1979

A tableau approach to power system analysis and design

Stephen W. Director
Carnegie Mellon University

R Sullivan

A TABLEAU APPROACH TO POWER SYSTEM ANALYSIS AND DESIGN

by

S.W. Director & R.L. Sullivan

DRC-18-1-79

January 1979

* Department of Electrical Engineering
Carnegie-MelIon University
Pittsburgh, PA 15213

** Department of Electrical Engineering
University of Florida
Gainesville, FL 32611

This work was supported in part by NSF Grants ENG-75-23078, ENG-75-02564, and ENG-77-20895.

Abstract

The tableau formulation of electronic circuit equations has been shown to be an efficient and useful formulation for circuit optimization. In this paper a tableau based formulation of the static power-flow equations for a power system is presented. Because of the explicit appearance of all quantities of interest the tableau formulation allows the general statement, and solution, of a variety of power system optimization and design problems without the need to resort to restrictive assumptions.

Introduction

Due to the increasing interest in the management of energy, the development of computer aided design tools for solving power system design problems is becoming of greater importance. As a result there has been renewed interest in applying optimization methods which have proven useful for electronic circuit design to power system problems.[1,3] The electronic circuit design problem is handled by recasting at into an optimization problem by forming an objective function which embodies the design objectives.[4] The designable parameters are then adjusted iteratively by some suitable nonlinear programing algorithm (see for example [5]) to minimize the objective function. This procedure requires the efficient evaluation of the objective function, which requires a circuit simulation, and the gradient of the objective function with respect to the designable parameters. Development of the sparse tableau approach for circuit simulation [6] and the adjoint approach for evaluating gradients[7] greatly aided the development of optimization procedures for circuit design.

Much of the work in applying these procedures developed for circuit design to power system design was hampered by the restrictive assumptions that were often made. Typical of these assumptions were that the transmission lines were lossless and voltage profiles were flat, clearly an unrealistic situation. These assumptions were made so as to alter the equations which describe a power system, i.e. the load flow equations, usually written in terms of power and voltage, so that they resemble the equations which describe an electronic circuit, usually written in terms of current and voltage.

In this paper we develop a tableau based formulation of the load flow equations for a power system. In addition to being able to take good advantage of highly efficient sparse matrix methods,[8,9] this formulation explicitly

displays all variables of interest in a variety of power system design problems. Thus an efficient procedure for computing the gradient of any objective function with respect to the designable parameters can be developed without making unrealistic assumptions.

In order to facilitate the presentation of the tableau formulation, we **introduce a consistent set of notation in the next section. Explicit development of the tableau formulation of the power-flow equations is given in Section III. In Section IV we present** a **general model of the power system design problem and discuss its solution. Specific power system design and analysis problems are discussed in Section** V.

II. Notation

In order to provide a general framework within which to describe power system design, as well as analysis, we introduce the following, somewhat unorthodox, notation. However, the use of this notation will greatly simplify the mathematical development of Sections III and IV.

Let the power system under consideration have a total of b branches of which there are n source branches (including slack[1], generator and load branches[2]) and t transmission-line element branches? For convenience we will number these branches in the following order:

slack branch	:	1
generator branches	:	2,3,...,g
loads	:	g+1,g+2,...,n
transmission line elements	:	n+1,n+2,...,b.

Further, we assume without loss of generality, that each bus[4] has exactly one source connected to it so that there are n buses[5].

Associated with each branch k $(k = 1,2,\ldots,b)$ are a complex-valued branch voltage, V_k, and branch current, J_k; a real and reactive power which enters the branch, denoted by P_k^E and Q_k^E, respectively; and a real and reactive power which leaves the branch, denoted by P_k^L and Q_k^L, respectively. We define the direction of entering power to be the same as the direction of entering current. Thus if branch k is connected from bus μ to bus ν

$$P_k^E = \mathrm{Re}\{V_\mu J_k^*\} \qquad (1a)$$

$$Q_k^E = \mathrm{Im}\{V_\mu J_k^*\} \qquad (1b)$$

$$P_k^L = \mathrm{Re}\{V_\nu J_k^*\} \qquad (1c)$$

$$Q_k^L = \mathrm{Im}\{V_\nu J_k^*\} \qquad (1d)$$

Note that the entering and leaving quantities as defined above can be either positive or negative. If, for example, $P_k^E < 0$ and $P_k^L < 0$, the power is actually entering branch k from bus ν and leaving thru bus μ. Observe that if $P_k^E < 0$ and $P_k^L \geq 0$ then branch k is generating power, while if $P_k^E > 0$ and $P_k^L \leq 0$ then branch k is absorbing power.

