}^{s-t} u(m_s, c_s, x_s)],
\]

subject to \((m_s, c_s, x_s, A_s) \in S(s)\) for \(s = t, t + 1, \ldots, \infty\), and also subject to the transversality condition

\[
\lim_{s \to \infty} E_t(\frac{1}{1+\xi})^{s-t} A_s = 0.
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u(m_t, c_t, x_t) + E_t\left[ \sum_{s=t+1}^{\infty} \left( \frac{1}{1 + \xi} \right)^{s-t} \nu(m_s, c_s, x_s) \right],
\]

subject to \((m_s, c_s, x_s, A_s) \in S(s)\) for \(s = t, t + 1, \ldots, \infty\), and also subject to the transversality condition

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\]

subject to \((m_s, c_s, x_s, A_s) \in S(s)\) for \(s = t, t + 1, \ldots, \infty\), and also subject to the transversality condition

\[
\lim_{s \to \infty} E_t\left( \frac{1}{1+\xi}^{s-t} A_s \right) = 0.
\]
Existence of an Augmented Monetary Aggregate for the Consumer

We assume that the utility function, \( u \), is blockwise weakly separable in \( (m_s, c_s) \) and in \( x_s \):

\[
u(m_s, c_s, x_s) = F[M(m_s, c_s), X(x_s)].\]
So Euler Equations are:

\[
E_s\left[ \frac{\partial V}{\partial m_{is}} - \rho \frac{p^*_s(R_s - r_{is})}{p^*_{s+1}} \frac{\partial V}{\partial X_{s+1}} \right] = 0,
\]

\[
E_s\left[ \frac{\partial V}{\partial c_{js}} - \rho \frac{p^*_s(e_{js} - R_s)}{p^*_{s+1}} \frac{\partial V}{\partial X_{s+1}} \right] = 0,
\]

and

\[
E_s\left[ \frac{\partial V}{\partial X_s} - \rho \frac{p^*_s(1 + R_s)}{p^*_{s+1}} \frac{\partial V}{\partial X_{s+1}} \right] = 0.
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and

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\]
New Generalized Augmented Divisia Index Under Risk

Definition The contemporaneous risk-adjusted real user cost price of the services of $m_{it}^a$ is $\mathcal{P}_{it}^a$, defined such that

$$\mathcal{P}_{it}^a = \frac{\partial V}{\partial m_{it}^a} \cdot \frac{\partial X_t}{\partial V}, \quad i = 1, 2, ..., n + k.$$
Theorem 1(a) The risk adjusted real user cost of the services of monetary asset $i$ under risk is $P_{it}^m = \pi_{it} + \psi_{it}$, where

$$\pi_{it} = \frac{E_t R_t^* - E_t r_{it}^*}{1 + E_t R_t},$$

and

$$\psi_{it} = \rho (1 - \pi_{it}) \frac{\text{Cov} \left( R_t^*, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{\text{Cov} \left( r_{it}^*, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}}.$$
**Theorem 1(a)** The risk adjusted real user cost of the services of monetary asset $i$ under risk is $\mathcal{P}_it^m = \pi_{it} + \psi_{it}$, where

$$\pi_{it} = \frac{E_t R^*_t - E_t r^*_it}{1 + E_t R_t},$$

and

$$\psi_{it} = \rho (1 - \pi_{it}) \frac{\text{Cov} \left( R^*_t, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{\text{Cov} \left( r^*_it, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}}.$$
Theorem 1(b) The risk adjusted real user cost of the services of credit card type $j$ under risk is $\mathcal{P}^c_{jt} = \tilde{\pi}_{jt} + \tilde{\psi}_{jt}$, where

$$\tilde{\pi}_{jt} = \frac{E_t e^*_{jt} - E_t R^*_t}{1 + E_t R_t},$$

and

$$\tilde{\psi}_{jt} = \rho \frac{\text{Cov} \left( e^*_{jt}, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}} - \rho (1 + \tilde{\pi}_{jt}) \frac{\text{Cov} \left( R^*_t, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}}.$$

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Theorem 1(b) The risk adjusted real user cost of the services of credit card type $j$ under risk is $\mathcal{P}_{jt} = \tilde{\pi}_{jt} + \tilde{\psi}_{jt}$, where

$$\tilde{\pi}_{jt} = \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t},$$

and

$$\tilde{\psi}_{jt} = \rho \frac{\text{Cov} \left( e_{jt}^*, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}} - \rho (1 + \tilde{\pi}_{jt}) \frac{\text{Cov} \left( R_t^*, \frac{\partial V}{\partial X_{t+1}} \right)}{\frac{\partial V}{\partial X_t}}.$$
Theorem 2 In the share equations, \( \omega_{it} = \pi^{a}_{it} m^{a}_{it} / \pi^{a'}_{t} m^{a}_{t} \), we replace the user costs, \( \pi^{a}_{t} = (\pi'_{t}, \tilde{\pi}'_{t})' \), by the risk-adjusted user costs, \( \mathcal{P}^{a}_{it} \), to produce the risk adjusted shares,

\[
S_{it} = \mathcal{P}^{a}_{it} m^{a}_{it} / \sum_{j=1}^{n+k} \mathcal{P}^{a}_{jt} m^{a}_{jt}.
\]

Under our weak-separability assumption, \( V(m_{s}, c_{s}, X_{s}) = F[M(m_{s}, c_{s}), X_{s}] \), and our assumption that the monetary aggregation function \( M \) is linearly homogeneous, the following generalized augmented Divisia index holds under risk:

\[
d \log M_{t} = \sum_{i=1}^{n+k} S_{it} d \log m^{a}_{it}.
\]
Conclusion

- Economic aggregation and index number theory measure service flows, independently of whether from assets or liabilities.
- So the transactions services of credit cards could be included in monetary aggregates.
- This paper has provided theory solving that long overlooked problem.
- Six possible approaches exist to incorporating credit card services into monetary aggregates: Method 1 or Method 2 under risk neutrality, Method 1 or Method 2 under CCAPM risk, or Method 1 or Method 2 under intertemporally nonseparable risk.
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