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ABSTRACT

This paper formulates the minimum utility calculation for a heat exchanger network synthesis, problem as a "transportation problem" from linear programming, thus allowing one to develop an effective interactive computing aid for this problem. The approach is to linearize cooling/heating curves and partition the problem only at potential pinch points. Thus formulated both thermodynamic and user imposed constraints are readily included, the latter permitting selected stream/stream matches to be disallowed in total or in part.

By altering the formulation of the objective function, the paper also shows how to solve a minimum utility <u>cost</u> problem, where each utility is available at a single temperature level. A simple one dimensional search procedure may be required to handle each utility which passes through a temperature change when being used.

Extending the partitioning procedure permits the formulation to accommodate match dependent approach temperatures, an extension needed when indirect heat transfer through a third fluid only is allowed for some matches.

Introduction

Two independently written manuscripts (Cerda and Westerberg (1979) and Mason and Linnhoff (1980)) were merged and significantly extended to produce this paper. Both had discovered the "transportation model" for the minimum utility calculation for the heat exchanger network synthesis problem.

In the last 13 years many papers have appeared which deal with the synthesis of cost effective heat exchanger networks to integrate chemical processes thermally. In the recent process synthesis review paper of Nishida et al (1981) 20% of the 190 papers listed are on this topic alone.

As pointed out in that and other earlier papers, a most significant contribution of this entire work is the insight by Hohmann (1971) and later by Linnhoff and Flower (1978) which permits one to establish the thermodynamic limit for minimum required utilities to accomplish all the specified heating and cooling for such a problem. This thermodynamic limit involves locating "pinch" points within such networks where a minimum approach temperature exists. This minimum utility limit is almost always attained by the better network designs found for such problems and thus is a very worthwhile target. Unfortunately, industry has typically implemented solutions using substantially more than the minimum required utilities — often 30% or more in excess (Linhoff and Turner (1980)).

In this paper we show how to formulate the minimum utility calculation as a classical "transportation problem" from linear programming, a problem for which very efficient solution algorithms exist. The approach is to linearize heating and cooling curves to any desired degree of accuracy. We will argue that only corner points and end points can be potential temperature "pinch^{11} points. The temperatures of these points partition the streams into substreams for which one can readily write the requisite thermodynamic constraints. Extending insights by Grimes (1980) and Cerda (1981), we show that many — often half or more — of the points can be eliminated as pinch point candidates, substantially reducing the size of the transportation problem which must be solved.

The designer frequently wishes to preclude matches being allowed between certain streams, and it would be useful for him to discover if these constraints seriously affect the minimum utility requirements for a process. The. transportation problem formulation readily accommodates such constraints. The designer may have several utilities available at different temperature levels and costs. Simple adjustment of the costs used in the objective function and some minor added partitioning permit one to find a solution having a minimum total utility cost. We also show that each utility which is not available at a constant temperature level may require an added one dimensional search.

Lastly we show how to generalize the temperature partitioning task if one wishes to assign a different minimum allowed approach temperature to each stream/stream match. Limiting the transfer of heat between any two streams to indirect transfer through a third fluid requires this type of calculation. The number of partitions can grow enormously. If the partitioning is not done completely, the calculation will yield an upper bound (and probably a good one) to the required minimum utilities.

The paper gives an effective algorithm to find a first, and often optimal, solution to the transportation problem, one which can be implemented by hand if desired. It also describes the classical transportation algorithm by Dantzig (1963), principally to show where in the solution "tableau" one discovers the thermodynamic pinch point(s) for all the problems described above.

The first two authors extend the use of transportation like models to aid in synthesizing minimum utility/minimum match networks in parts 2 and 3 of this paper.

Problem Definition

We are given a set of hot and cold process streams among which we wish to exchange heat to bring each from its inlet to its target temperature. In general additional heating and cooling in the form of utilities are needed to accomplish this task. Since the utilities used are costly, we wish to calculate the least amount needed which can then serve as a target to the design of a heat exchanger network to accomplish our task.

We assume sufficient information is given for each stream to allow us to calculate a heating or cooling curve for it as it passes through the exchanger network. We are given inlet and outlet temperatures; we must guess the likely pressure trajectory. Then we calculate enthalpy along this trajectory, plotting T (ordinate) versus enthalpy flow (flow rate times specific enthalpy, abscissa). Also given for the problem is a minimum ΔT driving force ΔT_{min} to be allowed in any heat exchange.

Example Problem

We shall illustrate the ideas throughout this paper with the example four stream problem whose data are given in Table 1. Figure 1 shows the cooling and heating curves for each of these streams.

Cold	T interval	Apparent S	**F C_P**	**Q - F C_P AT**
Stream, c_1	100-140	1.0	2.0	80
	140-180	1.1	2.2	**88**
Flow - 2	180-190	5.0 } 2 **phase**	10.0	100
	190-200	**4.0** } **region**	8.0	80
	200-250	0.5	**1.0**	50
			Total	398
Cold				
Stream, c_2	140-180	1.3	3.9	156
	180-225	1.5	4.5	202.5
				358.5
Flow • 3				
Hot	$300-200^+$	0.6	0.6	-60
Stream, h_1	200^+-200"	* **(phase** change)	•	-100
	200"-140	1.2	1.2	-72
Flow - 1				232
Hot				
Stream, h_2	280-100	0.8	3.2	-576
Flow - 4				

Table 1. Data for 4 Stream Example Problem. A T_{min} is 20° for the problem.

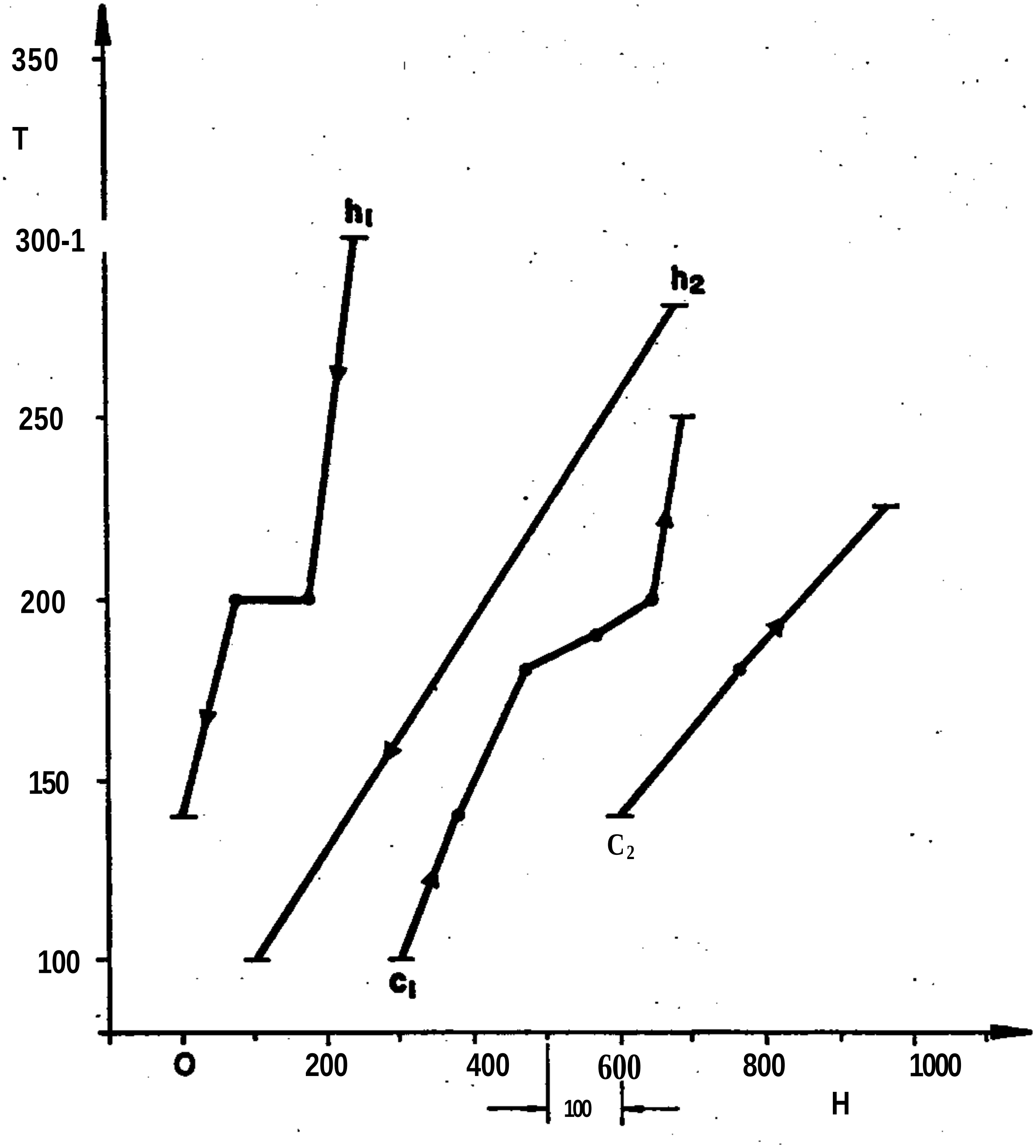


Figure I

Cooling Curves for Streams h_1 and h_2 and Heating Corves for Streams c_1 and c_2 for Example Problem.