In general we will avoid expressing network equations in terms of complex quantities, preferring to use, when necessary, two equations: one which describes the real part and one which describes the imaginary part. Thus, we express the kth branch voltage and current as

$$V_k = e_k + jf_k$$

and

$$I_k = c_k + jd_k$$

where e_k, f_k, c_k, and d_k are real. It is also convenient to make the following definitions

V : **the b-vector of complex branch voltages**

I : **the b-vector of complex branch currents**

P^E : **the b-vector of entering real branch powers**

Q^E : **the b-vector of entering reactive branch powers**

P^L : **the b-vector of leaving real branch powers**

Q^L : **the b-vector of leaving reactive branch powers**

and

$$e \equiv \mathrm{Re}\,\{V\} \tag{2a}$$

$$f \equiv \mathrm{Im}\,\{V\} \tag{2b}$$

$$c \equiv \mathrm{Re}\,\{I\} \tag{2c}$$

$$d \equiv \mathrm{Im}\,\{I\} \tag{2d}$$

Because of the branch numbering scheme assumed above, the complex **branch** voltage vector can be partitioned as follows

$$V = \begin{bmatrix} V_n \\ \hline V_t \end{bmatrix}$$

where

$$Y_n \wedge \ (V^V 2 " - " ^V n)^T$$

are complex bus (or source) voltages, and

$$V_t \equiv \left(V_{n+1}, V_{n+2}, \ldots, V_b\right)^T$$

are complex transmission line voltages. V_n may be further partitioned as follows

$$V_n = \begin{bmatrix} V_s \\ \hline V_g \\ \hline V_\ell \end{bmatrix}$$

where

$$V_s \equiv V_1$$

is the complex slack voltage,

$$\underset{\sim}{V}_g \equiv (V_2, V_3, \ldots, V_g)$$

is the complex generator voltage vector, and

$$\underset{\sim}{V}_\ell \equiv (V_{g+1}, V_{g+2}, \ldots, V_n)$$

is the complex load voltage vector. A similar partitioning will be used in the sequel for the other vectors defined above.

In some cases, we will find it more natural to express bus voltages in terms of magnitudes and phases. In such cases we use the following notation

$$V_k = E_k e^{j\delta_k} \tag{3}$$

where E_k is magnitude and δ_k is the phase angle in radians. By defining the quantities

$$\alpha_k \equiv \cos\delta_k \tag{4a}$$

and

$$\beta_k \equiv \sin\delta_k \tag{4b}$$

and recalling (2), we can express (3) as

$$e_k = E_k \alpha_k \tag{5a}$$

and

$$f_k = E_k \beta_k \tag{5b}$$

with

$$\alpha_k^2 + \beta_k^2 = 1 \tag{6}$$

In vector notation, (5a) and (5b) can be written as

$$\underset{\sim}{e}_n = \underset{\sim}{E}_n \times \underset{\sim}{\alpha}_n \tag{7a}$$

and

$$f_n = E_n \times U_n \tag{7b}$$

where "x" is used to indicate componentwise multiplication, i.e., if x and y are m vectors, then $x \times y$ is the m vector whose kth component is $x_k y_k$. Note that we can express (6) as

$$A_n^2 + B_n^2 = 1 \tag{8}$$

where $a^2 = a \times a$, etc., and 1 is a column vector with all components equal to 1. (In the sequal we will use U to denote the unit matrix. Note that given an n-vector x, $x \times U = \mathrm{diag}\,(x_1, x_2, \ldots, x_n)$.)

III. Power System Equations

The equations which describe a power system can be separated into two types: those which characterize the elements or "lines" and those which characterize the interconnections. We consider the element relationships first. There are basically three types of sources: slacks, generators and loads. The slack branch is characterized by

$$E_s = E_s \quad \bullet \tag{9}$$

and

$$\delta_s = \hat{\delta}_s \tag{10}$$

or

$$\alpha_s = \cos \hat{\delta}_s \tag{11}$$

where E_s and $\hat{\delta}_s$ are given quantities. (Note δ_s is constrained by Eq. (8).)

Generator branches are characterized by

$$\underset{\sim}{E}_g = \underset{\sim}{\mathcal{E}}_g \tag{12}$$

and

$$\underset{\sim}{P}^E_g = \underset{\sim}{\mathcal{P}}^E_g \tag{13}$$

where $\underset{\sim}{\mathcal{E}}_g$ and $\underset{\sim}{\mathcal{P}}^E_g$ are given and $\underset{\sim}{\mathcal{P}}^E_g < 0$. Load branches are characterized by

$$\underset{\sim}{P}^E_\ell = \underset{\sim}{\mathcal{P}}^E_\ell \tag{14}$$

and

$$\underset{\sim}{Q}^E_\ell = \underset{\sim}{\mathcal{Q}}^E_\ell \tag{15}$$

where $\underset{\sim}{\mathcal{P}}^E_\ell$ and $\underset{\sim}{\mathcal{Q}}^E_\ell$ are given and $\underset{\sim}{\mathcal{P}}^E_\ell > 0$. Finally, transmission line branches are characterized by

$$\underset{\sim}{I}_t = \underset{\sim}{Y}\,\underset{\sim}{V}_t \tag{16}$$

where $\underset{\sim}{Y}$ is a (txt) diagonal complex-valued matrix. $\underset{\sim}{Y}$ can be written as

$$\underset{\sim}{Y} = \underset{\sim}{G} + j\underset{\sim}{B}$$

where $\underset{\sim}{G}$ and $\underset{\sim}{B}$ are real (txt) diagonal matrices. Therefore (16) can also be expressed as

$$\underset{\sim}{c}_t = \underset{\sim}{G}\,\underset{\sim}{e}_t - \underset{\sim}{B}\underset{\sim}{f}_t \tag{18a}$$

and

$$\underset{\sim}{d}_t = \underset{\sim}{B}\,\underset{\sim}{e}_t + \underset{\sim}{G}\underset{\sim}{f}_t \tag{18b}$$

