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A SCREENING MODEL FOR LONG RANGE PLANNING AT THE POOL LEVEL

by

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A Scmafmg Model for Lomg Ramgt Plammiag at tke Pool Lerel

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ABSTRACT

Thb paper develops a malO-period, multi-utility model for ^aiqiag macro effects over exteaded horiaoas of 20 to 40 years* Plants are aggregated into categories and lamped at load ceaters, which may be iatercoaaected by lossy fines. There may be more than oae load ceater per utility. The geaermtioa, traasmiasioa aad operation planning problems for this set of load ceaters is formulated as a linear programming problem. A decomposHioa aad means-end analysis method b msed to sohre the problem. The results are useful for rnmimWt macro effects •· generation expansion by plant category; control technology retrofits; changes required in tie line capacities; loag term, iafter-atifity energy exchanges; kmg term fuel scheduling; aad the imparts of babble constraints for emissions such as SO_2.

1. INTRODUCTION

The running of a utility involves collaboration with other utifities. Activities contributing to, aad factors affecting, these eoUaboratioa* include

- remote siting aad sharing of generating plants.
- inter-utility energy flows.
- bubble constraints oa embsioas (ceilings oa the total emissions produced by the plants in a region that could encompass several utilities). Though such constraints are not now in effect, they are being seriously considered by regulatory bodies and could soon be adopted.

The possibility of collaboration increases the number of alternatives available to planners, For instance, some of the alternatives available to reduce the total SO_2 emissions produced by a utility are: (1) switch to lower sulfur fads, (2) retrofit the coal burning plants with scrubbers, and (3) purchase energy from other utilities. To determine the optimal mix of these alternatives over an extended time horizon, one needs to simultaneously consider all the utilities that could collaborate over the entire horisoa. Thb, of course, results in a very large optimirstioa problem. To make it computationally tractable, we have adopted the following measures:

- aggregation to reduce the number of variables.
- representation of all the relevant phenomena by linear modeb so that the overall problem becomes oae of linear programming.
- the use of a special decomposition aad means-end aaalysb to solve the linear programmiig problem (even though linear, thb problem b too large to be conveniently tackled by more direct methods).

The net result of this approach b a screening model, that is, a model that provides a comprehensive, bat relatively undetailed, view of the activities of multiple, interacting utilities over multiple time periods. The most natural application of the screening model b to power poob because poob are the bask nrgiaiiarioasJ units for promoting interactions amoug utifities. Bat the model can abo be applied to other groupings of utilities. la fact, siace it works off a database that contains information oa aU the grarrating units in the continental US, it can be applied to any subset of the generating units.

la function, the screening model b best suited to providing inertiawa of the activities of utifities. These otettieu's can be used for high-level decision making or to provide inputs, targets, aad guidelines for more detailed planning modeb that consider only oae utility at a time. The alternatives to using screening modeb are either to treat utilities as if they were independent with no interactions, or to guess the interactions in advance. Neither b an attractive alternative.

The remainder of the paper b organised as follows. Section 2 formulates the linear programming problem. Section 3 describes the decomposition and means-end aaaJysb used to sohre the linear programming problem. Section 4 presents an example.

2. FORMULATION OF THE LINEAR PROGRAMMING PROBLEM

2.1. Assumptions

1. The net exports of electrical energy from the group of utilities considered to the rest of the country are known in advance.
2. Operating costs and emissions are linear functions of operating level of generating plants aad pollution control technologies
3. Capital costs for both generating plants aad pollution control technologies are linear functions of their sites
4. Plants are aggregated into 10 categories.
5. Fuel sJternatives for each plant category are aggregated into at most 3 categories
0. Load demand points are aggregated into load centers.
7. Transmission losses between load centers are linear functions of power flow.
8. Retirement years of generating units are known in advance.

2.2. Tho Model

The multi-period multi-utifity planning (MUP) problem b formulated as a linear programming model using aggregated, rather than the indhridoa), plant and fed categories. Instead of directly solring thb problem as a single linear programming problem, which ì» potentially unmanageable, the problem b handled instead by a heuristic decomposition technique.