Solution

Hohmann (1971) presented a straightforward method to solve the minimum utility problem. He developed two curves — one the "super cooling curve" formed by merging the curves for all the hot process streams and **one the** "super heating curve" which merges the curves for all the cold process streams. On a T versus enthalpy flow diagram, these curves can be moved arbitrarily to the right or left and thus placed so the super cooling curve is below the super heating curve. The cooling curve is moved toward the heating curve until there is a minimum vertical distance occurring between the curves which equals the minimum allowed AT driving force the designer will permit in any heat exchanger. Figure 2 illustrates for our example problem with AT$_{min}$ = 20^{o}. This point of rainimum AT is termed a "pinch point" for the problem. By construction the curves are in exact heat balance where they are vertically above and below each other. If these super streams existed and were placed in a countercurrent heat exchanger, the temperatures of each side would follow the opposing trajectories shown. The pinch point precludes further exchange. The heating of the cold streams yet to be done, if any, represents the minimum hot utilities needed and the cooling of the hot streams yet to be done, minimum cold utilities. Both are identified in Figure 2.

Linnhoff and Flower (1978) note that no heat can pass across the pinch for a minimum utility solution. One can prove this observation easily by examining Figure 2. Suppose one attempted to use heat from the merged hot process stream above the pinch to heat the merged cold stream below the pinch. Such a move would bring the merged cold, stream below the pinch closer to the hot at the pinch, causing one to have to move the cold stream to the left to regain AT$_{inin}$ as the driving force at the pinch.

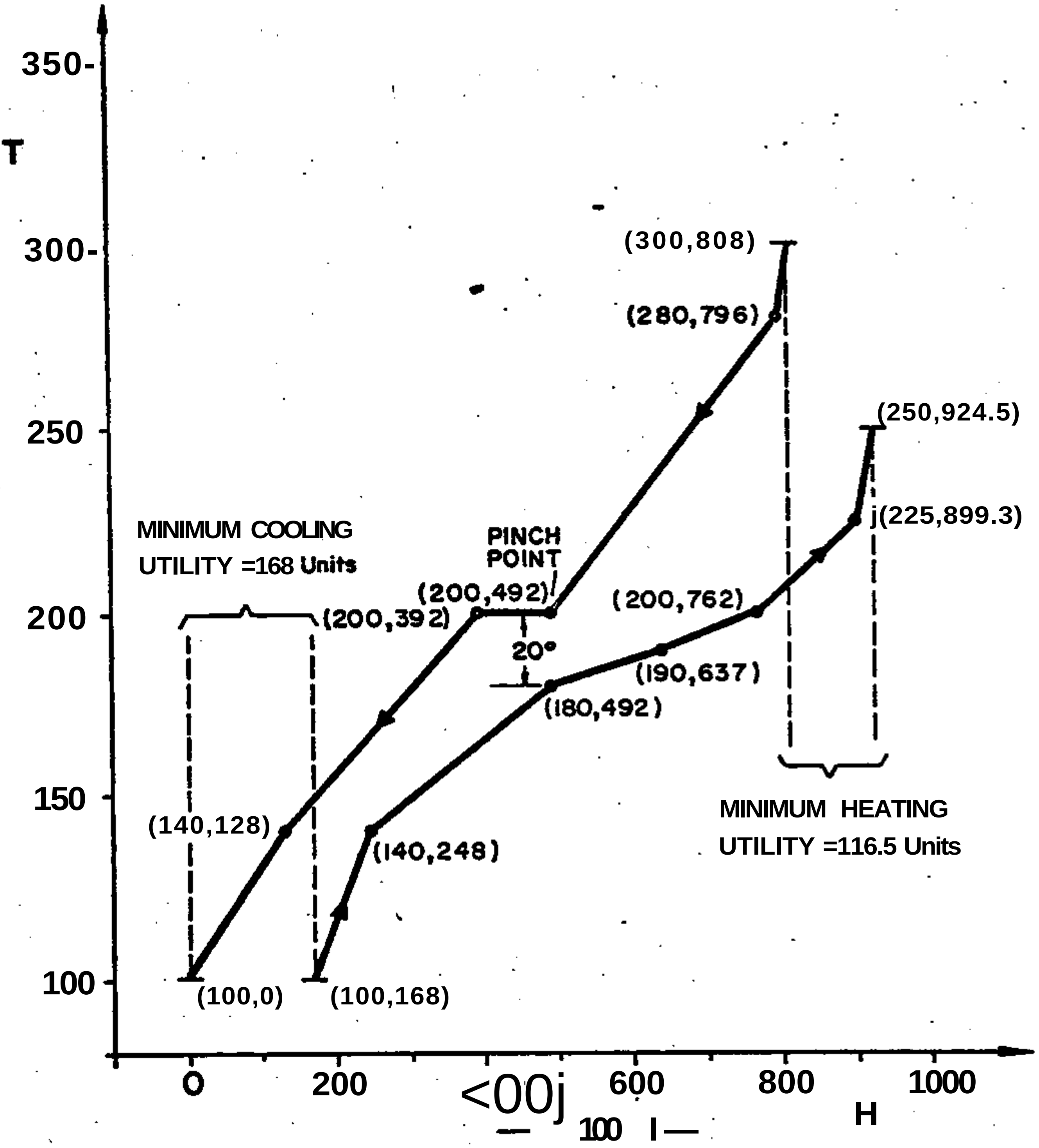


Figure 2

Merged Heating and Cooling Super Curves for Example Problem.

By moving the streams in this manner relative to each other, one must be increasing the requirement for utilities.

We wish to automate and generalize the Hohmann procedure. Using their problem table formulation, Linnhoff and Flower (1978) show how to solve the minimum utility problem if each stream is represented by segments of constant heat capacity* versus temperature. We take their ideas as our starting point, describing the task to be accomplished from a somewhat different viewpoint. This viewpoint will give us significant problem reduction insights.

We too shall assume that the cooling curve for each stream can be approximated by straight line segments. This assumption is actually very realistic and can always be made in a safe manner by linearizing below the curve for hot streams and above for cold streams. Keeping the linearized curves at least AT_{min} apart will guarantee" the actual streams are that far apart. Most streams, even those undergoing phase change, require only a few segments to approximate their heating or cooling curves reasonably.

Co/ineji Point and Pinch Points*

If the streams are all linearized as described, then the super curves of Hohmann are also built up of straight line segments as we see in Figure 2. Our goal will be to locate the pinch point for any given problem. Clearly we can state the following: 1) if it exists the pinch point occurs at a "corner" point for either of the two merged super curves, 2) not all corner points can be pinch points.

Corner points are where the super curves change slope. Clearly only a corner point where one curve approaches and then breaks away from the other curve can be a pinch point candidate. We can write the following relationships to test a corner point to see if it is a candidate pinch point.

Cold Curve Corner Point j

Candidate only if
$$\sum_{i \in I^+_{c,j}} (FC_p)_i > \sum_{i \in I^-_{c,j}} (FC_p)_i \qquad (1)$$

Hot Curve Corner Point ℓ

Candidate only if
$$\sum_{i \in I^+_{H,\ell}} (FC_p)_i < \sum_{i \in I^-_{H,\ell}} (FC_p)_i \qquad (2)$$

where sets $I^+_{c,j}$, $I^-_{c,j}$ are the cold streams contributing to the merged heating curve just above and below corner point j, respectively, and sets $I^+_{H,\ell}$ and $I^-_{H,\ell}$ are similarly defined for the merged hot cooling curve at corner point ℓ.

The above tests are generalizations of an observation by Grimes (1980), where he notes that if all streams are represented as single straight lines, then only stream inlet temperatures need be considered to solve the minimum utility problem. For this case corner points along a merged super curve will only occur where streams enter or leave the curve. Where a stream enters, the above tests will keep that temperature as a candidate pinch point; where it leaves, the point will be rejected.

