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A stochastic model for PLC systems

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by

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ABSTRACT

The designers of PLC (power line carrier) systems for distribution networks must contend with two sources of uncertainty. First, certain topological parameters, such as transformer locations, cannot be determined accurately. Second, certain other network parameters, such as loads, equipment characteristics and circuit configurations, vary with time. A previous paper [2] has shown that signal attenuations and error rates of PLC systems are very sensitive to these uncertainties. Therefore, it is critical that they be taken Into account in the design process. We develop stochastic models to aid In doing this. Specifically, the models predict the statistics'of signal attenuations and error rates In arbitrary PLC systems. This is done In two stages. First, a quadratic approximation to the network's propagation characteristics is developed. Second, this approximation and Monte Carlo sampling are used to obtain the requisite statistics.

An example is used to point out that conventional deterministic models can grevlously underestimate error rates and thus, to point out the need for stochastic models of the type described In the paper.

INTRODUCTION

Load Management and Distribution Automation schemes usually require two-way communications between a centralized control facility and a large set of points dispersed over a distribution network. PLC (Power Line Carrier) would appear to offer several advantages as a coamunication medium [1], [4], [6]. However, its performance to date has been disappointing. It is not uncommon for PLC systems under demonstration to completely fail In their attempts to reach some points and to have high error rates with others. By and large, they seem to under per form their designers' expectations. The most likely cause is the widely used assumption that power networks provide a deterministic environment for high frequency signals. There is some merit to this assumption when one is dealing with transmission networks. Distribution networks, however, are quite different. Their high frequency (over 1 kHz) behavior is subject to several uncertainties. In a previous paper [2] we discussed the nature of these uncertainties in some detail. Here we will only review their more important features.

Uncertainties

The uncertainties may be divided into two main categories: short term and long term.

In the short term (the period over which no Important pieces of equipment are upgraded or replaced) one must contend with two subcategories, namely:

(a) anatomical uncertainties which arise because it is Impractical and sometimes impossible, to precisely determine the values of all the relevant feeder parameters. Lengths of line segments, the locations of transformers and their electrical characteristics, etc. can only be approximated unless heroic data gathering measures are undertaken.

(b) temporal uncertainties which arise because certain quantities, such as noise levels and loads, vary with the time-of-day, week and season. For Instance, distribution transformers have resonances that are affected by their secondary loading. As this load varies, there can be profound changes in the Impedances the transformers present to carrier signals [2].

The result is that large numbers of the parameters used In modeling efforts must be treated as random variables. This in turn makes the outputs of the models random variables, as is illustrated In Fig. 1.

The long term Impacts are of the same sort but much more pronounced since one must contend not only with, the short term uncertainties but also with those associated with changes in feeder structure and equipment. Unless a PLC system is designed to weather these evolutionary changes its useful life expectancy will be short.

The Need for Stochastic Models

We conclude from the last section that the attributes used to measure PLC system performance will be random variables because of "temporal uncertainties" and that estimates of these attributes obtained from models will be additionally random because of "anatomical uncertainties." Therefore, minimum acceptable performance levels must be specified In statistical terms. An example of such a specification is: the expected number of error free messages received should be at least 90Z of those sent. To accommodate such specifications we need stochastic nodels able to estimate the moments and other statistical properties of the performance attributes. More specifically, the essence of a design procedure to accommodate statistically corrected performance criteria is as follows.

(a) Identify the uncertain parameters and determine the ranges over which they can vary as well as their joint probability distributions over these ranges.

(b) Select a nominal PLC design and evaluate its performance statistics using stochastic models.

(c) From the results Identify the points which are unsuitable for PLC communication, either because they are exceedingly sensitive to the uncertainties or because the signal levels reaching them are too low. Arrange for these points to be reached by other means like telephone lines.

(d) Using the stochastic models interactively, adjust the PLC system's decision variables so that the remaining points perform adequately. (Some typical decision variables are carrier frequency, repeater locations, receiver sensitivities and the main transmitter locations.)

