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Deriving the Risk Adjusted User Cost for the CFS US Divisia Monetary Aggregates

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Deriving the Risk Adjusted User Costs for the CFS US Divisia Monetary Aggregate

Ryan S Mattson

The Center for Financial Stability and West Texas A&M University

07/24/2015
Introduction

Goals:
1. Briefly define the current Divisia aggregates used at the CFS,
2. Describe the methodology and theory of risk adjustment,
3. Demonstrate the current results and paths for improvement.

How do we include risk in the construction of monetary aggregates?
User Cost

\[ \pi_{i,j} = \frac{r_i^B - r_{i,j}}{1 + r_i^B} \]  

(1)

Expenditure Share Weight

\[ s_{i,j} = \frac{\pi_{i,j} m_{i,j}}{\sum_{j=1}^{K}} \]  

(2)

Divisia Index (Tornqvist Theil Approximation)

\[ \log(M_i) - \log(M_{i-1}) = \sum_{j=1}^{K} \bar{s}_{i,j}(\log(m_{i,j}) - \log(m_{i-1,j})) \]  

(3)
The Benchmark Rate

\[ \pi_{i,j} = \frac{r_i^B - r_{i,j}}{1 + r_i^B} \] (4)

- Non-liquid asset,
- for DM4 determined by average bank loan rate\(^1\).

\(^1\)Offenbacher and Schachar (2011)
Derivation

\[ DM4 = M_i = M_{i-1} \prod_{j}^{K} \left( \frac{m_{i,j}}{m_{i-1,j}} \right)^{\bar{s}_{i,j}} \] (5)

- Based on Tornqvist-Theil approximation
- \( \bar{s}_{i,j} = \frac{s_{i,j} + s_{i-1,j}}{2} \)
- Seasonally adjusted \( m_{i,j} \)
- Then take Year over year growth rate.
The CFS Data

Broad aggregates constructed using:

- Large denomination time deposits,
- Overnight repurchase agreements,
- Commercial paper,
- Short term (3-month) treasury bills, and
- the assets included in M2.

Divided into DM3, DM4-, and DM4.
### Figure 6 - Components of CFS Divisia M4 - Highlights for May 2015

Components in this table build from the narrowest aggregate at the top to the broadest at the bottom. For example, the darkest green in the left column includes components of DM1. The next tier shows the components included in DM2, but not within DM1.

<table>
<thead>
<tr>
<th>Components</th>
<th>Weighted Contribution To DM4 Money Annual Growth Rate*</th>
<th>Divisia Growth-Rate Weights (Average of Last 12 Months)</th>
<th>Unweighted Year-Over-Year Percentage Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DM1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Currency</td>
<td>0.5%</td>
<td>6.8%</td>
<td>6.9%</td>
</tr>
<tr>
<td>Traveler's Checks</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-15.2%</td>
</tr>
<tr>
<td>Demand Deposits **</td>
<td>1.0%</td>
<td>10.4%</td>
<td>9.8%</td>
</tr>
<tr>
<td>OCDs at Commercial Banks **</td>
<td>0.1%</td>
<td>2.4%</td>
<td>5.5%</td>
</tr>
<tr>
<td>OCDs at Thrift Institutions **</td>
<td>0.2%</td>
<td>1.9%</td>
<td>8.6%</td>
</tr>
<tr>
<td><strong>Added into DM2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savings Deposits at Commercial Banks **</td>
<td>2.0%</td>
<td>29.5%</td>
<td>7.1%</td>
</tr>
<tr>
<td>Savings Deposits at Thrift Institutions **</td>
<td>0.2%</td>
<td>4.7%</td>
<td>4.7%</td>
</tr>
<tr>
<td>Retail Money-Market Funds</td>
<td>-0.1%</td>
<td>3.4%</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Small Time Deposits at Commercial Banks</td>
<td>-0.3%</td>
<td>2.0%</td>
<td>-12.7%</td>
</tr>
<tr>
<td>Small Time Deposits at Thrifts</td>
<td>-0.1%</td>
<td>0.6%</td>
<td>-10.2%</td>
</tr>
<tr>
<td><strong>Added into DM3</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Institutional Money-Market Funds</td>
<td>0.3%</td>
<td>9.7%</td>
<td>3.0%</td>
</tr>
<tr>
<td>Large Time Deposits</td>
<td>0.1%</td>
<td>8.6%</td>
<td>1.7%</td>
</tr>
<tr>
<td>Repurchase Agreements</td>
<td>-0.8%</td>
<td>7.1%</td>
<td>-10.4%</td>
</tr>
<tr>
<td><strong>Added into DM4</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial Paper</td>
<td>-0.3%</td>
<td>5.1%</td>
<td>-6.6%</td>
</tr>
<tr>
<td>T-Bills</td>
<td>0.0%</td>
<td>7.8%</td>
<td>-0.3%</td>
</tr>
</tbody>
</table>

Note: The row labels in the first column are shaded to show which components are included into which aggregates. Each data column is shaded on a gradient from dark green (maximum) to dark red (minimum). See the figures 10-15 for these calculations over time and a chart version of the above table.

* Average of last 12 monthly weighted contributions to CFS Divisia M4 growth rates (annual rates). Unlike the other percent calculations in this report, the weighted contributions use continuous percent changes computed from natural logs.

** Sweeps adjusted.
The User Cost Risk Literature

The User Costs are risk neutral.

- Barnett and Wu (2005) - CAPM with intertemporal nonseparability.