Cerda (1980) notes that no temperature need be considered if it is out of range, i.e. if it is along the merged stream and is more than ΔT_{min} above or below any of the temperatures spanned by the other. We can use this test to reject corner points as candidate pinch points also.

These two rejection tests will frequently eliminate about half of the corner points, which, as we shall see, will reduce our problem size to about 25% of its apparent original size, a significant reduction.

	T	$(\sum FC_p)^+$	$(\sum FC_p)^-$	Disposition
Hot	300			Reject. Too hot. (alia Cerda)
	280			Reject. Too hot.
	200^+	3.8	•"	Keep.
	200"	-	4.4	Reject.
	140	4.4	3.2	Reject.
	100	3.2	0	Reject.
Cold	100	0	2.0	Keep.
	140	2.0	6.1	Keep.
	180	6.1	14.5	Keep.
	190	14.5	12.5	Reject.
	200	12.5	5.5	Reject.
	225	5.5	1.0	Reject.
	250	1.0	0	Reject.

Table 2. Corner Points for Super Curves in Figure 2 and their Disposition as Candidate Pinch Points.

Table 2 lists all corner points for our example problem and whether they need be accepted or can be rejected as candidate pinch points. Note only one hot and three cold corner points out of 13 total need be kept.

Problem Partitioning

The problem can now be partitioned at the candidate pinch point temperatures. The hot candidate points are first projected onto the cold super stream and vice versa. As noted by Linnhoff and Flower (1978), this projection is offset by AT_{min}, thus the hot candidate pinch points project down AT_{min} onto the cold stream and the cold project up AT_{min} onto the hot stream. Table 3 lists the hot stream and cold stream intervals created by this partitioning.

Interval	Hot Stream			Cold Streams		
1	j»	to	120°	[illegible]	to	100°
2	120°	to	160°	100°	to	140°
3	160°	to	200*	140°	to	180°
4	200⁺	to	•	180°	to	•

Table 3. Temperature Intervals Created by Partitioning at Candidate Finch Points. AT_{min} = 20°. Temperatures not underlined are caused by projection from other stream.

Note we project the cold stream candidate pinch point at 100° onto the hot stream at 120°, the 140° onto the hot at 160° and so forth.

We now show that this partitioning is done as described to permit us to write thermodynamic constraints for our problem. We note that heat can be exchanged among and within the intervals as follows.

1) Hot interval is above (hotter than) the cold interval —- Heat can always be transferred from a hot stream at a hotter interval to a cold stream at a lower one. For example, heat in interval 4 for a hot stream can always transfer to interval 3 or below for the cold stream.

2) Hot interval is below (colder than) cold interval — No heat can transfer from the hot interval to the cold one because the hot interval is everywhere too cold, except for perhaps the hottest point which, after removal of an infinitesimal amount of heat is more than ΔT_{min} colder than every temperature for the cold interval. For example heat in hot interval 3 cannot transfer to cold interval 4.

3) Hot interval is the same as the cold interval — Heat can always be transferred between the merged streams within the same interval to the extent it is available as needed, i.e.

$$q \leq \text{Min (heat available, heat needed)}$$

for the interval with equality always possible.

Isolate the interval and move the cold super stream to be below the hot until it pinches. From the manner in which the intervals are defined, the hot end or the cold end of the interval must be pinched. At the pinch end, both curves are vertically aligned — i.e. both start together at the pinch. Moving away from the pinch, the curves are in heat balance vertically and everywhere at least ΔT_{min} apart. Thus one can transfer heat until one or the other of the two curves is satisfied. QED.

Transportation Problem Formulation

We can now model the minimum utility calculation as follows. Let c_{ik} be cold stream i in interval k and $h_{j\ell}$ be hot stream j in interval ℓ. Define a_{ik} as the heat needed by c_{ik}, which can be readily calculated after partitioning. For example the heat needed by cold stream c_1 in interval 3 (140^o to 180^o) is a_{13}= 88 units (see Table 1). Similarly define $b_{j\ell}$ as the heat available from stream $h_{j\ell}$. Let $q_{ik,j\ell}$ be the heat transferred from h.. to c... The q., .. are to be calculated. Assume there are L intervals (equals 4 for our example problem - see Table 3).

Let there be C-l cold process streams and H-l hot process streams in our problem. Then the cold utility will be the C^{th} cold stream and the hot, the H^{th} hot stream. Assume the heat needed by the cold utility is at the lowest level in the problem. Also assume it is in sufficient quantity to satisfy all the hot process stream cooling needs, i.e. we require

$$\begin{matrix} H-l & L \\ & \\ j-1 & X-l \end{matrix} \qquad (3)$$

Assume similarly that the hot utility is available at the highest level and is in sufficient quantity to satisfy by itself all the cold stream heating requirements.

$$SL' \underset{i-l}{\overset{C-l}{X}} \underset{k-l}{\overset{L}{X}} 'l_k \qquad (4)$$

Lastly assume the problem is in heat balance overall.

$$\begin{matrix} & & C-l & L & & & & H-l & L & & \\ *\sim* & + &) & / & *n. & " & *_m & + &) & / & {}^{b},£ \\ Cl & & « & « & ik & & HL & fcj & & " & JJ» \\ & & i-l & k-l & & & & j-1 & X-l & & \end{matrix} \qquad (5)$$

The above simply say, choose both $a\hat{}._{l}$ and $b-.._{HL}$ to be large numbers. Then adjust them so the entire problem is heat balanced.

We can now write our transportation model for the minimum utility problem as follows.

$$\underset{q_{ik,j\ell}}{\text{Min}} \quad \sum_{i=1}^{C} \sum_{k=1}^{L} \sum_{j=1}^{H} \sum_{\ell=1}^{L} c_{ik,j\ell}\, q_{ik,j\ell} \qquad (6)$$

Subject to

$$\sum_{j=1}^{H} \sum_{\ell=1}^{L} \qquad\qquad k\text{-}l_t 2,\text{---}_f L \qquad (7)$$

$$\sum_{i=1}^{C} \sum_{k=1}^{L} q_{ik,j\ell} = b_{j\ell} \qquad \begin{matrix} j=1,2,\cdots,H \\ \ell=1,2,\cdots,L \end{matrix} \qquad (8)$$

$$q_{ik,j\ell} \geq 0 \quad \text{for all } i,j,k \text{ and } \ell \qquad (9)$$

where

$$c_{ik,j\ell} = \begin{cases} 0 & \text{for i and j are both process streams and match is allowed, i.e. } k \wedge X_{\#} \\ 0 & \text{for i and j are both utility streams } (1 - C_9\, J - H). \\ 1 & \text{only i or only j is a utility stream} \\ M & \text{otherwise, where M is a very large (think infinity) number.} \end{cases} \qquad (10)$$

Equation (7) says that the heat required by cold stream i in interval k must be satisfied by transferring heat from somewhere among the hot streams. Equation (8) is a similar statement for hot stream j in interval £______it must give up its heat somewhere to other streams. (9) says all heats transferred must be nonnegative, that is no heat can flow from a cold stream to a hot one. (6) is the objective function to be minimized, with cost coefficients defined by (10). No cost is associated with an allowed process stream - process stream match or from the hot utility to the cold utility (this latter match would never be implemented in a network). Utility-process stream matches are given a nominal cost per unit of heat in the match so they will be used only if the free matches do not solve the problem. Thermodynamically disallowed matches are given a near infinite cost to preclude their being part of any optimal solution.

The above is a classical transportation problem for which a very efficient solution algorithm exists (see Dantzig (1963) for example). It is usually visualized by setting up a "tableau", as illustrated in Figure 3 for our example problem. The columns are for the hot substreams and the rows for the cold substreams.

Each entry is a "cell" which can contain 3 numbers. The upper right is the cost coefficient, $C_{IK,Jfs}$ ... The bottom number is the assigned $<l\text{-}\wedge_{IK,J*}$ •• f°r tke match; the upper left we will discuss momentarily. For each row a_{ik} is given to the far left and for each column b_{j*} to the very top. We place the hot utility column (labeled H) to the far right and the cold utility column (labeled C) to the bottom. Cells have been marked "I" if they are thermodynamically infeasible, i.e. if k > for entry $q_{IK,jt}$ ...

