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A STRUCTURAL OPTIMIZATION APPROACH IN PROCESS SYNTHESIS. PART II: HEAT RECOVERY NETWORKS

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Abstract

Several formulations of the transshipment model from Operations Research are proposed for the optimal synthesis of heat exchanger networks. The linear programming versions are used for predicting the minimum utility cost, and can handle restricted matches and multiple utilities. The mixed-integer programming version yields minimum utility cost networks in which the number of units is minimized, while allowing stream splitting and selection of most preferred matches. It is shown that the transshipment models can also be incorporated easily within a mixed-integer programming approach for synthesizing chemical processing systems. Several numerical examples are presented which show that the proposed models are computationally very efficient.

Scope

A major component affecting the overall performance of processing systems is the heat recovery network. The task of a heat recovery network is to exchange the available heat of all process streams in order to reduce the consumption of heating and cooling utilities. Since the cost of utilities is usually the dominant item, there is a great incentive to design heat recovery networks that integrate efficiently process streams.

There are several practical considerations involved in the synthesis of efficient heat recovery networks. First of all, there are different utilities that can be used for providing the necessary heating and cooling of process streams. For example fuel, steam at different levels (high, medium and low pressure) and hot water can be employed as heating utilities, while cooling water and refrigerants can be used as cooling utilities. Since all these utilities have different cost per unit heat, it is very important to synthesize heat exchanger networks for which the utility cost is at a minimum. Another important aspect in the design of heat recovery networks is stream splitting which is often necessary to attain better heat integration in the network. In some cases this can also be achieved by allowing multiple matches between certain pairs of streams, which then lead to cyclic networks. Finally, another consideration is the specification of forbidden matches between certain pairs of process streams. This restriction usually arises in practice because of the plant layout, safety requirements, or process control difficulties. Therefore, it is important that a procedure for the synthesis of efficient heat recovery networks should account for all the above considerations.

The main difficulty in the heat recovery problem is its inherent combinatorial nature since usually there is an enormous number of possible networks. In the last fifteen years a large number of methods have been proposed for tackling this synthesis problem. A review of all the previous research work is beyond the scope of this paper, and the reader can find an excellent coverage on this subject in a recent journal review by Nishida et al# [13]. The purpose of this paper is to present an approach for the systematic synthesis of heat recovery networks which is based on different transshipment model3. These models provide efficient procedures for synthesis, and can be incorporated in a natural form within the HELP formulation for the synthesis of total processing systems that is given in the third part of this series of papers [15].

Conclusions and Significance

The transshipment models that have been presented in this paper provide a systematic framework for the optimal synthesis of heat exchanger networks. The models for minimum utility cost have the advantage of involving linear programming problems of small size which can be solved with little computational effort. Also, as has been shown these models can be extended readily in order to be incorporated in synthesis procedures of processing systems that are based on mixed-integer programming. The model for minimum number of units which involves a mixed-integer linear programming problem, can be used for deriving network configurations that involve stream splitting and selection of most preferred matches. For the latter aspect a special weighting scheme was developed which allows the designer to specify different levels of priority for the matches. The numerical examples have shown the proposed approach to be very efficient and powerful.

Introduction

As discussed in the first part of this series of papers [14], the heat recovery network is one of the crucial components in a total processing system since its task is to exchange heat among the streams of the chemical process in order to reduce the consumption of utilities.

The heat recovery network synthesis problem that will be considered in this paper can be stated as follows. In a processing system there is a set $H = \{i | i=1,NH\}$ of hot streams that have to be cooled, and a set $C = \{j | j=1,NC\}$ of cold streams that have to be heated. Each hot stream i has mass flowrate F_i, heat capacity $(c_p)_i$, and has to be cooled from supply temperature T_i^s to target temperature T_i^t. Similarly, each cold stream j has mass flowrate F_j, heat capacity $(c_p)_j$ and has to be heated from supply temperature T_j^s to target temperature T_j^t. Since the total heat content of the hot and cold streams is usually unequal, and because of thermodynamic constraints in the transfer of heat, auxiliary heating and cooling is assumed to be available from a set $S = \{m | m=1,NS\}$ of hot utilities (e.g. fuel, steam), and a set $W = \{n | n=1,NW\}$ of cold utilities (e.g. cooling water, refrigeration). The objective of the synthesis problem is then to develop a network of countercurrent heat exchangers that satisfies the specifications at minimum investment and operating cost (e.g. in annualized form).

Due to the large number of possible network configurations and to the nonlinearities involved in the investment cost function of the heat exchangers, the main approach that has emerged in the last few years is to develop design objectives that will simplify and reduce the size of this synthesis problem. Although these objectives cannot guarantee rigorous cost minimization, they have the property of generating networks with maximum heat recovery which often corresponds to optimal or near optimal solutions. The most important objectives can be summarized in three major results that can be used for the design of energy efficient networks. The first two objectives were first identified by Hohmann [10] and later by Linnhoff and Flower [11], while the third one was proposed by Umeda et al. [16].

Minimum Utility Consumption. This is the most important design objective for an efficient heat exchanger network, since it corresponds to the maximum heat integration that can be attained in a feasible network for a fixed minimum temperature approach. Also, since the cost of utilities is commonly the dominant cost item, this objective allows the elimination of many network configurations which are inefficient and costly. The prediction of minimum utilities can be performed prior to developing the actual heat recovery network structure [11] . This design objective can be further refined as the prediction of minimum utility cost. This is necessary because in actual networks there is usually a variety of hot and cold utilities employed, and each utility is priced at different cost (i.e. fuel, heating steam at different pressure levels, hot water, cooling water, refrigerants, etc).

Minimum Number of Units. Another important objective is determining the minimum number of heat exchanger units that is required in the network. This objective attempts to minimize indirectly the investment cost of the network since the cost of each exchanger is assumed to be a concave function of the area. As noted by Hohmann [10], the minimum number of units is usually one less than the total number of process streams and necessary utilities.

Modification of Pinch Points. A pinch point can be regarded as a bottleneck that prevents further heat integration in a network. An example of a pinch point is shown in Fig. 1, in which the composite hot and cold streams of a process are plotted in a temperature/enthalpy diagram. Note that the presence of the pinch point limits the maximum heat integration that is possible. Therefore, it is important to identify the location of pinch points prior to developing a network, in order to consider changes in the process that can eliminate or modify these bottlenecks so as to enhance heat integration [16].

The first two design objectives have been used in previous methods for the synthesis of efficient heat exchanger networks. Flower and Linnhoff [6] proposed the thermodynamic-combinatorial algorithm (TC) which will generate all minimum utility usage networks with the minimum number of heat exchanger units and with no stream splitting. The first step of the TC algorithm is to divide the entire temperature range of the streams into temperature intervals according to partitioning rules that allow feasible heat exchange. Next, the minimum heating and cooling utilities are predicted using the procedure of the problem table as given in Linnhoff and Flower [11]. The last step is to generate all networks that require minimum utilities and have the fewest number of units.

