Endogenous Credit and Investment Cycles with Asset Price Volatility

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Can credit market frictions be a source of aggregate fluctuations?

- Financial sector perturbations vs. amplifications

We study the influence of limited commitment on the emergence of indeterminacy and sunspots.

- With limited commitment asset prices can jump to accommodate for changes in expectations.
- Fluctuations in macroeconomic outcomes and asset prices are tied to aggregate shifts in market expectations.
The Environment

- \( t = 1, 2, \ldots \)
- Two goods: capital good, consumption good.
- Each period:
  - Perfectly Competitive sector \( Y_t = F(K_t, L_t) \), where \( F \) linearly homogeneous.
  - Capital used in production fully depreciates.
  - New capital is produced,
  - Consumption is non storable
Agents and endowments

- 2 types of agents: 
  \[
  \begin{cases} 
  \alpha & \text{workers} \\
  1 - \alpha & \text{entrepreneurs}
  \end{cases}
  \]

- Workers
  - 1 unit of time in day and work for the representative firm receiving a wage \(w_t\).

- Entrepreneurs
  - No time (Don’t work)
  - Capital producing technology
  \[
  x_t \rightarrow (x_t)^\nu
  \]
  time \(t\) consumption  \rightarrow  time \(t+1\) capital

- Assume \(1 > \nu > 0\).
- Workers can not operate this technology but buy claims on next period capital.
Credit markets

Every period, a *competitive* credit market opens.

- Entrepreneurs and workers can trade claims (bonds) on future capital
- $q_t$ price of each bond: consumption goods needed at time $t$ to purchase a claim on 1 unit of capital at time $t + 1$
- $k_{t+1}^W$ and $k_{t+1}^E$ denote workers and entrepreneurs net demand for bonds
- Limited commitment: entrepreneurs can repudiate their debt and divert resources from their investment technology.
Workers problem

- Recursive formulation of worker’s problem:

\[ U_t(k_t^W) = \max_{(c_t^W, k_{t+1}^W)} \log(c_t^W) + \beta E_t \left[ U_{t+1}(k_{t+1}^W) \right] \]

subject to \[ c_t^W + q_t k_{t+1}^W = w_t + r_t k_t^W \]
Entrepreneur’s limited commitment

- Entrepreneurs can divert resources and consume after credit market closes
- Hidden consumption yields \((1 - \theta)\) utility for each unit of consumption
- Upon default, assume perpetual exclusion from credit markets.
- Entrepreneurs deviation payoff:
  \[
  \tilde{V}_t(S^E_t, k^E_{t+1})
  \]
Entrepreneur’s problem

Recursive formulation of entrepreneur’s problem:

\[ V_t(k_t^E, x_{t-1}) = \max_{(c_t^E, x_t, k_{t+1}^E) \in \mathbb{R}_+^2 \times \mathbb{R}} \quad c_t^E + \gamma E_t \left[ V_{t+1}(k_{t+1}^E, x_t) \right] \]

s.t.
\[ c_t^E + q_t k_{t+1}^E + x_t = r_t[x_{t-1}^v + k_t^E] \]
\[ q_t k_{t+1}^E + x_t \geq 0 \]
\[ \gamma E_t \left[ V_{t+1}(k_{t+1}^E, x_t) \right] \geq \tilde{V}_t(S_t^E, k_{t+1}^E) \]
Equilibrium

Definition
An equilibrium with limited commitment consists in sequences \((c_t^W, k_{t+1}^W)_{t=0}^\infty, (c_t^E, k_{t+1}^E, x_t)_{t=0}^\infty, (q_t, r_t, w_t)_{t=0}^\infty\) such that

- \((c_t^W, k_{t+1}^W)_{t=0}^\infty\) and \((c_t^E, k_{t+1}^E, x_t)_{t=0}^\infty\) solve workers and entrepreneurs problem respectively;

- Capital and labor earn their marginal product;

- Credit market clears
  \[\alpha k_{t+1}^W + (1 - \alpha)k_{t+1}^E = 0\]

- Capital evolves according with
  \[K_{t+1} = (1 - \alpha)x_t^{v'}\]
Stationary Equilibria

- Workers and entrepreneurs trade bonds
  - No default constraint is slack, and $S_t^E = 0$. ▶ FB
  - No default constraint binds, and $S_t^E = 0$. ▶ BNC
  - No default constraint binds, and $S_t^E > 0$. ▶ BC

- Workers and entrepreneurs do not trade bonds, and $S_t^E > 0$. ▶ NB
Stationary equilibria

\[ \nu(\theta) \]

\[ \nu(\theta) \]

\[ \overline{\nu}(\theta) \]

\[ \nu(\theta) \]
First Best Local Dynamics

- The price of new capital is a function of $x_t$:

  $$q_t \equiv q[x_t] = \frac{1}{\nu x_t^{\nu - 1}}$$

- In the end, the dynamics of the system in the first best are:

  $$E_t\left(\frac{c_{t+1}^W}{c_t^W}\right) = \frac{\beta f'(z[x_t])}{q[x_t]}$$

  $$c_t^W + q[x_t]k_W[x_t] = f(z[x_{t-1}]) - z[x_{t-1}]f'(z[x_{t-1}])$$

  $$+ f'(z[x_{t-1}])k_W[x_{t-1}]$$

- Investment is a predetermined variable, consumption is a non-predetermined variable.

