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# CAPITAL GAINS TAXES AND PORTFOLIO REBALANCING

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The major friction that investors face in rebalancing their portfolios is capital gains taxes, which are triggered by the sale of assets. In this article, we examine the impact of an investor's capital gains tax liability and existing risk exposure upon the optimal portfolio and rebalancing decisions. We capture the trade-off over the investor's lifetime between the tax costs and diversification benefits of trading. We find that the investor's incentive to re-diversify the portfolio declines with the size of the capital gain and the investor's age. Unlike conventional financial advice, the reset of the capital gains tax bases and the resulting elimination of the capital gains tax liability at death, suggests that the optimal equity proportion of the investor's portfolio increases as he ages.

**in this issue**

## >>> INTRODUCTION

The major friction that investors face in rebalancing their portfolios is capital gains taxes.[1] An investor who has only capital losses can, without the payment of a capital gains tax, rebalance his portfolio to the *unconstrained optimal* level of risk given the assumed structure of future tax liabilities. However, for an investor *with embedded capital gains*, the optimal amount of rebalancing depends upon the magnitude of these gains and the extent of deviation of the asset holdings from the optimal portfolio. The presence of these capital gains liabilities influences the investor's optimal rebalancing decision as the investor's optimal portfolio choice reflects the trade-off between (a) realizing a certain amount of capital gains immediately and using the liquidated funds to rebalance his portfolio and (b) deferring the realization of the remainder of the capital gains.

In this article, we examine the impact of such factors as an investor's capital gains tax liability and existing risk exposure upon the optimal portfolio and rebalancing decisions in a taxable account. We capture the trade-off over the investor's lifetime between the tax costs and diversification benefits of trading. We show that an investor's incentive to re-diversify his portfolio declines with the size of the capital gain and his age. Unlike conventional financial advice, the reset of the capital gains tax bases and the resulting elimination of the capital gains tax liability at death suggest that the optimal equity proportion of the investor's portfolio increases as he ages.

This article draws on Dammon, Spatt and Zhang (2001a), where we formalize and solve the problem facing an investor who derives utility (well-being) from his intertemporal consumption levels and final bequest. We formulate the portfolio-rebalancing problem confronting an individual investing in taxable accounts with an eye towards solving for the investor's optimal intertemporal portfolio. In this presentation we summarize the model framework and the features of its solution.[2]

## >>> MODEL FRAMEWORK

### The Tax Environment

There are several crucial features of the tax environment that our model captures. The most important feature is that the payment of capital gains taxes is triggered by the sale of assets rather than the total increase in the market value of the investor's assets (as in "accrual taxation" or "marking to market" the positions at the end of each tax year).[3] Our analysis focuses upon the impact of taxation of capital gains at their sale/realization on the investor's optimal rebalancing and asset allocation choices. It is this feature of our setting that significantly enhances its richness as the investor's optimal portfolio depends upon variables that reflect the extent of the investor's embedded gains and his existing portfolio holdings.

Another important feature of the tax environment in our model is the assumption that at death the investor's tax bases on risky assets are "reset" to their current market values, thereby eliminating the capital gain tax liability. This reset of the tax bases follows the actual tax code in the United States and has an especially important effect on the investor's optimal portfolio and rebalancing decisions as the investor ages and his mortality risk increases.

Because we focus upon long-term asset allocation issues and try to reflect the presence of portfolio-offset rules that limit the degree of asymmetry between the effective treatment of short-term and long-term realizations, we assume a constant tax rate for all capital gain and loss realizations. Eliminating any distinction between short-term and long-term realizations also helps simplify the analysis, as we do not need to condition the solutions upon how long the investor has held the asset.[4]

To summarize, the key features of our modeling of capital gains taxation are: (a) capital gains taxes are triggered by the sale of assets; (b) any capital gains tax liability is forgiven at the investor's death as the investor's tax basis is reset to the asset's market value; and (c) the capital gains tax rate does not depend upon how long the investor has previously held the asset.

### The Investor's Problem

In order to focus upon the impact of the investor's age and increasing mortality, we assume that the investor has a limited (finite) life expectancy and that the probability of survival in each period is given by an assumed observed mortality schedule. In much of the analysis the investor's income is derived *only* from

financial assets (i.e., by assumption there is no labor income). The investor can trade two assets—a risk-free asset whose return is constant over time and a risky security. The pre-tax capital gain return on the risky asset is assumed to follow a binomial process whose return is independent over time. The pre-tax dividend yield is assumed constant over time.

We assume that there are no trading costs and short sales are not permitted. Furthermore, nominal dividends and interest payments are taxed at the tax rate on ordinary income, while realized capital gains and losses are taxed at a capital gains tax rate (which can be lower than the ordinary income tax rate). To calculate the investor's capital gains tax we assume that the investor's tax basis is the weighted *average* purchase price of these shares. When the risky asset's current price is below the investor's tax basis, the investor sells shares to realize the tax loss and immediately repurchases the optimal number of shares of the risky asset.[5] When the risky asset's current price is above the investor's tax basis, the new basis reflects the weighted average of the prior basis and the current acquisition price of any new shares acquired.

