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Differences in Estimation and Mathematical Problem Solving Between Autistic Children and Neurotypical Children

Sara Haas
Carnegie Mellon University

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Dr. Robert Siegler, Advisor

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Abstract

The goal of this study was to observe the differences in math learning and problem solving between typically developing children and children with autism. Strategy selection involves different networks of brain processes including executive functions to identify the problem and task at hand, and determine the most efficient strategy to use. By observing the frequencies of different strategies being used, we are hoping to learn more about the mechanisms used for strategy selection in the autistic group. Additionally, participants were shown number lines and asked to estimate where a particular number fell on the number line. No previous research has investigated the differences between autistic individuals and neurotypical individuals in numerical estimation, and this study aims to see if in fact there are differences. During the sessions, children were presented with simple addition and subtraction problems to complete, as well as the number line tasks. The researcher asked the child how he or she solved the problems. This study will help us learn more about how children with autism differ from typically developing children, so that further studies can be conducted with the aim of improving mathematical curricula. At this point there has been an observable difference noted between the two populations, and as the study continues to include more participants, we are hoping that these results will become statistically significant. As this research continues and expands, we are hoping it will serve as a basis for enhancing mathematical curricula and help individuals with autism better develop proper mathematical strategy utilization.

Differences in Estimation and Mathematical Problem Solving Between Autistic Children and Neurotypical Children

The process of mathematical problem solving has not been widely researched in the autistic population. Many parents, clinicians, and educators however, report that children with autism have difficulties mastering multiple strategies, and correctly applying these strategies. (Schoenfeld, 1992). Solving single digit arithmetic problems only requires the use of one or possibly two strategies, but as students progress to more advanced math, it is necessary to learn multiple strategies to solve problems. For example, when "carrying" in double-digit addition, there is an additional strategical step necessary to correctly solve the problem that is not needed in single digit addition. More advanced math requires combining basic knowledge of strategies, with other higher-level strategies. Learning how to execute strategies, deciding when to use them, and identifying the correct strategies are important success in math. In order to initiate curricular enhancements as well as conduct extensive research studies, there must be empirical data to support these common observations.

This study aims to verify and document that individuals with autism differ in strategy preference. We hypothesize that because of documented deficits in the ability to use executive functions, individuals with autism would choose strategies that put the least amount of strain on executive processes. Another aim is to explore how comprehensibly they are able to explain how to correctly obtain an answer to a given addition or subtraction problem. This type of explanation would require executive processes, and we would expect to see difficulty in their explanation in accordance with the documented executive deficits. Additionally, we are interested to see if there are any observable differences in number line representation.

There are several reasons why individuals with autism struggle with math in a school setting. Many times they have difficulty explaining their answer, or showing their work, which is necessary to receive full credit for a problem. Qualities of autism include narrow repetitive stereotyped activities, as well as rigidity and constriction affecting thought, memory, and actions (DeLong, 1999). Memory in autism is also item-wise, hyper-specific and rote (DeLong, 1999).. Many parents and educators have observed these qualities and taught math to these individuals in a drilling manor to obtain the most successful results of math learning. It is of course imperative for a child to have extensive practice with arithmetic problems, but memorization of arithmetic facts cannot compensate for practice applying higher-level mathematical strategies. Because individuals with autism, have difficulty with executive function tasks, educators should focus on providing practice in areas that require them to use and develop better executive functioning, as opposed to enhancing their natural inclination to use rote memorization to solve math problems.

The specific interests and rigidity of thought processes of children with autism have been observed by many educators to pose difficulty teaching different subjects. This rigidity can translate itself into a language barrier. Educators need to be very cognizant of how individuals with autism respond to teaching tools such as verbal teaching, visual teaching, and practice mechanisms. Understanding and learning effective ways to communicate is necessary to aid and improve their understanding. If an educator does not take the time to learn how an individual with autism responds to verbal cues, or constantly teaches verbally instead of providing visual or written examples, this could be a hindering factor in the ability of children with autism to learn difficult math concepts.

As an example, many parents of children with autism find that their child is able to solve problems pertaining to their interest, but when given the same "problem" in a context outside of their area of interest have immense difficulty. This rigidity is an issue that many researchers seek to improve. Some, such as Temple Grandin, argue, that educators should allow individuals with autism to focus on special interests, and others aim to widen the children's interests and abilities. Additionally, there is an emotional piece that is often unaccounted for; all of the frustration and emotional consequences when constantly working against one's inner workings can have long term consequences. The children with autism whom I have studied so far have had a tendency to count using their pen or pencil in small increments to get the correct placement of the number for the number line task. They have a method that works for them, and they do not want to use another method. When I attempted to have them guess, they exhibited much frustration, and it took "tricking" them to complete the task properly.