The MUP problem b defined an:

Giren:

1. A Time horison divided into several periods.
2. The demand in each utifity for each period, given in the form of a load enrre.
3. The cost, availability, heating ral»e ami pollution content of a set of representative f neb for each utility in each period.
4. The cost* and other characteristics of a limited number of types of generating plants for each utility in each period*
5. The costs, efficiencies and other characteristic! of a set of pollution control technologies for each utifity is each period.
0. The transmission lines between the utifity nodes, their capacities and loss characteristics in each period.
7. The emission caps for inSvidnal utilities and/or the emission caps for the whole region (mntaťwing the many ntilities).

Find:

1. The inter-utility energy transfers in each sub-period.
2. The type, timing, sine and location (by utility) of generation expansions ami poflutiou control retrofit*.
3. Utility emission caps, if they were not specified in the input. Abo, the marginal cost of SO2 abatement.
4. The types and amounts of the fmeb used in each period and each utility.

Thb problem b formulated as a Linear Programming (LP) problem*

2.3. Notation

The terms used in the linear programming formulation are defined below:

Symbol	Description
	Indices/Subscripts
i	Type of generating plant/CT plant
k	Temporary index for time (k—1 to t)
m	Segments of the load eurre
t	Time period
j	Fuel type (j b removed if no fuel selection b allowed, e.g. nuclear plants)
n	Utility in the region
	Generating Plant Characteristics for utiDtr n
C_{itn}	Capital cost (S/MW)
$fuel_{itj}$	Operating cost with fuel type j (S/MWHr)
P_{i0n}	Initial plant capacity (MW)
;	Retired capacity in period t (MW)
AV_{in}	Arailability limit
CF_{in}	Capacity factor limit
e_{itjn}	pollution coefficient (tons of SOg/MWHr)
	Control Technology Characteristics for utility n
V_{itn}	Capital cost of CT type i (S/MW)
U_{itjn}	Operating cost (S/ton of SO_2 remored)
X_{i0n}	Initial CT capacity (MW)
$\bar{r}_{it}$	Retired CT capacity in period t (MW)
k_{itjn}	Conversion factor from tons of SO_2 to MW (MW/ton)
	Exogenous Variables
d	Discount Factor
L_{mtn}	Segments of load cunre (MW) for utility n
h_{mt}	Duration of segment m of load curve (hours)
Th_t	Total hours in period t
η_{trn}	Efficiency of the transmission line between utilities r and n
$Emax_t$	Regional emission constraint in period t (tons)
	Decision Variables for utility n
P_{itn}	Generation addition of type i in period t (MW)
θ_{itmjn}	Operating lerel of generating plant i (MW)
x_{itn}	CT addition of type i in period t (MW)

w_{itjn} Aetna! CT atifisasioa ia period t (tons of SO, removed)

O_{mtrn} Power outflow from atifity r to atifity a ia segment mL of the load duration curve

1.4. Linear Progmmaiiag Formialarioai

The following famalntiiM of the MUP problem applies to a region containing N atifitiai aad for aa extended planning horizon (typically 20 to 40 yean hi length). The horizon is divided into periods and the lond carve for each period b divided into segments. Then the total print worth of all capital and operatic expenses ia all the atifities ia the region b minimiieH over the horizon to yield:

Minimize

$$\sum_{n}\sum_{t}\Big(\sum_{i} C_{itn}p_{itn}+\sum_{ijmj} \theta_{itmjn}h_{mt}fuel_{itjn}+\sum_{i}(V_{itn}x_{itn}+\sum_{j} U_{itjn}w_{itjn})\Big)$$

subject to:

1. Demand constraints (generation should beat least equal to demand):

$$\sum_{i,j}\theta_{itmjn}+\sum_{r\neq n}\eta_{trn}O_{mtrn}-\sum_{r\neq n}O_{mtnr}\geq L_{mtn},$$

for aO m, t, a

2. Generation constraints (generated power should not exceed power capacity):

$$AV_{in}\sum_{k=1}^{t}p_{ikn}-\sum_{j}\theta_{itmjn}\geq AV_{in}[\sum_{k=1}^{t}R_{ikn}-P_{i0n}],$$

for all i, m, t, a

3. Capacity factor constraints (generated energy should not exceed energy capacity):

$$CF_{in}(Th)_t\sum_{k=1}^{t}p_{ikn}-\sum_{m,j}h_{mt}\theta_{itmjn}\geq CF_{in}(Th)_t[\sum_{k=1}^{t}R_{ikn}-P_{i0n}],$$

for all i, t, n

4. CT (Control Technology) constraints (CT usage should not exceed capacity):

$$\sum_{k=1}^{t}x_{ikn}-\sum_{j}k_{itjn}w_{itjn}\geq \sum_{k=1}^{t} r*T^X*m'$$

for all i, t, n

5. CT constraints (CT cannot remove more SO_2 than is produced)

$$\sum_{m}\theta_{itmjn}h_{mt}e_{itjn}-w_{itjn}\leq 0$$

for all j, i, t, n

6. Regional emissions constraints on SO_2:

$$\sum_{n,i,m,j}\theta_{itmjn}h_{mt}e_{itjn}-\sum_{n,i,j}\leq(Emax)_t,$$

for all t

7. Noa-negntirity of all rariables.

8. Other constraints may be imposed ae needed, for instance:

- Upper limits oa the reaoml efficiencies of poOatioa control technologies.
- Upper torfs on the aawwats of plant capacities that may be installed in a ntffity or ia the region.
- Upper torfs on the amounts of fads that any be need by n particular ntifity.
- Upper Haute oa inter-utility transmission line capacities aad availabilities.
- Constraints oa SO, at the iadmdanl atifitj te»eL
- Coastraiats oa sereral other emissions such as NO_x at the etilHy nad/or regional lereL

ia order to minimize the usual b st-period effects (i.c. oaderstated capital expeaditare), it is assumed that, alter the final period, the system will operate indefinitely at those levels. To do this,, the cost coefficieate for the operations variables (# aad w) ia the final period are multiplied by l/(l-d).

3. A NEW PROCEDURE TO SOLVE THE MUP PROBLEM

3.1. Overview

Even with the varioas aggregations aad Bacariiatioas, for most realistic appbention, the MUP model can be qrirte large. Thesiseof the problem (the number of variables aad aamber of constraints) b proportional to NXT, where N ͅ» the lumber of wtifitiea in the region, and T is the nnmber of periods in the time horison. For a representative set of valves of N aad T ($N \geq 10$, and $T \geq 15$), the problem becomes very large, and existiag LP codes womld have troable solving it ia any reasonable amovat of time. (The aamber of constraints wonld exceed WOO, aad the anmber of variables womld exceed 15000.) Therefore, a different soimtion procedure b developed for the MUP problem.

The MUP problem decomposes, in a nntmral way, into several single-period problems with the capital variables (generation capacity aad CT capacity) being the control variables. That b, with fixed capital variable values, the problem caa be divided into several smaller single-period electric power dispatch type problems, ia which the plant operating leveb and inter-ntiltty flow are the unknown variables.

A few words about the decomposition are appropriate here. There are several standard ways to decompose an LP problem [1-5]. Unfortunately, the general decomposition methods offer little in the way of improved running time or diminished storage requirements over the various sparse-matrix implementations of the Simplex algorithm. Instead of using one of these, knowledge of the problem and a "means-ends" approach has been used to obtain a new decomposition method.

Using the bounds on the capital variables as controls, the decomposition, for each period,

1. allocates the total generation plant expansions for the region to the individual utilities,

2. allocates the energy generation for the region to the individual utilities *(It. determines* inter-utility energy transfers),

3. and subdivides the regions emission caps among the individual utilities.

In making these allocations and subdivisions, the algorithm takes the inter-utility transmission losses and capacities into account.

Theoretically, it may be necessary to reiterate over the periods until the solutions converge; however, in general no more than one or two iterations ought to be required for a satisfactory solution. Details of the decomposition method follow.