The objective of the investor's optimization problem is to maximize his discounted expected utility of lifetime consumption, including the utility of his bequest at death.[6] Expected utility is the weighted average utility reflecting the probability of living through the respective dates. The treatment of bequest is consistent with the reset provision of the U.S. tax code under which the capital gains tax is forgiven at death. At the beginning of each period, the investor allocates his wealth among consumption, the risk-free and risky assets, and the payment of capital gains taxes resulting from a sale of the risky asset. The investor is assumed to have a power utility function so that he possesses constant relative risk-averse preferences. (Under such a utility function, the investor will allocate the same percentage of wealth to the risky assets regardless of his wealth level.)

In solving the investor's optimization problem, we have simplified the model in order to maintain computational tractability. For a detailed discussion of these simplifications of the model, see Appendix.

## Base-case Parameters for Numerical Solutions

In the numerical analysis we assume that the investor makes annual decisions starting at age 20 and lives for up to another 80 years. The annual mortality rates are taken from the 2000 U.S. Life Tables for the total population.[7]

For our base-case scenario we assume that the risk-free (pre-tax) interest rate is 6% annually, the nominal annual dividend yield is assumed to be a constant 2%, and the annual inflation rate is 3.5%. The nominal capital gains return on the stock follows a binomial process with an annual mean return of 7% and standard deviation of 20%.[8] The tax rate on dividends and interest is 36%, while the tax rate on capital gains is set at 20%. The investor is assumed to have a risk aversion parameter of 3.0 in the power utility formulation with an annual discount factor of .96.[9]

## >>> FINDINGS

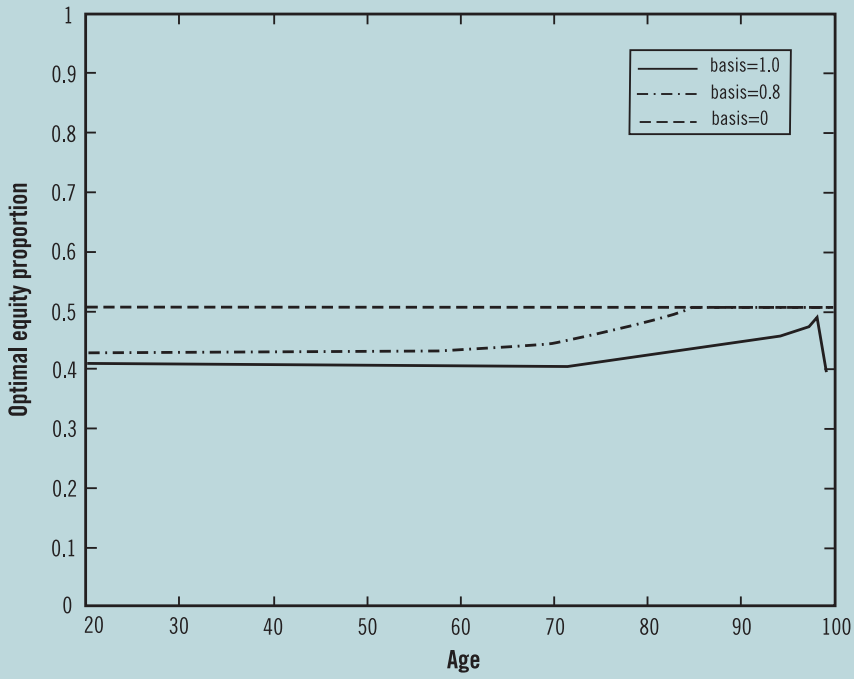
## The "Aging Effect"

Using our base-case parameters the optimal equity

### Consumption Decisions

Our setting can be used to solve for the investor's optimal consumption decisions. For a given level of total wealth, the larger the total embedded capital gain, the larger is the investor's implicit tax liability and the less wealthy is the investor on an after-tax basis. As suggested by options pricing theory, the optimal consumption-wealth ratio and the sensitivity of the value of the tax-timing option decline as the size of the investor's capital gain increases. The optimal consumption-wealth ratio also becomes less sensitive to the size of the gain for elderly investors due to the impending reset of the tax basis at death. However, for the purposes of this presentation we focus upon the investor's portfolio holdings rather than consumption decisions.

**Figure 1 Optimal Stock Holding As a Function of the Investor's Age**




Note: The basis-price ratio is set at 0, 0.8, and 1, respectively.
The initial stock holding as a fraction of beginning-of-period wealth is set at 50 percent.

holding in the presence of capital gains taxes is illustrated in Figure 1, which depicts the optimal stock proportion as a function of the investor's age at several different basis-price ratios, such as 1 (zero gain), 0.8 (25% gain) and 0 (entire position is gain). For our base-case parameters the investor's preferred equity allocation (i.e., when the basis-price ratio equals one) is approximately 41% if the investor is between 20 and 76 (it is higher for elderly investors). The figures are drawn for an assumed initial stock holding of 50% of the investor's beginning-of-period wealth, so that the investor is initially overexposed to equity.