An additional goal of the study is to see whether or not there is a difference in numerical estimation between the neurotypical and autistic groups, and whether the accepted research of there being a strong correlation between numerical estimation and arithmetic problems can be seen in the autistic population as well. Because individuals with autism have exemplified superior visual-spatial capabilities, one would expect their number line representation to have a stronger linear representation than that of typical individuals. Studies of spatial capabilities showed that individuals with autism possess a superior ability to learn the spatial layout of the environment from a map (Caron et al 2004). Although these studies have not specifically dealt with number line estimation, they have revealed that individuals with autism have very good spatial representation, an ability they could apply to apply to number line estimation.

There are several reasons why individuals with autism would experience difficulty learning math, and problem solving. One of the most profound reasons supported by research is atypical neurological problem solving circuitry. Research in this area has been conducted through various methods including functional magnetic resonance imaging (fMRI) techniques, allowing researchers to map brain activation at specific time points, onto anatomical images of the brain. In a Tower of London problem-solving study conducted by Newman, Carpenter, Varma, and Just (2003), brain activation and functional connectivity of autistic individuals was compared with control subjects. Functional connectivity refers to the quantities and characteristics of synaptic fiber connections that run through the brain connecting different areas. An individual benefits most when these fibers efficiently conduct signals and connect areas of the brain that coordinate together. The functional connectivity theory proposed by Just and his team has aimed to evaluate connectivity within and between many brain networks to account for deficits seen in autism. The Tower of London study revealed that synchronization and connectivity between frontal and parietal areas of activation was much less in individuals with autism than in controls. This suggests a deficit in the integration of information between the frontal and parietal areas of the brain in autism. In the Tower of London task, communication between these two regions is necessary, as the parietal lobe is responsible for visio-spatial related tasks, and the frontal lobes are important for strategy execution, planning, self-regulation, and much more. According to Happe (1999), “Planning and monitoring of behavior, set shifting, inhibiting automatic actions, and holding information online in working memory are all included among executive functions”.

Another finding from the Tower of London study, was that more severe ADOS (a clinical assessment used to diagnose autism) scores were correlated with less functional connectivity. The more severe the case of autism is, the more deficits reported in problem solving, theory of mind tasks, and IQ. In our study, we aim to replicate a correlation between weakness of problem solving and severity of ADOS score or comparable assessment.

Additionally, fMRI studies have revealed that there are many short running fiber connections in the frontal regions of the brain, but very few long fiber connections extending to regions such as the parietal region, located towards the rear of the brain (Newman, Carpenter, Varma, & Just, 2003). Having an abundance of circuits that go to and from all brain regions is important to facilitate the fast direct flow of information. Also in the Tower of London study, a positive correlation was found between size of the genus area (located in the corpus callosum) and functional connectivity. The corpus callosum, located in the middle of the brain, is important for communication between the right and left hemisphere, as well as for communication between the different lobes of the brain. Fibers from the occipital, frontal, parietal and temporal regions all run through the corpus callosum, particularly through the genus and splenium areas located within it. Because a larger corpus callosum and genus area contain more fiber tracts, larger corpus callosum size correlates with higher functional connectivity in autism.

In the behavioral data from the Tower of London study, individuals with autism spent a longer time on the difficult problems than the controls. The more difficult a problem is, the more the brain recruits the frontal lobes. Individuals rely more on strategies and algorithms when solving novel complex problems, resulting in greater activation across the frontal region of the brain than with a simpler familiar problem, (Delazer, Domahs, Bartha, Brenneis, Lochy, Trieb, & Benke, 2003). Due to functional connectivity deficits, one could hypothesize that solving took longer for the autistic population because the connections between the regions necessary for problem solving, the front-parietal network, do not communicate as effectively as in controls. Easier problems that did not require as much working memory capacity and connectivity did not show performance differences across the control and autistic groups.