3.2. Assumptions

Following assumptions about the MUP problem have been made when developing a heuristic decomposition method:

1. Electric power demand is expected to increase over time, hence the total generation capacity requirements are expected to increase with time.

2. The time horizon of study is generally such that, capacity brought online during the period of study will be operable at least until the end of the horizon.

3. The allowable emission limits are expected to decrease (or at least not increase) over time, hence CT capacity requirement are expected to increase with time.

4. The final period of study is a steady utility model, hence it is likely that the profile of capital additions for the solution of just the last period problem would be similar (and in many instances identical) to that for last period portion of the exact solution to MUP.

These assumptions together with other features of the MUP problem suggest the use of a means-end algorithm using the capital variables as the control variables. This algorithm essentially tries to find the optimal capital configuration for the final period (which is a steady-state problem), and works back toward the beginning periods.

3.3. A Decomposition Algorithm

Besides the existing data, such as present capacities, demands, variable costs, and emission limits, the inputs to each of the single-period problems are the various capital costs and upper and lower bounds on the variables corresponding to new capital construction. These capital variables refer to total new capacity that is online during that period, which may be built during or before that period. Hence a capital variable $\bar{p}_{it}$ (the subscript n referring to utility is suppressed for this discussion) refers to all new capital construction of type i in the periods up to and including t. It is the bounds and costs for these variables that are the main controls for the dynamic programming algorithm.

Before proceeding with the steps of the algorithm, let us examine how the bounds and costs for the variables are computed. For the final period problem (which is to be solved first), the lower bounds are at 0 and the upper bounds are maximum amount of generating capacity that can be added for all the periods. For example, in a ten period problem, if 1000 MW can be added in each period, the upper bound would be 10000 MW. The cost for the capital variable would simply be the discounted cost of construction in the final period. For any other period, t, the upper bound is the minimum of the maximum amount of generation that can be built up to and including that period and the value of $\bar{p}_{i,t+1}$, the amount online in the next period. The lower bound is the maximum of 0 and $\bar{p}_{i,t+1} - MK_{i,t+1}$ where $MK_{i,t+1}$ is the maximum amount of capacity that can be constructed in period t+1 alone. For example, if $\bar{p}_{j,t+1}$ were equal to 7500, and the maximum amount of construction allowed in period i+1 were 1000, then $\bar{p}_{jt}$ would be 6500, for if less than 6500 MW were constructed by period t, then it 7500 could not be constructed by period t+1. These bounds are derived from the minimal conditions for feasibility of capital additions. The cost for the capital variables is given by the discounted cost of construction in period t minus that for period t+1. That is, $\bar{p}_{i,t+1}$ is the amount of new capacity online in period t+1, and this cost is the cost of bringing the capacity online one period earlier.

The decomposition algorithm used to solve the MUP problem is as follows:

STEP 0. (Initialisation) Set t = T (the final period).

STEP 1. Compute the costs and upper and lower bounds for the capital variables (as above).

STEP 2. Solve the single-period problem for period t. Save the values of the capital variables.

STEP 3. Set t = t-1. If t=0, Stop. Otherwise go to STEP 1.

In general, this algorithm yields a near-optimal solution to the MUP problem, in a very reasonable time. It is possible to modify the algorithm to incorporate multiple passes in time, modifying the bounds on the capital variables using information from previous passes and the current pass. It is also possible, and not very difficult, to test for optimality for the entire problem using the dual variables for the capital upper bound constraints for the single-period problems.