An interesting feature of our solution is that the investor's optimal exposure to equity tends to increase with age, particularly at late ages. This is a result of the structure of capital gains taxation and more specifically the reset of the investor's tax basis at death. This reset at death increases the attractiveness of retaining highly appreciated positions, especially when the investor's life expectancy is relatively short. With a short horizon, the cost of not being fully rebalanced is small, while the tax benefit of deferral is high. In this sense the extent to which the investor scales back his position is influenced by the interaction between his age and the size of the capital gain.

Analogously, the investor will find it attractive to add relatively more equity to his portfolio as he ages (and his life expectancy falls) because of the option to realize losses, while retaining appreciated positions until the reset at death. This observation goes beyond the standard insight of estate planners that elderly investors may wish to retain highly appreciated positions to defer the capital gains tax liability (e.g., until death). In addition to addressing the investor's behavior once he possesses highly appreciated positions, our argument also points to the benefit of holding more equity to create the option of being able to defer the appreciated equity position (e.g., until the reset at death) and realize positions with capital losses. This is illustrated by the curve corresponding to the basis-

price ratio of one in Figure 1, which describes the investor's holdings (and purchase) of equity when the investor is not restricted by existing capital gains.[10,11]

The conventional advice on asset allocation is that an investor should reduce his equity exposure as he ages. In fact, some mutual fund organizations even promote the heuristic rule that an investor's percentage exposure to equity should be 100 – *A*, where *A* is the investor's age (in years). The underlying focus of this perspective concerns the shortening horizon over which the investor will utilize his funds as he ages and the absence of nonfinancial income during the retirement years as well as the declining value of his human capital wealth over time during the working years.

It is not apparent that models of risk bearing without taxes and other frictions should lead to strong age effects about portfolio composition during the retirement years. While the investor's age and the strength of his bequest motive will significantly influence the investor's consumption decision (i.e., the shape of his consumption path over time), the effect of age on the allocation decision will not be pronounced as long as the investor's risk aversion stays constant over time.[12] Of course, if the investor becomes more risk averse as he ages, then the relative demand for the risky assets would decline with the investor's age. Yet, many investors with substantial wealth view themselves as managing their funds at the margin for the benefit of their heirs, emphasizing both that risk aversion would not increase as they age as well as the importance of managing the capital gains tax liability efficiently.[13]

Even if the investor does not have a strong bequest motive, the investor can still find it attractive to borrow to help finance consumption in his latter years in order to defer some of the capital gains liability until it is eliminated at death (repaying the indebtedness through his estate). More generally, our analysis highlights the value to the investor of borrowing in his latter years to obtain liquidity, while deferring the realization of substantial appreciated positions until the investor's death. While our analysis so far abstracts from the stochastic structure of labor income, this simplification does not affect behavior during the investor's retirement years, i.e., after he has ceased earning significant labor income.

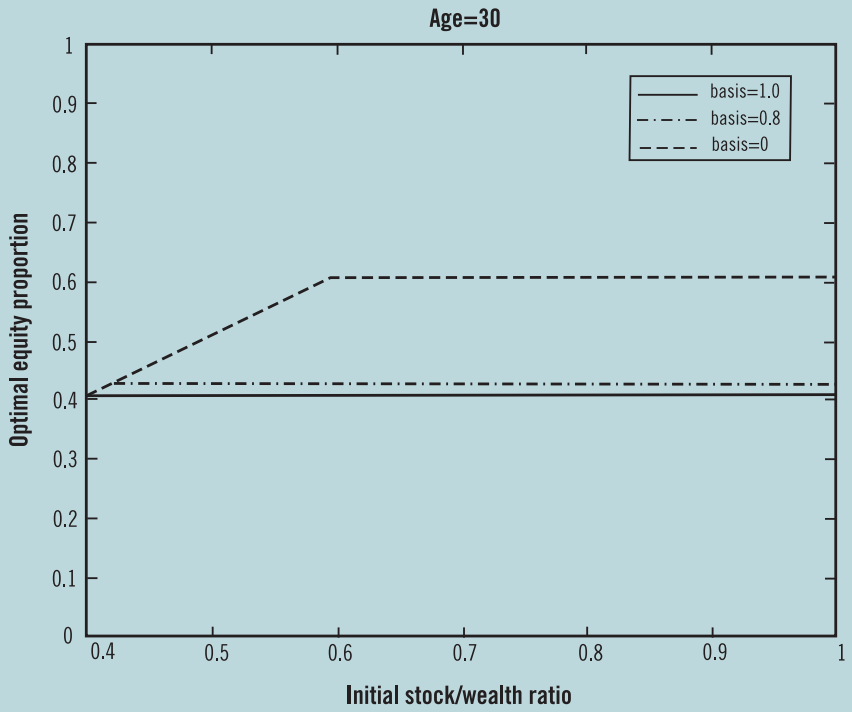
## The Size of the Gain and Portfolio Rebalancing

