Beyond laissez-faire: The case of heterogeneous beliefs

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More and more trade in financial markets...

Trade seems to grow without bounds...

…with peaks before financial crises
Potentially harmful trading:

Speculative trading based on differences in beliefs

Agents take advantage of other agents having different beliefs

Non harmful trading:

Trading based on differences in insurance needs

Agents trade because of differences in endowments and taste
An illustration

\[ \begin{align*}
  t = 0 & \quad s = 1 \\
  t = 1 & \quad s = 2
\end{align*} \]
An Illustration

Laura

\[ u_L(c) = c \]

\begin{align*}
    s = 1 & : t = 0 \\
    s = 2 & : t = 1
\end{align*}

\[ 2/3 \quad 1/3 \]

\[ 0 \quad 2 \]

Welfare:
\[ \begin{cases} 
    Laura : \frac{2}{3} \times 1 + \frac{1}{3} < \frac{2}{3} \times 2 \\
    Bob : \frac{1}{3} \times 1 + \frac{2}{3} < \frac{2}{3} \times 2
\end{cases} \]
\[u_L(c) = c\quad u_B(c) = c\]
AN ILLUSTRATION

Laura

\[ u_L(c) = c \]

Bob

\[ u_B(c) = c \]

\[ t = 0 \]

\[ t = 1 \]

\[ s = 1 \quad 0 \rightarrow 1 \quad 2 \rightarrow 1 \]

\[ s = 2 \quad 2 \rightarrow 1 \quad 0 \rightarrow 1 \]

\[ \frac{2}{3} \]

\[ \frac{1}{3} \]

\[ \frac{1}{3} \]

\[ \frac{2}{3} \]
An illustration

Laura

Bob

\begin{align*}
\text{Laura:} & \quad \frac{2}{3} \times 1 + \frac{1}{3} \times 1 < \frac{2}{3} \times 2 \\
\text{Bob:} & \quad \frac{1}{3} \times 1 + \frac{2}{3} \times 2 < \frac{2}{3} \times 2
\end{align*}

Welfare:

\begin{align*}
\text{Laura:} & \quad \frac{2}{3} \times 1 + \frac{1}{3} \times 1 < \frac{2}{3} \times 2 \\
\text{Bob:} & \quad \frac{1}{3} \times 1 + \frac{2}{3} \times 2 < \frac{2}{3} \times 2
\end{align*}
Spurious Unanimity:
Mongin (1997)

Spurious Trade:
Blume, Cogley, Easley, Sargent and Tsyrennikov (2014)
Brunnermeier, Simsek and Xiong (2014)
Duffie (2014)
Gayer, Gilboa, Samet and Schmeidler (2013)
Gilboa, Samuelson and Schmeidler (2014)
We consider: general equilibrium economies with heterogeneous beliefs

We study: trade
- A trade is seen as an improvement by everybody involved
- Everybody uses her beliefs to evaluate the trade

Our problem: evaluation of trade by a regulator
- Banning of speculative trade
- Authorization of non-speculative trade

Our approach: axiomatic
- Axioms for evaluation of improvements
- Characterization of rule satisfying axioms
States

\{1, 2, \ldots, S\} – states:

- \(\Delta\) – probability distributions on \(\{1, 2, \ldots, S\}\)

\[
\begin{align*}
&\bullet s = 1 \\
&\bullet s = 2 \\
&\vdots \\
&\bullet s = S
\end{align*}
\]

Probabilities are unknown

\(t = 0\) \hspace{1cm} \(t = 1\)

Everything is planned at date \(t = 0\) and happens at date \(t = 1\).
Goods and consumers

\{1, 2, \ldots, \ell\} – goods in every state

\{1, 2, \ldots, m\} – consumers \((X_i, u_i, \pi_i)\):

- \(X_i \subset \mathbb{R}^\ell\) – consumption set
- \(u_i : X_i \to \mathbb{R}\) – elementary utility function
- \(\pi_i = (\pi_i^1, \ldots, \pi_i^S) \in \triangle\) – probabilistic belief on \{1, 2, \ldots, S\}
Goods and Consumers

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Expected utility of consumer \(i\) for consumption plan \(x_i\):

\[ U_i(x_i) = \pi_i^1 u_i(x_i^1) + \pi_i^2 u_i(x_i^2) + \ldots + \pi_i^S u_i(x_i^S) \]

Expected utility of consumer \(i\) depends on her belief \(\pi_i\)!
A economy: $E = \left\{ \left( X_i, u_i, \pi_i \right) \right\} \times \left\{ Y_j \right\} \times \left\{ r \right\}$ with $\sum_i X_i \cap \left\{ \sum_j Y_j \right\} \neq \emptyset$

Ownership of firms and total resources does not play any role.
FIRMS, total RESOURCES and ECONOMIES

