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Calculating Remittiturs

Joseph B. Kadane
Carnegie Mellon University, kadane@stat.cmu.edu

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Abstract

This note reviews several methods proposed for calculating the amount of an appropriate remittitur using a sample of "comparable" cases. It recommends a simple quantile estimate.

A remittitur is a post-trial procedure that may arise after a plaintiff has established liability and is awarded damages by the jury. Under U.S. law, the defendant may move for a new trial, or, in the alternative, a reduction in the award, called a remittitur, on the grounds that the damages awarded by the jury are excessive. If the judge grants defendant's motion for a remittitur, the plaintiff is given the choice between a new trial and a reduced damage award set by the judge. What criteria should be used to decide whether a remittitur should be imposed, and how much should it be?

In 1986, the New York Legislature mandated that a remittitur be granted when the jury's award "deviates materially from what would be reasonable compensation."[2] In 1997, Judge Weinstein addressed the issue of how to make this phrase operational, in the case of Geressy vs. Digital Equipment Corporation.[3]

Briefly, Patricia Geressy sued the Digital Equipment Corporation (DEC) on the grounds that she was not warned that the DEC computer keyboard she used could cause repetitive stress injuries. The remittitur issue in the case concerned the jury award of $3,490,000 for pain and suffering.[4]

Judge Weinstein's method involves three steps:

1. He identifies 27 cases he regards as comparable to Geressy's.

2. He decides that the 95^{th} percentile of the distribution is the appropriate point in the distribution to take as the upper limit of permissible awards.

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[1]Leonard J. Savage University Professor of Statistics and Social Sciences, Emeritus, at Carnegie Mellon University. This note is in honor of Professor Joseph Gastwirth.

[2]CPLR §5501 (c)

[3]980 F. Supp. 640

[4]Her case was returned for a new trial anyway because of newly discovered evidence suggesting that her injury may have predated her computer usage.

3. Imposing the assumption of a normal (Gaussian) distribution, he calculates the average of the 27 comparable pain and suffering cases to be $\overline{x} = \$747,372$ and the estimated standard deviation to be $s = \$606,873$. If X has a Normal distribution with mean μ and standard deviation σ, then the probability that X is less than $\mu+2\sigma$ is 0.95. Then Judge Weinstein proposes that the upper limit for the pain and suffering award should be $\overline{x} + 2s = \$747,372 + 2(\$606,873) = \$1,961,118$, which he rounds to \$2,000,000. Since the jury awarded \$3,490,000 for pain and suffering, Judge Weinstein proposes a remittitur of \$3,490,000 - \$2,000,000 = \$1,490,000.

Each of these steps deserves comment. The first is an advance in the method for arriving at remittiturs, since it grounds the matter in many previously-decided cases. There is no question that the judgments in this matter are not easy. As Judge Weinstein writes "The task is difficult... Cases with similar causal agents, similarly-named diagnoses, or similar reductions in the quality of life might serve as benchmarks."[5]

There are limitations as well. The cases that are decided by juries are a non-random sample of those filed in court. Many cases settle, often under conditions of confidentiality. If those are typically stronger cases, the universe of decided cases will be typically weaker, leading to too low a view of an appropriate award. Similarly, if weaker cases tend to settle without trial, again a bias results, here leading to too high a view of the award. According to experienced plaintiff's lawyers, the best cases, with the most severe damages, are settled confidentially before trial, for extremely large sums. Strict confidentiality agreements prohibit the sizable awards in such cases from being included in Weinstein's database. Nonetheless, the procedure seems fair, if difficult. Each side can propose cases it believes to be comparable, and can argue the comparability of the other's list. Of course, each proposed comparable case will have points of similarity and points of difference with the case at issue, and confidential settlement amounts are not includable. It would seem reasonable, however, that case awards be adjusted for inflation, so that all amounts are in current dollars and represent current purchasing power.

