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Wavelet Benchmarking with Seasonal Adjustment

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Wavelet Benchmarking with Seasonal Adjustment
SEM, Paris

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University of California, Irvine

22nd July 2015
Quarterly Component of UK GDP Data

Quarterly Component of GDP Data

GDP Component Value

Quarter Index

1997 Q1
1998 Q1
1999 Q1
2000 Q1
2001 Q1
2002 Q1
2003 Q1
2004 Q1
2005 Q1
2006 Q1
2007 Q1
2008 Q1
2009 Q1
2010 Q1
2011 Q1
Introduction to Benchmarking

Annual Component of UK GDP Data and Annualised Quarterly Component of UK GDP Data

- Annual Component of GDP Data
- Annualised Component of Quarterly GDP Data

GDP Component Value

Year Index


400 600 800 1000 1200 1400 1600

Sayal (Cambridge)
Wavelet Benchmarking
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Benchmarking Framework

- $Y^H_O, Y^L_O = Y^L_T$ - Observed
- $Y^H_T$ - Unobserved
- $Y^L_T = JY^H_T$ - Benchmark Condition (J “Annualising” Matrix)

$Y^H_O$ violates benchmark constraint:

$$JY^H_O \neq Y^L_O$$ (1)

A - Benchmarking Matrix

$$\hat{Y}^H_T = A \begin{pmatrix} Y^H_O \\ Y^L_O \end{pmatrix}$$ (2)

$$J\hat{Y}^H_T = Y^L_O$$ (3)

- $\hat{Y}^H_T$ - Benchmarked Series
Dagum and Cholette Adjusted Quarterly Component of UK GDP Data

- Dagum and Cholette Quarterly Adjusted Component of GDP Data
- Quarterly form of Annual Component of GDP Data

Quarterly form of Annual Component of GDP Data

GDP Component Value

Quarter Index

1997 Q1, 1998 Q1, 1999 Q1, 2000 Q1, 2001 Q1, 2002 Q1, 2003 Q1, 2004 Q1, 2005 Q1, 2006 Q1, 2007 Q1, 2008 Q1, 2009 Q1, 2010 Q1, 2011 Q1
Annualised Versions of Denton and Dagum and Cholette Benchmarked Series of Component of UK GDP Data

- Annual Component of GDP Data
- Denton Annualised Component of GDP Data
- Dagum and Cholette Annualised Component of GDP Data
Haar Wavelets

**EFW:** \( \phi^{-1,1}(t) = \frac{1}{\sqrt{n}} 1(1 \leq t \leq n) \)

**MW:** \( \phi^{0,1}(t) = \begin{cases} 
1 & 1 \leq t \leq \frac{n}{2} \\
-1 & \frac{n}{2} < t \leq n \\
0 & \text{otherwise}
\end{cases} \), \( \phi^{j,k}(t) = 2^{j/2} \phi^{0,1}(2^j t - k) \)

![Graphs of Haar Wavelets](image)
Figure: Unbalanced Haar wavelet segmentation for dataset of length 600 for frequency levels $-1, 0, 1, 2$. Wavelets have been rescaled for illustration purposes.
Wavelets and Benchmarking

- $\varphi^{-1,1}(t)$ - Elementary Father Wavelet, $\varphi^{j,k}(t)$ - Mother Wavelet
- $Y^H_O$ (Noisy), $Y^L_O$ (Non Noisy) - Observed

$$Y^H_O(t) = \tilde{d}^H \varphi^{-1,1}(t) + \sum_{j=0}^{H-1} \sum_{k} \tilde{c}^H_{j,k} \varphi^{j,k}(t)$$ (4)

$$= \tilde{Y}^L_O(t) + \sum_{j=L}^{H-1} \sum_{k} \tilde{c}^H_{j,k} \varphi^{j,k}(t)$$ (5)