A similar strategy for the synthesis of heat recovery networks has been proposed by Cerda et al. [3],and Cerda and Westerberg [4], [5]. In the initial phase the minimum utility usage problem is considered* The temperature range of all the streams is partitioned into temperature intervals, but then the problem is modeled as a transportation problem where all possible routes are considered in which heat is shipped from the hot streams to the cold streams. Since heat can only flow from a hot stream at a higher temperature to a cold stream at a lower temperature, large cost coefficients are assigned to routes that are thermodynamically infeasible. This linear programming transportation model can be modified if necessary to account for restricted matches among certain streams, and is solved using the northwest corner algorithm. The next phase is to determine minimum utility networks involving the least number of units. This is done by reformulating the transportation problem as a mixed-integer linear program (MILP), and then relaxing the integrality constraints in order to solve it as a linear program. The final structure of the heat recovery network is derived often by hand, and stream splitting can be performed if necessary.

Several example problems have been solved successfully using the two synthesis methods described above. Although these methods cannot guarantee minimum cost of the heat exchanger networks, they obtain efficient designs that are in most cases optimal or near optimal solutions* Therefore, the design objectives of minimum utilities and minimum number of units provide very powerful targets in the synthesis of heat recovery networks•

In this paper a number of transshipment models will be proposed for the synthesis of heat recovery networks. The linear programming versions can be used for predicting the minimum utility cost with and without restricted matches. The mixed-integer version can be used for developing networks that involve a minimum number of heat exchanger units with possible splitting and mixing of streams, and where preferences.can be assigned to the matches. The main advantage of these models is that they can be solved with very little or modest computational effort. Also, these models can easily be connected with the MILP model proposed by Papoulias and Grossmann [15] for the synthesis of chemical processing systems. Therefore, by incorporating the transshipment models it is possible to account for changes in the chemical process that enhance heat integration and can lead to improved heat recovery networks* This objective cannot be attained using the TC method of Flowers and Linnhoff [6] which does not account for process stream changes that alter the heat exchanger network. The transportation model of Cerda et al. [3] can be incorporated in principle within an optimization framework that allows for process stream changes. However, the large size of the transportation model makes it difficult to include it in an integrated system, and the northwest corner algorithm cannot be used any more in this case for obtaining the optimal solution.

The Transshipment Model

One of the models that is widely used in the field of Operations Research to solve network problems is the transshipment model (see Garfinkel and Nemhauser, [7]; Hillier and Lieberman, [9]). The transshipment model is a variation of the well known transportation problem, and deals with the optimum allocation of resources. In particular, the transportation model seeks to determine the optimum network for transporting a commodity (e.g. a product) from sources (e.g. plants) directly to destinations (e.g. markets). On the other hand, the transshipment model investigates the optimum network for shipping the same commodity, but from sources to intermediate nodes (e.g. warehouses) and then to destinations.

The following analogy with the transshipment model can be made for the heat recovery problem. Heat can be regarded as a commodity that is shipped from hot streams to cold streams through temperature intervals that account for thermodynamic constraints in the transfer of heat. In particular the second law of thermodynamics requires that heat flows only from higher to lower temperatures, and therefore these thermodynamic constraints have to be accounted In the network model. This can actually be done by partitioning the entire temperature range into temperature intervals according to rules proposed by Linnhoff and Flower [11] , Grimes [8], and Cerda et al. [3]. these partitioning procedures guarantee the feasible transfer of heat in each interval of the network, given the minimum temperature approach ATmin^ In this way, as shown in Fig. 2, it can be considered that heat flows from hot streams to the corresponding temperature interval, and then to cold streams in the same interval with the remainder going to the next lower temperature interval. Therefore, the transshipment model for the heat recovery network has the hot streams and heating utilities as sources, the temperature intervals as the intermediate nodes and the cold streams and cooling utilities as the destinations. The heat flow pattern for each temperature interval shown in Fig. 3 is then as follows:

a) Heat flows into a particular interval from all the hot streams and heating utilities whose temperature range includes the temperature interval.

b) Heat flows out of a particular interval to the cold streams and cooling utilities whose temperature range includes the temperature interval.

c) Heat flows out of a particular interval to the next lower temperature interval. This heat is the residual (excess) heat that cannot be utilized in the present interval, and consequently has to flow to a lower temperature interval.

d) Heat flows into a particular temperature interval from the previous interval that is at higher temperature. This heat is the residual (excess) heat that cannot be utilized in the higher temperature interval.

It should be noted that this network flow pattern is a special case of the general transshipment model [7], [9], since all the flows of heat from hot streams to temperature intervals,and from temperature intervals to cold streams are normally fixed. In such a case the only variables in this transshipment network are the residual heat flows from one temperature interval to the next lover temperature interval, and the flowrates of hot and cold utilities.

There are different mathematical formulations of the transshipment model that can be employed for the systematic synthesis of heat recovery networks. As it will be shown in the next sections, these formulations can be used for predicting the minimum utility cost and for deriving networks with minimum number of units.

Minimum Utility Cost Problem

One of the design objectives employed in the synthesis of heat exchanger networks is to determine the minimum utility cost for a set of hot and cold process streams. This problem will be formulated as a transshipment problem assuming that there are no restricted matches among any pair of streams.

The first step is to partition the entire temperature range of all streams into K temperature intervals for which any suitable partitioning method can be used ([3],[8],[11]). The intervals are labeled from the highest level (k=l) down to the lowest level (k«K) of temperature, with each interval k (k=l,2,...,K) having a temperature change of AT_{fe}. The following sets are defined in order to identify the location of all streams and utilities relative to the temperature intervals:

Hfc " {*- l^hot stream i is present in interval k}

C_k - {j |cold stream j is present in interval kl

S_{fc} • |m I hot utility n is present in interval k) (1)

W_{fc} * jn | cold utility m is present in interval k}

Let Q^H_{ik} be the heat load of hot stream i entering temperature interval k. This heat load is given by,

$$Q^H_{ik} = F_i(c_p)_{ik}\,\Delta T^i_k \qquad (2)$$

where ΔT^i_k is the temperature change of stream i in interval k. Similarly the heat load Q^C_{jk} flowing to cold stream j from temperature interval k is calculated as,

$$Q^C_{jk} = W_{jk} \, \Delta T^j_k \qquad (3)$$

All utilities are placed in the appropriate intervals depending on their inlet and outlet temperatures. If Δh_{mk} is the enthalpy change of hot utility m in temperature interval k then the heat load Q^S_{mk} entering interval k is given by,

$$Q^S_{mk} = F^S_m \, \Delta h_{mk} \qquad (4)$$

Similarly, the heat load Q^W_{nk} of cold utility in temperature interval k is,

$$Q^W_{nk} = F^W_n \, \Delta h_{nk} \qquad (5)$$

By denoting the residual heat flowing out of interval k as R_k, and by performing a total heat balance on each interval k (see Fig. 3), the transshipment model for minimum utility cost is given by

$$\text{minimize } Z = \sum_{m \in S} s_m F^S_m + \sum_{n \in W} w_n F^W_n$$

$$\text{s.t.} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (P1)$$

$$R_k - R_{k-1} - \sum_{m \in S_k} F^S_m \Delta h_{mk} + \sum_{n \in W_k} F^W_n \Delta h_{nk} = \sum_{i \in H_k} Q^H_{ik} - \sum_{j \in C_k} Q^C_{jk} \qquad k = 1,2,\ldots K$$

$$F^S_m \geq 0 \quad m \in S, \quad F^W_n \geq 0 \quad n \in W$$

$$R_0 = R_K = 0, \quad R_k \geq 0 \quad k = 1,2,\ldots K-1$$

where s_m, w_n, are the unit costs for the hot and cold utilities. In the case that these cost coefficients are set to one the above formulation will yield a solution for minimum utility consumption. Such a solution will be equivalent to a minimum cost solution if only one type of heating and one type of cooling utility is considered. The optimal values of the hot and cold utility flowrates (F^S_m, m = 1,NS, and F^W_n, n = 1,NW) and the residual heat load IL_k of each interval k can easily be determined by solving the linear programming problem (Pi). The occurrence of any pinch points takes place between the temperature intervals with no residual heat flow, or equivalently at the point where the residual heat load R_k is equal to zero. Note that since the residuals IL_k do not correspond to any particular stream, but rather to the aggregate of the hot streams, the formulation in (Pi) is equivalent to merging the streams in composite hot and cold streams as in Fig. 1.