In order to describe the equations of interconnection we define the (nxb) bus incidence matrix $\underset{\sim}{A} = [a_{\mu k}]$

$$a_{\mu k} = \begin{cases} +1 & \text{if the current in branch } k \text{ leaves bus } \mu \\ -1 & \text{if the current in branch } k \text{ enters bus } \mu \\ \;\;0 & \text{if branch } k \text{ does not touch bus } \mu \end{cases}$$

Similarly we define the entering bus incidence and leaving bus incidence matrices:

$\underset{\sim}{A}^E = \left[a^E_{\mu k}\right]$ and $\underset{\sim}{A}^L = \left[a^L_{\mu k}\right]$ where

$$a^E_{\mu k} = \begin{cases} +1 & \text{if the current in branch k enters bus } \mu \\ 0 & \text{otherwise} \end{cases}$$

and

$$a^L_{\mu k} = \begin{cases} +1 & \text{if the current in branch k leaves bus } \mu \\ 0 & \text{otherwise} \end{cases}$$

Thus

$$\underset{\sim}{A} = \underset{\sim}{A}^L - \underset{\sim}{A}^E$$

Note that the columns of $\underset{\sim}{A}$ may be partitioned as follows:

$$\underset{\sim}{A} = \left[\underset{\sim}{A}_n \;\vdots\; \underset{\sim}{A}_t \right]$$

where $\underset{\sim}{A}_n$ contains the first n columns of $\underset{\sim}{A}$ and $\underset{\sim}{A}_t$ contains the last b-n columns of $\underset{\sim}{A}$, or

$$\underset{\sim}{A} = \left[\underset{\sim}{A}_s \;\vdots\; \underset{\sim}{A}_g \;\vdots\; \underset{\sim}{A}_\ell \;\vdots\; \underset{\sim}{A}_t \right]$$

where $\underset{\sim}{A}_s$ contains the first column of $\underset{\sim}{A}$, $\underset{\sim}{A}_g$ contains columns 2 through g of $\underset{\sim}{A}$, and $\underset{\sim}{A}_\ell$ contains columns g+1 through n of $\underset{\sim}{A}$. Similar partitioning of the matrices $\underset{\sim}{A}^E$ and $\underset{\sim}{A}^L$ can be made and are used in the sequel.

Kirchhoff's current law can now be expressed as

$$\underset{\sim}{A}\,\underset{\sim}{1} = \underset{\sim}{0} \tag{19}$$

or, in a form more suitable for our purposes, as

$$\underset{\sim}{A}_n \dot{\underset{\sim}{c}}_n + \underset{\sim}{A}_t \dot{\underset{\sim}{c}}_t = \underset{\sim}{0} \tag{20a}$$

and

$$\underset{\sim}{A}_n \underset{\sim}{d}_n + \underset{\sim}{A}_t \underset{\sim}{d}_t = \underset{\sim}{0} \tag{20b}$$

Kirchhoff's voltage law can be expressed as

$$V_t - A_t^T V_n = 0 \tag{21}$$

or, in terms of real and imaginary parts

$$e_t - A_t^T e_n = 0 \tag{22a}$$

and

$$f_t - A_t^T f_n = 0 \tag{22b}$$

The entering and leaving bus incidence matrices may be used to describe the relationship between entering and leaving branch powers, branch currents and bus voltages (see Eq. 1) as follows:

$$P_t^E = c_t \times \left(\left(A_t^E\right)^T e_n\right) - d_t \times \left(\left(A_t^E\right)^T f_n\right) \tag{23a}$$

$$Q_t^E = d_t \times \left(\left(A_t^E\right)^T e_n\right) + c_t \times \left(\left(A_t^E\right)^T f_n\right) \tag{23b}$$

$$P_t^L = c_t \times \left(\left(A_t^L\right)^T e_n\right) - d_t \times \left(\left(A_t^L\right)^T f_n\right) \tag{23c}$$

and

$$Q_t^E = d_t \times \left(\left(A_t^L\right)^T e_n\right) + c_t \times \left(\left(A_t^L\right)^T f_n\right) \tag{23d}$$

The final set of equations we need to complete the tableau formulation are a form of the static power-flow equations (SPFE) called the mismatch equations. These equations express the fact that the sum of all the powers entering a bus minus the sum of all the powers leaving a bus must equal zero. These equations are conveniently written as

$$A_s P_s^E + A_g P_g^E + A_\ell^E P_\ell^E + A_t^E P_t^E - A_t^L P_t^L = 0 \tag{24a}$$

and

$$A_s Q_s^E + A_g Q_g^E + A_\ell^E Q_\ell^E + A_t^E Q_t^E - A_t^L Q_t^L = 0 \tag{24b}$$

where we have recognized that P^, (£, f_g, (£, P^, and q\ are all zero because one terminal of each generator is grounded. In summary, we have