EWB: $\hat{Y}^H_O(t) = Y^L_O(t) + \sum_{j=L}^{H-1} \sum_{k} \tilde{c}^H_{j,k} \varphi^{j,k}(t)$ (6)

with $d^L_k, c^L_{j,k}, \tilde{d}^H_k, \tilde{c}^H_{j,k} \in \mathbb{R}$
Local linear model (with seasonal factor)

\[ y(t) = \mu(t) + \gamma(t) + \epsilon(t), \quad \epsilon(t), \quad (7) \]
\[ \mu(t) = \mu(t - 1) + \nu(t) + \varphi(t), \quad \varphi(t) \sim N(0, \sigma_{\varphi}^2), \quad (8) \]
\[ \nu(t) = \nu(t - 1) + \zeta(t), \quad \zeta(t) \sim N(0, \sigma_{\zeta}^2), \quad (9) \]
\[ \gamma(t) = - \sum_{j=1}^{k-1} \gamma(t + 1 - j) + \omega(t), \quad \omega(t) \sim N(0, \sigma_{\omega}^2) \quad (10) \]
\[ \epsilon(t) = \theta \epsilon(t - 1) + \delta(t) + \phi \delta(t), \quad \delta(t) \sim N(0, \sigma_{\delta}^2), \quad |\theta|, |\phi| < 1 \quad (11) \]

- \( \mu(t) \) - trend
- \( \nu(t) \) - slope
- \( \gamma(t) \) - seasonal
- \( \epsilon(t) \) - ARMA process
Boxplot Comparing Different Benchmarking Methods of 500 Simulations

MSE Value

Original
Elementary Wavelet
Denton 1
Denton 2
Denton Proportionate
Dagum and Cholette
Wavelet
Thresholding

True Wavelet Decomposition Coefficients

Noisy Wavelet Decomposition Coefficients
Thresholding

- Time Domain: $y_i = f(t_i) + \sigma \epsilon_i, \ i = 1, \ldots, n$
- Wavelet Domain: $y_{j,k} = w_{j,k} + \sigma \tilde{\epsilon}_{j,k}, \ j = 1, \ldots, J, \ k = 1, \ldots, k_j$

\[ \delta^H(y_{j,k}) = \begin{cases} 0, & \text{if } |y_{j,k}| \leq \lambda. \\
 y_{j,k}, & \text{if } |y_{j,k}| > \lambda. \end{cases} \]

\[ \delta^S(y_{j,k}) = \begin{cases} 0, & \text{if } |y_{j,k}| \leq \lambda. \\
 y_{j,k} - \lambda, & \text{if } y_{j,k} > \lambda. \\
 y_{j,k} + \lambda, & \text{if } y_{j,k} < -\lambda. \end{cases} \]
Alternative Seasonal Model

\[
\begin{pmatrix}
\gamma_{1,t+1} \\
\vdots \\
\gamma_{k,t+1}
\end{pmatrix}
= \begin{pmatrix}
\gamma_{1,t} \\
\vdots \\
\gamma_{k,t}
\end{pmatrix}
+ \begin{pmatrix}
\omega_{1,t} \\
\vdots \\
\omega_{k,t}
\end{pmatrix},
\]

or equivalently,

\[
\gamma_{t+1} = \gamma_t + \omega_t
\]

\[
\sum_{j=1}^{k} \gamma_{j,t+1} = \sum_{j=1}^{k} \gamma_{j,t} = \ldots = \sum_{j=1}^{k} \gamma_{j,0} = 0
\]

\[
\mathbb{E}(\omega_t) = \mathbf{0}_{k \times 1}, \quad \text{Var}(\omega_t) = \sigma^2\omega \left( \mathbf{I}_k - \frac{1}{k} \mathbf{I}_{k \times 1} \mathbf{I}'_{k \times 1} \right)
\]
Overview