Our solutions illustrate the important role of the size of the existing capital gain for the investor's portfolio rebalancing decision. If the investor is overexposed to equity, the investor will trade off the tax cost of selling some equity with the diversification benefit of the reduced exposure to the risky asset. The smaller the size of the gain, the smaller the tax costs of scaling back the equity exposure by a given amount. Consequently, when the size of the gain is small, the investor optimizes this trade-off by scaling back equity holding to a greater degree and getting closer to the unconstrained optimal exposure.[14] Analogously, it will also be optimal for the investor to scale back the exposure to a greater degree when the capital gains tax rate is particularly low. In contrast, when the size of the gain is very large (or the tax basis is very small), the investor may just scale back slightly or even retain the entire exposure (the investor acts as "locked in" and does not sell his position due to the tax cost of selling despite the risk-sharing benefits from reducing the equity exposure).

For a given size capital gain (i.e., fixing the basis-price ratio), the marginal cost to the investor of being overexposed to equity increases in the difference between the size of his position and the optimal position, while the marginal tax cost of rebalancing is constant in the position's size. Consequently, once the investor is substantially overexposed to equity, the trade-off he faces with capital gains taxes pushes him to the same exposure, i.e., the portfolio will not depend upon his previous holding of equity, though it depends upon the size of the investor's gain and age.

Many of these features are illustrated in Figure 2, which depicts the optimal stock holding at age 30 for our base-case parameters as a function of the initial proportion of equity in the portfolio for several different values of the basis-price ratio. The figure illustrates that the size of the investor's optimal equity holding increases with the fraction of beginning-of-period wealth invested in equity until the investor reaches his maximum exposure for each basis-price ratio. The optimal equity proportion also increases with the size of the gain (or lower basis-price ratio) in the figure as the investor scales back his exposure relatively less as the gain rises.

**Figure 2 Optimal Stock Holding As a Function of the Initial Stock Holdings**




Note: The basis-price ratio is set at 0, 0.8, and 1, respectively.

## Welfare Comparisons

The investor significantly enhances the utility value of his portfolio by efficiently managing his tax opportunities. We considered the welfare costs of not following the optimal policy. Specifically, we considered the welfare costs of the following inefficient alternatives: (a) realizing all capital gains and losses each period, or (b) following the buy-and-hold policy of never realizing capital gains or losses other than to finance consumption.[15] The welfare costs reflect how much additional wealth we need to provide the investor following an alternative realization policy so that he would be as well off as under the optimal policy. We can solve numerically for the optimal decision rules for each of these alternative realization policies and compare the investor's welfare.

The first alternative focuses upon the advantages of realizing losses without allowing the investor to defer gains. The welfare costs in this case increase in the size of the existing capital gain (since this alternative involves selling assets with embedded gains) and decrease in the investor's age (due to the shortening horizon). The welfare costs of the second alternative (the buy-and-hold policy) decrease in the investor's age (again due to the shortening horizon), but now increase in the size of the existing capital loss.

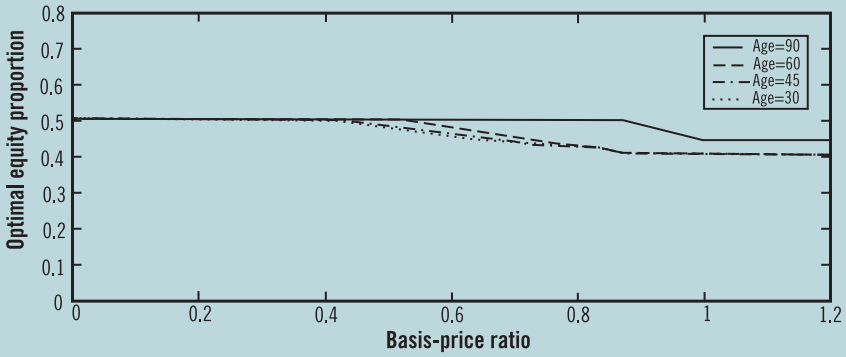
## Comparative Static Analysis

We examine the impact on our policy rules of varying the capital gains tax rate and the volatility of stock returns. Figure 3 illustrates the optimal equity exposure for our base-case parameters (upper panel), for a tax rate of 36% on capital gains and losses (middle panel) and for a standard deviation of equity returns of 30% (lower panel).[16] These figures are constructed assuming an initial equity proportion of 50 percent and holding all the other parameters at their base-case values.