$$\left.\begin{array}{l} \underline{A}_n \underline{c}_n + \underline{A}_t \underline{c}_t = \underline{0} \\ \underline{A}_n \underline{d}_n + \underline{A}_t \underline{d}_t = \underline{0} \end{array}\right\} \quad \text{2n KCL equations (see (20))} \qquad (25a)$$

$$\left.\begin{array}{l} \underline{e}_t - \underline{A}_t^T \underline{e}_n = \underline{0} \\ \underline{f}_t - \underline{A}_t^T \underline{f}_n = \underline{0} \end{array}\right\} \quad \text{2(b-n) KVL equations (see (22))} \qquad (25b)$$

$$\left.\begin{array}{l} \underline{c}_t \times \left[\left(\underline{A}_t^E\right)^T \underline{e}_n\right] - \underline{d}_t \times \left[\left(\underline{A}_t^E\right)^T \underline{f}_n\right] - \underline{P}_t^E = \underline{0} \\ \underline{d}_t \times \left[\left(\underline{A}_t^E\right)^T \underline{e}_n\right] + \underline{c}_t \times \left[\left(\underline{A}_t^E\right)^T \underline{f}_n\right] - \underline{Q}_t^E = \underline{0} \\ \underline{c}_t \times \left[\left(\underline{A}_t^L\right)^T \underline{e}_n\right] - \underline{d}_t \times \left[\left(\underline{A}_t^L\right)^T \underline{f}_n\right] - \underline{P}_t^L = \underline{0} \\ \underline{d}_t \times \left[\left(\underline{A}_t^L\right)^T \underline{e}_n\right] + \underline{c}_t \times \left[\left(\underline{A}_t^L\right)^T \underline{f}_n\right] - \underline{Q}_t^L = \underline{0} \end{array}\right\} \quad \begin{array}{l}\text{4(b-n) branch power equations} \\ \text{(see (23))}\end{array} \qquad (25c)$$

$$\left.\begin{array}{l} \underline{A}_s^E \underline{P}_s^E + \underline{A}_g \underline{P}_g^E + \underline{A}_\ell^E \underline{P}_\ell^E + \underline{A}_t^E \underline{P}_t^E - \underline{A}_t^L \underline{P}_t^L = \underline{0} \\ \underline{A}_s^E \underline{Q}_s^E + \underline{A}_g^E \underline{Q}_g^E + \underline{A}_\ell^E \underline{Q}_\ell^E + \underline{A}_t^E \underline{Q}_t^E - \underline{A}_t^L \underline{Q}_t^L = \underline{0} \end{array}\right\} \quad \begin{array}{l}\text{2n mismatch equations} \\ \text{(see (24))}\end{array} \qquad (25d)$$

$$\left.\begin{array}{l} \underline{e}_n - \underline{E}_n \times \underline{a}_n = 0 \\ \underline{f}_n - \underline{E}_n \times \underline{B}_n = 0 \\ \underline{a}_n^2 + \underline{B}_n^2 - 1 = 0 \end{array}\right\} \quad \begin{array}{l}\text{3n bus voltage equations} \\ \text{(see (7) and (8))}\end{array} \qquad (25,$$

$$\left.\begin{aligned}
&E_s - E_s = 0 \\
&a_s - \cos \hat{5}_s = 0 \\
&E_g - E_g - 0 \\
&P^E_g - \bar{J}^*_g = 0 \\
&P^E_\ell - P^E_\ell = 0 \\
&Q^E_t - Q^E_\ell = 0
\end{aligned}\right\} \quad \text{2n source relationships (see (9), (11), (12), (13), (14) and (15))} \tag{25f}$$

$$\left.\begin{aligned}
&c_t - G e_{fc} + B f_t - 0 \\
&d_t - B e_t - G f_t = 0
\end{aligned}\right\} \quad \text{2(b-n) transmission line equations (see (18))} \tag{25g}$$

Using vector notation the 8b+n equations, Eq. (25) can be conveniently expressed as follows:

$$W(x,p) = 0 \tag{26}$$

where x is the (8b+n) dependent state vector:

$$x^T = (c_n^T, c_t^T, d_n^T, d_t^T, e_n^T, e_t^T, f_n^T, f_t^T, E''_s, E''_g, E''_\ell, \alpha_s, \alpha_g, \alpha_\ell, \beta_s, \beta_g, \beta_\ell, P^E_s, (P^E_g)^T, (P^E_\ell)^T, (P^E_t)^T,$$
$$Q^E_s, (Q^E_g)^T, (Q^E_\ell)^T, (Q^E_t)^T, (P^L_t)^T, (Q^L_t)^T) \tag{27}$$

p is the independent parameter vector:

$$p^T = (E_s, E_g, \alpha_s, P^E_g, P^E_\ell, Q^E_\ell, g, b) \tag{28}$$

where the vectors $\underset{\sim}{g}$ and $\underset{\sim}{b}$ contain the diagonal elements of $\underset{\sim}{G}$ and $\underset{\sim}{B}$, respectively, i.e., $\underset{\sim}{G} = \underset{\sim}{g} \times \underset{\sim}{U}$ and $\underset{\sim}{B} = \underset{\sim}{b} \times \underset{\sim}{U}$.

