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**BARRIER FUNCTIONS FOR DESIGN CENTERING  
AND SECURITY ENHANCEMENT**

by

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BARRIER FUNCTIONS FOR DESIGN CENTERING AND SECURITY ENHANCEMENT

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ABSTRACT

This paper does two things. First, it relates the notions of power system security to the concepts of design centering. A benefit is a family of metrics for measuring security. Such metrics are badly needed - security is too ephemeral to lend itself readily to measurement. Second, the paper develops a method for solving centering problems and thereby, extremizing the aforementioned security metrics. The method uses Barrier functions to translate centering problems into unconstrained minimization problems. It appears to be simpler and more widely applicable than methods based on inscribing norm bodies in approximations to feasible regions. Snail test problems indicate that the Barrier-function method has a performance edge over inscribed norm body methods. We expect, this edge to increase with problem size.

INTRODUCTION

Dispatching

The dispatching problem for power systems is concerned with minimizing operating costs subject to constraints reflecting security concerns and permissible power flows. These flows are embodied in a network model. In its simplest form this model assumes that the transmission lines have constant efficiencies. The resulting strategies are called "Economic Dispatch." When more detailed network models are used the strategies are called "Optimum Power Flows" (OPF's).

Dispatching algorithms are used in two contexts. The first is in Control Centers where their function is to readjust the outputs of generators to track load (power demand) in real time on a minute-by-minute basis. The second context is that of off-line planning studies.

Network sizes are of the order of 1000 nodes and 100 generators.

Security

The term security as used here refers to a power system's ability to meet its loads without unduly stressing its components or allowing the voltages at key nodes to stray from pre-

scribed ranges. At issue are the effects of random events (disturbances, contingencies) such as the sudden failure of a generator or transmission line. Such events produce sudden changes in the network's configuration. Flurries of transient activity follow. Some apparatus may be overloaded and taken out of service resulting in further configurational changes. When steady state conditions are eventually reestablished the system may suffer from a deficiency of generation or transmission capacity and be unable to supply all its loads till repairs are made.

Security is more a vague notion than a well defined attribute. There are, as yet, no good and simple metrics for measuring it. There is, however, a lot of ongoing research. Its range is too broad for us to do it justice here. Instead, we will confine ourselves to mentioning some key ideas.

Most approaches to assessing security begin with the assembly of a list of possible contingencies. It is usual to assume that these can occur only "singly" that is, if a contingency occurs then all the repairs it necessitates will be completed before another contingency occurs. The most straightforward way of proceeding further is to rank the contingencies by some importance criteria and calculate the complete responses of the network to each of the more important contingencies. These responses are then checked against some acceptability criteria. The more unacceptable responses there are, the less secure the system is. We will refer to this process as a Simulation Based Assessment (SBA).

Calculating power system transient responses can take many hours on a big computer. Therefore, it is impossible to use SBA's in real-time dispatching and inconvenient to use them in off-line studies. An alternative is to forego response calculations in favor of some more easily obtainable clues or fragments of information\*. If well chosen, these clues or fragments might contain almost as much useful information as the entire responses. The fragments can be organized into algebraic constraints reflecting security concerns and the constraints appended to dispatching problems. We will refer to such approaches as Fragmentary Information Heuristics or FIH's.

## Paper Organization

Some of the latest thinking on FIH's leads to a formulation that Stott (1) has called "Contingency Constrained Dispatch." We will discuss this formulation in the next section. Then we will suggest some improvements based on centering concepts, develop a security metric and describe an algorithm for extremizing its value.

### CONTINGENCY CONSTRAINED OPTIMUM POWER FLOWS

#### Some Notation

Consider a network with initial configuration  $N_0$ . Suppose there are  $J$  possible contingencies. Assume that they can occur only "singly."