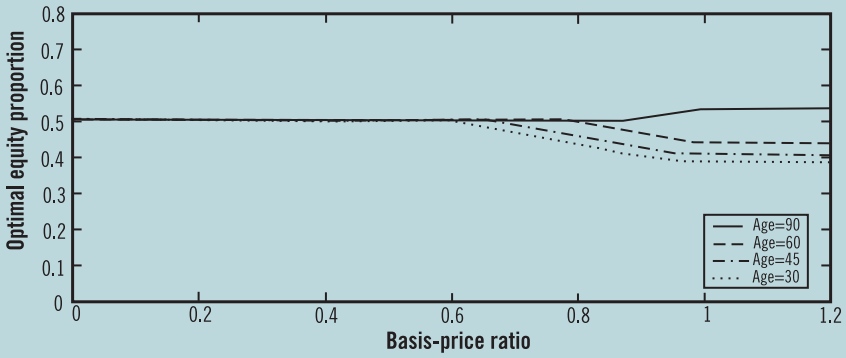
An increase in the capital gains tax rate from the base-case 20% to 36% (middle panel) reduces the size of

## Figure 3 Optimal Stock Holding As a Function of the Basis-price Ratio: Three Scenarios

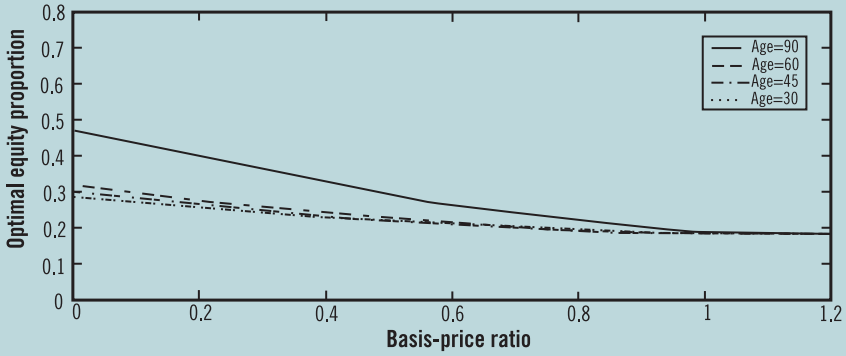
PANEL A. BASELINE CASE




PANEL B. CAPITAL GAINS TAX IS RAISED FROM 20 PERCENT TO 36 PERCENT




PANEL C. STOCK RETURN VOLATILITY IS RAISED FROM 20 PERCENT TO 30 PERCENT




Note:
1) The investor's age is set at 30, 45, 60, and 90, respectively.
2) The initial stock holding as a function of beginning-of-period wealth is set at 50 percent for each panel.

the capital gain that the investor is willing to realize, especially for young investors who derive the greatest benefit from rebalancing.[17] In addition, in the breakeven and loss region the investor's holding of equity is more sensitive to the investor's age when the capital gains tax rate is higher.

An increase in the volatility of the stock from the base-case 20% to 30% (lower panel) has a dramatic impact on the investor's equity holdings. Not surprisingly, the increase in riskiness of equity reduces the optimal holdings for all investors depicted in the figure despite the increase in the value of the tax-timing option. However, in some circumstances elderly investors with large gains can retain their equity exposure to avoid the capital gains tax at death. The benefits of rebalancing are greater in a more volatile environment so that young and middle-aged investors will cut their exposure to equity, even when their embedded gains are large. The findings associated with an increase in volatility are similar to those that would arise from increasing the investor's risk aversion.

## Mandatory Capital Gains Taxation at Death

An interesting benchmark for comparison of our asset allocation results is the treatment of capital gains at death in Canada. Under the Canadian law the ability to defer the capital gains tax ends at death. Unrealized capital gains are automatically taxed in Canada at death rather than benefiting from either (a) a step-up in the investor's tax basis to eliminate the capital gains tax liability, or (b) allowing the investor to defer the tax payment by delaying the sale of the equity and retaining the prior tax basis.

Using our base-case parameters, an initial equity proportion of 50 percent of the investor's beginning-of-period wealth, and full taxation of capital gains at death, we illustrate in Figure 4 the optimal exposure to equity under the Canadian law. The upper panel of Figure 4 depicts the optimal stock proportion as a function of the investor's age at several different basis-price ratios, such as 1 (zero gain), 0.8 (25% gain) and 0 (entire position is gain). Notice that the optimal equity exposure is now relatively insensitive to the investor's age because there is no opportunity to escape the capital gains tax at death and so the tax benefit to deferral of capital gains largely reflects the time value of money and is nearly constant over time. This figure strongly suggests that the aging effects in portfolio holdings in our basic analysis are driven by the reset provision at death rather than other features of our setting such as the specific details of the bequest motive.

We illustrate in the lower panel of Figure 4 the impact of the size of the gain upon the investor's rebalancing decision by plotting the optimal exposure to equity as a function of the basis-price ratio for investors at ages 35 and 90 in our modified setting without the reset at death. This highlights the nature of the rebalancing decision across tax bases and the tradeoff between the tax costs and risk-sharing benefits, even absent the potential for eventual reset of the tax basis at death.