Thz JnJutiaL SoJjJutJjon

The transportation problem algorithm requires an initial feasible solution. If we are careful, this initial solution is frequently already optimal. A row and column reordering algorithm has proved very effective to help get a good initial solution. Simply reorder all process stream rows such that the number of infeasible cells decreases from top to bottom and all process stream columns such that they decrease from right to left. For ties, place the higher temperature cells toward the top and to the left. Figure 3 is ordered in that manner. If only thermodynamic constraints are involved, tie breaking is unnecessary.

	V	∞		200+o		160°		120°	-00	
		60	256	148	128	24	128	6/\	10,000	
aik	Hot Cold	h14	h24	h13	h23	h12	h22	h21	H	pik
230	00 c14	1° 60	1° 170	M |	M |	|M |	M |	M |	1	0
202.5	c24 180°	h	|» 86	M |	M |	|M |	M |	M |	1 116.5	0
88	'13	1"	| 0	0 88	0	|M |	M |	M |	1	-2
156	c23 140°	1°	|°	0 60	0 96	|M I 1	M |	M |	1	-2
80	c12 inn0	1°	|"	0	0 32	1° 24	0 24	M |	1	-2
10,051.5	C - oo	h	|'	1	1	h	1 104	1 64	0 9883.5	-1
	γjℓ	0	0	2	2	2	2	2	1	

Pinch

Pinch

FIGURE 3

Transportation Problem Tableau for Example Problem.
Tableau shows Initial feasible (and optimal) solution.

Once reordered, we apply the following slightly modified "Northwest Algorithm" to get our initial feasible solution.

1. Start in the upper left (northwest) corner.
2. Move from left to right in the uppermost row to the first column having a cost less than M, finding the cell corresponding to row c_{ik}, column $h_{j\ell}$.
3. Assign $q_{ik,j\ell} = \mathrm{Min}(a_{ik}, b_{j\ell})$ to the cell.
4. Decrement both a_{ik} and $b_{j\ell}$ by $q_{ik,j\ell}$.
5. Cross out the row or column which has its heating or cooling requirement a_{ik} or $b_{j\ell}$ reduced to zero.
6. Repeat from step 2 until all rows and columns are deleted.

In Figure 3, we start with row c_{14} and column h_{14}. We assign $q_{14,14} = 60 = \mathrm{Min}(230, 60)$ to the cell and cross out column h_{14}. a_{14} is now equal to 170(= 230-60). Starting again at step 2, we identify row c_{14} again and column h_{24}. We assign 170 units to this cell, cross out row c_{14} and reduce b_{24} to 86. The rest of the tableau is filled out the same way. Note row 2 has to go all the way to the hot utility to complete its need for heat.

If only thermodynamic constraints are involved and if ΔT_{min} is the same for all matches, then one can readily demonstrate the above is repeating the same calculations needed for the problem table of Linnhoff and Flower (1978). Thus the initial solution is always optimal for such a problem. We can read off the minimum utility requirements as 116.5 units of heating and 104 + 64 = 168 units of cooling, which agrees with the Hohmann calculation we did in Figure 2. The 9883.5 units of heating by the hot utility and assigned to the cold utility is a "dummy" number and is ignored.

To locate the pinch most easily, we should first discuss how to solve a transportation problem, which we shall do momentarily.

We might note the reduction of the problem size resulting from only including the temperatures which are potential pinch points when partitioning. The partitioning of Linnhoff* and Flower (1978) would have included every corner point in the problem, i.e. hot temperatures 300°, 280°, 200°, 140° and 100° and cold temperatures, 100°, 140°, 180°, 190°, 200°, 225° and 250°. The combined set of hot temperatures (after projecting the cold onto the hot) gives the following list: 100°, 120°, 140°, 160°, 200"^{o} , 200*^{o} , 210°, 220°, 245°, 270°, 280° and 300°. A corresponding list 20° colder exists for the cold streams. For our example problem we would create a tableau having 13 cold substreams plus the cold utility and 18 hot substreams plus the hot utility to give a tableau with 14 x 19 ss 266 cells versus (see Figure 3) a tableau with 48 cells. Here the reduced problem is only 18% the size of the full one. As we shall see a calculation is needed for every cell if we need to check for optimality so the reduction is real in terms of work required for solving.

Non Thzxmodynamlc Constraints

With a mathematical formulation for the minimum utility problem, we can add certain types of constraints trivially. One can readily . add constraints to preclude the exchange of heat between selected process streams, either in part or totally. For example a match may be undesirable because the two streams would be unsafe if mixed accidentally because of a leak in an exchanger. Other reasons for rejecting a match are that the streams may be physically too far apart and both vapor, thus requiring expensive piping to get them together, or the exchange may be a problem for control or startup.

The engineer could first solve the minimum utility problem with only thermodynamic constraints. He could then selectively preclude matches or part matches and discover the impact, on the minimum utilities required. If the impact is too high, he can reconsider the validity of the constraint.

To add user imposed constraints, we' repeat the same procedure we used earlier. The difference is that we can only merge hot or cold streams over the temperature ranges where they are treated identically. Also the initialization algorithm is no longer guaranteed to yield an optimal solution. We illustrate these ideas by example. We shall solve our example again but this time disallowing heat exchange between c_1 and c_2 above the bubble point (180°) of c_1 . To be safe we disallow any exchange above 175°.

We now must treat c_1 and c_2 differently (and thus unmerged) above 175°. The corner points are found for c_1 and c_2 merged up to 175° then found individually for c_1 and c_2 above that point. Also we must treat h_1 and h_2 differently—here we could limit this different treatment to above 195°. The resulting candidate pinch points will be found to be: cold—100°, 140°, 175° and 180° and hot—200°+, and 195°. Projecting the temperatures gives the final hot stream partitioning temperatures of -∞, 120°, 160°, 195°, 200°+, and ∞ Cold stream partitioning temperatures are 20° colder. Figure 4 is the solution tableau for our problem, showing the first feasible solution found by using the modified Northwest Algorithm. Three cells are disallowed over those not permitted because of thermodynamics, and they are marked with a "D" and given a cost of "M". If this solution is optimal, and we shall see in a moment that it is, then minimum hot utilities are increased from 116.5 to 170 (by 53.5 units). Cold utilities, by heat balance, must also increase by 53.5 units, which they do. Thus the restriction causes a 37.6% increase in total utilities used. One can now ask if it is worth that increase.

		∞		$200^{+\circ}$		195°		160°		120°	-∞	
	$b_{j\ell}$	60	256	106	16	42	112	24	128	64	10000	
a_{ik}	Cold \ Hot	h_{15}	h_{25}	h_{14}	h_{24}	h_{13}	h_{23}	h_{12}	h_{22}	h_{11}	H	ρ_{ik}
230	c_{15} ∞	0 0 - 60	2 M D	2 M I	2 M I	2 M I	2 M I	2 M I	2 M I	2 M I	1 1 + 170	0 Pinch
202.5	c_{25} 180°	-2 0	0 0 202.5	0 M I	0 M I	0 M I	0 M I	0 M I	0 M I	0 M I	-1 1	-2
11	c_{14}	-2 0	0 M D	0 0 11	0 M D	0 M I	0 M I	0 M I	0 M I	0 M I	-1 1	-2
19.5	c_{24} 175°	-2 0 +	0 0 - 19.5	0 0	0 0	0 M I	0 M I	0 M I	0 M I	0 M I	-1 1	-2
77	c_{13}	-2 0	0 0 + 34	0 0 - 43	0 0	0 0	0 0	0 M I	0 M I	0 M I	-1 1	-2
136.5	c_{23} 140°	-2 0	0 0	0 0 + 52	0 0 16	0 0 42	0 0 - 26.5	0 M I	0 M I	0 M I	-1 1	-2
80	c_{12} 100°	-2 0	0 0	0 0	0 0	0 0	0 0 80	0 0	0 0	0 M I	-1 1	-2
10051.5	C -∞	-1 1	1 1	1 1	1 1	1 1	1 1 + 5.5	1 1 24	1 1 128	1 1 64	0 0 - 9830	-1
	$\gamma_{j\ell}$	0	2	2	2	2	2	2	2	2	1	

FIGURE 4

Transportation Problem Tableau for Example Problem where No Heat Can Be Exchanged between c_1 and h_2 above 175°.

We need to decide if the solution is optimal. To do so we give the steps for solving a transportation problem <u>without justification</u>. The algorithm will be seen to be very simple, and we shall show how to find the pinch points in the result.

To solve a transportation problem, given a first feasible solution, proceed as follows.