Many types of math (addition, subtraction, multiplication) involve many of the same areas of problem solving as in the Tower of London. The frontal parietal network is activated during almost all mathematical problem-solving tasks. Grabner, Ansari, Reishofer, Stern, Ebner, and Neuper (2007) conducted a study to evaluate the correlation between mathematical competence and parietal activation during mental calculation. Results indicated, “older children less strongly engage auxiliary cognitive function such as executive processes and working memory while solving arithmetic problems, and more strongly rely on task-specific parietal brain regions” (Grabner et al, 2007). The difference in activation between younger and older children reflects the gradual change in brain activation pattern due to learning and practice effects. When children first learn math, they must use frontal areas in the brain involved in decision-making and working memory. Initial arithmetic learning requires mastering and choosing from several strategies, such as counting on fingers, counting verbally, and using retrieval methods. Through practice and experience with different strategies, individuals improve in their ability to identify and utilize superior strategies. The decreased activation in the frontal regions of the brain, and increased activation in the parietal and temporal regions, is seen when individuals rely on retrieval for answers. Retrieval means that the child is able to come up with the answer, without actually calculating it. Instead, they access the answer from long-term memory (this area is thought to be located in the temporal lobe, adjacent to the parietal lobe). Even though many have the ability of using a retrieval strategy (activation patterns are shifted to reflect that of someone who has had a lot of experience with the math problem), individuals do not implement this strategy until they are completely confident. Geary and Brown (1991) conducted a study that revealed that perfectionists tend to use a back up method to check their retrieval and only use retrieval independently when they have no doubt. Although this method tends to take longer, it ensures accuracy in their answers.

In addition to functions of the frontal lobes already mentioned, specific areas within that lobe have been identified to be important in working memory. One study by Ischebeck, Zamarian, Siedentopf, Koppelstatter, Benke, Felber and Delazer (2006), found that stronger activation is observed within the left inferior frontal gyrus and left precentral gyrus for complex calculations in comparison to simple calculations. Similar activation patterns were observed in studies specifically investigating working memory, suggesting that these activations are due to the working memory demands imposed by complex problem solving and calculations (Ischebeck, Zamarian, Egger, Schocke, & Delazer, 2007). Simpler calculations rely on retrieval methods, whereas more complex calculations and problem solving can require multiple strategies and calculations in conjunction with retrieval. Therefore, the frontal lobes, greatly responsible for working memory, are more engaged when an individual must solve more complex calculation problems.

Adding to the importance of working memory in complex problem solving, large working memory capacity has been linked to faster information processing, and higher performance. The learning of arithmetic in particular requires sufficient working memory capacity to hold the original problem in memory, while computing the answer so that the problem and answer can be associated (Siegler, 2003). Delazer, Domahs, Bartha, Brenneis, Lochy, Trieb and Benke’s (2003) study also explained that working memory functions are crucial in complex mental calculation for “storing intermediate results, for updating intermediate results and for retrieving them.” Like Ischebeck et al.’s (2007) working memory study mentioned earlier, Delazar et al.’s (2003) study showed activation in the left inferior frontal gyrus for more complex math problems that required greater working memory space. In the study, when subjects were shown a problem that they had been trained on, there was less activation in this area than when they were shown a novel problem. This difference in activation may be accounted for by higher working memory demands for the untrained as compared to the trained condition. The importance of the left inferior frontal area in complex calculation was also stressed by Gruber, Indefrey, Steinmetz and Kleinschmidt (2001), who attributed increased activation in theses areas to linguistic and working memory functions, as well as to the application of rules.

Additionally, the retrieval process in problem solving involves carrying out the subgoal of “searching long term memory based upon the retrieval cue while actively maintaining the overall task goal (the episodic context to be recovered) and then comparing the outputs from long term memory with the task goal to determine whether the information actually satisfies the goal” (Braver, & Bongiolatti, 2002). This further explains why individuals with autism would have difficulty applying appropriate strategies and answers stored in long-term memory to novel problems. Constraints on working memory due to poor communication between parietal and frontal lobe regions, makes sub goal processing much harder for individuals with autism.

The parietal lobe likewise makes significant contributions to numerical processing. This region is active in number comparison, approximation, and counting. The region is not only number specific, but also supports “visio-spatial processes, attention and spatial working memory related to numerical processing” (Delazer et al., 2003). Individuals with autism are believed to think very visually, and have even presented parietal activation for non-visual tasks. In a study by Just (2008), individuals with autism evoked parietal activation when presented with sentences that were assessed as non-imagery. Because individuals with autism have trouble accessing the functions of the frontal lobe, these individuals likely rely heavily upon the parietal lobe, especially in mathematical tasks. Many parents and those familiar with autistic individuals report that the population tends to give explanations of experiences (such as solving a multiplication problem) using visual descriptions (Grandin, 1995). Visual explanations may be a consequence of the sparse long running fibers within the brain making it difficult to integrate visual experiences with their executive processes. In a study by Ring, Baron-Cohen, Wheelwright, Williams, Brammer, Andrew, and Bullmore (1999) that focused on frontal lobe inadequacies and reliance upon the parietal region, subjects were shown an image, and told to find the embedded figure in the image. Individuals with autism were faster than controls to recognize the embedded figure. During the embedded figures task, the group with autism probably employed an approach characterized by few demands on the parts of the frontal lobe largely responsible for working memory, and greater reliance on regions involved in object perception. To compensate for lagging working memory, the parietal region seemed to be utilized differentially to controls.