Representative Utility Function

Maximize

\[ E_t \left[ \sum_{s=0}^{\infty} \alpha^s U(m_{t+s}, c_{t+s}, c(t+s-1), \ldots, c(t+s-n)) \right] \]  

\[ W_t = p_t^* c_t + \sum_{i=1}^{M} p_t^* m_{i,t} + \sum_{j=1}^{K} p_t^* k_{j,t} = p_t^* c_t + p_t^* A_t \]  

\[ W_{t+1} = \sum_{i=1}^{M} R_{i,(t+1)} p_t^* m_{i,t} + \sum_{j=1}^{K} \tilde{R}_{(t+1),j} p_t^* k_{j,t} + Y_{t+1} \]  

\[ \lim_{s \to \infty} \alpha^s p_t^* A_{t+s} = 0 \]
Contemporaneous User Cost

\[ \pi_{i,t} = \frac{\partial U_t / \partial m_{i,t}}{\lambda_t} \]  \hspace{1cm} (10)

\[ \lambda_t = E_t \left[ \sum_{l=0}^{n} \beta^l \frac{\partial U(t+l)}{\partial c_t} \right] \]  \hspace{1cm} (11)

Price kernel marginal utility growth of consumption:

\[ Q_{(t+1)} = \beta \left( \frac{\lambda_{(t+1)}}{\lambda_t} \right) > 0 \]  \hspace{1cm} (12)
The Risk Adjusted User Cost BW (2005)

IF there is an arbitrary locally risk free non-money asset with $r^f_t$, if the pricing kernel is linearly approximated with the real return on wealth portfolio $A_t$, $r_{A,t+1}$, such that $Q_{t+1} = a_t - b_t r_{A,t+1}$, and if user costs and certainty equivalent user costs of the monetary assets and wealth portfolios are $\pi_{i,t}$, $\pi^e_t = \frac{r^f_t - E_t[r_{i,t+1}]}{r^f_t}$, $\Pi_{A,t}$, and $\Pi^e_t = \frac{r^f_t - E_t[r_{A,t+1}]}{r^f_t}$, then for $\beta_{i,t} = \frac{Cov_t(r_{A,t+1}, r_{l,t+1})}{Var_t(r_{A,t+1})}$, the risk adjusted user cost of $i$ at $t$ is:

$$\pi_{i,t} - \pi^e_{i,t} = \beta_{i,t}(\Pi_{A,t} - \Pi^e_{A,t})$$  \hspace{1cm} (13)

So we approximate:

$$\hat{\pi}_{i,t} = \pi^e_{i,t} + \beta_{i,t}(\hat{\Pi}_{A,t} - \Pi^e_{A,t})$$  \hspace{1cm} (14)
The Approximation Method.

Problem: Need the covariance between the kernel and asset return, but the kernel is not observable.

Proposed linear approximation:

\[ Q_{(t+1)} = a(z_t) - b(z_t)r_{A,(t+1)} \]  \hspace{1cm} (15)

Conditioning variables \( z_t = [1 \ \Delta c_t] \).

Used regression estimate \( E_t[Q_{t+1}] = \frac{1}{r_t} = a(z_t) - b(z_t)r_{A,t+1} \)
Estimating Portfolio User Cost, Risk Premium

With Conditional CAPM estimates of $b(z_t)$, the $\Pi_{A,t}$ is estimated:

$$\Pi_{A,t} = \Pi_{A,t}^e + b(z_t) \text{Var}_t(r_{A,t+1})$$  \hspace{1cm} (16)

Can also get the covariance of the pricing kernel:

$$\text{Cov}_t(Q_{t+1}, r_{A,t+1}) = -b(z_t) \text{Var}_t(r_{A,t+1})$$  \hspace{1cm} (17)

And completes the estimation: $\beta_{i,t} = \frac{\text{Cov}_t(r_{(A,(t+1)),i,(t+1)})}{\text{Var}_t(r_{A,(t+1)})}$ risk premium.
Introduction
The CFS Divisia Aggregates
Risk Adjusted User Costs
User Costs and Aggregates

Risk Premium

CURR DD OCDC SAVC STDC IMMF TB CP LTD RP

Mattson (CFS) CFS Risk Adjustment, SEM 2015 15 / 22
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The CFS Divisia Aggregates

Risk Adjusted User Costs

User Costs and Aggregates
### Introduction
The CFS Divisia Aggregates

### Risk Adjusted User Costs

#### User Costs and Aggregates

<table>
<thead>
<tr>
<th>Date</th>
<th>RA RMF</th>
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<tr>
<td>1/1/1992</td>
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<td>7/1/1992</td>
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<td>7/1/1996</td>
<td>0.09</td>
</tr>
<tr>
<td>1/1/1997</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Retail Money Market Funds**

![Graph of Retail Money Market Funds](image)

- **RA RMF**: Blue line
- **RMF**: Red line

*Source: Mattson (CFS) CFS Risk Adjustment, SEM 2015*
Introduction

The CFS Divisia Aggregates

Risk Adjusted User Costs

User Costs and Aggregates

Savings and Institutional Money Market Mutual Funds

(Risk Adjusted) User Cost

Savings and Institutional Money Market Mutual Funds

RA SAVC

RA IMMF

Mattson (CFS)
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User Costs and Aggregates

Overnight and Continuing Repurchase Agreements
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User Costs and Aggregates

Risk Adjusted Repo and Small Time Deposits (Commercial Banks)
CFS Risk Free DM4 and Risk Adjusted DM4

% Change From One Year Ago
CFS Risk Free DM4 and Risk Adjusted DM4
Recession Risk Adj. DM4 CFS DM4

Recession
Risk Adj. DM4
CFS DM4
Conclusions

By the CAPM estimation teh risk adjusted DM4 tracks the risk free fairly well.

User costs face large changes; reflect movement to security.

Potential for other approximation techniques.