1. We must first establish for each row a "row cost^{11}, P_{ik} and for each column a "column cost", $Y_{j\ell}$. We show row and column costs along the right side and bottom of the tableau. Start with the top row and assign it a row cost of zero. (We set y_{15} to zero.)

2. For any row c_{ik} for which a row cost is already assigned, find an active cell (q_{u} .. > 0) in that row. Assign a column cost Y.. for the column corresponding to the active cell, such that

$$Y_{j\ell} + \rho_{ik} = c_{ik,j\ell} \qquad (11)$$

(Set Y_{15} to 0 so 0 + 0 = 0.)

3. Repeat step 2 for assigned columns to set row costs.

4. Repeat steps 2 and 3 as needed until all row and column costs are set. (Set Y_H to 1, set P_C to -1, set Y_{23} to 2, etc.) Row and column costs resulting using this algorithm are shown in Figure 7. Continue as follows.

5. For every cell (or at least every inactive cell) write

$$\xi_{ik,j\ell} - P_{ik} + Y_{j\ell} \qquad (12)$$

into the upper left corner of the cell.

6. If no cell exists where $£_{ik,j\ell} > C_{ik,j\ell}$, exit. The current tableau is optimal. Otherwise continue.

For our example, the tableau is found to be optimal. The steps needed if not optimal are as follows.

7. For any cell with $£_{ik,j\ell} > C_{ik,j\ell}$, find a loop of active cells which this cell completes by moving alternatively down rows and across columns. Such a loop will exist.

(Pretend cell (c_{24}, h_{15}) is a candidate cell. A loop would traverse the cells (clockwise) ($c_2$4> h_{15})t (c_{15}, ^$_{15}$)> (°i5» ^{H}^» ^^{c}» ^{H}^» (°> ^{h}23^' (c_{23}, h_{23}), (c_{23}, h_{14}), (c_{13}, h_{14}), (c_{13}, h_{25}), (c_{24}, h_{25}), and again (c_{24}, h_{15}).)

8. Mark the first cell (i.e. cell (c_{24}, h_{15})) with a "+", the second cell with a "-", the third with a "+", alternating "+" with "-" around the loop. Note one must have an even number of unique cells in such a loop so, when we reencounter the first cell, it will again be marked with a ^{lf}+".

9. Find $q_{\cdot f}$ .. of minimum value associated with a "-^{11} cell. Call it q . .
^{M}ik,jt Tnin

(For our example q_{min} = Min(60, 9830, 26.5, 43, 19.5) = 19.5.)

10. Add q_{min} to all "+" cells and subtract it from all ^{lf}-^{tf} cells. Doing this step assumes each row and column remains in heat balance, that our initially inactive cell is now active and another cell (the one originally set at q_{min}) i$_s$ now inactive — breaking the loop.

We add 19.5 to all the "+" cells and subtract it from all "-" cells. Cell (c^$_{-4}$, h«$_{25}$) becomes inactive.

We would now have a new and better solution to our problem (if we had had to continue past step 7). Repeat from step 1, establishing row and column costs again, etc.

*J<ien*jJ!ying, the. Pinch Point*

The row and column costs identify the pinch points for our problem. If row cost p_{ik} is different from $p_{i,k+1}$ for stream i then the minimum utility problem pinches at the temperature which partitions the problem between cold intervals c_{ik} and $c_{i,k+1}$« Similarly we can spot the pinch points by looking at the column costs, $Y_{j\ell}$.

For the problem in Figure 4, the pinch points are between c_{15}/c_{14} (i.e. at cold stream temperature 180°)• The change from 0 to 2 in Y., for $h_{ic}//h_{iy}$ gives the same result — a pinch at hot temperature 200°.

The proof follows from observing as we did earlier that no heat crosses the pinch point. All $C_{..}$., are zero for active matches among process streams so where one is zigzagging back and forth among hot and cold substreams, the corresponding p_{ik} and $y_{j\ell}$ become the negative of one another and do not change value. The pattern is broken at the pinch point. One cannot carry the value of a row or column cost directly across the pinch because no heat crosses the pinch. The row and column costs on the other side of the pinch point must be generated by first passing through the cell in the lower right belonging to the interchange of heat between the hot and cold utilities. One then sets these row and column costs by zigzagging back up to cells just below the pinch. Passing through this zero cost cell changes the p_{ik} and $y_{j\ell}$ by the sum of the costs assigned to the utility/process stream matches (here 1 + 1 = 2).

The row and column costs have been developed in Figure 6 also; the pinch is between levels 4 and 3, corresponding to a cold stream temperature of 180° and hot of 200*, the same as above.

Minimum Utility Cost Problem

Often several different hot and cold utilities will exist in a problem. For example steam may be available at several different pressures and thus at several different condensing temperatures. Aside from cooling water one may also have brine or one may propose to "raise" steam with excess heat at prescribed pressures. We can deal directly with this problem as a transportation problem if all heating and cooling can be treated as occurring at point temperature sources — i.e. each operate at a single temperature. Condensing steam is readily handled, therefore. Unfortunately cooling water is not a point source in terms of temperature as it is heated when it passes through the process. We shall first assume point temperature sources for all utilities and show how to set up a minimum utility cost problem as a transportation problem. We shall then discuss how the problem must be solved for nonpoint sources.

For (temperature) "point utility sources", add the temperatures for the utilities to the candidate hot and cold pinch points used to partition the problem. Change the costs C., ., 1K,JX> for utility-process stream matches to reflect the per unit cost of the utility involved. When initializing using the Northwest Algorithm, always use the least expensive utility possible when utilities are needed. The "left to right" search along a row and top to bottom search along a column will work if the least cost utilities are listed to the left or to the top of the more expensive ones. Otherwise, solve as before.

We note that the actual C M ikfjX used for utility costs need only set a rank ordering among the hot utility stream costs or the cold utility stream costs. Assume utility streams cost us money. Therefore, for a minimum cost utility problem, one will never use more than the minimum total amount of utilities found in our earlier formulation. The only question is how to divide the utility heating and cooling requirements among the utilities available. Clearly we will use the least expensive hot utility until no more hot utility is needed or until it can no longer be used thermodynamically——i.e. until it pinches with the cold process streams to which it is supplying heat. Being the least expensive is all we need to know, not its exact cost. The argument should now be obvious.

Thus we need only assign relative costs to utilities, with these relative costs usually reflecting the temperature level. Hotter hot utilities are generally more expensive than colder ones; similarly, colder cold utilites are generally more expensive than hotter ones. The peculiar case of "raising^{19} steam is handled by still assuming that the steam raising "utility^{11} costs money but less than cooling with cooling water. If the cost is made less than zero (i.e. reflects making a profit) the problem solution may no longer involve minimum total utility usage, and if it does not, the solution will in fact be unbounded. One will have unfortunately set the costs so it is profitable to turn a hot utility into a source of heat to generate steam, an unlikely real world situation or at least one superfluous to the problem at hand.

Figure 5 shows the tableau for our example problem if we have two sources of heating——one at 205° and one at 300$^+$ degrees. Only thermodynamic constraints are considered. Mote, two pinch points exist, one at (205°/185°) and one at (200$^+$/180°). Grimes (1980) observed that there must be one pinch point for each utility past the first in a minimum utility cost problem.

Also note that we use 63 units of the more expensive utility, H_2, and 53.5.of the less expensive colder utility, H_1. Costs assumed for H_1 and H_2 were only to rank order them; i.e. H_1 has a cost of 1 and H_2 of 2.

	V	∞	205°		200°+o		160*		120°	-∞			
		S7	240	3	16	148	128	24	128	64	5000	5000	
a_{ik}	Hot / Cold	his	"25	h_{14}	h_{24}	h_{13}	"23	h_{12}	h_{22}	h_{21}	H_1	H_2	p_{ik}
180	c_{15} ∞	1° 57	0 123	M	M	M	M	M	M	M	M	1 2	0
180	c_{25} 1^5°	1 «	lo 117	M	M	M	M	M	M	N	M	1^2 63	0 Pinch
150	c_{14}	1°	0	0 3	1° 16	M	M	M	M	M	1i 31	1^2	-1 Pinch
22.5	c_{24} 180°	1°	1°	o	°	M	1."	M	M	M	1 22.5	2	-1
88	c_{13}	1°	1°	0	«	° 88	1°	M	M	M	1	1^2	-3
156	c_{23} 140°	1°	1°	10	°	° 60	1 0 96	M	M	M	1	1^2	-3
80	c_{12} 100°	1°	1 »	o	0	»	1° 32	° 24	° 24	M	1	1^2	-3
10051.5	C	11	11	11	1	'	1^1	1	1 104	M 64	0 4883.5	1° 5000	-2
	$\gamma_{j\ell}$ -∞	0	0	1	1	3	3	3	3	3	2	2	

Pinch Pinch

FIGURE 5

Minimum Utility Cost Solution Example.