In addition to its role in visual processing, the parietal lobe has an important role for learning math. The Ischebeck et al. (2007) study aimed to identify early practice effects of arithmetic on brain activation. In one fMRI session, Ischebeck et al. (2007) were able to show changes in brain activation patterns of typical subjects when learning math. In the study, as the amount of training in complex multiplication increased, the parietal-frontal areas decreased in activation, and the parietal-temporal regions increased in activation, specifically the left angular gyrus. The parietal lobe is very important in calculation, as activation within posterior parietal occipital areas has been observed in visio-spatial imagery and is assumed to reflect visualization involved in complex calculation. (Ischebeck et al, 2007). Shifts in activation from parietal regions to temporal regions could be taken to indicate a shift from a more calculation related generation of the result to a more language-related retrieval as the result becomes rooted in semantic long-term memory. It therefore makes sense that the angular gyrus is activated during retrieval, as it lies in the parietal lobe adjacent to the temporal lobe. The angular gyrus has several functions including the understanding of metaphors, which appears to be done by associating different sensory experiences (seeing, feeling, hearing touching) with each other and with prior knowledge. The angular gyrus additionally mediates the mapping between symbols and numerical magnitudes (Grabner et al, 2007), which makes it very important in the retrieval process in well-rehearsed calculation problems. It has been contended that stronger activation in the angular gyrus for trained problems might reflect increasing reliance on automatic verbal fact retrieval as a consequence of training (Grabner et al, 2007). Studies by Ischebeck et al (2007); Delazer et al (2003); and Qin, Carter, Silk, Stenger, Fissell, Goode, and Anderson (2004), have all shown that the better learned and repeated a mathematical process is, the more the angular gyrus is activated relative to other areas. Like many other processes, the activation level of the angular gyrus shows an upside down U shaped curve over the course of learning. There is little activation when the problem is first being addressed, followed by increased activation upon mastery, and finally decreasing activation with increasing expertise. It is important to note that activation levels in the angular gyrus, are not related to the difficulty of the problem, but rather the amount of practice the individual has had with the problem (Ishcebeck et al, 2007).

As mentioned already, the angular gyrus is a structure in the brain whose activation is involved in mathematical processing. This structure is also important in the mastery of reading, and fMRI studies in this domain could have implications for improving mathematical computation. In a study by Meyler, Keller, Cherkassky, Gabrieli, and Just (2008), poor readers were given intensive supplementary instruction to improve their reading. After 100 hours of instruction, their brain activation patterns seemed to approach that of normal readers, with a large increase in activation in the angular gyrus. Additionally, the study used Diffusion Tensor Imaging techniques. Diffusion Tensor Imaging (DTI) measures the rate, flow and direction per voxel (usually 3mm cubed) of how water diffuses through white matter in the brain. DTI can also provide structural information about the brain. Through DTI measurements, the study was able to show a volume increase in fiber-tract-filled white matter, subsequent to the long-term instruction subjects received. Although this study did not involve individuals with autism, and did not study arithmetic instruction, the study shows that brain activation patterns can be improved to reflect normalized activation. Moreover, neural plasticity of the brain allows instructional interventions to increase white matter volume. Although some studies have shown abnormal pruning in an autistic brain, and difference in anatomical size throughout development (Happe, 1999), there is still potential to alter the natural course of development with effective intervention. Early intervention programs have been shown to have high success rates in behavioral improvement compared with the absence of early intervention (Eikeseth, Smith, Jahr, & Eldevik, 2002). The angular gyrus is an area shown in several studies to change activation with exposure to experience, making the potential benefits of early intervention to aid autistic children develop effective math skill sets very plausible.