The above transshipment model (Pi) is an alternative formulation to the reduced transportation model proposed by Cerda et al. [3] for predicting minimum utility usage in a heat recovery network without any restricted stream matches. However, it should be noted that the size of the transshipment model is considerably smaller than the reduced transportation model that has NS + Ntf + [(K)(K+l)/2] variables and 2K rows. The size of the transshipment model (Pi) is:

a) Number of variables - NS + NW + K - 1

b) Number of rows • K

i This leads to a linear program of small size even for large number of streams. For example, given 20 process streams, 3 hot utilities and 1 cold utility the maximum number of temperature intervals is 23 according to the partitioning procedure by Grimes [8]. Therefore, the maximum size of the transshipment model is only 26 variables and 23 rows, while the maximum size of the reduced transportation model of Cerda et al. [3] is 280 variables and 46 rows.

Minimum Utility Cost with Restricted Matches

A practical consideration in the design of some heat recovery networks is the specification of forbidden matches between certain pairs of process streams. This case typically arises because of safety and control considerations, or because a pair of streams is located too far apart in the plant. Since the transshipment model (P1) does not account for restricted matches, it is necessary to develop a new formulation.

The model for restricted matches is conceptually similar to (P1), but the difference is that only the unrestricted hot and cold streams can be merged since the restricted streams must be treated separately. This is necessary because the forbidden matches must be prevented from exchanging heat in the formulation of heat balances in each interval.

In order to derive the model, assume that the set of restricted matches is specified for only some process streams and is given by

$$P = \{(i,j) \mid i \in H,\ j \in C,\ \text{match between } i \text{ and } j \text{ is forbidden}\} \tag{6}$$

The streams involved in the set P can then be identified by the subsets

$$HP = \{i \mid i \in P\}, \quad CP = \{j \mid j \in P\} \tag{7}$$

The remaining process and utility streams can be considered to be merged in hot stream h, and cold stream c. If the partitioning of temperatures is performed on all the original process and utility streams as in problem (P1), the heat content of the merged hot and cold streams in each interval k will then be given by

$$Q^H_{hk} = \sum_{\substack{i \in H \\ i \notin HP}} Q^H_{ik} + \sum_{m \in S_k} F^s_m \,\Delta h_{mk}, \quad Q^C_{ck} = \sum_{\substack{j \in C \\ j \notin CP}} Q^C_{jk} + \sum_{n \in W_k} F^W_n \,\Delta h_{nk} \tag{8}$$

Since the reduced set of streams to be analyzed is given by

$$H' = \{i \mid i=h,\ i \in HP\}, \quad C' = \{j \mid j=c,\ j \in CP\} \tag{9}$$

each hot stream $i \in H'$ will be assigned an individual heat residual R_{ik} as shown in Fig. 4. Also, the heat exchanged between hot stream $i \in H'$ and cold stream $j \in C'$ in the temperature interval k will be denoted as Q_{ijk}. It then clearly follows that for the forbidden matches $Q_{ijk} = 0$, $(i,j) \in P$, It should be noted that there is the possibility that a hot stream $i \in H$ will exchange heat with cold stream $j \in C$, $(i,j) \in P$, in an interval k where stream i is actually not present. This can happen if hot stream i is present at a higher temperature interval $k' < k$, so that the exchange of heat takes place through the residual $R_{ik'}$. Therefore, it is convenient to define the subsets of streams for potential heat exchange in each interval k, which are given by

$$
\begin{aligned}
H_k' &= \{ i \mid i \in H, \text{ stream } i \text{ is present in interval } k' \le k \} \\
C_k' &= \{ j \mid j \in C, \text{ stream } j \text{ is present in interval } k \}
\end{aligned} \qquad (10)
$$

By performing individual heat balances for the reduced set of hot and cold streams in each interval (see Fig. 4), the minimum utility cost problem for restricted matches will be given by the following transshipment model,

$$\text{minimize } Z = \sum_{m \in S} c_m^S F_m^S + \sum_{n \in W} c_n^W F_n^W$$

s.t.

$$R_{i,k} - R_{i,k-1} + \sum_{j \in C_k'} Q_{ijk} = Q_{ik}^H \qquad i \in H_k', \quad k=1,2,\ldots K$$

$$\sum_{i \in H_k'} Q_{ijk} = Q_{jk}^C \qquad j \in C_k', \quad k=1,2,\ldots K$$

$$
\left.
\begin{aligned}
Q_{hk}^H &= \sum_{\substack{i \in H \\ i \notin H_P}} Q_{ik}^H + \sum_{m \in S_k} F_m^S \Delta h_{mk} \\
Q_{ck}^C &= \sum_{\substack{j \in C \\ j \notin C_P}} Q_{jk}^C + \sum_{n \in W_k} F_n^W \Delta h_{nk}
\end{aligned}
\right\} \quad k=1,2,\ldots K
$$

$$Q_{ijk} = 0 \quad (i,j) \in P, \quad Q_{ijk} \ge 0 \quad i \in H_k', \; j \in C_k', \quad k=1,2,\ldots K$$

$$F_m^S \ge 0 \quad m \in S, \quad F_n^W \ge 0 \quad n \in W$$

$$R_{i,0} = R_{i,K} = 0, \quad R_{i,k} \ge 0 \quad k=1,2,\ldots K-1, \quad i \in H'$$

It should be noted that in the implementation of this model the variables Q^H_{hk}, Q^C_{ck} can be eliminated with the third and fourth equality constraints in (RPl). Therefore, the variables for this model are F^*_m, P^*_n, Q_{ijk}, and R_{ik}. Although the actual size of the transshipment model is dependent on the particular problem data, it is possible to calculate the following upper bounds on the number of variables and rows:

a) Maximum number of variables = NS + NW + (NHP + 1)[(NCP + 2)K - 1]

b) Maximum number of rows = (NHP + 1) (NCP + 1) K

where NHP is the number of restricted hot streams, and NCP is the number of restricted cold streams* For the example of 20 process streams, 3 hot utilities, 1 cold utility and 1 restricted match, and with 23 temperature intervals, the maximum size of the restricted transshipment model is 140 variables and 92 rows. The number of actual variables however, will be lower since many of the variables Q_{ijk} will be set to zero either because some matches are forbidden, or otherwise because they dre thermodynamically infeasible. it should also be noted that the size of model (RPl) is much, smaller than the transportation model for restricted matches proposed by Cerda et al. [3].