\{1, 2, \ldots, n\} – firms \(Y_j\)

- \(Y_j \subset (\mathbb{R}^\ell)^S\) – production set

\(r \in (\mathbb{R}^\ell)^S\) – total resources

An economy: \(E = ((X_i, u_i, \pi_i)_i, (Y_j)_j, r)\) with

\[\sum_i X_i \cap \left(\{r\} + \sum_j Y_j\right) \neq \emptyset\]

Ownership of firms and total resources does not play any role
Allocations, trades and improvements

An allocation: \((x, y)\) with

- \(x = (x_1, \ldots, x_m)\) and \(y = (y_1, \ldots, y_n)\)
- \(x_i \in X_i\) for all \(i\) and \(y_j \in Y_j\) for all \(j\)
- \(\sum_i x_i = r + \sum_j y_j\)
An allocation: \((x, y)\) with
- \(x = (x_1, \ldots, x_m)\) and \(y = (y_1, \ldots, y_n)\)
- \(x_i \in X_i\) for all \(i\) and \(y_j \in Y_j\) for all \(j\)
- \(\sum_i x_i = r + \sum_j y_j\)

A trade: \((\mu, \nu)\) with
- \(\mu = (\mu_1, \ldots, \mu_m)\) and \(\nu = (\nu_1, \ldots, \nu_n)\)
- \((x, y) + (\mu, \nu)\) is an allocation
Allocations, trades and improvements

An allocation: \((x, y)\) with

- \(x = (x_1, \ldots, x_m)\) and \(y = (y_1, \ldots, y_n)\)
- \(x_i \in X_i\) for all \(i\) and \(y_j \in Y_j\) for all \(j\)
- \(\sum_i x_i = r + \sum_j y_j\)

A trade: \((\mu, \nu)\) with

- \(\mu = (\mu_1, \ldots, \mu_m)\) and \(\nu = (\nu_1, \ldots, \nu_n)\)
- \((x, y) + (\mu, \nu)\) is an allocation

An improvement: an allocation and a trade \([(x, y), (\mu, \nu)]\) with

- \(u_i(x_i + \mu_i) > u_i(x_i)\) for all \(i\) with \(\mu_i \neq 0\).
The regulator evaluates improvements

The regulator authorizes or bans improvements
THE regulator

The regulator evaluates improvements

The regulator authorizes or bans improvements

The regulator knows nothing about probabilities

All consumers increase their utilities by the trade...
...but consumers have different beliefs...
...so some consumers have mistaken beliefs
Evaluation map for regulator $\Gamma$ maps economies and improvements to authorization decisions

$$\Gamma(E, (x, y), (\mu, \nu)) = \begin{cases} 
0 & \text{Ban } (\mu, \nu) \\
1 & \text{Authorize } (\mu, \nu)
\end{cases}$$

What kind of properties should the evaluation map have?
MAHAUT AND TANCREDE IN PARIS

\[ \begin{align*}
    s &= 1 \\
    s &= 2 \\
    t &= 0 \\
    t &= 1
\end{align*} \]
Mahaut and Tancrede in Paris

\begin{align*}
    s = 1 & \quad u_M(c) = c \\
    s = 2 & \quad u_T(c) = \ln(c)
\end{align*}

Welfare:
\begin{align*}
    Mahaut & : \frac{2}{3} \times 1 + \frac{1}{3} \times 2 < \frac{2}{3} \\
    Tancrede & : \frac{2}{3} \ln(2) + \frac{1}{3} \ln(0.1) < 1
\end{align*}
Mahaut and Tancrede in Paris

Mahaut: \[ u_M(c) = c \]
Tancrede: \[ u_T(c) = \ln(c) \]

\[ s = 1 \]  
\[ s = 2 \]

\[ t = 0 \]  
\[ t = 1 \]

\[ \frac{2}{3} \]
\[ \frac{2}{3} \]
\[ \frac{1}{3} \]
\[ \frac{1}{3} \]

\[ \frac{2}{3} \times 1 + \frac{1}{3} \times 1 < \frac{2}{3} \times 2 \]

\[ \frac{2}{3} \ln(2) + \frac{1}{3} \ln(0.1) < \frac{1}{3} \ln(1.1) \]
**Mahaut and Tancrede in Paris**

- **Mahaut**: $u_M(c) = c$
- **Tancrede**: $u_T(c) = \ln(c)$

<table>
<thead>
<tr>
<th>$s = 1$</th>
<th>$1 \rightarrow 2$</th>
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- $t = 0$
- $t = 1$
Mahaut and Tancrede in Paris

- Mahaut: \( u_M(c) = c \)
- Tancrede: \( u_T(c) = \ln(c) \)

\[
\begin{align*}
\text{Mahaut} & : \quad \frac{2}{3} \times 1 + \frac{1}{3} \times 1 < \frac{2}{3} \times 2 \\
\text{Tancrede} & : \quad \frac{2}{3} \ln(2) + \frac{1}{3} \ln(0.1) < \frac{1}{3} \ln(1.1)
\end{align*}
\]
Mahaut and Tancrede don’t speculate

Mahaut and Tancrede have identical beliefs

Mahaut and Tancrede do not engage in speculative trade!!!