Qin et al's (2004), study further emphasized the importance of early intervention by demonstrating that before adolescence there is a more evident practice effect when exposed to algebra. The fMRI study demonstrated a gradual transition from chiefly frontal lobe activation to parietal activation as the child participants gained more experience with algebraic problems. Adults learning algebra did not show the same intense practice effect in the parietal region of the brain. “The process of retrieval of a memory is the process of selecting the correct memory trace in competition with other memory traces. Evidence from the parietal region seems to suggest that children are capable of speeding up the process of identifying the correct memory trace with practice.” (Qin et al, 2004). This finding reiterates the importance of teaching mathematics as early as possible. Most curricula do not introduce algebra until students have reached adolescence, leaving very little time before the brain’s neuroplasticity significantly decreases. Imaging studies have shown that during the adolescent years, white matter volume continues to increase, and grey matter in regions such as the parietal, decreases (Paus, Zijdenbos, Worsley, Collins, Blumenthal, Giedd, Rapoport, & Evans, 1999). Because myelination (insulation of fiber tracts to enhance conduction) of fiber tracts and synaptic pruning drastically slow down subsequent to adolescence, it is advisable to teach abstract concepts such as algebra well before adolescence

To date math research in autism has primarily focused on autistic savants. Chiang and Lin (2007) note that most individuals with Asperger’s syndrome and high functioning autism tend to have average mathematical capabilities, and tend to include many gifted individuals. It is hard to use studies of savants or Chiang and Lin’s (2007) results to conclude whether or not individuals with autism struggle or succeed in math, as one focuses on an extreme of the population and the other uses standardized tests as measurements of competencies, which have been problematic in the autistic population due to the structure of the tests.

This study’s purpose is to compare effectiveness of strategies employed by typical and autistic individuals in mathematical problem solving. Based on the research presented above, we could expect the autistic individual to do just as well as control subjects on simple problems. For complex problems requiring multiple steps and strategies, we would expect the autistic group to take longer than the control group, but not necessarily expect difference in accuracy. According to Caron et al (2004), “operations that require conscious manipulation of information such as planning or switching from one mental set to another are impaired in autism”. Frustration might be observed as well, as the individual with autism would have to work much harder than the typical participant. If we were to conduct an fMRI study comparing practice effect activation patterns of typical and autistic participants, activation would likely overlap, but would take longer to shift from frontal to parietal in the autistic participant group. This would occur because of lower working memory capabilities in the frontal lobes and less connectivity within the frontal-parietal network in individuals with autism.

It is imperative that educators take into account the biological mechanisms that help individuals learn in different domains, such as math. In the United States, far more attention and repetition is given to easier problems, and little attention is given to difficult problems (National Mathematics Advisory Panel, 2008). Especially for individuals with autism, this unequal balance of teaching easy and complex problems is counterproductive. Individuals with autism require more practice applying strategies and knowledge to novel problems and situations. In theory, the more practice they have with challenging problems, the more demand they place on the frontal-parietal network to more efficiently correspond. Such practice should help strengthen synaptic firing, and possibly create more longitudinal synaptic connections. Due to the ideal neuroplasticity in a young child’s brain, the earlier children with autism receive more focused and intensive attention of this sort, the greater potential they have for improving their brain’s developmental course. This study hopes to encourage further research to support advantageous changes in mathematical curricula, as well as yielding empirical data reflecting strengths and weaknesses among autistic children, compared to controls, in mathematical strategy selection and problem solving.

Methods

Participants

Three participants with a high functioning autism diagnosis (confirmed by ADOS documentation or comparable assessment) between ages five and seven were recruited through the Watson Institute located in Sewickley, PA. Autistic participants recruited spent more time daily in a general classroom than a special education program. The control group consisted of thirteen participants without an autism diagnosis between all ages five in kindergarten. Three of the children were recruited from Pleasant Valley Elementary School, and the other ten were recruited from the Carnegie Mellon Children's school. Participants and their parents submitted a signed Carnegie Mellon IRB approved consent form. Participants received a Carnegie Mellon T-Shirt for their participation in the study.

Apparatus

For this study, minimal supplies were used. Each participant was given a generic #2 pencil with full eraser to complete the study. The study consisted of 1 sheet of paper (back and front) for mathematical problems as well as 20 number line estimation problems, each on their own piece (or side) of paper. The built in laptop camera was used to record the study session.