Minimum Number of Heat Exchanger Units

The objective of the previous transshipment models is to determine the minimum utility cost and location of any pinch points in a heat recovery network. However, because there are often many minimum utility cost networks, a desirable objective is to obtain from among these networks one that has the minimum number of heat exchanger units since this will usually correspond to an optimal or near optimal solution*

At this point it is assumed that the minimum utility cost will have been determined with either of the transshipment models (PI) or (RPl).

Since the utility flowrates and their corresponding heat contents **will** then be known, the utility streams can be added to the sets of process streams so as to define the augmented sets $H = \{H,S\}$ and $C = \{C,W\}$ of hot and cold streams. Also, in general the optimal solution to (P1) or (RP1) will indicate the existence of one or more pinch points in the network, in which case the problem can be partitioned in subnetworks as no heat will flow across each pinch point [8]. More specifically, if NL - 1 pinch points occur, the K temperature intervals can be partitioned in NL sets of intervals above and below each pinch point that define the boundary of the subnetworks. The subsets of temperature intervals corresponding to each subnetwork ℓ, will be denoted by SN_ℓ, $\ell = 1,2,\ldots NL$.

In order to satisfy the minimum utility cost solution only the streams within each subnetwork ℓ should be allowed to exchange heat as otherwise heat would be transferred across the pinch points, and hence, the utility usage increased. It is therefore convenient to denote as $H_\ell \subseteq H$ and $C_\ell \subseteq C$ the hot and cold streams present in subnetwork ℓ. Following a similar treatment as in the problem of restricted matches (RP1), the heat residuals of the hot streams $i \in H_\ell$ will be represented by R_{ik}, $k \in SN_\ell$, $\ell = 1,NL$, while the heat exchanged between the streams in the subnetwork will be represented by Q_{ijk}, $i \in H_{\ell k}$, $j \in C_{\ell k}$, $k \in SN_\ell$ where

$$
\begin{aligned}
H_{\ell k} &= \{i \mid i \in H_\ell, \text{ stream } i \text{ is present in interval } \bar{k} \le k;\ \bar{k}, k \in SN_\ell\} \\
C_{\ell k} &= \{j \mid j \in C_\ell, \text{ stream } j \text{ is present in interval } k \in SN_\ell\}
\end{aligned} \tag{11}
$$

The 0-1 binary variable $y_{ij\ell}$ can then be introduced to denote the existence of a match between streams $i \in H_\ell$ and $j \in C_\ell$ in subnetwork ℓ. It is assumed here that each one of these potential matches is associated to a potential heat exchanger unit* Since the total heat exchanged between the given pair of streams is given by the sum of their heat exchange taken over the intervals of the subnetwork, the binary variables can be related to the variables Q_{ijk} through the inequalities

$$\sum_{k\in SN_\ell} Q_{ijk} - U_{ij\ell}\, y_{ij\ell} \le 0 \qquad i\in H_\ell,\ j\in C_\ell,\ \ell = 1,2,\ldots NL \tag{12}$$

$$\text{where} \quad U_{ij\ell} \le \min\left\{ \sum_{k\in SN_\ell} \hat{Q}^H_{ik} \,,\ \sum_{k\in SN_\ell} \hat{Q}^C_{jk} \right\} \tag{13}$$

corresponds to the upper bound on the heat that can be exchanged* Note that when the binary variable $y_{ij\ell}$ in (12) takes a zero-value no heat can be exchanged, but when it is set to a value of one any amount of heat that does not exceed $U_{ij\ell}$ can be exchanged. The problem of minimizing the number of units in the heat exchanger network can then be formulated as the following mixed-integer transshipment problem,

$$\text{minimize } Z = \sum_{\ell=1}^{NL} \sum_{i\in H_\ell} \sum_{j\in C_\ell} e_{ij\ell}\, y_{ij\ell}$$

s.t. (P2)

$$\left.\begin{array}{ll} R_{ik} - R_{i,k-1} + \sum\limits_{j\in C_{\ell k}} Q_{ijk} = \hat{Q}^H_{ik} & i \in H_{\ell k} \\ \sum\limits_{i\in H_{\ell k}} Q_{ijk} = \hat{Q}^C_{jk} & j \in C_{\ell k} \end{array}\right| \begin{array}{l} k \in SN_\ell \\ \ell = 1,2,\ldots NL \end{array}$$

$$\sum_{k\in SN_\ell} Q_{ijk} - U_{ij\ell}\, y_{ij\ell} \le 0 \quad i \in H_\ell,\ j \in C_\ell \quad \ell = 1,2,\ldots NL$$

$$\left.\begin{array}{ll} R_{ik} \ge 0 & i \in H_{\ell k} \\ Q_{ijk} \ge 0 & i \in H_{\ell k} \\ & j \in C_{\ell k} \end{array}\right| \begin{array}{l} k \in SN_\ell \\ \ell = 1,2,\ldots NL \end{array}$$

$$y_{ij\ell} = 0,1 \quad i \in H_\ell,\ \ j \in C_\ell,\ \ \ell = 1,2,\ldots NL.$$

Note that in the objective function (P2), each binary variable $y_{ij\ell}$, is multiplied by the weight $e_{ij\ell}$, that can acc<>unt for the cost or preference of the match between the steams. Since cost coefficients are difficult to derive because of the nonlinearities that are involved in the temperatures, it is more practical to think of these weights e_{ij}..£ as coefficients that reflect preferences of the matches. If there are no particular preferences all the weights can be set to one, in which case problem (P2) will provide a solution with minimum number of units. However, since very often there will exist more than one such solution the objective of selecting preferred matches becomes important. This is particularly true when for the various pairs of matches there are significant differences in heat transfer coefficients, materials of construction, or when pairs of streams are located in different sections of the plant. In such cases, weights can be derived so that the optimal solution in (P2) exhibits always the minimum possible number of heat exchangers, but if there is a choice, the preferred stream matches are selected. The derivation and formulas for these weights are given in the Appendix for the cases when levels of priority are assigned either to individual matches or to groups of matches. It should also be noted that forbidden matches can be handled readily in (P2) by setting the variables y^i = °> (*>J)*Pf 4=1,2,...NL. Clearly these forbidden matches would have to be the same as the ones specified in the model (RPl).

The MILP given by (P2) can be solved either in its full form, or otherwise it can be decomposed into the NL smaller subproblems for each subnetwork, and in each case standard branch and bound enumeration codes can be used. Cerda and Westerberg [4] developed also a MILP based on the transportation model for deriving networks with minimum number of units, but they propose to use several LP relaxations to avoid solving the original mixed-integer problem.

The Transshipment Model for Integrated Systems

Another important objective in the synthesis of heat recovery networks is to consider feasible changes in the structure or parameters of the chemical plant that can improve heat integration [16]. As it was described in the introduction section, these improvements correspond to the modification or elimination of pinch points in the heat recovery network in order to further reduce the utility cost* However, these improvements are only feasible whenever it is possible to alter certain process stream flowrates and/or temperatures. Therefore, the above requirement necessitates that the heat recovery network model should be included within a mathematical framework for the synthesis of a chemical processing system, such as the MILP approach proposed by Papoulias and Grossmann [14], [15]. In order to accomplish this task, it is proposed to incorporate the heat recovery network in the chemical processing system with the minimum utility cost models (Pi) or (RPl), since this will ensure that the optimal design of the integrated system is energy efficient. The actual derivation of the network structure can then be performed separately using the MILP model (P2) after optimizing the integrated system.