Let:  $N_j$  be the configuration in which the network finds itself immediately after the occurrence of the  $j$ -th contingency

$Y$  be a vector of network input and state variables

$Y_j$  be a steady state solution for the  $j$ -th configuration. That is,  $Y_j$  is a solution of the power flow equations (Kirchoff's laws for steady state conditions on  $N_j$ ):

$$G_j(Y) = 0, j = 0, 1, \dots, J \quad (1)$$

$H_j(Y) \geq 0, j = 0, 1, \dots, J$  be constraints that must be met for configuration  $N_j$  to be feasible. (These constraints are equipment capacities and operating limitations that if violated will cause  $N_j$  to switch to another configuration)

$R_j$  be the associated feasible region for  $N_j$ , i.e.:

$$R_j = \{Y | H_j(Y) \geq 0\} \quad (2)$$

$f(Y)$  be the cost function chosen as the objective in dispatching

#### Steady State Solutions as Indicators of Security

The general form of the contingency constrained OPF\* is:

$$\begin{aligned} \text{(CCOPF):} \quad & \text{Min } f(Y_0) \\ & Y_0 \\ \text{st} \quad & Y_j \in R_j \quad \forall j \end{aligned}$$

\*Most other Fragmentary Information Heuristics lead to problems of the same structure but with the  $R$ 's and  $Y$ 's defined slightly differently.

The rationale is as follows: If  $N_j$  has a feasible steady state solution (if  $Y_j \in R_j$ ) then one may at least entertain the hope that the transients precipitated by the  $j$ -th contingency will settle to this solution. In contrast, if  $Y_j \notin R_j$  then  $N_j$  cannot exist for long and the configuration that follows it will almost certainly be less desirable (since it is unlikely that failures or unscheduled removals of equipment will leave the system better off).

#### Some Remarks on Problem Size

For a 1000 node network,  $Y$  is approximately of dimension 2000. If we consider 9 contingencies, (CCOPF) would have about 20,000 equality constraints (for the steady state solutions of the 10 configurations). 99 contingencies would result in 200,000 equality constraints. In addition, there would be the inequality constraints delineating the feasible regions,  $R_j$ . Clearly, (CCOPF) is a formidable problem. Procedures for its solution are still very much in the research stage.

#### A SECURITY METRIC BASED ON CENTERING

In this section we will propose some improvements to the ideas underlying Contingency Constrained OPF's.

Suppose that the  $j$ -th contingency has occurred so that the network is in configuration  $N_j$ . Let  $\Gamma$  be the trajectory followed by  $Y$ . The network will remain in configuration  $N_j$  if  $\Gamma$  remains within  $R_j$  (see Fig. 1) The chances of this happening are increased if  $Y_0$  and  $Y_j$  are inside  $R_j$  and far away from its boundaries.

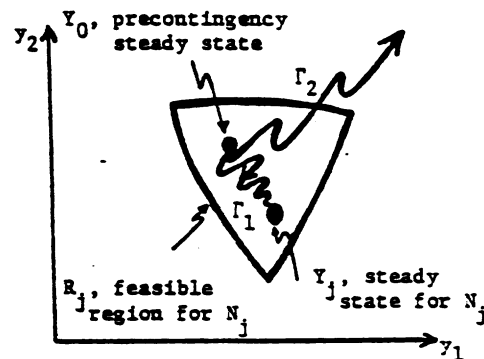


Fig. 1. Trajectory  $\Gamma_1$  stays within  $R_j$ . If it occurs configuration  $N_j$  is preserved. Trajectory  $\Gamma_2$  goes outside  $R_j$ . If it were to occur,  $N_j$  would cease to exist when  $\Gamma_2$  crossed the boundary of  $R_j$ .

This line of reasoning suggests a family of security metrics,  $S_{pq}$ , of the form:

$$S_{pq} = \sum_{j=1}^J a_j \left[ \left\| Y_j^* - Y_0 \right\|^p + c_j \left\| Y_j^* - Y_j \right\|^q \right] \quad (3)$$

where  $p$  and  $q$  are arbitrary positive constants whose values the user selects

$Y_j^*$  is a center of  $R_j$  (a point in some sense as far away from the boundaries of  $R_j$  as possible)

$a_j$  is a constant reflecting the relative importance of the  $j$ -th contingency. The probability of this contingency occurring would be a good choice for  $a_j$ .

$c_j$  is another weight

The metric,  $S_{pq}$ , reflects distance from a goal - being at the centers of all the possible feasible regions and thus, being as secure as possible. However,  $S_{pq}$  gives no indication of absolute security (the most secure point could be very secure or quite insecure). However, the use of  $S_{pq}$  will, as we shall see, allow certain computations to be done in parallel. Later, after an operating point has been obtained for the system, the values of other metrics reflecting distance from the boundaries (rather than from the centers) can be calculated.

