Discovering SIFIs, a Temporal Complex Approach

Alessandro Spelta  
*Catholic University of Milan, alessandro.spelta@unicatt.it*

Pablo Kaltwasser  
*University of Leuven, pablo.roviraKaltwasser@kuleuven.be*

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Discovering SIFIs, a temporal-complex approach
A tensor decomposition

Pablo Rovira Kaltwasser\textsuperscript{1,2} and Alessandro Spelta\textsuperscript{3,4}
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\textsuperscript{1}University of Leuven, Department of Economics
\textsuperscript{2}Luxembourg Central Bank
\textsuperscript{3}Catholic University of Milan, Department of Economics
\textsuperscript{4}Complexity Lab in Economics
Motivation: the macroprudential approach

- The note on “Macroprudential policy frameworks” produced jointly by the Financial Stability Board (FSB), the International Monetary Fund (IMF) and the Bank for International Settlements (BIS) in February 2011 defines macroprudential policy as “a policy that uses primarily prudential tools to limit systemic or system-wide financial risk, thereby limiting the incidence of disruptions in the provision of key financial services that can have serious consequences for the real economy”.

- Financial stability does not depend solely on the soundness of the individual components that make up the financial system but it also depends on complex interactions and interdependencies between these components.
Motivation: the macroprudential approach

The systemic risks, as defined by macroprudential policy, emerges in two dimensions:

- **the time dimension**, i.e. the rise of financial imbalances over time and the procyclicality of the financial system.
- **the cross-sectional dimension**, across firms and markets, i.e. common exposures, risk concentrations, linkages and interdependencies across entities and sectors.

Can we look at the two dimension simultaneously? Is it possible to **rank banks** according to their **systemic importance** (cross-sectional dimension) in an evolving network and to develop an **early warning** signal (temporal dimension) for the evolution of the **traded volume**?

Yes!!! If we work with **tensor**...
The temporal dimension

- Addressing to systemic risk in the time dimension requires measuring and monitoring of the financial cycle.

- "financial cycle": a systemic phenomenon whereby perceptions of value, risk, and financing constraints contribute to the slow build-up and (usually) more rapid decline in credit and asset valuations across the financial system.

- As Andrew Crockett said: "The received wisdom is that risk increases in recessions and falls in booms. In contrast, it may be more helpful to think of risk as increasing during upswings, as financial imbalances build up, and materializing in recessions."

- Thus it is useful to forecast upswings and recessions via an early warning signal.
The cross-sectional dimension

- The primary cross-sectional macroprudential tool at the international level for addressing risks at **global systemically important financial institutions** (G-SIFIs) was developed by the FSB, the Basel Committee on Banking Supervision (BCBS) and other international groupings.

- **Global systemically important banks** (G-SIBs), currently numbering 29, are selected through a methodology that refers to:
  - Size
  - Interconnectedness
  - Cross-border activity
  - Lack of available substitutes
  - Complexity

- **Interconnectedness** aims at capturing the impact that an institution’s bilateral exposures can have on other institutions in the financial network.
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- **Interconnectedness** aims at capturing the impact that an institution’s bilateral exposures can have on other institutions in the financial network.
  - It is related to the detection of the most central "player" in a network.
The interbank market is represented by a directed network with “nodes” being individual banks and each “edge” being a loan from one bank to another;
Interconnectedness of balance sheets: channels of systemic risk

- Borrowing and lending represent different sources of risk for financial institutions. While debtors may transmit risks to the system through their creditors, creditors may pose risks onto the system if they suddenly decide to stop providing liquidity to their debtors.

- Shocks originate on both sides of banks' balance sheet.
  - We define default/devaluation shock a shock which hits the assets of the bank.
  - We define illiquidity shock a shock which hits the liabilities of the bank.
Interconnectedness of balance sheets: channels of systemic risk

- **Default/devaluation contagion**: default/devaluation of one bank may trigger default/devaluation of another bank

![Diagram](image)
Interconnectedness of balance sheets: channels of systemic risk

- **Default/devaluation contagion**: default/devaluation of one bank may trigger default/devaluation of another bank

```
\begin{align*}
\text{Devaluation Effect} & \quad \text{Illiquidity Effect}
\end{align*}
```
Interconnectedness of balance sheets: channels of systemic risk

- **Iliquidity contagion**: liquidity shortage of one bank may trigger illiquidity of another bank
Interconnectedness of balance sheets: channels of systemic risk

- **Illiquidity contagion**: liquidity shortage of one bank may trigger illiquidity of another bank
The Basel III approach to SIFIs

- In the case of the interbank (IB) market, interconnectedness can be captured by the weighted sum of IB assets and liabilities i.e aggregate interbank exposures.
- We distinguish between Systemically Important Borrowers (SIB) and Systemically Important Lenders (SIL) via IB liabilities and assets.