The transshipment models for minimum utility cost can be extended easily so that they can be incorporated within a MILP synthesis model for arbitrary chemical processing systems. All that is required is.to treat the flowrate compositions of the process streams as variables that will account for the interactions between the chemical processing plant and the heat recovery network. Also, if it desired to investigate several discrete inlet and outlet temperatures for some of the process streams in a MILP formulation [14], individual streams can be defined for each temperature condition. If the set of components for each stream i is denoted by $D_i = \{d\}$, by performing the temperature partitioning on all the streams, it follows from (P1) that the minimum utility cost transshipment model for unrestricted matches will be given by

$$\text{minimize } Z = \sum_{m\in S} s_m F_m^S + \sum_{n\in W} w_n F_n^W$$

s.t.

$$\sum_{i\in H_k} \sum_{d\in D_i} F_{id}(c_p)_{idk}\Delta T_k + \sum_{m\in S_k} F_m^S \Delta h_{mk} + R_{k-1} \qquad \text{(P3)}$$

$$- \sum_{j\in C_k} \sum_{d\in D_j} F_{jd}(c_p)_{jdk}\Delta T_k - \sum_{n\in W_k} F_n^W \Delta h_{nk} - R_k = 0 \qquad k = 1,2,\dots K$$

$$F_m^S \geq 0 \quad m\in S, \qquad F_n^W \geq 0 \quad n\in W$$

$$R_o = R_K = 0, \quad R_k \geq 0 \quad k = 1,2,\dots K-1$$

The above formulation is a linear programming model and can be included in a MILP model for the chemical processing plant by adding the objective function and the constraint set of (P3). Also, it is possible to add an annualized investment cost for all heat exchangers by considering this cost to be proportional to the total heat transferred in the network. A numerical example for the synthesis of a chemical processing plant and its associated heat recovery network is presented by Papoulias and Grossmann [15]. Clearly, the model for restricted matches can be derived similarly as (P3).

The Synthesis Procedure

The transshipment models developed previously in the paper can be used in the following procedure for the synthesis of heat exchanger networks:

Step 1. Develop Temperature Intervals

In the first step the entire temperature range of all the streams is partitioned into temperature intervals. The partitioning methods proposed by Linnhoff and Flower [11], Grimes [8] and Cerda [3] can be used for this purpose. However, the first procedure yields approximately twice as many intervals for the same problem when compared with the other two methods. Therefore, the partitioning procedure first proposed by Grimes and then modified by Cerda is the most efficient to use, since the resulting model has fewer temperature intervals which in turn reduces the size of the transshipment models. In particular, the following rules are applied in the method by Grimes [8]:

Rule 1. Decrease the supply temperature of each hot stream/utility by the specified minimum temperature approach ΔT_{min}.

Rule 2. Place the decreased supply temperatures of all hot streams/utilities as well as the original supply temperatures of all cold streams/utilities in a list. These temperatures that define the partition for cold streams, are arranged in order of decreasing values. The temperatures of the hot streams/utilities in the list will then be given by increasing the temperatures of the cold streams/utilities by ΔT_{min}. Note that the highest temperature (first entry in the list) should correspond to a hot utility, and the lowest temperature (last entry in the list) should correspond to a cold utility to ensure that heating at the highest level and cooling at the lowest level are always available.

Rule 3. The temperature intervals k are numbered in increasing order, $k = 1,2,\ldots K$, starting from the highest pair of temperatures on this list.

The number of partitioned temperature intervals K with the above procedure will then be given by, $K \leq NH + NC + NW + NS - 1$.

Step 2. Prediction of Minimum Utility **Cost**

In this step the minimum utility cost for a given problem is determined using either the transshipment model (Pi), or otherwise when there **are** restricted matches the model (RP1). These two transshipment models **are** essentially network flow problems that can be solved efficiently with special purpose algorithms (Bradley et al. [1]) or with standard linear programming codes. The optimal solution of the transshipment models (Pi) and (RP1) does not give the actual*network design (i.e. actual matches among streams), but rather has embedded all networks that exhibit minimum utility cost. Therefore this step of the synthesis strategy reduces significantly the number of heat recovery networks from further consideration without excluding any energy efficient designs.

Step 3. Improving Heat Integration in the Network

This step is associated with the elimination or modification of bottlenecks (pinch points) in the heat recovery network, in order to further reduce the utility cost. This improvement can be only done if it is possible to alter certain stream flowrates and/or temperatures. Consequently, the implementation of this step is optional since in many cases it is not allowed to alter any problem data* In the case that certain stream flowrates and temperatures are allowed to vary, the minimum utility cost problem can be formulated as the transshipment model (P3). However this transshipment model should be connected and solved simultaneously with the MILP model of the chemical processing system that accounts for all variations in process stream flowrates and temperatures as discussed in Papoulias and Grossmann [15].

Step 4. Selecting Networks having Minimum Number of **Units**

After predicting the minimum utility cost the transshipment model (P2) is employed to determine the minimum number of units and the actual matches that should take place in the network. As noted before, weights can **also** be used to assign preferences to the matches. Problem (P2) involves the solution of a MILP which can be solved with standard branch and bound codes [7], [9], and with the option of decomposing the problem into subnetworks.

It is important to point out that the MILP transshipment problem (P2) does not provide directly the heat exchanger network configuration. However, the optimal solution of (P2) contains all the necessary information to derive the network by hand. Specifically, the solution will indicate the pair of streams involved in each match, the corresponding amount of heat that is exchanged, and the temperature intervals over which the exchange of heat takes place. The derivation of the network configuration will often be a simple task since no work is required for merging manually heat exchanger units. Furthermore, since in the derivation of the model (P2) no assumption was made to forbid stream splitting or cyclic networks, by knowing for instance the temperature intervals over which the actual matches take place one can determine easily whether stream splitting is required. Clearly, there will be instances in which one or more different networks can be derived since the optimal solution of (P2) will not necessarily define a single configuration and/or parameters for the network. In this case a detailed analysis of the different networks could be performed to select the final solution.

Numerical Examples

The application of the transshipment models presented in this paper is demonstrated on four different example problems. The first problem has two cold and two hot process streams, and is referred in the literature as the 4SP1 problem [2], [8]. The data for the 4SP1 problem are shown in

Table 1. The minimum temperature approach required at all points of the network is specified as 10°C. In the absence of any restricted matches, the minimum utility consumption for problem 4SP1 is determined using the transshipment model (Pi), which is shown in Fig. 5 and consists of five temperature intervals. The optimal solution was obtained using the LINDO [12] computer code on a DEC-20 computer in less than 2 seconds* The minimum heating required is 128 KW and the minimum cooling is 250 KW. The pinch point for this problem occurs between the first and second temperature interval (249°C - 239°C). Next, the MILP model (P2) is used in order to obtain the minimum number of heat exchanger units for problem 4SP1. This MILP transshipment model has 7 binary variables, 21 continuous variables, 30 rows, and the optimal solution was obtained with the LINDO computer program in less than 6 seconds on a DEC-20 computer. In Fig. 6 the heat recovery network having the least number of units (5 units) is shown. The minimum utility usage and number of units is identical to the solution reported by Cerda [2].

The second numerical example is again problem 4SP1, but in this case the match between cold stream 1 and hot stream 2 is forbidden. The minimum utility consumption for this problem is determined using the restricted transshipment model (RP1) which was solved using LINDO in less than 2 seconds (DEC-20). The minimum heating required is 260 KW and the minimum cooling is 382 KW, which are identical to the values reported by Cerda [2]. The final network for the restricted 4SP1 problem having the least number of units (5 units) is shown in Fig. 7, and was obtained in less than 5 seconds.