Hj^ 1B available at 205° and Is less costly than H,.

*A/on Point Temp&i&tusie. ConsvOiairub**

A utility stream which provides its heat or cooling in total or in part as sensible heat or is mu It i component and passes through a phase change can significantly complicate the minimum utility cost solution procedure. Let us speak specifically about cooling water as our example utility of this type. Normally onB uses cooling water by heating it *ttam* some available inlet temperature (say 37°C) to an allowable exit temperature (say 50°C). The problem arises if cooling can be done at 37°C but 50°C is too hot. Then one must use more cooling water until its exit temperature is low enough to do the cooling needed. In the limit of a point-temperature source, one would use an infinite amount. If the cooling water cost is proportional to the amount used, then cost is affected by its exit temperature.

Two flows are significant for such a* utility: 1) the minimum flow which results if the entire temperature range (from 37°C to 50°C) can be used and 2) the maximum, flow such that the cost per unit of cooling makes it more expensive than a colder utility, say brine.

For such a utility, we can establish the flow per unit of heat as:

$$F/Q \text{ « } V\ J_{T_{in}}^{T_{out}} C_p\, d9$$

and for each we can plot cost versus T_{out} as shown in Figure 6 where C^ is the cost per unit flow. If T_{out} for cooling water falls below T^1, then one should switch to brine as a coolant.

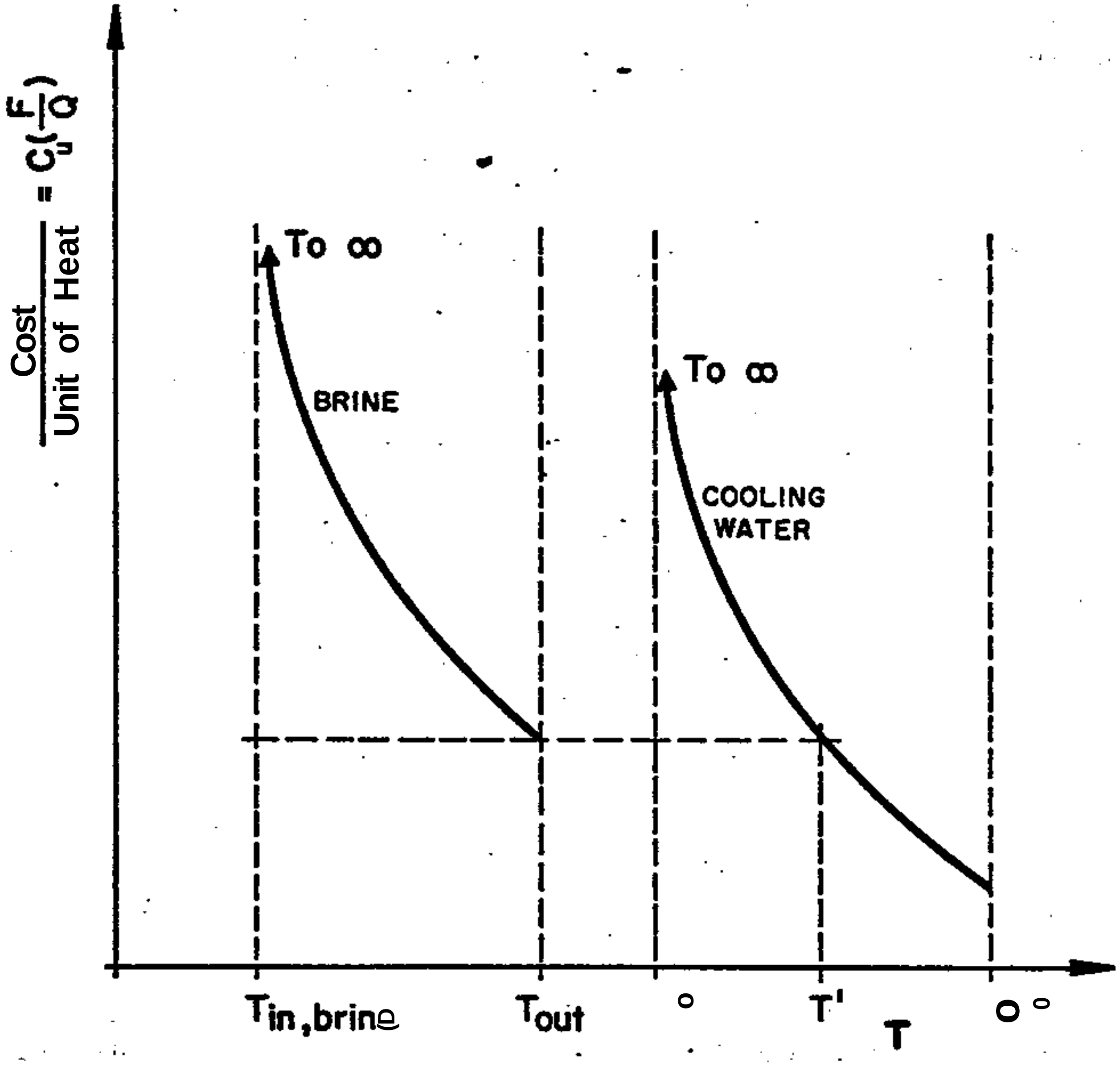


Figure 6

Cost Versus Outlet Temperature for Brine and Cooling Water per Unit of Cooling.

To solve a minimum utility cost problem with a non point-temperature utility, first solve the minimum utility cost problem as if its temperature everywhere were its inlet temperature________i.e. treat as a point-temperature source utility. Use as its cost/unit of heat, the cost resulting from allowing it to heat or cool through its maximum temperature range -— i.e. its least cost/unit of heat.

Next set the flow to that at which it ceases to be less costly than another utility — the flow corresponding to exit temperature T^f in Figure 6 for cooling water. Treat the utility as a required process stream with this flow, entering at its inlet and leaving at T^f; resolve the minimum utility problem to see the impact when using such a process stream. If the use of other utilities does not increase, then this utility should be used as a heating or cooling source in a minimum utility cost solution. If the usage increases for the other utilities, then it should be rejected as a utility; in our example, brine should become the cooling utility instead. The reason is obvious; its flow would have to increase beyond its maximum economic flow to be part of a minimum utility usage solution. It is thus too costly per unit of heating or cooling supplied.

Repeat the above for every non point-temperature source utility to select the active utilities. Then, one at a time, we have to set their flowrates as follows. The flows are bounded between $F_{m\,m}$. (entire tempera- $_{*}$ ture range is used) and F^{max}, another utility becomes less expensive. Figure 7 shows how the minimum utility usage should change versus flowrate for such a utility. Change the flow to its minimum, again treat as a required process stream and solve the minimum utility usage problem. If the usage does not increase, the minimum flow is the solution. Otherwise we have to search for the flow, F (see Figure 7). Increasing the flow