Observe that while the power system itself contains only linear elements, the equations which describe it, (25), are nonlinear because power enters nonlinearly.

Given values for $\underset{\sim}{p}$, Eq.(26) may be solved using a Newton-Raphson iteration scheme described by

$$x^{k+1} = x^k + \Delta x^k \tag{29}$$

where $\Delta \underset{\sim}{x}^k$ is the solution of the linear system

$$\underset{\sim}{J}^k \Delta \underset{\sim}{x}^k = -\underset{\sim}{f}(\underset{\sim}{x}^k, \underset{\sim}{p}) \tag{30}$$

where the (μ,ν^{th}) element of the $((8b+n) \times (8b+n))$ Jacobian matrix is given by

$$J_{\mu\nu} = \left.\frac{\partial f_\mu}{\partial x_\nu}\right|_{x = x^k} \tag{31}$$

and $\underset{\sim}{x}^k$ is the k^{th} estimate of the solution.

The general form of the jacobian is shown in Fig. 1. (Note - only the non-zero partitions of J are shown).

Observe that (30) is a highly sparse set of equations thus making its solution amenable to the application of sparse matrix methods including optimal ordering and code generation [8,9]. It should be emphasized that there is evidence to support the conjecture that when sparse matrix methods are employed, it is preferable to use them to solve a large sparse set of equations such as (25) [6,9] rather than employing a predetermined pivot order to obtain a denser reduced set of equations.

IV. Power System Design

It will be shown in the next section that many power network design problems can be recast into a nonlinear programming problem of the form:

$$\min_{\underset{\sim}{p}} \phi = \phi(\underset{\sim}{x},\underset{\sim}{p}) \qquad (32)$$

subject to

$$\underset{\sim}{N}(\underset{\sim}{x},\underset{\sim}{p}) = 0$$

where ϕ is a performance function which reflects the design objectives and is a function of $\underset{\sim}{x}$, given by (27), and the parameters $\underset{\sim}{p}$, and $\underset{\sim}{N}(\underset{\sim}{x},\underset{\sim}{p})$ is defined by (25). The typical approach to solving such a problem is to employ a minimization algorithm such as the Fletcher-Powell method [5]. However, to such an algorithm we need the gradient

$$\frac{d\phi}{d\underset{\sim}{p}}$$

An efficient procedure for computing this gradient is now derived.

We begin by forming the Lagrangian, i.e., by appending via Lagrange multipliers the network equations (26) to the objective function (32):

$$\hat{\phi}(\underset{\sim}{p}) = \phi(\underset{\sim}{x},\underset{\sim}{p}) + \underset{\sim}{\lambda}^T \underset{\sim}{N}(\underset{\sim}{x},\underset{\sim}{p}) \qquad (33)$$

where the language multiplier vector is partitioned as follows

$$\lambda^T = [\lambda_{n1}^T, \lambda_{n2}^T, \lambda_{t1}^T, \lambda_{t2}^T, \lambda_{t3}^T, \lambda_{t4}^T, \lambda_{t5}^T, \lambda_{t6}^T, \lambda_{n3}^T, \lambda_{n4}^T, \lambda_{n5}^T, \lambda_{n6}^T, \lambda_{n7}^T,$$
$$\lambda_{s1}^T, \lambda_{s2}^T, \lambda_{g1}^T, \lambda_{g2}^T, \lambda_{l1}^T, \lambda_{l2}^T, \lambda_{t7}^T, \lambda_{t8}^T]$$

where the first subscript is used to indicate the number of elements in each partition.

A necessary condition for $\hat{\Phi}$ to be at a minimum is that its first variation be zero:

$$\delta\hat{\Phi}(\underset{\sim}{p}) = \left[\frac{\partial\Phi}{\partial \underset{\sim}{x}} + \underset{\sim}{\lambda}^T \frac{\partial \underset{\sim}{N}}{\partial \underset{\sim}{x}}\right]\delta\underset{\sim}{x} + \left[\frac{\partial\Phi}{\partial \underset{\sim}{p}} + \underset{\sim}{\lambda}^T \frac{\partial \underset{\sim}{N}}{\partial \underset{\sim}{p}}\right]\delta\underset{\sim}{p} = 0 \tag{34}$$

where the structure of $\partial \underset{\sim}{N}/\partial \underset{\sim}{p}$ is shown in Fig. 2.

By choosing the Lagrange multipliers to satisfy the adjoint-Euler equations

$$\underset{\sim}{\lambda}^T \frac{\partial \underset{\sim}{N}}{\partial \underset{\sim}{x}} = -\frac{\partial \Phi}{\partial \underset{\sim}{x}} \tag{35}$$

(34) becomes

$$\delta\hat{\Phi}(\underset{\sim}{p}) = \left[\frac{\partial\Phi}{\partial \underset{\sim}{p}} + \underset{\sim}{\lambda}^T \frac{\partial \underset{\sim}{N}}{\partial \underset{\sim}{p}}\right]\delta\underset{\sim}{p} \tag{36}$$

from which we conclude that

$$\underset{\sim}{\nabla}\Phi(\underset{\sim}{p}) = \frac{\partial\Phi}{\partial \underset{\sim}{p}} + \underset{\sim}{\lambda}^T \frac{\partial \underset{\sim}{N}}{\partial \underset{\sim}{p}} \tag{37}$$