The next example is the 7SP4 problem, which has 6 hot streams and 1 cold stream. The minimum utility cost problem was solved first and the optimal heating and cooling utility requirements found are included with the problem data in Table 2. The minimum temperature approach required at all points in the network is specified to be 20°F, and there is a pinch point at 430°F - 410°F. The MILP model for problem 7SP4 has 14 binary variables, 54 continuous variables and 58 constraints. The optimal solution was obtained in 10 seconds with the LINDO code, and is shown in Fig. 8. This design represents a minimum utility cost network with the least number of heat exchangers (10 units). Note that splitting of the cold stream is required above and below the pinch point, and there are two cyclic matches in the network (C1-H2, C1-H4). Therefore, it is clear that the MILP model (P2) has embedded in it all minimum cost utility networks with or without stream splitting and with cyclic matches.

The last problem solved was the 10SP1 problem [2], which has 5 hot and 5 cold process streams as shown in Table 3. This particular problem requires cooling water as the only utility, since there is excess heat at all points of the heat recovery network and no pinch point occurs. In order to obtain the network structure requiring the minimum number of units the MILP model for 10SP1 is solved. This MILP model has 30 binary variables, 172 continuous variables and 119 constraints and was solved in less than 30 seconds using LINDO on a DEC-20 computer. The optimal solution corresponds to an unsplit network with 10 heat exchanger units as shown in Fig. 9. It is interesting to note that the MILP transportation formulation of Cerda and Westerberg [4] for the 1OSP1 problem, has the same number of binary variables and rows but requires 357 continuous variables.

Finally, to illustrate the application of weights for preferred matches in problem 10SP1, it was assumed that four different levels of priority were assigned to the 30 possible matches shown in Table 4. As can be seen the highest level of priority $p=1$, was assigned to the matches with cooling water because of the advantage of controlling directly the target temperatures of the hot streams. For the remaining matches it was assumed that the 10 process streams were located in three different sections of the plant. Therefore, the level of priority $p=2$ corresponds to matches that take place within each of the three sections in the plant, the level $p=3$ corresponds to the matches that take place between the adjacent sections, and the lowest level $p=4$ takes place between the two sections that are furthest apart. By using the weights shown in Table 4, (with $\alpha=6$ as discussed in the Appendix), the network that was obtained is shown in Fig. 10. Note that this network has 10 heat exchanger units and involves 3 matches with $p=1$, 5 matches with $p=2$, and one match for both $p=3$ and $p=4$ priority. Therefore, with respect to the configuration in Fig. 9, there are 3 more matches at the level $p=2$ and only one at the level $p=3$, thus resulting in a system that requires less integration among the three sections in the plant. The computer time requirements for this problem were considerably higher (9 min.) due to the existence of a large number of networks with minimum number of units. It should be noted that when the weights were set to one for simply obtaining a network with minimum number of units, the computer time was much smaller because the LINDO computer code would determine as the optimal solution the first network in the enumeration with 10 units.

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Appendix

Weighting scheme for preferred matches

In the formulation of the transshipment model (P2) for minimizing the number of units, the weights $e_{ij\ell}$ were included in the objective function to denote the preference or priority level of matching hot stream i and cold stream j in subnetwork ℓ. As shown in this Appendix, these weights can be selected so as to obtain networks with minimum number of units but containing stream matches with highest priority. This procedure is useful in the case when several or many networks exist with minimum number of units.

Firstly, assume that the designer specifies a different priority level p to each triplet (i,j,ℓ), $i \in H_\ell$, $j \in C_\ell$, $\ell = 1,2,...NL$, where $p = 1,2,...N$, and N is the cardinality of the triplets. The value $p=1$ will denote the highest priority level, whereas the value $p=N$ corresponds to the lowest priority. Therefore, p is a one-to-one mapping from each triplet (i,j,ℓ), and therefore one can define the weights γ_p in decreasing level of priority as

$$\gamma_p = \gamma_{p(i,j,\ell)} = e_{ij\ell} \qquad p = 1,2,...N \tag{A1}$$

In order to obtain the weights γ_p given specified priorities p, for each individual triplet, the following functional form is assumed

$$\gamma_p = \alpha + p/N \qquad p = 1,2,...N \tag{A2}$$

The parameter α in (A2) must be chosen so that it can be guaranteed that the weighted objective function in (P2) will lead to networks containing matches with highest priority, but with the smallest number that is possible. This can actually be achieved by requiring that the sum of the q + 1 weights of highest priority be larger than the sum of the q weights of lowest priority for any $q < N$; that is

$$\sum_{p=1}^{q+1} Y_p > \sum_{p=N-q+1}^{N} Y_p \quad \text{for} \quad 1 \leq q < \frac{N}{2} \tag{A3}$$

Since $$\sum_{p=1}^{q} p = (q)(q+1)/2 \tag{A4}$$

by substituting (A2) and (A4) in (A3), the inequality can be written as

$$(q+1)a + (q+1)(q+2)/2N > aN + (N+1)/2 - (N-q)a - (N-q)(N-q+1)/2N \tag{A5}$$

which in turn can be simplified **to yield,**

$$a > [q(N-q-1) - 1]/N \tag{A6}$$

Since the right hand side of (A6) is maximized at $q = (N-1)/2$, the inequality in (A6) will hold for all q, $q = 1,2,\ldots N-1$ if

$$a > [(N-1)^2 - 4]/4N \tag{A7}$$

Therefore, a choice of a satisfying (A7) will guarantee that the weights as given by (A2) will produce networks with minimum number of units but with the most preferable matches. It should be noted that if N is too large the proposed choice of a may be higher than needed since it is sufficient that (A6) holds for a valid upper bound q^U on the number of matches. Since the right hand side increases monotonically in q, for $q \leq (N-1)/2$, the choice of q^U is justified if it lies below this value. Also, since the right hand side in (A7) is smaller than $N/4$, a practical choice for large N is given by

$$a > \min\{q^U(N-q^U-1)/N,\ N/4\} \tag{A8}$$

In the case that priority levels are assigned to groups of matches and not to individual matches the procedure must be modified somewhat. For this case, assume that the N matches are partitioned in NG groups, where each group $G_p = \{(i,j,*)|p = (i,j,*)\}$, has cardinality σ_p, and priority level $p = 1,2,\ldots NG$. If an upper bound q^U is given, it is convenient to define the indices r and t as follows:

a) r is the largest integer such that $\sum_{p=1}^{r} \sigma_p \le q + 1$

b) t is the smallest integer such that $\sum_{p=t}^{NS} \sigma_p \le q^U$

Assuming that the weights γ_p are given as

$$\gamma_p = \alpha + p/NS \qquad p = 1,2,\ldots NS \tag{A9}$$

and by following a similar reasoning in imposing the inequality in (A3), the parameter α that must be chosen to guarantee minimum number of units with most preferred matches is given by

$$\alpha > (1/NS)\left[\sum_{p=t}^{NS} p\sigma_p + \left(q^U - \sum_{p=t}^{NS} \sigma_p\right)(t-1) - \sum_{p=1}^{r} p\,\sigma_p - \left(q^U + 1 - \sum_{p=1}^{r} \sigma_p\right)(r+1)\right] \tag{A10}$$

Finally, to test the validity of the upper bound q^U, it is necessary to check whether the right hand side in (A10) decreases when evaluated at $q^U - 1$. If the test fails, the upper bound q^U must be reduced.