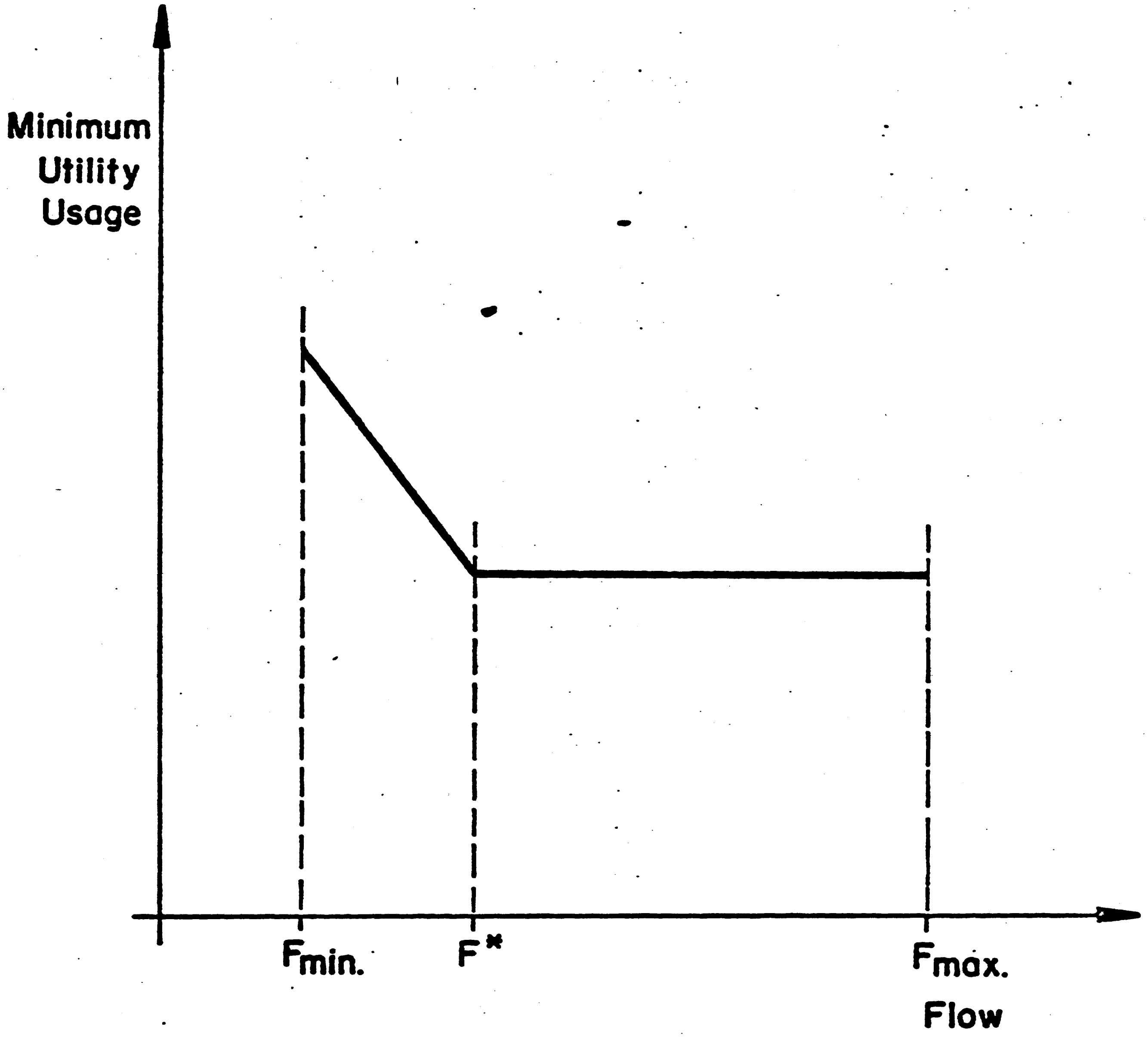


Figure 7

Effect of Varying Flowrate for Non Point Temperature Utility on Minimum Utility Usage.

will decrease total other utility usage for the problem up to flow F ; it will then have no effect. We seek therefore the flow F as our minimum utility cost solution.

The search should be done at low flows, e.g. at F_{min} and F_{min} + AF. Assuming a linear behavior these two solutions can be used to project to F^*, our next guess. The search can use a one dimensional secant method together with an interval reducing method; it will be rather quick. Fortunately each utility of this type can be dealt with separately, a significant problem decomposition.

Match Dependent AT_{min}

We now consider the last topic to be covered in this paper: how to solve the minimum utility usage or cost problem when AT_{min} is not the same for every match allowed. We shall discover first why this problem is an important one and then how to solve it.

Suppose we have two streams we will not allow in the same exchanger because a leak would lead to too dangerous a situation or because the streams are both vapor and far apart, leading to very costly piping requirements. We may want to know the impact of using a third fluid as illustrated in Figure 8 as a heating/cooling loop between them on utility usage.

We see that, if such a fluid could be found, it will exchange heat in two exchangers, thus doubling the required AT_{min} needed between our two original process streams. We could thus model the minimum utility usage, where some streams can only exchange heat indirectly, by simply doubling the required AT_{min} for them. Note there is a significant impact on exchanger area required over a direct exchange at the larger AT, essentially increasing it by a factor of 4 since the driving force is halved and two exchangers are needed.

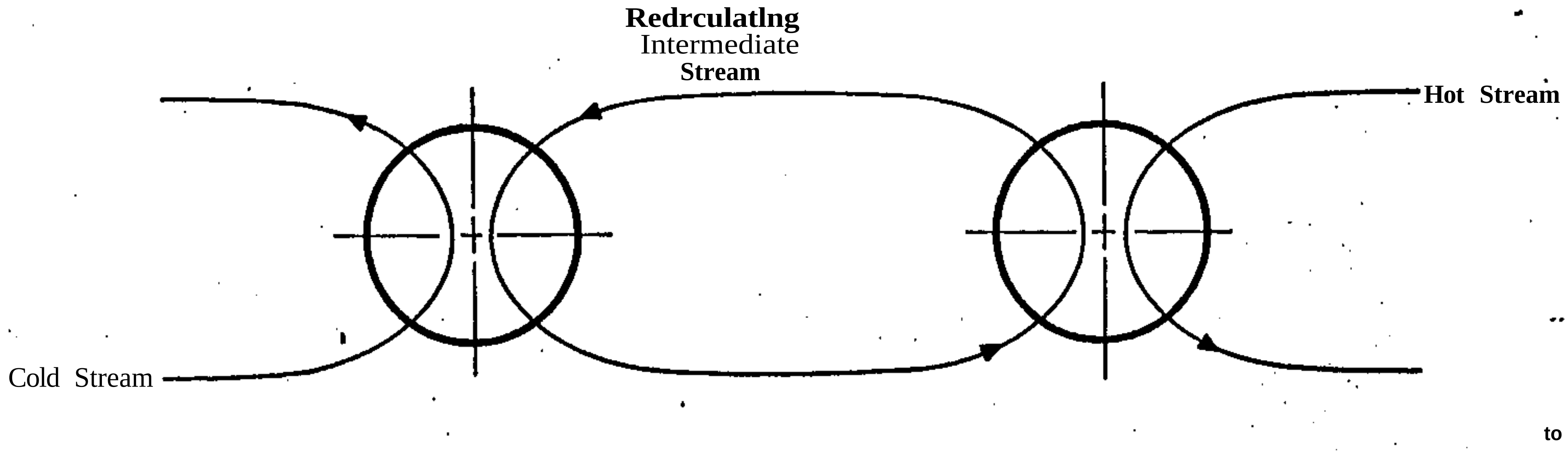


Figure 8

Indirect Transfer of Heat between a Hot and a Cold Stream.

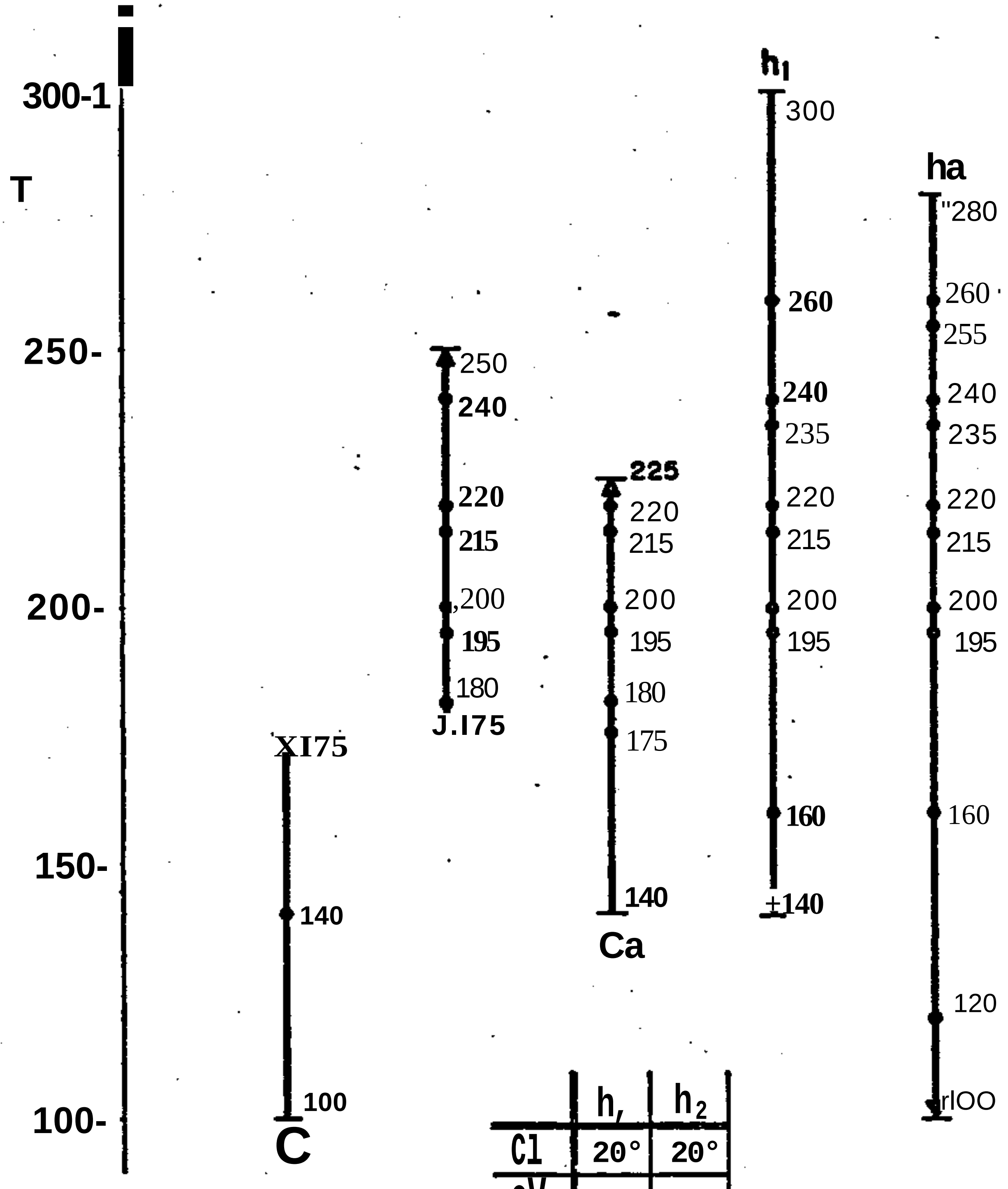
To solve we shall discover we only need to change the step where we project hot stream candidate pinch points onto cold streams and vice versa. The consequence is not negligible as we will create an enormous increase in the number of partitions for our problem.