The last step of this design procedure calls for some comment. Some design problems require formal optimization methods to find good values for their decision variables. However, this does not seem to be the case for PLC systems. It is easy to devise measures to raise or lower their performance attributes. Therefore, all that is needed to Home in on a good PLC design for a given distribution network is a stochastic model of the network and its communication equipment together with a designer who can address Intelligent "what if" questions to the model.

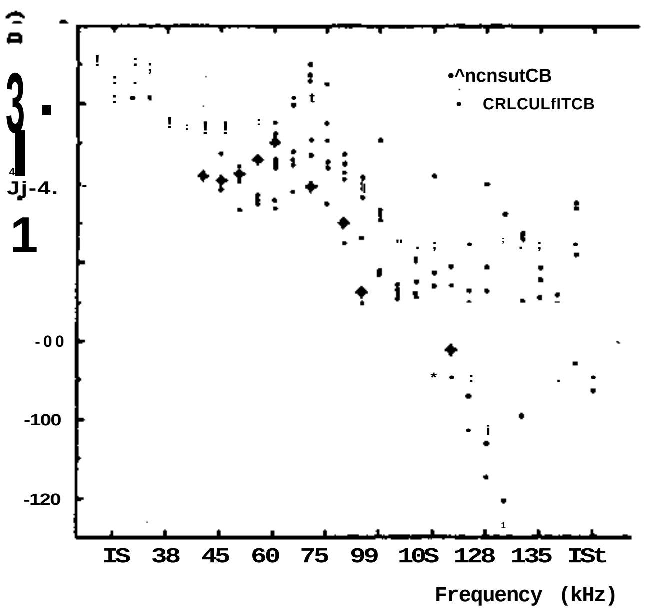

Fig. 1. Some measured and calculated attenuations for a representative feeder [2]. The measurements were made at one time. Different values were observed at other times but were not recorded. The calculated attenuations were obtained by using standard models with the values of uncertain parameters randomly selected from ranges typical of short term uncertainties. These parameters were assumed to be uniformly distributed over their ranges. Notice that the scatter of the calculated attenuations is fairly wide indicating that the attenuations are quite sensitive to parameter uncertainties.

Intent

When we speak of PLC performance here, we are thinking of whether receivers will be able to correctly Interpret the messages sent to them. Crucial factors are the signal and noise levels at each receiver's terminals. The signal levels can be determined using network theory. Noise, however, is best handled by more empirical means - tables and nomograms of noise data collected for representative feeders or measured on site [3]. We will assume that the signal and noise models can be separated and that their results can be added to give the signal-noise mix at a receiver's terminal.

We will not deal with the noise models here. Rather, we will concentrate on developing stochastic models for signal attenuation.

COMPONENT MODELS

Transmitters in PLC systems are connected either between two line conductors or between a conductor and ground. Their signals must find their way through and around large numbers of feeder components like line segments, capacitor banks and transformers, before they reach the receivers. The attenuations suffered by the signals can be calculated using lumped parameter, steady state models for the components. The forms of these models are well known [5], [7], [8] Less well known are the uncertainties in some of the model parameters. We will illustrate these uncertainties with the examples below.

Line Segments

A line segment (a stretch of line with uniform geometry and no Irregularities such as branch points or transformers except at its ends) can be represented by an Equivalent-II [7], [9] of the form shown in Fig. 2. Because line lengths and equipment locations cannot be precisely determined from feeder maps, the parameters of the Equivalent-II cannot be precisely calculated. These uncertainties are of the short term variety. Some representative values are given in Table III.

In the long term, the conductors of the segment could be replaced - a possibility that considerably increases the range of parameter uncertainty.