Hence the in and out weighted degree of an exposures matrix measure the total amount of borrowing (in-going) and lending (outgoing)

\[ k_{i,in}^w = \sum_{j=1}^{n} e_{i,j} \]
\[ k_{i,out}^w = \sum_{j=1}^{n} e_{i,j} \]
The Basel III approach to SIFIs

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\[ k_{i,in}^w = \sum_{j=1}^{n} e_{i,j} \qquad k_{i,out}^w = \sum_{i=1}^{n} e_{i,j} \]
Weakness pt-1

- This measure does not incorporate the **systemic dimension** of the bank’s exposures (it is a *local centrality*). An alternative measure is required in order to:
  - incorporate the exposures of the bank’s **peers**, and of the **peers of its peers**, etc.
  - award higher score to banks connected to *important* banks.

- **Feedback centrality** takes into account the whole **spatial distributions** of banks’ linkages.
Interconnectedness: behind first peers pt-1
Interconnectedness: behind first peers pt-2
Interconnectedness: behind first peers pt-3
The degree centrality is a static measure, a snapshot of the system, whereas financial networks are continuously evolving in time. As distress starts to propagate, the existing financial connections might dissolve and new ones might appear, modifying the way other reverberations of a crisis are channeled through the network.

- we take into account the influence of the network topology in the way shocks propagate and the reverse effects of banks’ defaults on the structure of interbank networks as well.

- Dynamic centrality takes into account the whole temporal distributions of banks’ linkages.
Interconnectedness: temporal network

- We calculate the banks’ importance for the whole periods not solely computing the banks’ importance at each time, but given a time sequence of transaction.

- Together with time’s importance i.e. how the network topology in a given period affects the results on banks’ importance.
We propose a novel measure of systemic risk able to determine the systemic importance of each bank taking into account not only their first order exposures but also higher order exposures in an evolving network. This measure also produce an early warning signal for the evolution of overall traded volume.

- specifically designed for directed networks to distinguish between Systemically Important Borrowers (SIB) and Systemically Important Lenders (SIL).
- feedback centrality measure: each adjacent node is assigned a centrality score proportional to a weighted sum of the scores of its neighbors;
- intrinsically dynamic: the weights depend on the amount of the transactions between banks and on the periods in which those transactions occur defined by the time score.
- the time score summarizes the strength of contemporaneous spillover effects in each time period i.e. the risk associated to the whole network in each period.
- The time score can be interpreted as an early warning indicator for the overall traded volume.
A temporal network is appropriately represented as a \textit{time-ordered sequence} of exposure matrices, each one describing the state of the financial network at a given point in time.

The exposure matrices are thus combined in a \textit{three-way tensor}.

We represent the interbank market dynamic as a \textit{three-way tensor} $\mathcal{E} \in \mathbb{R}^{I \times I \times K}$ where the generic element $\mathcal{E}_{ijk}$ represents the funds lent by bank $i$ to bank $j$ at time $k$. 
We propose the **TOPHITS** algorithm to produce three vectors. The first two contain indices of borrowing and lending **systemic importance** for each bank in the network. The third one represents the time scores for the periods considered.

The results can be easily interpreted in terms of **hubs** and **authorities** in presence of multiple linkages.

- The authority score quantifies the role of a node as a source of information. The hub score measures the acquaintance of a node.

The algorithm converges to a particular **tensor decomposition** technique, called **CP** decomposition.
Institutions with an high **authority** score are the main **SIB**, banks with a high **hub** score are the main **SIL**. The algorithm is based on the relationships between the set of relevant authority (**SIB**) and a set of relevant hub (**SIL**) that joint them in a linked structure.

**SIL** and **SIB** exhibit a *mutually reinforcing relationship*: a **SIL** is a bank that points to many good **SIB**; a **SIB** is an institutions that is pointed to by many **SIL**.