Thus to evaluate the gradient $\underset{\sim}{\nabla}\Phi(\underset{\sim}{p})$, we proceed as follows:

i) Solve the original system of equations (25) using the Newton-Raphson approach, eq. (30). (Sparse matrix methods should be employed here.)

ii) Upon convergence of the Newton-Raphson iteration in step (i), the adjoint-Euler equations can be solved for $\underset{\sim}{\lambda}$. Observe that these equations are a transposed version of (30) with a different right-hand side. Thus given the $\underset{\sim}{L}\,\underset{\sim}{U}$ factors of $\underset{\sim}{J}$ obtained in step (i), all that is required here is a <u>single forward-backward substitution</u> [10]. Thus little additional computational effort is required to obtain λ and $\underset{\sim}{x}$ has been found,

iii) Once $\underset{\sim}{\lambda}$ is obtained, (37) can be evaluated to determine $\underset{\sim}{\nabla}\Phi(\underset{\sim}{p})$.

Observe that in the above derivation no restrictions were placed upon what constitutes ϕ or p. Hence we can use this procedure to conveniently compute the sensitivities of such quantities as power lost in at line, $P_t^L = (P_t^E - P_t^E)$ with respect to a generator voltage E_g or power P_g^E.

4. Application of the Tableau Approach

In power system analysis and design a variety of problems must be addressed which involve analyzing and synthesizing power networks. The tableau approach presented in the last two sections can be effectively employed to solve many of these practical problems principally because all the variables of interest are explicitly displayed. In this section a representative collection of such problems will be discussed.

Optimal Power Flow Analysis

The so-called optimal power-flow problem [11] can be stated as follows: Determine P_g which minimizes the cost of serving the real power loads at the system busses (2,3,...,g). Mathematically, the problem can be stated as

$$\min_{P_g^E} \phi(P_g^E, P_g^B) = \phi_1 \left[P_1^E\right] + \phi_2 \left[P_2^E\right] + \dots + \phi_g \left[P_g^E\right]$$

subject to

$$W(x,p) = 0$$

where each of the ϕ_j's are polynomials in P_j^E, i.e.,

$$\Phi_j = k_1 + k_2 P_j^E + k_3 \left(P_j^E\right)^2 + \cdots \qquad j = 1,2,3,\dots,g$$

From (37) and (25) we see that the terms of the gradient if this objective function is

$$\frac{\partial \Phi}{\partial P_k^E} = \frac{\partial \Phi_k}{\partial P_k^E} - \underset{\sim}{\lambda}_{n3}^T \underset{\sim}{A}_{g_k}^E + \lambda_{g2_k}$$

where $\underset{\sim}{A}_{g_k}^E$ is the $(k-1)^{st}$ column of $\underset{\sim}{A}_g$ and λ_{g2_k} is the k^{th} element of $\underset{\sim}{\lambda}_{g2}$.

Minimum-Loss Compensation Problem

The minimum loss problem [12] generally refers to the problem of determining where to add reactive power support in a system to minimize the real power losses. Mathematically, the problem is

$$\min_{\underset{\sim}{b}} \Phi(\underset{\sim}{P}_t^E, \underset{\sim}{P}_t^L, \underset{\sim}{b}) = \sum_{k=1}^{t} \left(P_{t_k}^E - P_{t_k}^L\right)^2$$

subject to

$$\underset{\sim}{N}(\underset{\sim}{x}, \underset{\sim}{p}) = 0$$

The difference between $P_{t_k}^E$ and $P_{t_k}^L$ is the real power loss in the kth transmission line element. In this case $\underset{\sim}{b}$ is the shunt susceptance vector containing the shunt compensation to be used in minimizing system losses. If desired, the performance index Φ can be augmented by the addition of a term associated with the cost of compensation to obtain the minimum cost compensation design to achieve the minimum loss objective. From (37) and (25) the gradient is seen to be

$$\frac{\partial \Phi}{\partial \underset{\sim}{b}} = \left(\underset{\sim}{\lambda}_{t8} \times \underset{\sim}{f}_t\right)^T$$

Automated Network Design Problem

An important area of power system design involves minimizing a performance index which reflects line overloads or deteriorated bus voltages or both by adding new elements to the power network. The design problem can be stated mathematically as follows:

$$\min_{g,b} \Phi(c_c, d_c, e_n, f_n, g, b) = \begin{cases} \sum_{k=1}^{t} W_k \left(\left(c_{t_k}^2 + d_{t_k}^2 \right)^{1/2} - I_k^o \right)^2 & \text{if current overload is of concern} \\ \text{or} & \\ \sum_{k=1}^{t} \left(\left(e_{t_k}^2 + f_{t_k}^2 \right)^{1/2} - E_k^o \right)^2 & \text{if voltage profile degradation is of concern} \end{cases}$$

subject to

$$W(x,p) = 0$$

where $W_k = 0$ if $c_{c_k}^2 + d_{c_k}^2 < I_k^{o2}$ and 1 otherwise, I_k^o and E_k^o are desired quantities, and g and b are the conductances and susceptances of new elements that are added to the original system in order to minimize Φ. Since in practice g and b are functions of element type, size, etc., they obviously cannot assume arbitrary values, thus penalty functions must generally be added to Φ to insure a feasible solution. Furthermore, prior to initiating the design step the bus incidence matrices A_t^T and A_t must be updated to account for the newly added bus interconnections. The gradient expression is seen to be

$$\frac{\partial \Phi}{\partial g} = (\lambda_{t7} \times e_t)^T \text{ and } \frac{\partial \Phi}{\partial b} = (\lambda_{t8} \times f_t)^T$$