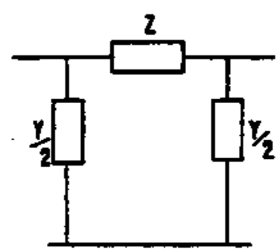

Fig. 2. Equivalent ir model of a multiphase line. Z and Y are matrices calculated as in [7] and of dimension equal to the number of equivalent conductors whose identities are to be preserved.

Distribution Transformers

A simple model for a distribution transformer is shown In Fig. 3. Short term uncertainties occur because the parameters (and sometimes the transformer type) are not known and because the load varies with time of day. Representative values of the parameters and load are shown in Table V. The range of Impedances the transformer can present to carrier signals on its primary side is indicated by the curves of Fig. 4.

SOME NOMENCLATURE

From the component models one can assemble an admittance matrix for the entire distribution network. This matrix can be quite large [10]. To keep it to a manageable size one must Invoke some scheme for aggregating components and eliminating nodes. Several such schemes are possible. Wasley [11]. has noted that when a line segment is of length 1/8 or less of the carrier wavelength, it makes little difference whether the segment is treated as a lumped or distributed device. This criterion can be used to select points along a feeder's length at which to aggregate transformers and other components.

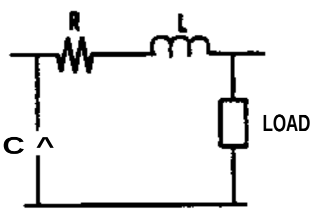

Fig. 3. A simple model of a distribution transformer.

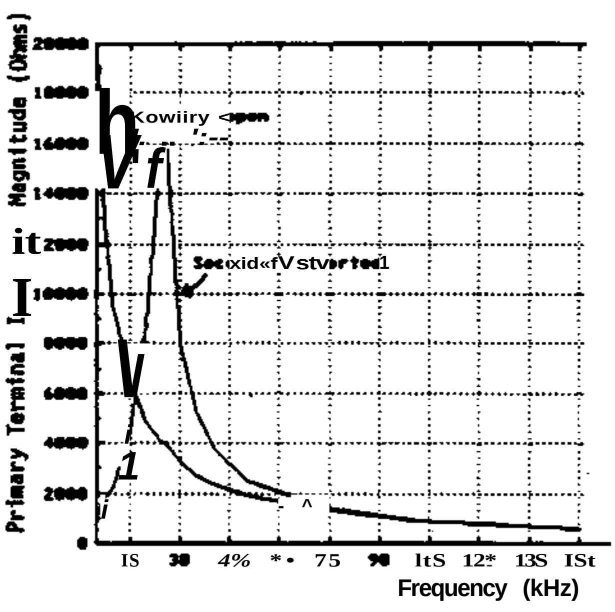

Fig. 4. Representative values of short and open circuit impedances for a distribution transformer. Light loading situations approach the open circuit case; at rated load the transformer behaves almost as if it were shorted.

Once the admittance matrix has been obtained one can write expressions relating the voltage of the injected signal to the voltages appearing at the receivers' terminals [2], [9]. These expressions have the form:

$$Q(V,X,e,w) = 0 \qquad (1)$$

where $V = (v_1, v_2, \ldots, v_m)^T$ is a vector of the magnitudes of the carrier voltages appearing at the receivers.

$X = (x_1, x_2, \ldots, x_p)^T$ is a vector of the network's uncertain parameters

e is a complex number representing the voltage of the injected signal

w is the signal's frequency

Q is a function vector of dimension m. Q is linear in V and e but nonlinear In X and w.

Let: x_i^L, x_i^u = upper and lower bounds on the values the uncertain parameter, x_i

$\Omega = \{X \mid x_i^L \le x_i \le x_i^u \ \forall\ i\}$, be the feasible set of the uncertain parameters

P(X) be the joint probability density function (pdf) of X over Ω

$E\{\cdot\}$ denote the expected (mean) value of $\{\cdot\}$

$Var\{\cdot\}$ denote the variance of $\{\cdot\}$

$Covar\{\cdot,\cdot\}$ denote the covariance of $\{\cdot,\cdot\}$

$X^\circ = E\{X\}$

$A = [\lambda_{ij}]$ be the covariance matrix of X, i.e., $\lambda_{ij} = Covar\{x_i, x_j\}$.