1. Transform High and Low Frequency Series to Wavelet domain

2. Elementary Benchmarking Step

3. Thresholding Step

4. Transform adjusted series to Time domain
### Simulated Non Dyadic Monthly and Quarterly Data

<table>
<thead>
<tr>
<th>Series Type</th>
<th>MSE</th>
<th>Revision Metric</th>
<th>Growth Rate Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original (Noisy)</td>
<td>6731.98</td>
<td>NA</td>
<td>13.11</td>
</tr>
<tr>
<td>Denton(_a,1)</td>
<td>3382.58</td>
<td>1.58</td>
<td>11.48</td>
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<tr>
<td>Denton(_a,2)</td>
<td>3529.47</td>
<td>2.76</td>
<td>11.74</td>
</tr>
<tr>
<td>Denton(_p,1)</td>
<td>3657.26</td>
<td>1.40</td>
<td>12.15</td>
</tr>
<tr>
<td>Cholette and Dagum</td>
<td>3378.83</td>
<td>1.51</td>
<td>11.48</td>
</tr>
<tr>
<td>Elementary Wavelet</td>
<td>3436.33</td>
<td>0.00</td>
<td>11.71</td>
</tr>
<tr>
<td>Wavelet Benchmarking</td>
<td>1856.62</td>
<td>0.81</td>
<td>8.26</td>
</tr>
</tbody>
</table>

**Table:** Average MSE and revision values of different benchmarking methods corresponding to 500 non dyadic monthly and quarterly simulated series. The quarterly and monthly series had lengths 70 and 210 respectively.
### Table: Average MSE and revision values of different benchmarking methods corresponding to 500 non dyadic monthly and quarterly simulated series. The quarterly and monthly series had lengths 10 and 30 respectively.

<table>
<thead>
<tr>
<th>Series Type</th>
<th>MSE</th>
<th>Revision Metric</th>
<th>Growth Rate Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original (Noisy)</td>
<td>2392.07</td>
<td>NA</td>
<td>20.39</td>
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<tr>
<td>Denton(_a,1)</td>
<td>926.92</td>
<td>2.94</td>
<td>17.68</td>
</tr>
<tr>
<td>Denton(_a,2)</td>
<td>1003.47</td>
<td>4.66</td>
<td>18.11</td>
</tr>
<tr>
<td>Denton(_p,1)</td>
<td>1069.29</td>
<td>2.87</td>
<td>18.66</td>
</tr>
<tr>
<td>Cholette and Dagum</td>
<td>922.37</td>
<td>2.75</td>
<td>17.71</td>
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<tr>
<td>Elementary Wavelet</td>
<td>996.02</td>
<td>0.00</td>
<td>18.75</td>
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<tr>
<td>Wavelet Benchmarking</td>
<td>566.46</td>
<td>0.95</td>
<td>13.18</td>
</tr>
</tbody>
</table>
Different Benchmarking Methods Applied to Component of UK GDP Data

Quarterly Component of GDP Data
Wavelet Benchmarked Series
Denton Benchmarked Series
Dagum and Cholette Benchmarked Series
Quarterly form of Annual Component of GDP Data
<table>
<thead>
<tr>
<th>Series Type</th>
<th>Revision Metric</th>
<th>Growth Rate Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denton_{a,1}</td>
<td>0.61</td>
<td>7.27</td>
</tr>
<tr>
<td>Denton_{a,2}</td>
<td>0.35</td>
<td>7.60</td>
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<tr>
<td>Denton_{p,1}</td>
<td>1.16</td>
<td>5.33</td>
</tr>
<tr>
<td>Cholette and Dagum</td>
<td>1.95</td>
<td>7.19</td>
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<tr>
<td>Elementary Wavelet</td>
<td>0.00</td>
<td>6.12</td>
</tr>
<tr>
<td>Wavelet Benchmarking</td>
<td>0.87</td>
<td>6.61</td>
</tr>
</tbody>
</table>

**Table**: Metric values for different benchmarked series corresponding to Official ONS data.
Summary

- Wavelet Domain - Natural Framework for Benchmarking
- Outperform Currently Used Methods

Further Research

- Non Binding Benchmarking
- Basis Selection
- Seasonal Adjustment in Wavelet Domain
References


Homesh Sayal and John A. D. Aston and Duncan Elliott and Hernando Ombao. An Introduction to Applications of Wavelet Benchmarking with Seasonal Adjustment. *arXiv:1410.7148*