Judge Weinstein's second step, the choice of the 95^{th} percentile, is an arbitrary one with a long history. The number .95, and its complement, .05,

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go back to the choice of the distinguished statistician R.A. Fisher to use that as the criterion for "significance" in testing statistical hypotheses (Fisher (1973)). At the time, computers were not available, so laboriously calculated statistical tables were used instead, and some criterion had to be established. There is no theory supporting the use of .05 and .01 as criteria. Indeed, two other important statisticians, H. Raiffa and R. Schlaiffer (Raiffa and Schlaifer (2000)) write that "actual decisions are made treating the numbers .05 and .01 with a respect usually reserved for the number 13." Nonetheless, some percentile needs to be chosen to operationalize the concept of "deviates materially." This is a legal issue, and there is no reason to reject Judge Weinstein's choice.

Weinstein's third step is the imposition on the data of the assumption of the Normal distribution. Using the Normal assumption, Judge Weinstein takes the 95^{th} percentile of the distribution of comparable claims, $\mu + 2\sigma$, to be the upper limit of damages for Geressy.

However the numbers μ and σ are not known with certainty. When the data $(x_1, \ldots, x_n)$ come for a Normal distribution, the estimates

$$\overline{x} = \sum_{i=1}^{n} x_i/n$$

and

$$s^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2/n,$$

for μ and σ^2 respectively have several attractive properties (Casella and Berger (1990)).

The question of whether the data follow a Normal (or Gaussian) distribution is an empirical question, not a legal one. A standard way to assess whether data are Normal is a quantile-quantile plot (Wilk and Gnanadesikan (1968)). If the data are approximately Normal, they should fall roughly on a straight line in such a plot. As Figure 1 shows, they do not.

To understand the inequalities that are discussed below, imagine that, for any given case, there is a (conceptual) infinite population of comparable cases, each with a finding of liability and some award (stated in current dollars, after adjusting for inflation). One can ask, among all such distributions for a legal upper limit guaranteeing the plaintiff at least a 95% probability of an appropriate award. The difficulty with this approach, as will be seen

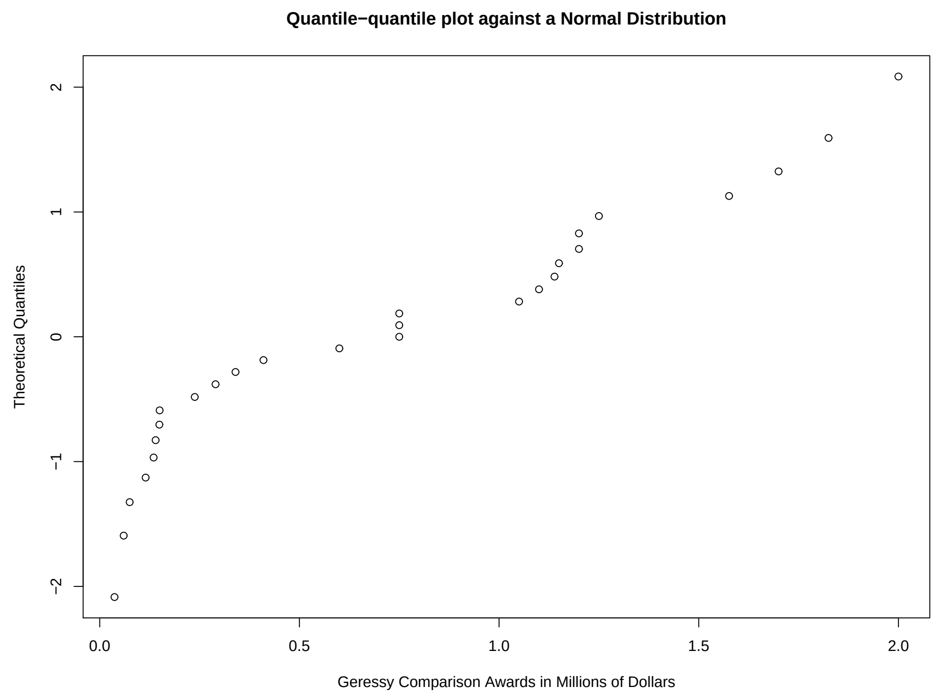

Figure 1: If the Geressy data follow a normal distribution, this plot would be a straight line (approximately).

below, is that it protects against distributions that are far from the observed data found by the judge to come from comparable cases.