PROBLEM STATEMENT

The main problem we consider here is as follows:

Given: the component models, the network configuration, Ω, P(X), A, e and w

Find: the first few moments (expected value, variance, etc.) of $v_1, v_2, \ldots, v_m$, the carrier voltages appearing at the receivers.

With these moments and some information on receivers and noise levels one can estimate error rates and other performance attributes.

The method for determining the moments of the voltage at any one receiver is the same as for all the other receivers. Therefore, we will consider only the k-th receiver and for the sake of convenience drop the subscript k from its voltage. That is, we set

$$v = v_k .$$

A BRIEF REVIEW OF METHODS FOR ANALYZING NETWORKS WITH UNCERTAIN PARAMETERS

Several methods have evolved for dealing with problems of the sort we are considering here. Perhaps the oldest are of the Monte Carlo (MC) variety [12], [13]. They rely on sampling the feasible region of the uncertain parameters, calculating the network's response for each sample and processing the responses through some estimator. Convergence to the right results are guaranteed but are usually very slow and computationally expensive.

Among the less expensive alternatives are:

(a) procedures that seek to map criteria for acceptable performance into the space of the uncertain parameters and then estimate the "intersection" between the region that meets these criteria and the feasible region, Ω [14], [15]. Such procedures seem able to handle only small numbers of uncertain parameters, typically 10 or less.

(b) procedures that improve the rate of convergence of MC methods through variance-reduction techniques such as "Importance Sampling" [16]. The difficulty here is to find a suitable variance reduction technique [20].

(c) procedures that decrease the cost of calculating a network's response through the use of streamlined techniques such as "large sensitivity algorithms" and "radial sampling" [17], [18]. These help but one must still contend with the slow convergence of MC methods.

The PLC problem can include hundreds of uncertain parameters. It is too large to be conveniently handled by any of the standard methods listed above. Therefore, we will synthesize a nonstandard method better suited to its needs.

A STATISTICAL ANALYSIS PROCEDURE FOR LARGE NETWORKS

This procedure has two main steps. First a quadratic approximation to v is developed. Then an MC sampling scheme will be used to construct a linear-mean-square (LMS) model from which results of improved accuracy can be obtained. The first step will be referred to as prediction, the second, as correction.

v is twice continuously differentiable w.r.t. X. Therefore, we can approximate it with a quadratic function, f, obtained by truncating the Taylor series expansion of v about X^o, as shown below:

$$v(X) \simeq f(X) = f(X^o+\Delta X) = v(X^o) + G^T \Delta X + \frac{1}{2}\Delta X^T H \Delta X \qquad (2)$$

where G and H are Gradient and Hessian matrices of v w.r.t. X. These matrices can be efficiently calculated by using the well known Adjoint Method which requires a single L-U factorization and some forward and backward substitutions. See [19] for further details.

As shown in Appendix A, the expected value of f is given by:

$$E\{f\} = f(X^o) + \frac{1}{2} \sum_i \sum_j h_{ij} \lambda_{ij} \qquad (3)$$

where the h's and λ's are elements of the Hessian and Covariance matrices respectively.

We are now ready to move on to the correction step. Consider the quantity Θ, defined as follows:

$$\Theta = v(X) - \alpha\,[f(X) - E\{f\}] \qquad (4)$$

where α is real.