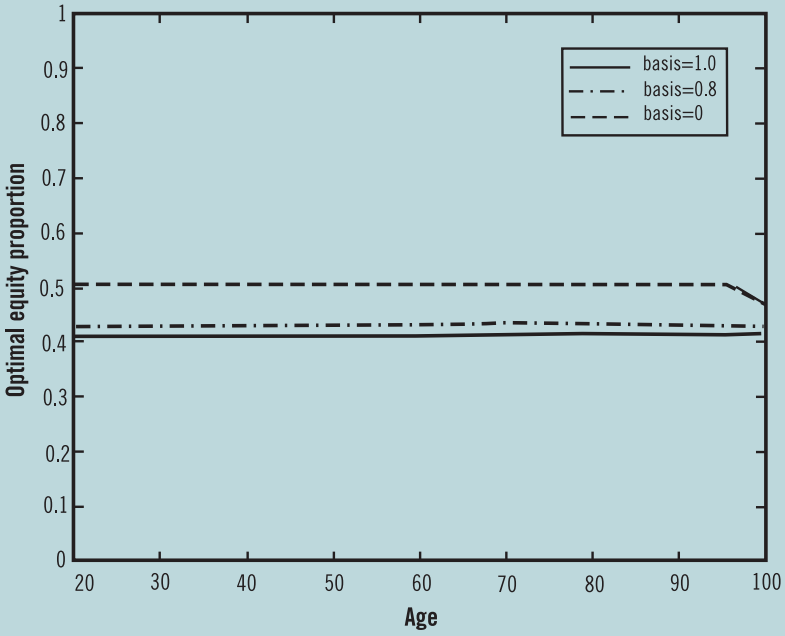
## Labor Income

While an important component of an investor's wealth and risk is determined by human capital, our earlier analysis did not incorporate labor (non-financial) income. Though ideally one would like to model labor income with its own stochastic specification, we introduce the investor's labor income as proportional to his wealth so as to avoid the need to increase the dimensionality of our model. This allows us to capture several fundamental aspects of the impact of non-financial income on asset allocation over the life cycle in the presence of taxes. For example, investors younger than age 65 (when we assume that the labor income terminates) substantially increase their optimal holding of equity relative to the case without labor income. This is in response to the increase in the investor's effective wealth and the modest risk of non-financial income. Consequently, human capital and the reset provision have the opposite effects on the optimal equity exposure as a function of the investor's age. Of course, the human capital effect is typically present only when the impact of the reset benefit provision is weakest (i.e., at young ages).

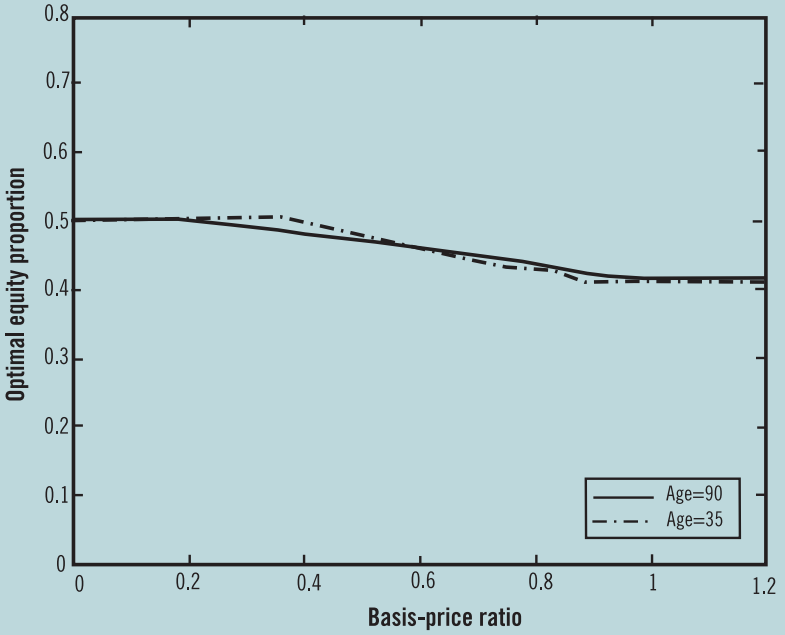
An important aspect of non-financial income is that by enhancing the investor's income stream only in his working years there is a greater flow of new savings. This greater flow of new savings helps limit the extent to which the investor becomes locked in and allows the investor to rebalance his portfolio without payment of capital gains taxes by altering the portfolio allocation of new investments. In practice,

## Figure 4 Optimal Stock Holding Under Mandatory Capital Gains Taxation At Death

PANEL A. OPTIMAL STOCK HOLDING AS A FUNCTION OF THE INVESTOR'S AGE




PANEL B. OPTIMAL STOCK HOLDING AS A FUNCTION OF THE BASIS-PRICE RATIO AT AGE 35 AND 90, RESPECTIVELY




Note:The initial stock holding as a fraction of beginning-of-period wealth is set at 50 percent for both panels.

incremental savings over time creates the ability to adjust the structure of one's portfolio.