- Only if the steady state is a sink, we will have indeterminacy.
First Best System Local Dynamics

Proposition

*If the no default constraint is slack, the stationary equilibrium with bonds is a saddle.*

Corollary

*There is a unique convergent path to the steady state. The dynamic model is locally determinate, so that endogenous fluctuations do not emerge.*
Local dynamics with a binding no-default constraint, bonds, and $S^E = 0$

- Investment is now pinned down by a forward looking no-default constraint:

$$ (1 - \theta) x_t = E_t \{ \gamma c_{t+1}^E + \gamma (1 - \theta) x_{t+1} \} $$

and the price is no-longer linked to fundamentals

$$ q_t = q_t(c_{t+1}^W, x_t, x_{t+1}) $$
We finally obtain

\[(1 - \theta)x_t = E_t \left\{ \gamma r(x_t) \left[ x_t^\nu - \frac{\alpha}{1 - \alpha} \frac{c_{t+1}^W + \frac{1-\alpha}{\alpha} x_{t+1} - w(x_t)}{r(x_t)} \right] \right. \]

\[+ \left. \gamma (1 - \theta)x_{t+1} \right\} \]

\[E_t \left\{ \frac{c_{t+1}^W}{c_t^W} - \beta \alpha \left[ c_{t+1}^W + \frac{1-\alpha}{\alpha} x_{t+1} - w_{t+1}[x_t] \right] \right. \]

\[\left. \frac{(1 - \alpha)x_t}{(1 - \alpha)x_t} \right\} = 0 \]

Both consumption and investment are non-predetermined variables.
Proposition

Assume that $\gamma \geq \min\{s, (1 - \theta)\beta\}$. Then the limited commitment steady state with bonds and no collateral is a saddle.

Corollary

The limited commitment steady state with bonds and no collateral is always locally indeterminate.
Idea

- There are an infinite number of combinations of initial values for both state variables such that
  - The system is on the stable arm,
  - The system will always converge to the steady state.
- Given an initial value $x_0$, investment converges to a unique steady state $x_{BNC}$ and consumption to $c_{BNC}^W$.
- However sunspots matter
Starting from the steady state, a forecasting error hits the economy.

$x$ and $c$ are tied by so that the economy comes back to the stable arm, guaranteeing convergence to the steady state.

Stochastic bounded equilibrium trajectories driven by expectations shocks are possible when the steady state is a saddle.
Sunspot equilibria

- Add forecasting errors $e_{t+1} = \tilde{x}_{t+1} - E_t(\tilde{x}_{t+1})$, with $E_t(e_{t+1}) = 0$

- Local sunspot equilibria can be obtained by considering that $e_{t+1}$ follows an i.i.d. stochastic process of bounded support with small variance
Simulation Exercise

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
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<tr>
<td>$\gamma$</td>
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<tr>
<td>$\sigma$</td>
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<td>$s$</td>
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<td>$\theta$</td>
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<tr>
<td>$\nu$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>0.0185</td>
</tr>
</tbody>
</table>

- Sunspot shocks $\mathcal{N}(0, \sigma_e)$ i.i.d.
- $\sigma_e$ chosen to match variance in consumption growth
Findings

- 500 draws of $\sigma^t$, compute average standard deviation of series.
- $\sigma_x > \sigma_{cW}$: Investment more volatile than consumption
- $\sigma_x > 3.1 \cdot \sigma_y$: Investment more volatile than output
- Changes in marginal productivity make $r$ countercyclical, $w$ pro cyclical.
- $\sigma_{kW} = 2.65 \cdot \sigma_x$
Simulated Series

Our model is able to generate price volatility in the credit market consistent with the one observed in the data. The model generates (out of 500 simulations) an average ratio of the standard deviation of the price of bonds $q_t$ and output equal to 8.05. This number compares with a value of 5.37, if we use the Moody's BAA Corporate Bond Yield index, which only includes higher rated corporate bonds. As we expect price variability to be higher for lower rated corporate bonds, the standard deviation of the price of all kinds of bonds will be higher than the Moody's index. This suggests that our results reflect the overall volatility in the prices of bonds, which has been difficult...
Volatility of $q_t$

- $\sigma_q = 8.05 \cdot \sigma_y$

- Because $\nu \approx 1$, without limited commitment

$$q_t^{FB} = \frac{1}{\nu x^{\nu-1}} \approx 1$$

- All the variability comes from the binding no-default constraint.

- Moody's BAA Corporate Bond Yield index: $\sigma_q = 5.37 \cdot \sigma_y$
Simulated Series

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Conclusions

- Developed a framework to study dynamics in lending, production, and investment, with capital and limited commitment.

- With perfect credit markets, prices are pinned down by technology.

- With limited commitment
  - Prices no longer pinned down by technology, but depend on expectations of future market conditions.
Conclusions

Limited commitment introduces the possibility of

- Indeterminacy
- Endogenous fluctuations in credit, investment etc, driven by self fulfilling prophecies.
- Bond price volatility, consistent with data, in response to shifts in expectations.
Conclusions

Policy recommendations?

- Changes in $\nu$ and $\theta$ can move the economy away from the indeterminacy region.

- What policy parameters can influence $\nu$ and $\theta$?