Notice that:

$$\mathrm{Var}\{\Theta\} = \mathrm{Var}\{v\} + \alpha^2\,\mathrm{Var}\{f\} - 2\alpha \,.\, \mathrm{Covar}\{v,f\} \qquad (5)$$

Let α^* be the value of α that minimizes $\mathrm{Var}\{\Theta\}$ and let Θ^* be the corresponding value of Θ. Then, by finding the stationary points of $\mathrm{Var}\{\Theta\}$ with respect to α we see that:

$$\alpha^* = \frac{\mathrm{Covar}\{f,v\}}{\mathrm{Var}\{f\}} = r\sqrt{\frac{\mathrm{Var}\{v\}}{\mathrm{Var}\{f\}}} \qquad (6)$$

$$\mathrm{Var}\{\Theta^*\} = \min_{\alpha}[\mathrm{Var}\{\Theta\}] = \mathrm{Var}\{v\}[1-r^2] \qquad (7)$$

where r is the correlation coefficient of f and v. Because f and v are strongly correlated (in fact f is a quadratic approximation to v), r is close to unity. Hence $\mathrm{Var}\{\Theta^*\}$ is much smaller than $\mathrm{Var}\{v\}$. Therefore, an MC procedure will converge to $E\{\Theta^*\}$ much faster than to $E\{v\}$. But, from (4) we see that:

$$E\{\Theta\} = E\{v\} - \alpha[E\{f\} - E\{f\}] = E\{v\} \;\forall\, \alpha \qquad (8)$$

This suggests the following MC procedure for calculating $E\{v\}$:

(a) Select N samples of X, namely $X_1, X_2, \ldots, X_N$.

(b) Calculate $v(X_i)$ and $f(X_i)$, $i \in N$.

(c) Calculate $E\{f\}$ from (3).

(d) Estimate α^*.

(e) Estimate $E\{v\}$ from:

$$E\{v\} = E\{\Theta^*\} \simeq \frac{1}{N} \sum_{i \in N} [v(X_i) - \alpha^* f(X_i)] + \alpha^* E\{f\} \qquad (9)$$

For comparable levels of accuracy this procedure will require far fewer samples than if $E\{v\}$ were calculated directly. However, it does require the value of α^* and this is not easy to calculate. Fortunately, the procedure is not very sensitive to the value of α. One may use $\alpha=1$ instead of $\alpha = \alpha^*$, in which case the procedure reduces to the method of Control Variates [20] and an increase in efficiency is achieved under fairly wide conditions, namely:

$$\mathrm{Covar}\{v,f\} > \frac{1}{2}\,\mathrm{Var}\{f\} \qquad (10)$$

Another and better approach Is to estimate the value of a* from the sample results In which case the procedure falls Into the class of linear-mean-square estimation (IMS) problems [21]. Notice that (4) can be rearranged into the form:

$$v - Bjf + B_2 + 9 \qquad (11)$$

where 8_1 and 6_2 are constants. In IMS procedures, 9 Is neglected and 8_1 and 8_2 are approximated by the quantities B_1, and B^ where [20]:

$$\bar{B}_1 = \frac{\sum_{i \in N} f(X_i)v(X_i) - N \bar{f} \bar{v}}{\sum_{i \in N} f^2(X_i) - N[\bar{f}]^2} \qquad (12)$$

$$\bar{B}_2 = \bar{v} - \bar{B}_1 \bar{f} \qquad (13)$$

$$1 \; I \; \frac{}{1} \sum_{i \in N} f(X_i)$$

$$\bar{v} = \frac{1}{N} \sum_{i \in N} v(X_i) \; .$$

The resulting LMS model is:

$$vOO*?\wedge f(X) + \bar{B}_2 \qquad (14)$$

In Summary

The steps involved in applying the prediction-correction process described above are:

(a) compute the gradient and Hessian of v at $X - X^\circ$

(b) using ft and P(X), generate N samples of X, naael y! h-h. . . . .h

(c) using a network simulator and (2), calculate $v(X_\pm)$ and $f(X_\pm)$ V l£N

(d) using (12) and (13) calculate ^ and B_2 the coefficients in the LMS model, (14)

(e) by sampling the LMS model, calculate the requisite moments of v(X). This is an Inexpensive process because $\bar{L}_1$ and B_5 are constants and evaluating f(X) requires only a moderate amount of matrix multiplication.