The trade should be authorized
Respecting Unanimity

Axiom Respecting Unanimity (RU)

Consider \((E, (x, y), (\mu, \nu))\).

If \(\pi_i = \pi_h\) for all \(i\) and \(h\) with \(\mu_i, \mu_h \neq 0\), then

\[
\Gamma(E, (x, y), (\mu, \nu)) = 1
\]

All rules in literature satisfy RU
Hannah and Lucas in Copenhagen

\[ \begin{align*}
    s &= 1 \\
    s &= 2 \\
    t &= 0 \\
    t &= 1
\end{align*} \]
Hannah and Lucas in Copenhagen

\[ u_M(c) = c \]

Welfare:
\[
\begin{align*}
\text{Hannah: } & \quad \frac{1}{3} \times 1 + \frac{2}{3} \times 2 < \frac{2}{3} \\
\text{Lucas: } & \quad \frac{1}{3} \ln(0.1) + \frac{2}{3} \ln(2) < \frac{1}{3} \ln(1.1)
\end{align*}
\]
Hannah and Lucas in Copenhagen

\[
s = 1 \quad s = 2
\]

\[
t = 0 \quad t = 1
\]

Hannah \quad Lucas
\[
u_M(c) = c \quad u_T(c) = \ln(c)
\]

\[
\begin{align*}
Hannah: & \quad 1/3 \times 1 + 2/3 \times 2 < 2/3 \\
Lucas: & \quad 1/3 \ln(0.1) + 2/3 \ln(2) < 1/3 \ln(1.1)
\end{align*}
\]
Hannah and Lucas in Copenhagen

- Hannah: $u_M(c) = c$
- Lucas: $u_T(c) = \ln(c)$

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$$s = 1$$

$$s = 2$$

$$t = 0$$

$$t = 1$$

Welfare:

- Hannah: $\frac{1}{3} \times 1 + \frac{2}{3} \times 2 < \frac{1}{3} \times 2$
- Lucas: $\frac{1}{3} \ln(0.1) + \frac{2}{3} \ln(2) < \frac{1}{3} \ln(1.1)$
Hannah and Lucas in Copenhagen

\[ \begin{array}{ccc}
Hannah & Lucas \\
1 \rightarrow 0 & 0.1 \rightarrow 1.1 \\
\end{array} \]

\[ u_M(c) = c \quad u_T(c) = \ln(c) \]

Welfare:
\[ \begin{align*}
\text{Hannah:} & \quad \frac{1}{3} \times 1 + \frac{2}{3} \times 1 < \frac{2}{3} \times 2 \\
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\end{align*} \]
Hannah and Lucas don’t speculate

Hannah and Lucas have identical beliefs

Hannah and Lucas do not engage in speculative trade!!!

The trade should be authorized according to RU
**Unification of markets**

Market in Paris with Mahaut and Tancrede trading: *Authorize*

Market in Copenhagen with Hannah and Lucas trading: *Authorize*
Unification of markets

Market in Paris with Mahaut and Tancrede trading: Authorize

Market in Copenhagen with Hannah and Lucas trading: Authorize

European market with all four agents trading:

Agents have different beliefs

There is no trade between agents with different beliefs

The aggregate trade should be authorized
None of the rules in the literature satisfies RNT
INDEPENDENCE OF IRRELEVANT AGENTS

Axiom Independence of Irrelevant Agents (IIA)

Consider \((\mathcal{E}, (x, y), (\mu, \nu))\) and \((\mathcal{E}', (x', y'), (\mu', \nu'))\).

If for every \(i \in \{ i \mid \mu_i \neq 0 \} \cup \{ i \mid \mu'_i \neq 0 \}\),

\[
(X_i, u_i, \pi_i) = (X'_i, u'_i, \pi'_i) \quad \text{and} \quad (x_i, \mu_i) = (x'_i, \mu'_i),
\]

then

\[
\Gamma(\mathcal{E}, (x, y), (\mu, \nu)) = \Gamma(\mathcal{E}', (x', y'), (\mu', \nu')).
\]

All rules in the literature satisfy IIA
An evaluation map satisfies RU, RNT and IIA \emph{if and only if} for all $(E, (x, y), (\mu, \nu))$, 
\[ \Gamma(E, (x, y), (\mu, \nu)) = 1. \]

Pareto Domination is the unique rule satisfying RU, RNT and IIA!
Final Remarks

Keep Calm

and

Use Pareto Optimality