Table 1. Data for Problem ASP1

Streams	Fc_p (KW/°C)	T^S (°C)	T* (°C)	Q080
Cl (Cold)	7.62	60	160	+762
HI (Hot)	8.79	160	93	-589
C2 (Cold)	6.08	116	260	+876
H2 (Hot)	10.55	249	138	-1171
S (Steam)		270	270	
CW (Cooling water)		38	82	

Table 2. Data for Problem 7SP4

Streams	Fc_P (Btu/°F)	I^s (°F)	T* (°F)	Q (Btu)
Cl (Cold)	47.0	60	710	+30,550
HI (Hot)	15.0	675	150	-7,875
H2 (Hot)	11.0	590	450	-1,540
H3 (Hot)	4.5	540	115	-1,912
H4 (Hot)	60.0	430	345	-5,100
H5 (Hot)	12.0	400	100	-3,600
H6 (Hot)	125.0	300	230	-8,750
CW (Water)	11.03	80	140	6,618
F (Fuel)	8,390.0	800	801	-8,390

Table 3. Data for Problem 10SP1

Streams	Fc_p (KW/°C)	T^S (°C)	T^t (°C)	Q (KW)
Cl (Cold)	7.62	60	160	+762
C2 (Cold)	6.08	116	222	+644
C3 (Cold)	8.44	38	221	+1545
C4 (Cold)	17.28	82	177	+1642
C5 (Cold)	13.90	93	205	+1557
H6 (Hot)	8.79	160	93	-589
H7 (Hot)	10.55	249	138	-1170
H8 (Hot)	14.77	227	66	-2378
H9 (Hot)	12.56	271	149	-1532
H10 (Hot)	17.73	199	66	-2358
W (Water)	42.66	38	82	1877

Table 4. Preferred Matches for Problem 10SP1

<table>
<tr><th></th><th>Cl</th><th>C2</th><th>C3</th><th>C4</th><th>C5</th><th>CW</th></tr>
<tr><td>H6</td><td colspan="2" rowspan="2">4</td><td rowspan="2">3</td><td colspan="2" rowspan="2">2</td><td rowspan="5">1</td></tr>
<tr><td>H7</td></tr>
<tr><td>H8</td><td colspan="2">3</td><td>2</td><td colspan="2">3</td></tr>
<tr><td>H9</td><td colspan="2" rowspan="2">2</td><td rowspan="2">3</td><td colspan="2" rowspan="2">4</td></tr>
<tr><td>H10</td></tr>
</table>

Level of priority	Weights
P - 1	6.25
p - 2	6.5
P - 3	6.75
p - 4	7

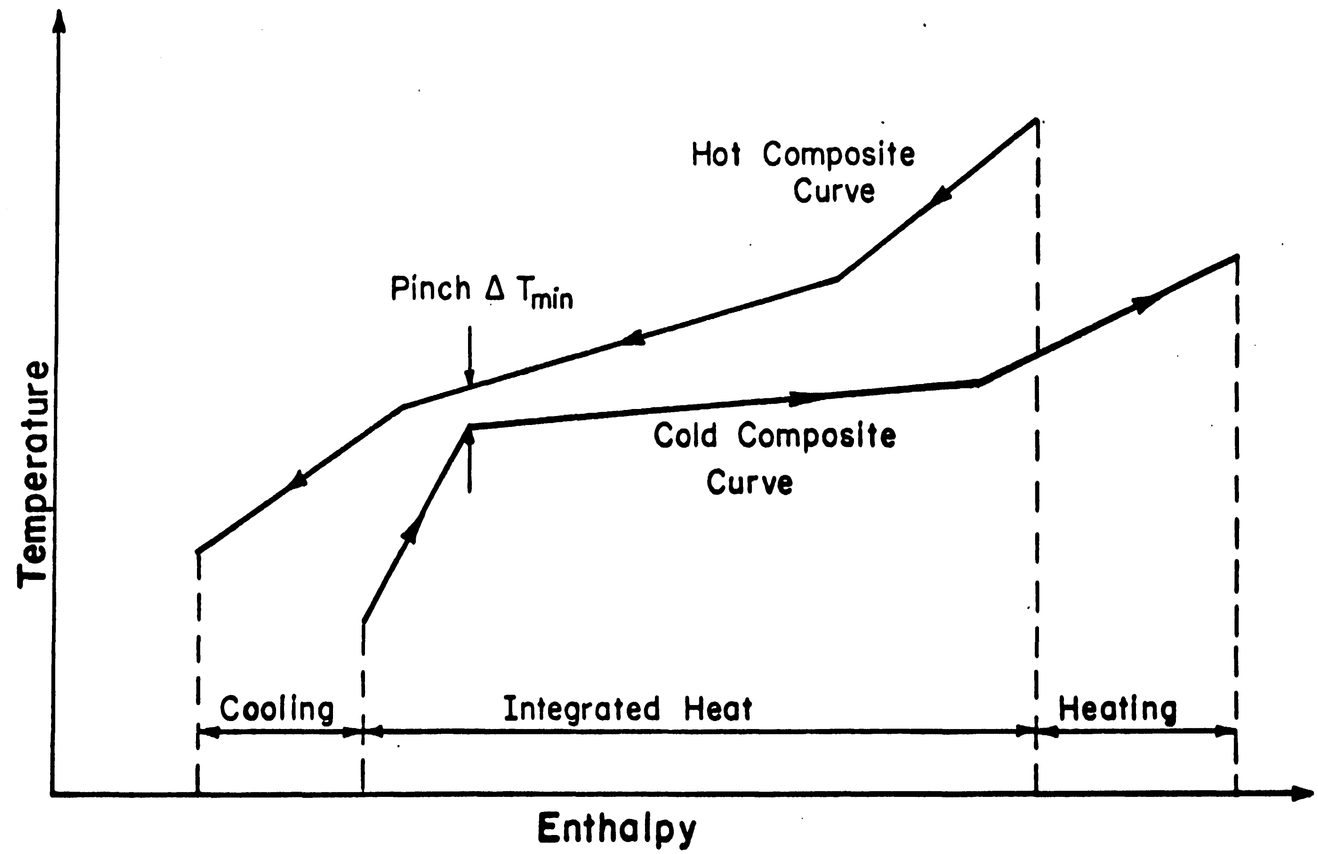

Hot Composite
Curve
Pinch Δ T_{min}
Cold Composite
Curve
Temperature
Cooling
Integrated Heat
Heating
Enthalpy