Remarks

• Consider a network with M nodes, p uncertain parameters and m receivers. The number of multiplications required in forming the gradients and Hessians for all the receivers is approximately $M^3 + m\,p\,M^2$. The computation of B_1 and B_2 takes about NM^3 multiplications. Therefore, the total operation count for the prediction-correction process is about $(N+1)M^3 + m\,p\,M^2$. In contrast, a straightforward Monte Carlo procedure takes about Ui^3 multiplications where L is the number of samples used. In our experience the two processes produce results of comparable accuracy when N is about 50 and L is about 1000. Therefore, unless the number of receivers and uncertain parameters is so large that their product exceeds 1000 M, the predictor-corrector procedure should produce substantial savings over a straightforward Monte Carlo procedure. It should be noted here that every receiver does not need to be examined separately. Instead, a single representative can be selected from each neighborhood of receivers.

APPLICATIONS - ESTIMATING RECEIVER PERFORMANCE

Receiver Models

Receivers can be modeled by threshold or binomial functions such as:

$$Y - 1 \text{ if } v \geq a$$
$$- 0 \text{ otherwise}$$

where v is the signal level at the receiver and a is its threshold for adequate reception. This threshold will vary with, the level of the noise present. When Y-l the reception is successful, when Y*0, unsuccesful.

The binomial model la simple but does not really reflect the behavior of receivers. Better models are obtained by using continuous functions like the function 0(v> whose value is the probability of error when the signal level is v. For example, suppose that the noise present at a receiver's terminal is white, additive and with variance R/2. If the modulation scheme uses PSK (phase shift keying) then the probability of a bit being In error [22] is:

$$0(v) - \tfrac{1}{2} \exp(-v^2/R) \qquad (15)$$

Performance Attributes on Indices for Receivers

Several indices may be constructed to measure receiver performance. Among the best and most obvious are E{y} and E{0}, the expected error rates for the binomial and continuous receiver models, respectively. Another index that is useful in worst-case-design-approaches is $0(v_{lower})$ where

$$v_{lower} - E\{v\} - K\,Var\{v\} \; .$$

By Making K large (say 3 or 4) we ensure that most of the signals appearing at the receiver exceed v_{lower}* If "* novdesign the system so that $0(v_{lower})$ is sufficiently small we ensure that most messages will be successfully received.

The predictor-corrector models described in the previous section provide the statistics of v and hence, provide the data to calculate Indices of the type mentioned above. Alternatively, if a continuous model is used for the receiver, 0 can be plugged into the predictor-corrector models in place of v and the Indices calculated directly.

An Example

Consider the system described in Appendix B. This system has 5 nodes. Suppose that signals are injected at node #1 and we examine their reception at node #5.

The statistics for voltage attenuations, calculated by the predictor-corrector method described previously, are shown in Table I. Notice that the scatter (4σ-Interval) is fairly large and varies with frequency. Deterministic models can give no indication of this scatter. Their limitations can be further illustrated by considering error rates. Suppose we use a standard, deterministic model [5],[10] [11] in its most usual way, i.e. with the uncertain parameters assumed to be constant at their expected values. Suppose further, that we assume the noise is Gaussian, additive and 12 db below the signal level, calculated by the deterministic model. (12-13 db are typical signal/noise ratios). Let the receiver be of the PSK type so that error rates are given by equation (15).

The error rates calculated by the deterministic and stochastic models are compared in Table II. Their differences reflect the well known fact that:

$$E\{\phi(X)\} \neq \phi(E\{X\})$$

Notice that the deterministic model is consistently low, sometimes by as much as a 3 orders of magnitude (10^3). This explains, at least in part, why field measurements tend to give far higher error rates than are predicted by deterministic models.