## >>> CONCLUDING COMMENTS

Our framework provides a tractable way to capture the trade-off between the payment of capital gains taxes and portfolio rebalancing. We demonstrate how an investor's optimal portfolio holdings depend upon the investor's age, existing capital gain and portfolio holdings. For an investor with embedded capital gains, the incentive to rebalance his portfolio by selling stock is inversely related to his age and the size of the gain. Elderly investors have a strong incentive to defer the realization of existing capital gains and to increase their ownership of equity as they age due to the forgiveness of capital gains taxes at death under the current U.S. tax code.

In addition to the substantive qualitative insights that emerge from our analysis, our study also makes a methodological contribution. Our work provides a tractable way to incorporate a realistic treatment of capital gains taxes in the analysis of asset allocation. Most directly, we have extended the analysis to incorporate two risky securities in Dammon, Spatt and Zhang (2001b).[18]

Building upon our framework we also have introduced in Dammon, Spatt and Zhang (2002a) the opportunity for tax-deferred investing into an integrated asset allocation setting.[19] Our work emphasizes the value of optimal asset *location* (which assets to hold in which account) in addition to optimal asset allocation. The implications of our framework for estate planning issues, including the titling of securities by a married couple and the location of exposures between trusts and personal assets, are explored in Dammon, Spatt and Zhang (2002b).

## >>> ACKNOWLEDGEMENTS

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## >>> APPENDIX

The inherent difficulty in solving the underlying problem is that the investor's decisions depend upon the investor's existing portfolio holdings, specifically his holdings of the risky asset and its respective tax basis. In this sense the investor's underlying expected utility maximization problem is fundamentally dynamic. We can recast this model recursively as a dynamic programming problem. While numerical methods can be used to solve such dynamic programming problems, it is crucial to the tractability of these problems that there are only a small number of underlying "state variables," upon which the optimal decision rules are functions.[20]

We identify several simplifications to help limit the number of state variables, so that the problem is tractable numerically. An important additional advantage of these simplifications is that they highlight the variables that are central to understanding the solutions, thereby focusing the reader upon the fundamental aspects of the problem.

In the underlying problem the state variables include the tax basis (and distinct purchase price) for the risky asset and the corresponding size of that position as well as the investor's wealth. To simplify the problem and highlight the key underlying intuitions, we restrict attention to a problem with a *single* risky asset and a risk-free asset.[21]

The *average basis* can be updated using the investor's prior average basis so that assuming an average basis rule greatly limits the dimensionality of the underlying state space and helps makes it tractable to obtain numerical solutions. Indeed, mutual funds report to investors the basis of their holdings using the average basis rule, which is a heuristic used by many investors. In Canada the actual tax liability is determined by an average basis rule. While this averaging rule is not optimal for the United States tax code (e.g., with constant tax rates over time it is optimal for the investor to realize the highest basis positions first), most of the qualitative features that emerge from our solutions reflect robust features of the solution to the investor's optimization problem. In addition, DeMiguel and Uppal (2002) present numerical results suggesting that the value of the investor's optimal solution is not substantially larger under the optimal policy than when using the averaging rule.

Finally, specializing the investor's preferences to constant relative risk aversion (power utility) ensures that the resulting demands for the risky (and risk-free) asset as well as consumption are proportional to wealth. Consequently, we can treat the beginning-of-period wealth as the numeraire to normalize wealth out of the consumption and portfolio choice problems and solve for the fraction of the investor's wealth that is allocated to current consumption, the risky asset and the risk-free asset. Imposing the overall budget constraint, this leads to two independent choice variables. The optimal asset allocation (the proportion of assets allocated to the risky security) and consumption decisions are functions of the investor's current asset allocation, average tax basis and age. As a result, the problem can be expressed with two continuous state variables as well as age.

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## ENDNOTES

[1] The magnitude of direct trading costs has declined substantially in recent years as reflected by the huge drop in commissions and the dramatic reduction in tick size brought about by decimalization. In most situations facing taxable investors the major component of trading costs is capital gains taxes.

[2] This article examines investing in the investor's taxable account, but does not incorporate tax-deferred investing. Tax-deferred investing in a setting with capital gains taxation in the taxable account is examined in Dammon, Spatt and Zhang (2002a) and briefly referenced in the concluding comments of this article.

[3] The effect of taxing capital gains upon asset realizations was first explored in Constantinides (1983, 1984). Both these papers focus upon the value of harvesting capital losses. Constantinides (1984) also addresses the optimality of harvesting gains at preferential (long-term) capital gains tax rates under some circumstances.

[4] Our assumption of symmetric treatment of short-term and long-term realizations is particularly natural in light of the interpretation of a period being one year when we parameterize and implement our model. An interesting dynamic analysis of optimal gain and loss harvesting in the presence of asymmetric treatment of short-term and long-term realizations is given in Dammon and Spatt (1996), though they do not address the intertemporal portfolio problem of a risk-averse investor. The value of realizing long-term gains to create the option of realizing future short-term losses was first explored in Constantinides (1984).