#### Usage

We can use  $S_{pq}$  in two ways. First, we can directly extend (CCOPF) to give:

$$\begin{aligned} & \text{Min } f(Y_0) \\ & Y_0 \\ \text{st } & S_{pq} \leq \theta \end{aligned}$$

where  $\theta$  is some tolerance. Alternatively we could use a multiobjective formulation (2), (3) in the conflicting objectives of cost and  $S_{pq}$ . In either case we would need tools to solve

large centering problems to find the  $Y_j^*$ 's.

These tools will be discussed in the next section.

#### BARRIER FUNCTIONS FOR CENTERING

Much of the ongoing work on centering concentrates on inscribing the largest possible norm body in an approximation to the feasible region (4). We need less complex approaches for our problem and will develop them with the aid of transformation procedures (penalty and barrier functions). Originally, transforma-

tion procedures were created to replace constrained problems with sequences of unconstrained problems (5). Used in this way, they have several disadvantages. However, as we shall see, they are eminently well suited to the centering problem.

Consider the connected region  $R_j$  (defined as before) and the unconstrained minimization problem:

$$(P1): \quad \text{Min}_Y b_j(Y)$$

where:  $b_j$  is a Barrier function given by:

$$b_j(Y) = \sum_i \phi_i(h_{ij}(Y))$$

$\phi_i(t)$  is defined continuously on the interval  $t > 0$  such that  $\phi_i(t)$  decreases monotonically with increase in  $t$  and  $\phi_i(t) \rightarrow \infty$  as  $t \rightarrow 0_+$

$h_{ij}$  is the  $i$ -th element of  $H_j$ , the vector function that delineates  $R_j$

The effect of the  $\phi_i$  terms is to push the solution of (P1) towards the interior of  $R_j$ . That is, they have a centering effect.

By selecting different barrier functions one would obtain different solutions to (P1), i.e., different centers. It is not immediately clear if these could be made to coincide with the centers obtained by inscribing norm bodies in  $R_j$ . However, the "barrier-function-centers" are just as intuitively appealing as the "norm body centers."

In solving (P1) with an iterative algorithm one must either begin with a feasible point or push the barriers back to include the starting point. This is equivalent to using barriers

$$b_j' = \sum_i \phi(h_{ij}(Y) + \alpha_i)$$

and an expanded region:

$$R_j' = \{Y | h_{ij}(Y) + \alpha_i \geq 0\}$$

where the  $\alpha$ 's are chosen to be large enough so the starting point is in  $R_j'$ .

Alternatively, one could use functions intermediate to penalty and barrier functions, i.e., functions that slope in the right direction but are finite at finite boundaries of  $R_j$ .

## AN ALGORITHM

In this section we will outline an algorithm for maximizing security in the sense of minimizing  $S_{pq}$ . First we will find the centers (defined in the barrier function sense) of the feasible regions. Next we will perform the simpler task of determining a precontingency, steady state operating point that minimizes  $S_{pq}$ .

The centers of the feasible regions will be found with the aid of a variable metric method developed by Powell (6) and extended by Talukdar and Giras (7).

To begin with,  $Y$  is partitioned into two vectors:  $U$ , a vector of decision variables (whose steady state values the dispatcher can assign) and  $X$ , a state vector. We denote by  $U_j$ ,  $X_j$  the steady state values of  $U$  and  $X$  for configuration  $N_j$ . We assume that we can solve the equality constraints, (1), for  $X_j$  given  $U_j$ . We denote the center of  $R_j$  by  $(U_j^*, X_j^*)$ .

For each  $j$ , i.e., for each contingency and its feasible region:

1. Select a starting value,  $U_j^0$
2. Set  $k = 0$
3. Find  $\gamma_j^k(U_j)$ , an approximation to  $X_j$ , the steady state solution for  $N_j$ . This approximation is found (7) by applying Newton's Method to the steady state relations

$$G_j(U_j^0, X_j) = 0$$

The tolerance on the approximation is tightened (i.e., more Newton iterations are used) as  $k$  increases. Initially, only one iteration may be needed but later two or three may be required.