3.4. Single Period Problem Formulation

A formulation of the single-period problem follows:

For each period t,

Minimize

$$\sum_{n} \{[\sum_{i} (\bar{C}_{itn} \bar{p}_{itn} + \sum_{i,m,j} f_{itmjn} h_{mt} fuel_{itjn} + \sum_{i} (\bar{V}_{itn} \bar{z}_{itn} + \sum_{j} U_{itjn} w_{itjn})]\}$$

subject to:

1. Demand coastraiats (generation should be at least equal to demand):

$$\sum_{i,j} e_{itmjn} + \sum_{r \neq n} \pi_{trn} O_{mtrn} - \sum_{r \neq n} O_{mtnr} \geq L_{mtn},$$

for ail m, •

2. Geacratioa coastraiats (generated power _·.__u___^_______. saosjq BO* cxccefl power capacity):

$$AV_{in} \bar{p}_{itn} - \sum_{j} e_{itmjn} \geq AV_{in} [\sum_{k=1}^{t} R_{ikn} - P_{i0n}],$$

for aO i, m, a

3. Capacity factor constraints (generated energy should not taeeedeiMrcr capacity):

$$CF_{in}(Th)_t \bar{p}_{itn} - \sum_{m,j} h_{mt} e_{itmjn} \geq$$

$$CF_{in}(Th)_t [\sum_{k=1}^{t} R_{ikn} - P_{i0n}],$$

for aU i, a

4. CT (Control Technology h constraints (CT usage should not exceed capacity):

$$\bar{z}_{itn} - \sum_{j} k_{itjn} w_{itjn} \wedge \sum_{k=1}^{t} {}^{T}ikm - X_{i0n},$$

for all i, a

5. CT constraints (CT cannot remove more SOj than is produced)

$$\sum_{m} e_{itmjn} h_{mt} e_{itjn} - w_{itjn} \leq 0$$

for all j, i, t, a

0. Regional embsioas constraints on SO_2:

$$\sum_{n,i,m,j} e_{itmjn} h_{mt} e_{itjn} - \sum_{n,i,j} w_{itjn} \leq Em**_t$$

7. Noa-negativity coastraiats oa all variables, aad

$$\bar{p}_{i,t,n} \leq \bar{p}_{i,t+1,n}$$

for aU t,n, aad for $t \neq T$

8. Other constrainti may be imposed as needed, for instance:

- Upper Emits oa the removal efficiencies of pofiutioa control technologies.
- Upper fimtes oa the amoaats of plant capacities that may be installed in the utility or the region.
- Upper Bmtts oa the amounts of fads that may be ased by a particmlar mtilhy.
- Upper limits oa inter-utility traasaiission lines capacity aad availability.
- Coastraiats oa SO, at atility level.
- Constraints oa several other emissions such as TSP aad NO_z at atility and/or repoaal lerel.

The special notation ased here that is different from that in the formulation in Section 2 is:

$$\bar{p}_{itn} = \sum_{k=1}^{t} p_{ikn}, \text{ for all } i$$

$$\bar{z}_{itn} = \sum_{k=1}^{t} z_{ikn}, \text{ for all } i$$

$$\bar{C}_{itn} = C_{i,t,n} - C_{i,t+1,n} d^{t+1}/d^{t}, \text{ for } t \neq T$$

$$\bar{V}_{itn} = V_{i,t,n} - V_{i,t+1,n} d^{t+1}/d^{t}, \text{ for } t \neq T$$

The soratioa procedure is implemented using the XMP linear programming package {6j.

4. AN EXAMPLE

la this section, we present some of the resvhs ohtaiard from a study of seren utilities. Snch resalu are at least as seasHrre to the input data as they are to the methodology. Oar objectives here are to illustrate the sort of omtpat the MUP methodology caa generate. We have neither the space to present all the iapat data nor to comment oa its accuracy. Therefore, we will not identify the utilities involved.

Information oa the existing and announced generating nates for the utilities were obtained from the Unit Inventory File [7] that has been compiled for all the nates in the U.S. under another part of the project that supported this work. Much of the information oa costs aad characteristics of fads aad control technologies was abo obtained from databases aad models developed for this project. The remainder of the input information, particularly on emissions ceilings aad demand growths aad demand shapes is conjectured.