In a recent article, Haug and Steinmeyer (2008) propose using Tchebycheff's[6] inequality. This inequality states that among all distributions X having a mean μ and a standard deviation σ, the probability of the event that $\mu - k\sigma < X < \mu + k\sigma$ is at least $1-(1/k)^2$, for every k. Applying this inequality to find the 95^{th} percentile, Haug and Steinmeyer solve $1-(1/k)^2 = .95$ to find $k \approx 4.5$. Therefore, they propose that the upper limit of a damage award should in general be $\mu + 4.5\sigma$.

When the data do not follow the Normal (*i.e.* Gaussian) distribution,

[6]Tchebycheff was Russian. Often in English his name is transliterated as Chebycheff, so literature may be found under either spelling.

the estimates $\overline{x}$ for μ and s^2 for σ^2 have much less to recommend them than they do when the data do follow a Normal distribution. Thus it is unclear, in the non-Normal case, such as the Geressy data, that $\overline{x}$ and s^2 summarize all that is useful about the observed data.

Notwithstanding these weaknesses, Haug and Steinmeyer substitute the estimates $\overline{x}$ and s for μ and σ, and propose $\overline{x} + (4.5)s$ as the upper legal limit on a damage award. In the Geressy case, $\overline{x} + (4.5)s = \$747,372 + (4.5)(\$606,873) = \$3,478,300$, which would imply virtually no remittitur.

An inequality such as Tchebycheff's is called sharp if there is some distribution X that satisfies it with equality. Examining that distribution can be instructive, as it can lead to insight into whether the inequality is useful in the context to which it is being applied. Tchebycheff's inequality is sharp, and the special distribution that makes it so takes only three values as follows:

$$\begin{array}{cc} & \textbf{value} \quad\quad \textbf{probability} \end{array}$$

$$X = \begin{cases} \mu - k\sigma & 1/2k^2 \\ \mu & 1 - 1/k^2 \\ \mu + k\sigma & 1/2k^2 \end{cases}$$

To check that this distribution satisfies the requirements that its mean is μ, and its variance is σ^2, I write

$$\begin{aligned} E(X) &= \tfrac{1}{2k^2}(\mu - k\sigma) + (1 - 1/k^2)\mu + (1/2k^2)(\mu + k\sigma) \\ &= \mu + \tfrac{1}{2k^2}(-k\sigma + k\sigma) = \mu \\ E(X-\mu)^2 &= \tfrac{1}{2k^2}(-k\sigma)^2 + \tfrac{1}{2k^2}(k\sigma)^2 = \sigma^2. \end{aligned}$$

Finally $P\{\mu - k\sigma < X < \mu + k\sigma\} = 1 - 1/k^2$, which shows that Tchbycheff's inequality is sharp, *i.e.*, it cannot be improved upon within the class of distributions it considers.

So with this as background, we assess the suitability of using Tchebycheff's inequality in the context of remittiturs. A first, and somewhat theoretical matter, is that not all probability distributions have means μ and variances σ^2, and Tchebycheff's inequality applies only to those that do. In particular, distributions with a high upper tail may not have such moments.

More importantly, Tchebycheff's inequality does not honor the constraint on the problem that awards must not be negative. For example, substituting the estimates from the cases judged comparable to Geressy's, the lower value of the distribution that makes the Tchebycheff inequality sharp is $\overline{x}-(4.5)s = \$747,372-(4.5)(\$606,873) = -\$1,983,556$. Hence, Tchebycheff's inequality is protecting against distributions that do not satisfy the natural constraints of the problem, leading to a bound that is too low.

On the other hand, by protecting against all possible distributions with a given mean and variance, rather than attending more closely to the data on awards in comparable cases, the Tchebycheff approach can be argued to be giving too much protection to the plaintiff and hence not enough to the defendant. The only way to justify the Tchebycheff approach, then, is to hope that these two effects will somehow cancel each other. In some cases they may do so, but this seems to be a fragile basis on which to make legal policy.

There is another bound, called Markov's bound, that does satisfy the constraint that awards cannot be negative. Formally stated, among all distributions Y with a mean μ and satisfying $P\{Y < 0\} = 0$, the probability of the event that $Y \geq t$ is less than or equal to μ/t, for all $t \geq \mu$.