To explain is best done by example. Suppose we resolve our problem where c_t and h_2 were allowed to exchange heat only below 175°. We how state that they can indirectly exchange heat above the cold stream temperature of 175°. We shall model this possibility by requiring a 40° minimum driving force above 175° for c_1 between streams c_1 and iu. The candidate pinch points for the streams are almost the same as before: c_1 — 100°, 175°, 180°; c_2 —— 140°; i^ —— 200°$^+$; and in addition h_2 —— 280° since h_2 is now less than 40° (= 2AT $_{m\cdot m}$) hotter on entry than c_1 is on exit (250°)•

Figure 9 shows the required temperature projections for this problem. We break c« into c_1^1 and c_1^{lf} at 175° for convenience. It is best to explain the projections one at a time. We start with the inlet temperature for c_1 at 100°. Below 175° for c_1' the AT_{mm} between it and h_2 is only 20° so we project the 100° onto h_2 at 120°.

Next consider 140° on c^« This temperature projects onto both h_1 and h_2 at 20° higher or at 160°• The 160° on both h_1 and h_2 project back onto c_1 at 140°. So much for the easy ones.

Now consider 175° on c_1''. It projects onto h_1 at 195° and onto h_2 at 215° (i.e. 40° higher, not 20°). The 195° on h_1 projects onto c_2 at 175°. The 215° on lu projects back onto c_2 at 195° which projects onto h_1 at 215° which projects onto c_1 at 195°. Unfortunately we are "off to the races" now because 195° on c_1 projects onto h_2 at 235° which projects onto c_2 at 215°, back to \i at 235° and onto c_1 at 215°. The 215° on c_1 continues: 255° on h_2, 235° on c_2, 255° on h_1, and, panting, it stops



	h_1	h_2
C_1	20°	20°
CV	20°	40°
C_2	20°	20°

Figure 9

The Temperature Projection Step for Example Problem Allowing Indirect Heat Transfer between C_i and C_2 above 175° on h_1.

since c_2 has no portion at 235° for h_1 to project onto. The 280° inlet temperature for h_2 projects as follows: 240° onto c_1, 260° onto h_1. The 200° temperature on h_1 projects as: 180° on c_1 and c_2, 180° on c_1 to 220° on h_2 to 200° on c_2, 220° on h_1 to 200° on c_1, etc.

Figure 10 shows the resulting intervals for this problem as well as an initial feasible solution. The temperature levels are identified by their ranges rather than by a second subscript as labeling them by a second subscript is no longer obviously done. Utility usage is back to the minimum found for the unconstrained problem (Figure 3) so this initial feasible solution must also be optimal. The use of indirect heat transfer has therefore returned our utility requirements back to their original minimum value.

The row and column costs (p_{IK} and Y_{j*}) are also shown so we can locate the pinch point for this problem. The p_{ik} change values when c_1 and c_o cross 180° and $y_{\cdot m}$ when $h_\cdot$ and h_o cross 200°; thus this point is the pinch point for the problem.

If one chooses to stop the projecting of temperatures back and forth, say only up to a single repeat reflection on a stream, then, if one is careful about identifying infeasible cells in Figure 10 as those for which at least a 20° driving force is not available, the solution found will be an upper bound on the minimum utility usage. This bounding follows because more partitioning leads <u>only</u> to more chances for heat exchange between streams.

Discussion

Three earlier works formulated the heat exchanger network synthesis problem as a problem involving a linear programming model () These earlier formulations led to an "Assignment" or "Set Covering" problem

Tableau for Indirect Heat Transfer Example

	$b_{j\ell}$		24	64	12	16	43	3	16	9	48	3	16	9	48	106	16	42	112	24	128	64	10000	
a_{ik}	Cold \ Hot	T Interval	h_1(260-300)	h_2(260-280)	h_1(240-260)	h_2(255-260)	h_2(240-255)	h_1(235-240)	h_2(235-240)	h_1(220-235)	h_2(220-235)	h_1(215-220)	h_2(215-220)	h_1(200-215)	h_2(200-215)	h_1(195-200)	h_2(195-200)	h_1(160-195)	h_2(160-195)	h_1(140-160)	h_2(120-160)	h_2(100-120)	H	ρ_{ik}
10	c''_1	240-250	10	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I		0
20	c''_1	220-240	14	6		I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I		0
22.5	c_2	220-225		22.5				I	I	I	I	I	I	I	I	I	I	I	I	I	I	I		0
5	c''_1	215-220		5			I		I	I	I	I	I	I	I	I	I	I	I	I	I	I		0
22.5	c_2	215-220		22.5						I	I	I	I	I	I	I	I	I	I	I	I	I		0
15	c''_1	200-215		8	7				I		I	I	I	I	I	I	I	I	I	I	I	I		0
67.5	c_2	200-215			5	16	46.5					I	I	I	I	I	I	I	I	I	I	I		0
40	c''_1	195-200					1.5	3	16	9	I	3	I	I	I	I	I	I	I	I	I	I	7.5	0
22.5	c_2	195-200									22.5			I	I	I	I	I	I	I	I	I		0
140	c''_1	180-195									22.5		I	9	I	I	I	I	I	I	I	I	105.5	0
67.5	c_2	180-195											16		48	I	I	I	I	I	I	I	3.5	0
11	c''_1	175-180													I	11	I	I	I	I	I	I		-2
19.5	c_2	175-180														19.5		I	I	I	I	I		-2
77	c'_1	140-175														75.5	1.5			I	I	I		-2
136.5	c_2	140-175															14.5	42	80	I	I	I		-2
80	c'_1	100-140																	32	24	24	I		-2
10051.5	C																				104	64	9883.5	-1
		$\gamma_{j\ell}$	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	1	

FIGURE 10

rather than a "Transportation" problem. The Assignment problem is well known and also has a very efficient solution algorithm available to solve it.

The approach was to partition each stream into small equal portions involving "Q" units of heat each, rather like slicing a carrot into small equal sized bits. Constraints preclude matches not possible thermodynamically. The solution has every hot bit of Q heat units matched to exactly one cold bit of Q units for another stream. The notion of a pinch point was not mentioned in this approach. Also the assignment problems created are very large relative to those created here, and it is unable to determine the precise minimum utility for two reasons: 1) the inaccuracies caused by the "slicing" and 2) the pinch point will likely appear in the middle of a slice. Thus, while we can advocate solving moderately large problems by hand, they cannot.

The partitioning generated here is caused by the corners in the cooling curves —— admittedly some are there due to approximating the curves, but this partitioning seems the more natural one.

The handling of utilities which are not available at a single fixed temperature for the minimum cost problem and the handling of match dependent ΔT_{min}'s are new with this work.