A bank will be a **SIL** if the institutions to which it lends are systemically important borrowers.

A bank will be a **SIB** if the banks from which it borrows are systemically important lenders.

- Two banks will be ranked differently as hubs/authority even if they lend/borrow the same amount of funds, depending on the behavior of their borrowers/lenders.
The authority score: the "devaluation sink" that would appear if a bank were insolvent.

The hub score: the "liquidity sink" that would materialize if a bank suddenly interrupted providing credit.

A bank has a high hub score if it lends funds to borrowers that also borrow from banks that may not be able provide credit anymore.

A bank has a high authority score if it borrows from institutions that also lend to banks that may not be able to repay their debt.
The Economic interpretation pt.1.2

The role of SIBs:
Spread of devaluation shock
Receiving of liquidity shock

The role of SILs:
Spread of liquidity shock
Receiving of devaluation shock
The \textit{time score} attributes a certain score to each period depending on the interaction between hubs, authorities in that period and on the size of the links that brings together couples of hub and authority.

- For a given distribution of hubs and authorities over time, the time score will be higher the higher the traded volume.
- For a given distribution of traded volume over time the time score will be higher the higher the size of hubs and authority.

If $i \stackrel{k,w}{\rightarrow} j$ means that bank $i$ lends to bank $j$ an amount $w$ at time $k$, then:

- The \textit{hub} (SIL) score of bank $i$ is the sum of \textit{authority} (SIB) scores for the bank that $i$ points to, multiplied by the size of the transaction $w$ and by the corresponding \textit{time score} $k$ of the period in which these transactions occur.
- The \textit{authority} (SIB) score of bank $j$ is the sum of \textit{hub} (SIL) scores of all the banks that point to $j$, multiplied by the size of the transaction $w$ and by the corresponding \textit{time scores} $k$ of the period in which these transactions occur.
- The \textit{time score} of period $k$ is the sum of \textit{hub} (SIL) scores for bank $i$ multiplied by the \textit{authority} (SIB) scores for bank $j$ and by the size of the transaction $w$ over all transactions $i \stackrel{w}{\rightarrow} j$ that involve period $k$. 
Economic interpretation pt.3
Economic interpretation pt.3
Economic Interpretation pt.3
The **hub** (SIL) score of bank $i$ is the sum of **authority** (SIB) scores for banks that $i$ points to, multiplied by the size of the transaction $w$ and by the corresponding **time score** $k$ of the period in which these transactions occur.

\[ h_{B1} = w_{B1,B2,T1} a_{B2} t_{T1} + w_{B1,B2,T2} a_{B2} t_{T2} + w_{B1,B3,T2} a_{B3} t_{T2} + w_{B1,B4,T1} a_{B4} t_{T1} \]
The **hub** (SIL) score of bank $i$ is the sum of **authority** (SIB) scores for banks that $i$ points to, multiplied by the size of the transaction $w$ and by the corresponding **time score** $k$ of the period in which these transactions occur.
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Economic interpretation pt.4

- The **authority** (SIB) score of bank $j$ is the sum of **hub** (SIL) scores of all banks that point to $j$, multiplied by the size of the transaction $w$ and by the corresponding **time score** $k$ of the period in which these transactions occur.

\[
\begin{align*}
a_{B_2} &= w_{B_1,B_2,T_1} h_{B_1,T_1} + w_{B_1,B_2,T_2} h_{B_1,T_2} + \ldots + w_{B_4,B_2,T_2} h_{B_4,T_2}
\end{align*}
\]
The **time score** of period $k$ is the sum of **hub** (SIL) scores for bank $i$ multiplied by the **authority** (SIB) scores for bank $j$ and by the size of the transaction $w$ over all transactions $i \xrightarrow{w} j$ that involve period $k$. 
The **time score** of period $k$ is the sum of **hub** (SIL) scores for bank $i$ multiplied by the **authority** (SIB) scores for bank $j$ and by the size of the transaction $w$ over all transactions $i \xrightarrow{w} j$ that involve period $k$. 

\[
\begin{align*}
t_{T1} &= w_{B1,B2,T1} \cdot a_{B2,h_{B1}} + w_{B1,B4,T1} \cdot a_{B4,h_{B1}} + \\
&\quad + w_{B2,B3,T1} \cdot a_{B3,h_{B2}} + w_{B2,B4,T1} \cdot a_{B4,h_{B2}}
\end{align*}
\]
The TOPHITS algorithm pt.1