The basic problem consists of seven utilities interconnected by lossy lines over a time horizon of 20 years that are divided into 10 periods (5 1-year periods, 2 2-year periods, 2 3-year periods, and one 5-year period) using a 2% per annum demiad growth. The basic groupings or pools (defined by the transmission lines) are utilities A, C, D, aad G comprising oae pool aad B, E, aad F comprising the other, with C abo rnaarrtinc to the second pooL The demand load curves used were ntimatH 3-segment load duration curves. The energy demands used for the seven utilities during the first period were approximately 12, 8, 12, 3, 24, 20, and 20 1000 GWHrs/Year. Since one of the reasons for power transfers is differing peak times, and that using load duration curves instead of a skyline curve implies simultaneous peaking, the solution can be expected to be biased toward peaking units. (I.e. the peak-shaving that power transfers can accomplish are not reflected in this example.)

Results of two runs are summarized below. The first is with an allowable emission level that is binding only during the final period; the second uses a 30% lower level that becomes binding during the final three periods. The differences in answers occur only during the final three periods (11 years) of the study. The only generation capacity added during the first 7 periods were those that were already scheduled (i.e. the model did not predict the need for other additions).

The net power transfers for four of the first seven periods are given in Table 4-1. Nuclear facilities were scheduled to come on line between 1985 and 1987 for utilities A and F, and this accounts

Table 4-1: Inter-Utility Power Transfers--First Seven Periods

Net Power Export GwHrs/Year

Utility	1985	1987	1989	1993
A	350	4090	3600	2827
B	-2624	-5010	-3821	-719
C	-5149	-5935	-6448	-4008
D	552	159	292	84
E	4281	3000	2745	955
F	-1258	2630	1546	-12
G	4057	1410	2368	1011

for the change in the transfer patterns during those periods. Utilities B and C were scheduled to have coal units come on line for the 1993 period, and this accounts for most of the pattern change there.

Table 4-2: Inter-Utility Power Transfers—Final Three Periods

Loose Pollution Standard

Net Power Export GwHrs/Year

Utility	1996	1999	2004
A	2221	4469	2126
B	-1675	-2152	-432
C	-2443	-251	-23
D	-102	-199	149
E	2481	2351	-221
F	-998	-3135	673
G	277	-783	-2228

Tight Pollution Standard

Net Power Export GwHrs/Year

Utility	1996	1999	2004
A	2221	2623	2126
B	-1539	-2016	-432
C	-2793	93	-98
D	-102	-311	149
E	2702	3362	-221
F	-998	-3135	673
G	673	-428	-2152

Tables 4-2 and 4-3 summarize the inter-utility transfers and generation additions, respectively, for the last three periods of the two runs. Even though four different generation technologies were permitted, (natural gas, oil, nuclear, and coal), only natural gas (for a peaking unit) and coal (for a base unit) were selected by MUP. The differences in the generation additions for the two runs are slight. Utility A brings on 17 more MW of peak capacity earlier for the tighter emission standard case, and utility C builds slightly less coal and more peak for the tighter emission case.

Table 4-3: Natural Gas/Coal Accumulative Additions MW

Loose Pollution Standard

Utility	1996	1999	2004
A	0/0	354/0	474/0
B	0/0	119/0	119/446
C	0/0	670/0	670/625
D	0/0	21/0	156/0
E	0/0	0/0	0/1469
F	0/0	0/0	0/1323
G	0/0	517/0	517/0

Tight Pollution Standard

Utility	1996	1999	2004
A	0/0	337/0	474/0
B	0/0	119/0	119/446
C	0/0	687/0	687/606
D	0/0	21/0	156/0
E	0/0	0/0	0/1469
F	0/0	0/0	0/1323
G	0/0	517/0	517/0

The differences in flows for the two examples can be explained primarily in terms of existing non-polluting generating units that were basically uneconomical to dispatch for the loose emission constraint case, but became useful when the emission standard became tighter, raising the marginal cost of power. This occurred for utility E, and to a lesser extent, utilities B,C, and G, for periods 8 and 9 (years 1996 and 2004). In the final period, these units were utilized for both runs. It is this that accounts for utility A's difference in exported power in 1999 (utility E replaced A for exports to some extent).

5. CONCLUSIONS

A tractable model for multi-utility planning has been developed using deterministic data. It is also possible to include some uncertainties that are common to electrical power generation problems. The sorts of uncertainties that are important can be included through another heuristic and chance constrained programming.

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