Procedure

Each study session lasted about 25-30 minutes (At Carnegie Mellon's children school there was a limitation of 20 minutes per child). After going over consent materials and before starting the study, each participant was asked if they were willing to complete some math problems. Upon the participant's oral agreement, the experimenter led the participant into the test room. The experimenter asked the student to sit down at the table/desk, and the experimenter then started the recording using the video camera. Each child did the arithmetic and number line tasks in a randomized order. The experimenter sat down next to the participant and put the first set of math problems face down in front of the student along with a pencil. The student was instructed to complete the problems as accurately as possible, and not to risk accuracy for speed. They were told that they could take as much time as they needed. Only one problem was displayed at a time to ensure that the child was only focusing on one task. Because many of the students had not seen math problems in the given format, the experimenter would read out the problem such as “This question is six plus four, can you get the answer for me?”. If the child still had problems, the experimenter would say, “If you start with six oranges, and someone gives you four more, how many do you have all together?” Upon the students’ completion of the first set of math problems, the experimenter informed the student that he or she could rest for a few minutes if needed, and continue when ready. Next, the experimenter put the second set of math problems face down in front of the student. The student was instructed to complete every problem as accurately as possible, and after each problem, the experimenter stopped and asked a few questions about the particular problem. These problems were also shown one at a time, as with the first set of ten math problems. The experimenter asked, “How did you get your answer?” and then “Pretend that I don’t know any math. Can you teach me how to get the answer?” After completion of all of the math problems, the experimenter told the student that he or she could take another break if needed and say when they were ready to continue. Next, the experimenter placed the 20 number line items in a stack face down in front of the participant. The experimenter explained the directions of the number line task first showing a blank number line, and saying the following “This is a special number line. This number line starts at 0 and ends at 100. Every single number between 0 and 100 is somewhere on this line. I am going to show you a number, and ask you to mark where you think that number goes on the line. Remember, you can make your mark on the line, anywhere you think that number goes on the number line." The experimenter then demonstrated putting hatch marks on various parts of the blank number line, and then instructed the participant to practice as well. Then the experimenter showed the participant the number lines with specific numbers to guess, each shown one by one. Once the student was done, the experimenter stopped recording, and congratulated the subject for doing a great job. The participant was then escorted back to their parent or class, and was given a Carnegie Mellon T-Shirt for participating in the study.

Design

In addition to the study, the autistic participants were asked to provide documentation of their autism diagnosis and IQ scores if applicable. For the actual study, on the first piece of paper there were 10 math problems in random order. These were a mixture of single digit addition and subtraction (the addition problems were grouped together, and the subtraction problems were grouped together). The second piece of paper contained another set of 10 math problems with the same characteristics. Each number line was on a scale of 0 to 100, 25cm in length, and above the number line was a typed number (different for each problem). The same mathematical problems were given to every participant for both the control group and autistic group, and the order that the two tasks were administered was randomly varied in both groups. The first set of math problems was scored for completeness and accuracy. The second set of math problems were coded based on playing back the video recording. Each answer was scored for accuracy. Additionally, for each problem, answers to the two questions mentioned in the methods section asked by the experimenter were coded from the video. The method by which they solved the math problem was recorded as the answer to question 1. Question 2 was graded by whether or not they were able to explain how to teach how to solve the problem. This question's aim is to see if individuals using methods such as recall and silent counting are able to explain how to solve the problem in a way the requires them to use executive function when explaining to the experimenter. For the number line task, a scatter plot was created for each subject graphing the actual number and the estimation. Using SPSS, the best-fit model was assessed. During the coding of the recordings, it was also noted if there were any significant fluctuations in affect at any point throughout the study: For example: frustration, confusion, or impatience.

Results

Number Line Task

An examination of the number line representations in our sample of 13 typical kindergarteners and 3 autistic children revealed a higher frequency of a logarithmic number line representation (60%) in the typical control group, than the autistic children. All three autistic children had linear number line representations with very high R-squared values of the linear equation *(M r-squared= .87)*. The average percentage of absolute error (absolute (actual-estimated)/length of number line *100)) for the typical group was 18%, where as in the autistic group it was only 1%. The statistical significance of this data cannot be determined due to the small population samples.

Arithmetic Problem Solving Task

An examination of addition and subtraction problem solving in these same populations revealed a higher frequency of recall in the autistic group than in the typical group. Across both groups, recall was used 39% of the time, followed by counting out loud (16%), counting on fingers (14%), counting in head (8%), and lastly guessing (4%). The remaining 18% of responses were not answered for various reasons including: time constraints and frustration level of child.