CONCLUDING REMARKS

Conclusions

Performance attributes for PLC systems, such as signal attenuations and receiver error rates, are best characterized by random variables because of the time dependent uncertainties in the network's structure and electrical characteristics. Models for estimating these attributes must contend with the additional uncertainties introduced by the difficulties in precisely determining the values of certain network parameters such as the lengths of line segments and the locations of transformers. In the short term these uncertainties can produce a significant scatter in the estimates of the performance attributes produced by the models. Long term effects (when the network changes as pieces of equipment are upgraded and reconfigured) can be only more pronounced.

This paper has developed stochastic models for estimating the expected values, scatter and other statistical properties, of a PLC system's performance attributes. The use of such stochastic models appears to be crucial to the proper design of PLC systems. Conventional deterministic models can provide very misleading information on system performance. For instance, in the example considered, a conventional deterministic model consistently underestimated the error rate by a factor in excess of 10 and sometimes as high as 10^3.

Conjectures

One may hypothesize that the extensive field tuning/modification that many PLC manufacturers have had to undertake when demonstrating their systems is due to their use of deterministic models in designing the systems. It is reasonable to expect that the tuning/modification process will have to be repeated many times as the distribution network's components are replaced and upgraded. Much of this could be eliminated if the impacts of short and long term uncertainties are predicted (with stochastic models of the sort described in the paper; and taken into account in the initial designs of the PLC systems.

Acknowledgement

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APPENDIX A

EXPRESSIONS FOR THE MOMENTS OF f (X)

Recall from (2) that f (X) is a quadratic approximation to v(X) and is given by:

$$f(X) = f(X^\circ) + G^T \Delta X + \frac{1}{2}\Delta X^T H \Delta X \qquad \text{(A1)}$$

The expected value of f is obtained as follows:

$$E\{f\} = f(X^\circ) + \frac{1}{2} E\{\Delta X^T H \Delta X\}$$
$$= f(X^\circ) + \frac{1}{2} \sum_i \sum_j V_{ij} \qquad \text{(A2)}$$

The variance is obtained as follows:

$$\mathrm{Var}\{f\} = E\{[f - E\{f\}]^2\}$$
$$= E\{[v(X^\circ) + G^T \Delta X + \frac{1}{2}\Delta X^T H \Delta X]^2\} .$$

The third moment, U_3, is obtained as follows:

$$U_3 = E\{[f - E\{f\}]^3\}$$
$$= E\{[v(X^\circ) + G^T \Delta X + \frac{1}{2}\Delta X^T H \Delta X]^3\} .$$

If all the x_i are Independently Gaussian the above expressions reduce to [23]:

$$\mathrm{Var}\{f\} = \sum_i G_i^2 \lambda_{ii} + \frac{1}{2}\sum_i h_{ii}^2 \lambda_{ii}^2 + \frac{1}{2}\sum_{i \neq j}\sum h_{ij}\lambda_{ii}\lambda_{jj}$$

$$\mu_3 = 3\{ \sum_i G_i^2 h_{ii}\lambda_{ii}^2 + \sum_{i \neq j}\sum G_i G_j h_{ij}\lambda_{ii}\lambda_{jj} \}$$

APPENDIX B

A SIMPLE SYSTEM

The arrangement of the system is shown In Fig. 5. Data for the line segments are given in Table III. These segments are modelled by Equivalent's whose matrices, obtained by BPA Line Constant Program, are shown In Table IV. Transformers are modelled by circuits of the form shown in Fig. 3 with data as in Table V. Transformers occur on the central conductor at nodes 2-3-4-5-6 in the network. The carrier signal is Injected into the center phase at node #1 and received on the same phase at node #5. All uncertain parameters are assumed to be Independent and Gaussian.

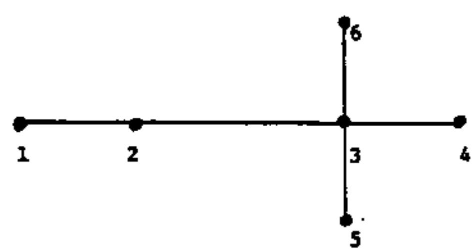

Fig. 5. One line diagram of a test system.