Contingency Analysis

The area of contingency analysis is concerned with testing a given design to insure that selected line outages do not result in component overloads or voltage profile problems. Strictly speaking, this problem has traditionally not been viewed as an optimization problem, but one of sensitivity analysis. Unfortunately, most of the methods currently in use involve either cumbersome perturbation methods where lines are actually removed, and the new set of equations $W(\tilde{x},\tilde{p})$ resolved or simplifying assumptions are made to transform the nonlinear equations into a linear subset [9]. Such equations are generally called the DC power-flow equations.

Using the tableau approach, the resulting equations (see Eq. (25)) are explicit functions of line loads, $\tilde{P}^L$ and $\tilde{Q}^L$; bus voltages, $\tilde{e}_n$ and $\tilde{f}_n$; and line admittances, $\tilde{g}$ and $\tilde{b}$. Thus a change in $\tilde{g}$ and $\tilde{b}$ can be easily related to changes in the line loadings and bus voltages, i.e.,

$$\tilde{0} = \tilde{N}(\tilde{x} + \Delta\tilde{x}, \tilde{p} + \Delta\tilde{p}) \approx \tilde{N}(\tilde{x},\tilde{p}) + \frac{\partial \tilde{N}}{\partial \tilde{x}} \Delta\tilde{x} + \frac{\partial \tilde{N}}{\partial \tilde{p}} \Delta\tilde{p}$$

But, $\tilde{W}(\tilde{x},\tilde{p}) = 0$, therefore

$$\frac{\partial W}{\partial \tilde{x}} \Delta x = - \frac{\partial W}{\partial \tilde{p}} \Delta p$$

or

$$\frac{\partial \tilde{x}}{\partial \tilde{p}} = - \left(\frac{\partial \tilde{N}}{\partial \tilde{x}}\right)^{-1} \frac{\partial \tilde{N}}{\partial \tilde{p}}$$

This is similar to Eq. (35) in Section IV except for the right-hand-side. Thus, in the course of solving any of the problems described earlier in this section, the relative impact that line outages may have on the voltages and power-flows can easily be obtained.

VI. Conclusion

In the paper, we have proposed a tableau formulation of the power-flow equations, and have shown how any one of a general class of problems encountered in power system analysis and design can be easily specified feasible due to the existence of highly efficient sparse matrix techniques. It should be noted that with this formulation, it is now possible to develop a generic computer code which can solve a variety of power system design problems. Such a code should reduce both the human and computer resources required in maintaining and using computer design tools.

References

1. F. Wu, "Nonlinear Resistive Circuit Models for Power System Steady State Analysis," Proceedings of the 14th Annual Allerton Conf., pp. 261-268, Sept, 1976.

2. Fischl, R., and Puntel, W. R. (1972), "Computer-Aided Design of Electric Power Transmission Networks," IEEE PAS Conference Paper, C720168-8.

3. Puntel, W. R., and et. al., (1973), "An Automated Method for Long-Range Planning of Transmission Networks," PICA Proc., pp. 38-46.

4. S. W. Director, "Survey of Circuit-Oriented Optimization Techniques," IEEE Trans. on Circuit Theory, Vol. CT-18, No. 1, January 1971.

5. R. Fletcher and M. J. D. Powell, "A Rapidly Convergent Descent Method for Minimization," Computer Journal, Vol. 6, pp. 163-168, 1963.

6. G. D. Hachtel, R. K. Brayton and F. G. Gustavson, "The Sparse Tableau Approach to Network Analysis and Design," IEEE Trans. on Circuit Theory, CT-18, No. 1, 101-113 (1971).

7. S. W. Director and R. A. Rohrer, "The Generalized Adjoint-Network and Network Sensitivities," IEEE Trans. on Circuit Theory, Vol. CT-16, pp. 318-323, August 1969.

8. D. J. Rose and Ralph A. Willoughby, eds., Sparse Matrices and their Applications, Plenum Press, 1972.

9. I. S. Duff, "A Survey of Sparse Matrix Research," Proc. of IEEE, Vol. 65, No. 4, pp. 500-535, April 1977.

10. S. W. Director, "LU Factorization in Network Sensitivity Computations," IEEE Trans. on Circuit Theory, Vol. CT-18, No. 1, pp. 184-185, Jan. 1971.

11. H. W. Dummel and W. F. Tinney, "Optimal Power-Flow Solutions," IEEE Trans. on PAS, Vol. PAS-87, pp. 1866-1876, 1968.

12. A. M. Sasson, "Non -Linear Programming Solutions for Load-Flow, Minimum-Loss, and Economic Dispatching Problems," IEEE Trans. on PAS, Vol. PAS-88, pp. 399-408, 1969.