The autistic population used recall to answer 100% of their answers, whereas the typical group used recall 30% of the time. Recall yielded the highest accuracy rate, 97%,, followed by counting on fingers (82%), counting out loud (81%), counting in head (62%) and guessing (0%). The average percent error amongst all participants for the first set of math problems was 11.52%. For the autistic group it was .19%, and the typical group was 14.61%. For the second set of math problems evaluating strategy, the average percent error amongst all participants was 5%. For the autistic group it was .42%, and the typical group was 6.77%. Additionally, strong correlations were present between percent of answers in the first set recalled and correctly answering the second set of strategy math problems ($r=.98$, $df=5$ p-value $< .001$), the percent error of the second set of math problems ($r=.95$, $df= 5$, p-value= .020), and the total percentage of correct answers for both sets ($r=.91$, $df =5$ p-value< .001). Counting out loud ($r=.91$, $df =5$ p-value= .007), and counting on fingers ($r=.91$, $df =5$ p-value= .005), were both also significantly correlated with the total percentage of correct answers for both sets. These two strategies were the only strategies that were significantly correlated with the child being able to "teach" the experimenter how to solve the math problem ($r=.748$ $df=5$, p-values of .016, and $r=.748$ $df=5$,.014 respectively). For correct answers on the second set of problems that required feedback regarding the individual's strategies, all strategies used were significantly correlated (counting out loud: p-value< .001, counting on fingers: p-value < .001, counting in head: p-value = .005), except for guessing. Due to small sample size, it was not possible to test the statistical significance of group differences between the autistic population group and the typical population group.

Discussion

Our results are consistent with previous studies data showing that young neurologically typical children generally have a logarithmic number line representation, as well as that recall (memorization of arithmetic facts) is representative of advanced math skill and yields more accurate answers. Although the number of autistic participants in this study was very limited, it is very interesting to note that they very frequently used the most advanced method of problem solving, yet 66% of the time could not explain how to complete the problem in a way other than reciting the problem with its answer. The only two methods that predicted the ability to explain a math problem were counting out loud, and counting on fingers. Because these two methods actively engage executive processes, it would make sense that their use would be correlated with the ability to explain the processes. Moreover,, explaining is also an executive task. If one uses recall to answer a problem, more executive functioning is necessary to translate the problem from a "representation" to an explanation. Especially for those who already notably challenged in executive functioning, explaining how to solve a math problem when one uses recall could be quite difficult.

It was also very interesting that all three autistic participants had very high r-squared values for a linear model of number line representation. Because individuals with autism are so visually oriented, many would argue that individuals with autism would solve this task as a "visual" task, and not as a task considering magnitudes. It would be very interesting to further study how these individuals are solving this problem, and further more how typical individuals solve number line estimations as well.

Many variables were accounted for during the statistical analysis. Although order of administration of the two tasks (number line and math problems) was varied, it was not randomized. Statistical tests were run to check for order effects, and showed that none of the dependent variables had statistically significant order effects. We found the same results when checking for gender effects, and diagnosis effects (autism or typical). It is expected that there would be no significant differences between these two groups statistically due to the small sample size of the autistic sample population. Age was controlled for and therefore no statistical analysis to check if this was a confounding variable were necessary. Due to the limitations of our study recruitment, we were unable to obtain IQ scores, as well as autism diagnostic scores for every participant. With more testing of participants in the, future these measures will be used as controls

Conclusion

The preliminary results inspire more research to be conducted to see if the prominence of recall in our autistic sample applies to the great population, as well as the difficulty accessing executive functions to explain to a novel math problem solver how to obtain the answer. Additionally, more research needs to be conducted to determine if the autistic population does develop a linear representation of a number line at an earlier age than typical children, and if so, why this is the case. In future research it will be important to take careful note as to how each participant understands the task. Because it has also been noted that individuals with autism have difficulty switching from one mental set to another (Caron et al, 2004), it would be interesting to conduct the arithmetic part of this study having the addition and subtraction questions randomly mixed together, instead of having the two types grouped together. This might reveal a difficulty switching between tasks, recognizing one is addition and one is subtraction. This study is ongoing and will continue to examine these factors.

References

Braver, T., & Bongiolatti. S. (2002). The Role of Frontopolar Cortex in Subgoal Processing during working memory. *NeuroImage* 15, 523-536.

Caron MJ, Mottron L, Rainville C, Chouinard S (2004) Do high functioning persons with autism present superior spatial abilities? *Neuropsychologia* 42: 467-481

Chiang, H.M. & Lin, Y.H. (2007) Mathematical ability of students with Asperger's syndrome and high-functioning autism: a review of literature. *Autism* **11**, 547–556

Delazer, M., Domahs, F., Bartha, L., Brenneis, C., Lochy, A., Trieb, T., & Benke, T. (2003). Learning Complex Arithmetic-an fMRI Study. *Brain Res Cogn Brain Res* 18, 76–88.