[5] Throughout our analysis we assume that there are no transaction costs and no wash sale restrictions on realizing losses (in practice, investors must defer the realization of the tax loss if they repurchase the same position within 30 days of realizing the loss). For example, wash sale rules are not very restrictive in practice given the existence of close substitute securities that can be repurchased immediately after a sale of the security in question without triggering wash-sale treatment. Consequently, it is optimal in our model for the investor to sell his entire position when he has a loss and then repurchase the optimal risky exposure.

[6] The bequest function at death is based upon the investor providing his beneficiary with a uniform real consumption stream over $H$ periods.

[7] While we are using an *aggregate* mortality curve for illustration, a particular investor should use his own life expectancy based upon gender and health information.

[8] The resulting risk premium is (1.07)(1.02)-1.06 = .0314 = 3.14%, which is well below the historical average. This reflects our desire that the investor optimally owns bonds as well as equity for reasonable levels of risk aversion. Our choice of risk premium is consistent with the perspectives of many financial economists about anticipated future risk premium, as illustrated by the analysis of Fama and French (2002).

[9] These base-case parameters are the same as in the base-case in Dammon, Spatt and Zhang (2001a), except that we have replaced the mortality schedule, the capital gains tax rate here is 20% rather than 36%, and the standard deviation is 20% rather than 20.7%.

[10] The curve corresponding to the basis-price ratio of one does not continue to rise at age 99 in Figure 1 because we do not allow the investor to realize losses at the time of death.

[11] Our analysis understates the value of holding equity in several respects. Because we focus upon a single security and treat its basis as the average acquisition cost, the investor does not have the ability to treat the tax basis of new shares as the acquisition price. Instead, the option value of these positions is limited because they are being blended with the investor's previously acquired appreciated positions. If the investor's prior appreciation is substantial, then our solutions do not illustrate fully the benefit that arises from newly acquired positions. However, if the basis-price ratio equals one then the increasing proportion of equity in the investor's portfolio as he gets older illustrates to a

greater degree the underlying tax benefit of increasing his exposure to equity.

[12] A simple example of this point is illustrated by the case without taxes in which the investor has an additive separable log utility function over his consumption each period as well as the annuitized amount of his real bequest. In each period the investor's allocates his portfolio to maximize the log of his one-period ahead wealth.

[13] Poterba (2001) documents that elderly investors with low embedded capital gains have a greater propensity to gift assets to their heirs during their lifetimes as compared to investors with substantial capital gains and equivalent wealth, reflecting the interest of investors with large capital gains in utilizing to a greater degree the reset of the tax bases at death.

[14] Our model also can produce situations in which the investor's optimal future proportion of equity is increasing in the size of his earlier capital gain when the investor initially owns less equity than his optimal level. When the investor is *underexposed* in his equity holding a larger gain can reduce the amount of additional equity purchased by the investor because the basis of his newly acquired shares will reflect the averaging with existing shares with substantial appreciation. This situation is illustrated by numerical solutions in Dammon, Spatt and Zhang (2001a). We do not emphasize this conclusion in the presentation here because it reflects an artifact of the average basis rule that we use to facilitate numerical solutions rather than a general feature of the optimal capital gains problem with risk-averse investors.

[15] An alternative context in which the welfare consequences of the capital gains tax is explored is in Chari, Golosov, and Tsyvinski (2002), who suggest that there are substantial distortionary consequences of capital gains taxation for entrepreneurial activity.

[16] Our base-case standard deviation is representative of the annual volatility of the market index, while increasing the standard deviation to 30% makes the standard deviation representative of that of individual stocks.

[17] At age 90 the investor actually adds equity when the capital gains tax rate is 36% due to the extent of the benefit of the reset provision at death for this tax rate.

[18] The introduction of each additional risky asset requires two additional continuous state variables (one for the asset's proportion and another for its basis). Related analyses of the asset allocation problem with two risky assets that utilize the framework developed in Dammon, Spatt and Zhang (2001a) are Gallmeyer, Kaniel and Tompaidis (2001) and Garlappi, Naik and Slive (2001). As discussed in the Appendix, the *curse of dimensionality* impedes greatly the introduction of additional state variables (and assets).

[19] Because the reset of the investor's tax basis at death does not arise within the tax-deferred account, the aging effects we emphasize here do not arise within that account.

[20] Since we want our numerical solution to be relatively smooth, we need to solve the underlying problem on a relatively dense grid. The resulting computational demands then expand exponentially with the number of state variables and our formulation suffers from the *curse of dimensionality*.

[21] Even using a risk-free asset and a single risky asset whose return follows a binomial process, Dybvig and Koo (1996) were able to solve the full-blown optimization problem for a risk-averse investor using the entire distribution of bases only for horizons up to four periods.

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