- The TOPHITS method produces three vectors that include hub and authority scores for banks as in HITS, and time scores for each period as well.
- These scores can be computed iteratively. Let $l$ denote the number of banks and $K$ the number of periods. The hub, authority, and time scores are updated as follows:

\[
\begin{align*}
    h_i^{(t+1)} &= \sum_{j \rightarrow k} a_j^{(t)} t_k^{(t)} & i = 1, \ldots, l \\
    a_j^{(t+1)} &= \sum_{i \rightarrow k} h_i^{(t+1)} t_k^{(t)} & j = 1, \ldots, l \\
    t_k^{(t+1)} &= \sum_{i \rightarrow j} a_i^{(t+1)} h_j^{(t+1)} & k = 1, \ldots, K
\end{align*}
\]
In tensor form

$$h^{(t+1)} = \mathcal{E} \times_2 a^{(t)} \times_3 t^{(t)}$$  \hspace{2cm} (1)

$$a^{(t+1)} = \mathcal{E} \times_1 h^{(t+1)} \times_3 t^{(t)}$$ \hspace{2cm} (2)

$$t^{(t+1)} = \mathcal{E} \times_1 h^{(t+1)} \times_2 a^{(t+1)}$$ \hspace{2cm} (3)

where $\mathcal{E} \times_i x$ indicates that the tensor $\mathcal{E}$ should be multiplied by the vector $x$ in dimension $i$, the HITS model is generalized to the **TOPHITS** model.
Equivalently

\[ h_i = \sum_{j=1}^{I} \sum_{k=1}^{K} \mathcal{E}_{ijk} a_j t_k \]  \hspace{1cm} (4)

\[ a_j = \sum_{i=1}^{I} \sum_{k=1}^{K} \mathcal{E}_{ijk} h_i t_k \]  \hspace{1cm} (5)

\[ t_k = \sum_{i=1}^{I} \sum_{j=1}^{J} \mathcal{E}_{ijk} a_i h_j \]  \hspace{1cm} (6)
The CP decomposition

- Authorities, hubs and time scores are calculated in according to a three way CP decomposition, that yields a rank-$R$ approximation of a tensor $\mathcal{E}$ of the form.

$$\mathcal{E} \approx [\sigma; U, V, W] \equiv \sum_{r=1}^{R} \sigma_r u_r \circ v_r \circ w_r \quad (7)$$

- If $R = 1$ we get a set of triplets $\{u_r, v_r, w_r\}$ where the $u$ and $v$ vectors contain hub (SIL) and authority (SIB) scores for the nodes as in HITS, and the vector $w$ contains the time score.

- Where $h = u_1$, $a = v_1$ and $t = w_1$ define the dominant time score,

$$\sigma_r = \|u_r\| \|v_r\| \|r_r\|.$$ 

- We solve the CP decomposition using non-negative tensor decomposition thus $u_r \geq 0$, $v_r \geq 0$, $w_r \geq 0$.

- For $R > 1$, $\{u_r, v_r, w_r\}$, we yield time score, hubs (SIL) and authorities (SIB) for the other subdominant factors (communities), tracking their activity through time.
A simple example pt-1: the spatial distribution
A simple example pt-1: the spatial distribution

<table>
<thead>
<tr>
<th>Banks</th>
<th>O. D.</th>
<th>Hub</th>
<th>I. D.</th>
<th>Auth.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
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<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0.22</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.43</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>0.82</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.51</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.51</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>0.51</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.21</td>
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<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.21</td>
</tr>
<tr>
<td>11</td>
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<td>2</td>
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<td>0</td>
<td>0</td>
<td>2</td>
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<tr>
<td>13</td>
<td>0</td>
<td>0</td>
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<td>0.21</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.21</td>
</tr>
</tbody>
</table>

- Even if banks 2 and 4 have the same out-degree, bank 4 has higher authority because of its neighbors’ importance. Due to the structure of lending and borrowing relationships, a shock can pass through bank 4 with higher probability than through bank 2.