DeLong G. R. Autism: new data suggest a new hypothesis. *Neurology* 1999; **52**(5): 911–916.

Eikeseth, S., Smith, T., Jahr, E., & Eldevik, S. (2002). Intensive behavioral treatment at school for 4- to 7-year-old children with autism: A 1-year comparison controlled study. *Behav Modif* 26, 49–68.

Geary, D. C., & Brown, S. C. (1991). Cognitive Addition: Strategy Choice and Speed-off Processing Differences in Gifted, Normal, and Mathematically Disabled Children. *Developmental Psychology* 27, 398–406.

Grabner R., Ansari D., Reishofer G., Stern E., Ebner F., & Neuper C. (2007). Individual

Differences in Mathematical Competence Predict Parietal Brain Activation During Mental Calculation. *NeurImage* 38, 346–356.

Grandin, T. (1995). *Thinking in pictures: And other reports from my life with autism*. New York: Doubleday.

Gruber, O., Indefrey, P., Steinmetz H., & Kleinschmidt, A. (2001). Dissociating neural correlates of cognitive components in mental calculation. *Cereb Cortex* 11, 350–359.

Happe, F. (1999). Autism: Cognitive Deficit or Cognitive Style? *Trends in Cognitive Sciences* 3, 216-222.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstatter, F., Benke, T., Felber S., & Delazer, M. (2006). How Specifically do we Learn? Imaging the Learning of Multiplication and Subtraction. *NeuroImage* 30,1365–1375.

Ischebeck, A., Zamarian, L., Egger, K., Schocke, M., Delazer, M. (2007). Imaging early practice effects in arithmetic. *NeuroImage* 36, 993-1003.

Just, M. A. (2008). What brain imaging can tell us about embodied meaning. *Symbols, Embodiment: Debates on Meaning and Cognition*, Manuel de Vega, Arthur M. Glenberg & Arthur C. Graesser (eds.), 75-84. Oxford University Press.

Lemaire, P., & Siegler, R.S. (1995).Four Aspects of Strategic Change: Contributions to Children's Learning of Multiplication. *Journal of Experimental Psychology: General* 124, 83–97.

Meyler, A., Keller, T. A., Cherkassky, V. L., Gabrieli, J. D. E., & Just, M. A. (2008). Modifying the Brain Activation of Poor Readers During Sentence Comprehension with Extended Remedial Instruction: A Longitudinal Study of Neuroplasticity. *Neuropsychologia* 46, 2580-2592.

National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Newman, S. D., Carpenter, P. A., Varma, S., & Just, M. A. (2003). Frontal and parietal participation in problem solving in the Tower of London: fMRI and computational modeling of planning and high-level perception. *Neuropsychologia* 41, 1668-1682.

Paus, T., Zijdenbos, A., Worsley, K., Collins, D.L.,Blumenthal, J., Giedd, J.N., Rapoport, J.L., & Evans, A.C. (1999). Structural maturation of neural pathways in children and adolescents: In vivo study. *Science* 283, 1908–1911.

Rickard, T. C., Romero, S. G., Basso, G., Wharton, C. M., Flitman, S., & Grafman, J. (2000). The calculating brain: An fMRI study. *Neuropsychologia* 38, 325–335.

Ring, H. A., Baron-Cohen, S., Wheelwright ,S., Williams, S. C., Brammer, M., Andrew, C., Bullmore, E. T. (1999). Cerebral correlates of preserved cognitive skills in autism: A functional MRI study of embedded figures task performance. *Brain* 122, 1305–1315.

Rittle-Johnson, B., Siegler, R.S., & Alibali, M. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology* 93, 346-362.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Handbook for research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Siegler, R.S. (1994). Cognitive Variability: A Key to Understanding Cognitive Development. *Current Directions in Psychological Science* 3, 1-5.

Siegler, R.S. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 289–303). Reston, VA: National Council of Teachers of Mathematics.

Snyder A. W., Paradox of the savant mind, *Nature*, 413 (2001), pp. 251-252.

Qin, Y., Carter, C.S., Silk, E.M., Stenger, V.A., Fissell, K., Goode, A., & Anderson, J.R. (2004). The change of the brain activation patterns as children learn algebra equation solving. *Proceedings of the National Academy of Sciences of the United States of America* 101, 5686–5691.