TABLE I Voltage Attenuation Statistics for Point #5 of the Example System

Frequency kHz	E{v} Pred.	E{v} Correct.	Var{v} Pred.	Var{v} Correct.	Third Moment Pred.	Third Moment Correct.	4a-Interval *
6.0	0.229	0.224	0.652E-3	0.733E-3	0.597E-5	0.189E-4	∓ 48Z
10.0	0.425	0.423	0.494E-3	0.403E-3	0.278E-5	0.552E-5	∓ 19Z
18.0	0.256	0.255	0.788E-3	0.537E-3	-0.267E-5	0.171E-5	∓ 36Z
20.0	0.132	0.128	0.925E-3	0.797E-3	0.171E-4	0.300E-5	∓ 86Z
30.0	0.110	0.113	0.147E-3	0.178E-3	+0.922E-6	-0.589E-7	∓ 47Z
40.0	0.131E-1	0.129E-1	0.105E-5	0.116E-5	0.339E-9	0.248E-9	∓ 33Z

*a^2 here is equivalent to Var{v}. Using the Chebyschev Inequality [21], the probability that v will be inside this interval is calculated from:

$$P\{|v-E\{v\}f \leq 4o\} \geq 1 - \frac{1}{4^2} - .94$$

In other words, at 6 kHz 94Z or more of the messages vill have carrier voltage levels within ±48Z of the mean carrier voltage, 0.224. At 10 kHz the scatter is much less. 94Z of the carrier voltages vill be within 19Z of the mean. And so on.

TABLE II Error Rates of a PSK Receiver at Point #5 of the Example System

Frequency kHz	Error Rate Predicted by a Deterministic Model * (Error/bit transmitted)	Expected Error Rate from the Stochastic Model (Error/bit transmitted) Predicted	Corrected
6.0	6.0 x	8.0 x 10"7	1.0 x io-6
10.0	6.0 x lO"8	1.0 x 10"7	1.0 x ID"7
20.0	6.0 x lO"8	5.0 x 10~*	2.0 x lO"4
30.0	6.0 x lO"8	3.0 x 10"4	4.0 x io-6
40.0	6.0 x Iff8	9.0 x 10"7	2.0 x ID"6

* The carrier voltage is calculated using X^o, the mean value of the uncertain parameter vector. The noise level ta assumed to be consistently 12 db below this carrier voltage so that the deterministic model yields the same signal-to-noise ratio at all frequencies and hence, the same error rate.

TABLE III. Data for the 3-Phase Lines of the Network in Fig. 5

Section	Line Type	Conductor	Nominal Length (mi)	Uncertainty in Length
1-2	Undg.	#750 MCM	0.80	±10Z
2-3	Aerial	#477 ACSR	0.90	±10Z
3-4	Aerial	#477 ACSR	0.60	±15Z
3-5	Aerial	#477 ACSR	0.60	±15Z
3-6	Aerial	#477 ACSR	0.30	±102

TABLE IV Equivalent Matrices for Phase Conductors

Capacitance Matrix (Farad/mile)

0.1559E-7		
-0.3997E-8	0.1760E-7	
-0.1998E-8	-0.3997E-8	0.1559E-7

Impedance Matrix (Ohm/mile) at 30 kHz.

0.1161E+2 + j 0.4131E+3		
0.8809E+1 + j 0.1275E+3	0.9365E+1 + j 0.3767E+3	
0.9999E+1 + j 0.1026E+3	0.8809E+1 + j 0.1275E+3	0.1161E+2 j 0.4131E+3

TABLE V Representative Parameter Values for a Distribution Transformer

Parameter	Lower Bound	**Upper Bound**
Capacitance C (nF)	0.995	1.770
Inductance L (mH)	26.0	**92.0**
Secondary Load (ohms) R	560.0	**5000.0**

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