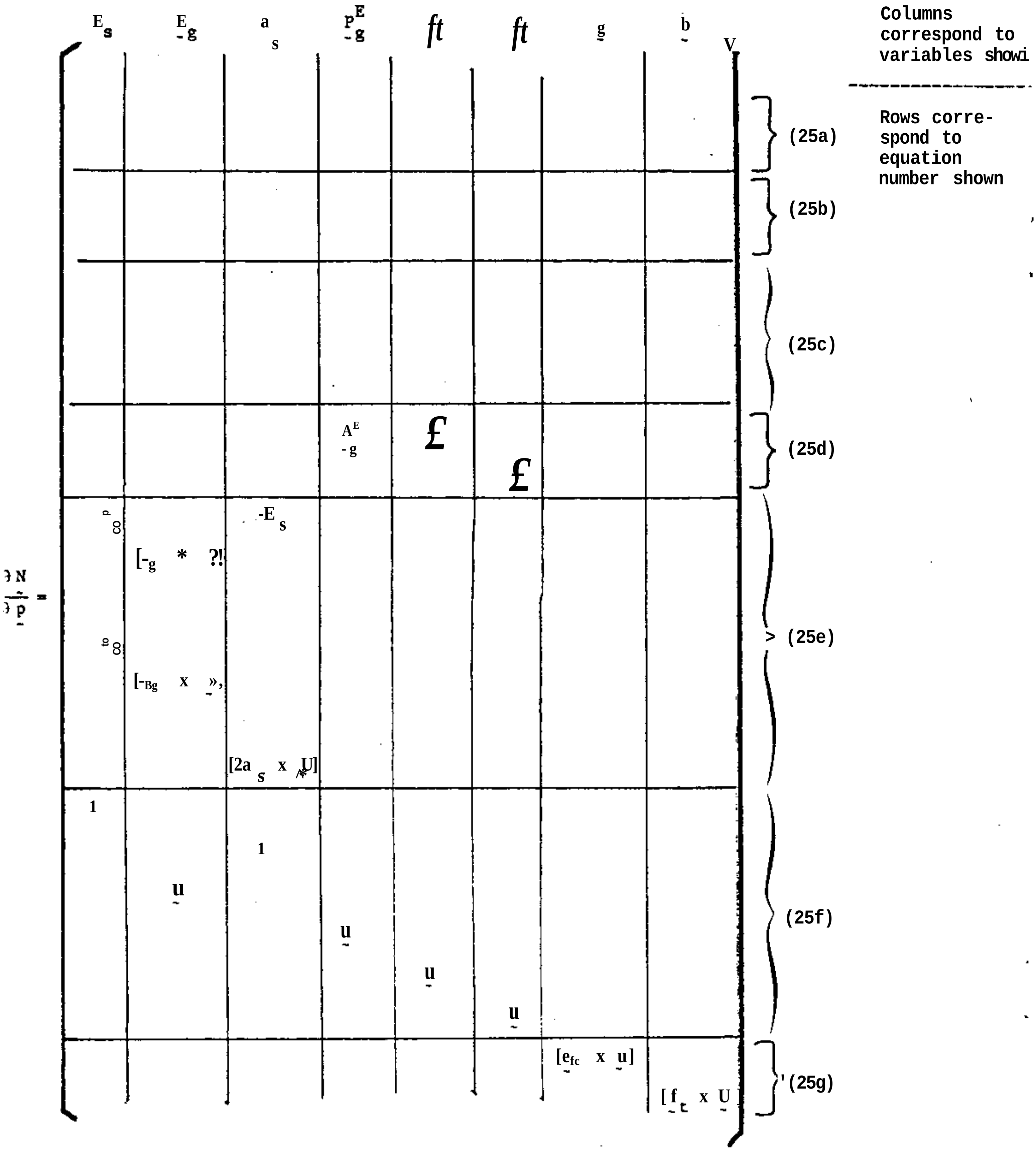
Footnotes

1. A slack branch is a generator branch whose real and reactive powers are dependent variables, i.e., its power will be determined by the network.
2. A load branch is a power consuming branch.
3. A transmission line can be modeled, for our purposes, as a pi network containing complex-valued impedances. t represents the total number of elements used to model all transmission lines.
4. A bus is point of interconnection of two or more elements, i.e., a node.
5. If there is more than one source connected to a bus, they can be combined to form a single source.

Figure Captions

1. The structure of the Jacobian in equation (30). Only nonzero partitions are shown. All other partitions contain zeros. Note that the columns of T correspond to the variables indicated on top. The superscript k means that all variables are evaluated,at the k^{th} iteration.

2. The structure of the matrix $3 \underset{\sim}{N}\wedge \underset{\sim}{p}$ in equation (34). Only nonzero partitions are shown. Note that the rows corresponding to equations (25e) have been separated into equations which correspond to the slack bus, the generator buses and the load buses.



Columns correspond to variables showi
Rows corre-
spond to
equation
number shown
(25a)
(25b)
(25c)
(25d)
(25e)
(25f)
(25g)

Fig. 2