4. Replace  $X_j$  with its approximation in the inequality constraints for configuration  $N_j$  to get:

$$H_j(U_j, \gamma_j^k(U_j)) \geq 0$$

5. Form the Barrier function,  $b_j$ , for the inequality constraints. One way is to set

$$b_j(U_j) = \sum_i 1/h_{ij}(U_j, \gamma_j^k(U_j))$$

6. Obtain  $U_j^{k+1}$ , an improved estimate to  $U_j^*$ , by taking a step with a descent algorithm, e.g. (6), applied to the problem:

$$\text{Min}_{U_j} \{b_j(U_j)\}$$

7. Check for convergence of  $U_j^{k+1}$ . If it has converged set  $U_j^* = U_j^{k+1}$  and solve the equality constraints for  $X_j^*$ . Otherwise, set  $k = k+1$  and return to 3.

Notice that the above process can be performed in parallel for each configuration.

Once the centers of the feasible region have been determined we can proceed to the problem of finding the most secure operating point. That is, the operating point that comes as close to the centers as possible in the sense of minimizing  $S_{pq}$ . The associated problem is:

$$\begin{aligned} \text{(SP):} \quad & \text{Min}_{Y_0, Y_1, \dots, Y_j} \{S_{pq}(Y_0, Y_1, \dots, Y_j)\} \\ \text{st} \quad & G_j(Y_j) = 0 \quad \forall j \\ & U_j = U_0 \quad \forall j \end{aligned}$$

The second set of equality constraints in this problem specifies that the steady state values of the decision variables for each configuration be the same as those for the initial configuration. The reason is that the dispatching algorithm is too slow to do anything about those decision variables in the time frame of the contingencies and their associated transients.

In problem (SP) we can replace the state variables with approximations to give:

$$\text{(SP2):} \quad \text{Min}_{U_0} \{S_{pq}([U_0, \gamma_0(U_0)], [U_0, \gamma_1(U_0)], \dots, [U_0, \gamma_j(U_0)])\}$$

where  $\gamma_j(U_j)$  is an approximation to  $X_j$  obtained by applying Newton's Method to

$$G_j([U_j, X_j]) = 0$$

As in step 3 of the centering algorithm, the approximations are tightened as the overall solution is approached (7).

Problem (SP2) is unconstrained because we assumed that we wanted to center w.r.t. all the inequality constraints and replaced them with barrier functions. Actually, one may wish to preserve some of the inequality constraints in which case they would appear, unchanged, in (SP2). If there are many of them one would be well advised to use a powerful constrained optimization procedure, such as (6), for solving (SP2).

## SOME TEST RESULTS

We have applied the algorithm to the 5 and 30 bus power system examples described in (7) to both maximize security and find the Pareto frontier of tradeoffs between it and operating cost. Space limitations prevent us from presenting the results here.

We have also tested the centering part of the algorithm on a number of small examples, two of which are shown in Figures 2 and 3. In both cases the center obtained by using barrier functions coincided with the Euclidean-norm-body-center. However, the Barrier function approach worked faster. For instance, for the problem of Fig. 3 the Barrier approach took 7 function evaluations and 21 gradient evaluations while a norm body approach (4) took 15 function evaluations and 45 gradient evaluations.

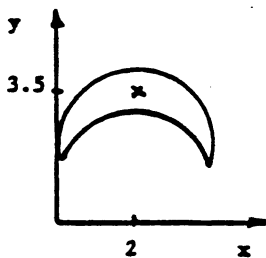


Fig. 2. The feasible region is delineated by the constraints:

$$(x-2)^2 + (y-2)^2 \leq 4$$

$$(x-2)^2 + (y-1)^2 \geq 4$$

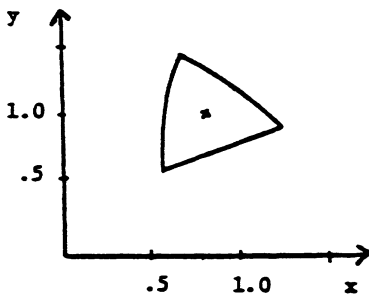


Fig. 3. The feasible region is delineated by:

$$e^{-x+1}((y-1)^2+1) \leq 1.5$$

$$e^{x-2y+1} \leq 1.5$$

$$x^2 + y^2 - 1 \leq 1.5$$

This example was taken from (4).

## CONCLUDING REMARKS

The ideas reported here are from work in progress. They seem to be basically sound but still need a good deal of investigation and development.

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