- The same applies to bank 1 for in-degree and hubness.
A simple example pt-3: the spatio-temporal distribution
A simple example pt-3: the spatio-temporal distribution
A Simple Example pt-5: the Spatio-Temporal Distribution
A simple example pt-3: the spatio-temporal distribution
A Simple Example pt-7: the Spatio-Temporal Distribution
A simple example pt-3: the spatio-temporal distribution

<table>
<thead>
<tr>
<th>Time</th>
<th>Av. Deg</th>
<th>Av. Hits</th>
<th>Tophits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.13</td>
<td>0.1</td>
<td>0.40</td>
</tr>
<tr>
<td>2</td>
<td>1.13</td>
<td>0.8</td>
<td>0.33</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
<td>0.13</td>
<td>0.29</td>
</tr>
<tr>
<td>4</td>
<td>1.91</td>
<td>0.13</td>
<td>0.57</td>
</tr>
<tr>
<td>5</td>
<td>2.26</td>
<td>0.8</td>
<td>0.65</td>
</tr>
</tbody>
</table>

- The average degree is not affected by the links’ **spatial** distribution change over nodes.
- The HITS is not affected by the links’ **temporal** distribution change over nodes.
- The **time score** of the TOPHITS takes into account both the distribution and the size (temporal distribution) of the links.
The dataset

- We consider a set of 354 banks, represented by their exposure to the rest of the reporting banks, measured on monthly basis.
- The generic element $e_{i,j}$, represents the amount of funds borrowed by bank $i$ from bank $j$.
- The e-Mid interbank market is described by a third order tensor $\mathcal{E} \in \mathbb{R}^{354 \times 354 \times 168}$.
- The interbank tensor is composed by 168 slices $E \in \mathbb{R}^{354 \times 354}$ representing monthly transactions from January 1999 to December 2012.
Results for $R=1$, Hubs, Authorities, Time scores
The time score as an early-warning signal

- The **traded volume** (TV) smoothly increases up to the peak of the first quarter 2007 and decreases afterwards.
- The **time score** (TS) reaches its maximum before the crisis and starts to decrease in 2006, before the abrupt fall of the third quarter of 2007.
- The **cross-correlation** suggests that TV has a delayed response to the TS, or perhaps a delayed response to a common stimulus (financial crisis) that affects both series.
  - The **time score** is a potential **early-warning signal** of the upcoming topological collapse.
The cross-correlation

- The **cross-correlation** function (ccf) of the two time series is the product-moment correlation as a function of the lag between the series.

\[
r_{TV,TS}(k) = \frac{c_{TV,TS}(k)}{\sqrt{c_{TV,TV}(0) c_{TS,TS}(0)}}
\]

where \( c(k) \) is the the **cross-covariance** function (ccvf) defined as

\[
c_{TV,TS}(k) = \frac{1}{N} \sum_{t=1}^{N-k} (TV_t - \overline{TV}) (TS_{t+k} - \overline{TS}) \quad ; k = 0, 1, ..., (N-1)
\]

\[
c_{TV,TS}(k) = \frac{1}{N} \sum_{t=1-k}^{N} (TV_t - \overline{TV}) (TS_{t+k} - \overline{TS}) \quad ; k = -1, ..., -(N-1)
\]
The cross-correlation is greater than 0.8 with $k = 0, \ldots, -12$.

$TS_{t+k}$ with $k$ negative leads to $TV_t$.

An above average value of $TS$ is likely to lead to an above average of $TV$ about 1 year later. A value of $TS$ below its average is associated with a likely above average $TV$ value about 1 year later.
The community structure

- The tensor factorization method is similar to the community detection techniques.
  - The number of components ($R$) we choose to approximate the tensor is the number of communities or activity patterns we extract.
  - Communities are retrieved independently. Thus overlapping communities are handled appropriately.
  - The importance of a central node in a community will ensure it to be ranked at the top of the community.

- The number $R$ of components is chosen on the basis of the desired level of accuracy:
  - If the number of components is low, only the strongest structures are represented.
  - Using a high number of components faces the risk of overfitting noise.
Choosing the number of community

- The Core Consistency Diagnostic or CORCONDIA is proposed to determine whether, given a fixed number of factors, the model is well explained by a CP decomposition or if it overfits noise.
- The CP decomposition up to six communities is a good approximation of the original data $\mathcal{E}$, having a Core Consistency Diagnostic around 100%.

![Graph showing the relationship between number of communities and Core Consistency Diagnostic]
Results: the community structure
What’s next???

- The introduction of the temporal dimension can be exploited to develop appropriate **predictions** of the systemic importance for each financial institution.

THANKS

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