Again, Markov's inequality is sharp; the distribution showing this puts all its probability on two points as follows:

$$\begin{array}{cc} \textbf{value} & \textbf{probability} \end{array}$$

$$Y = \begin{cases} 0 & 1-\mu/t \\ t & \mu/t \end{cases}.$$

Again, I check that Y has the required properties to make Markov's inequality sharp. Clearly $E(Y) = 0(1-\mu/t) + t(\mu/t) = \mu$, and $P\{Y < 0\} = 0$ so this distribution Y satisfies the constraints of the Markov inequality. Furthermore,

$$P\{Y \geq t\} = \mu/t.$$

Showing again that the Markov inequality is sharp.

Again, not all distributions have means μ, so the Markov inequality does not apply to them, but at least it does satisfy the constraint that damage awards cannot be negative. Among distributions with means, again accepting $\overline{x}$ as an estimate for μ, the estimated upper limit for remittiturs would be $\overline{x}/.05 = 20\overline{x}$. Hence, applied to the Geressy case, this would imply an upper limit of $(20)(\$606,873) = \$12,137,460$, far greater than any of the other limits considered so far.

Because the Markov inequality corrects the defect of the Tchebycheff approach by restricting the set of possible damage awards to be non-negative, it is not surprising that it leads to an absurdly high limit. The distribution that makes it sharp would have 95% of the damage awards be $0, and this is simply far from the case, as Figure 1 shows.

The Markov inequality does permit an analysis of the bounds found using the Tchebycheff inequality, when the restriction is imposed that awards cannot be negative. Taking the bound to be $\mu + (4.5)\sigma$ from the Tchebycheff inequality, the probability that this is exceeded by a distribution that puts probability 0 on negative values is bounded by $\frac{\mu}{\mu+(4.5)\sigma}$. Substituting the Geressy estimates, 606,873/3,478,300 = 17.4%, far above the permitted 5%.

Neither the Tchebycheff nor the Markov inequality approaches seem satisfactory. The distributions that make them sharp are far from the data in the Geressy case, and far from the likely distributions of damages in any imaginable cases. It is necessary to start over.

The problem of finding an estimate of the 95^{th} percentile of a distribution without specifying a particular form for the distribution is known in statistics as quantile estimation. To understand how this works, consider in general a sample of size n from some (unspecified) continuous distribution. Let $X_{i:n}$ be the i^{th} largest in the sample.

Intuitively, the intervals $(-\infty, X_{1:n})(X_{1:n}, X_{2:n}) \ldots (X_{n-1:n}, X_{n:n})(X_{n:n}, \infty)$ each contain the same probability, and there are $(n+1)$ of them. Thus we would expect the largest in a sample of 19 to define the upper limit of a 5% interval, and hence the size of an appropriate limit to a jury award. Similarly, a sample of size 39 would divide the line into 40 intervals of probability $1/40 = .025$ each, so the second largest award would be a good estimate. What about sample sizes between 19 and 39, such as Geressy's? The proposal is to use a linear extrapolation, as follows:

In general, let

$$Q(p) = (1-g)X_{j:n} + gX_{j+1:n},$$

where $j = [p(n+1)], g = p(n+1) - j$, and $[x]$ is the largest integer smaller than x. The quantity $Q(p)$ is a standard quantile estimate for the p^{th} quantile, found for example in Davis and Steinberg (1986).

In the Geressy case, $p = .95$ and $n = 27$. Then $j = [(.95)(28)] = [26.6] = 26$, and $g = p(n+1) - j = .6$. The largest award, $X_{27:27}$, is $2,000,000, and the second largest award, $X_{26:27}$, is $1,825,000.

Then

$$\begin{aligned} Q(.95) &= (.4)(\$1,825,000) + (.6)(\$2,000,000) \\ &= \$1,930,000, \end{aligned}$$

remarkably close to Judge Weinstein's $1,961,119. This estimate is simple to calculate, and is valid regardless of the shape of the underlying distribution. It does not even require that the distribution have a variance or a mean. Consequently it would seem to be a reasonable recommendation for general use. Its chief drawback is that to estimate the 95^{th} percentile (or .95 quantile) requires a sample size of comparable cases of at least 19. But without such a sample size, an estimate of the 95^{th} percentile would have to rely on drastic assumptions about the upper tail of the distribution of awards. It is precisely such an assumption that the theory of quantile estimation seeks to avoid.

Conclusion:

This note proposes to calculate a remittitur using quantile estimator to estimate the 95^{th} percentile of a sample of comparable cases.

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