Fig. 1

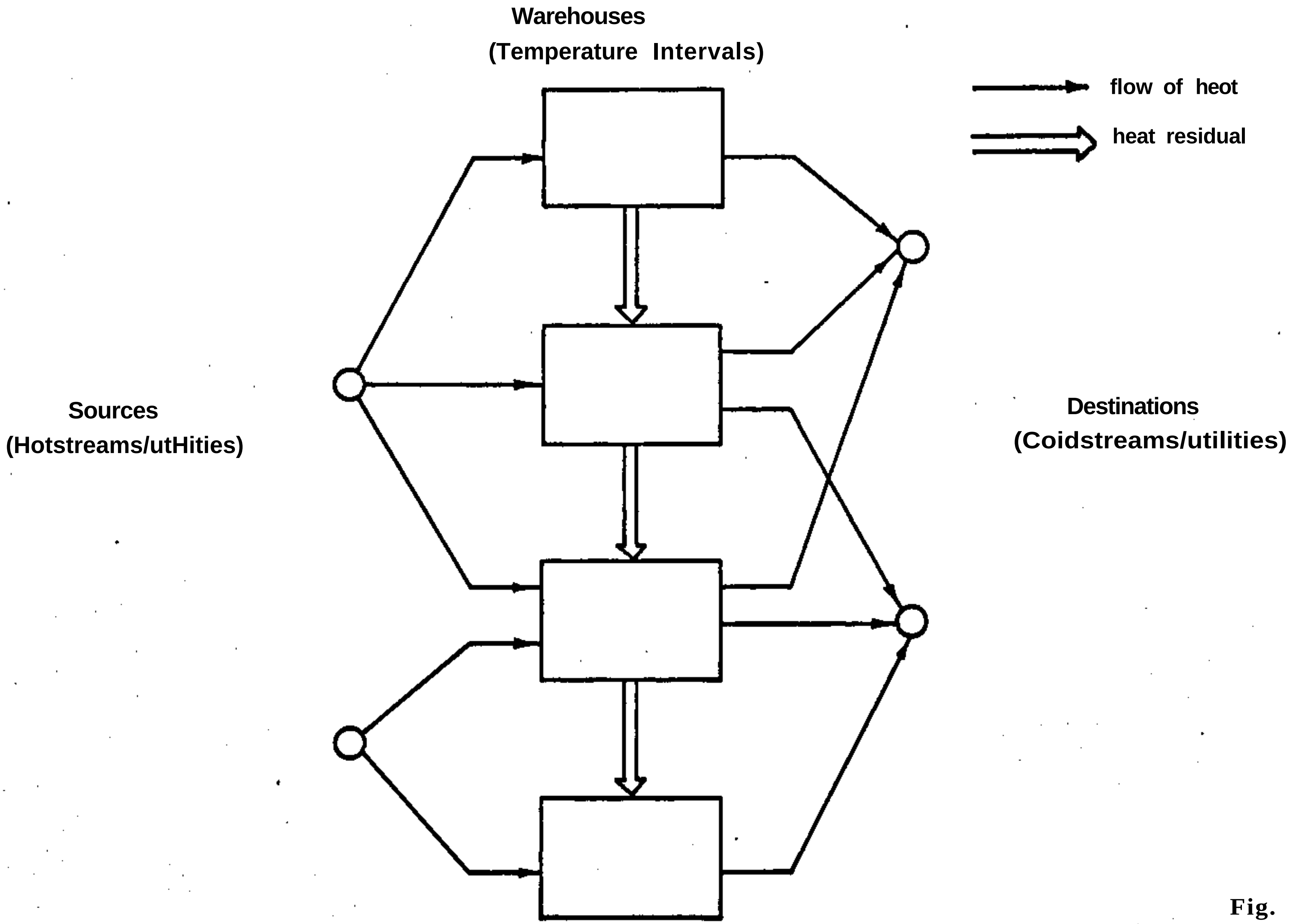
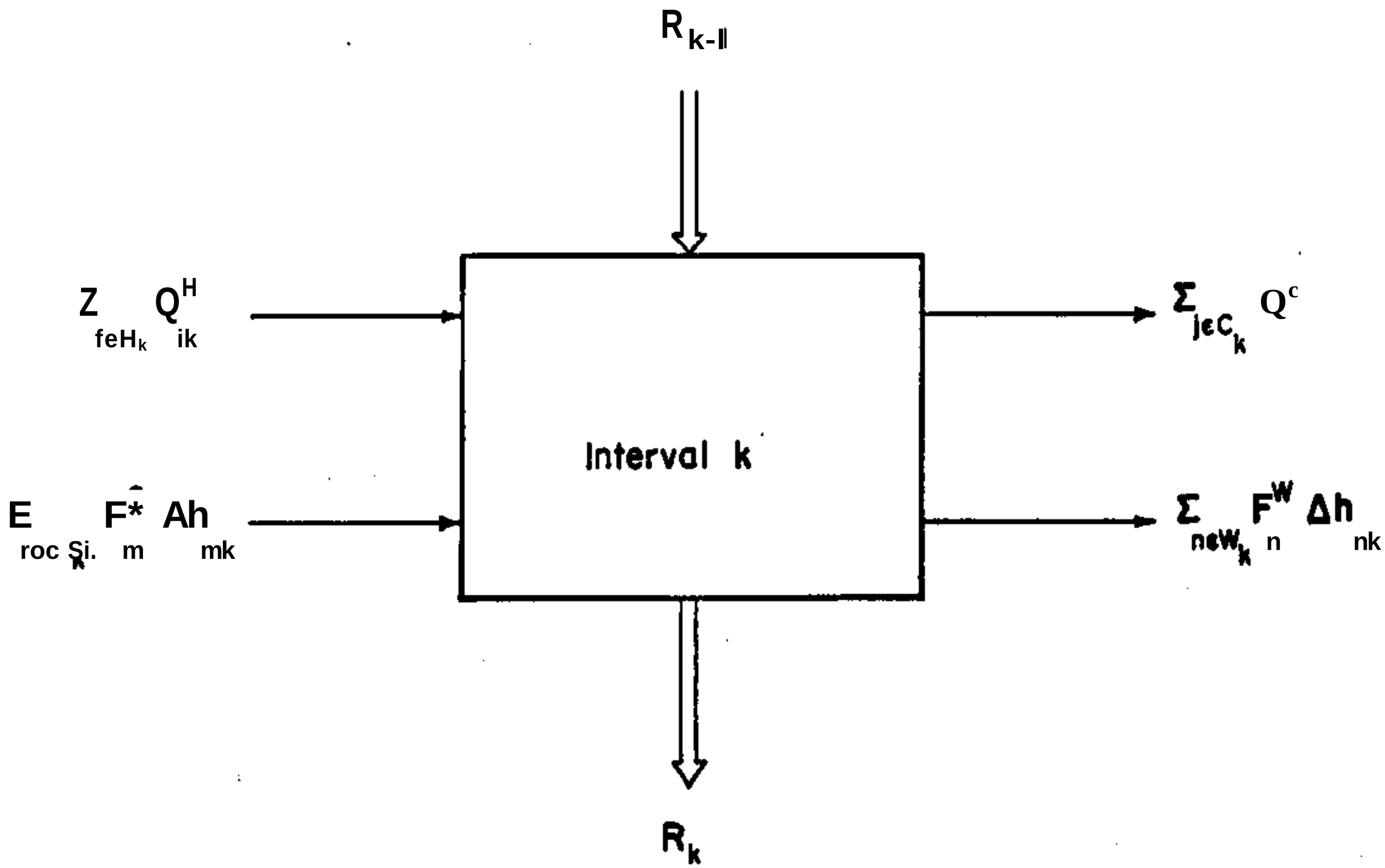

Warehouses
(Temperature Intervals)
flow of heot
heat residual
Sources
(Hotstreams/utHities)
Destinations
(Coidstreams/utilities)

Fig. 2



R_{k-1}
$\sum_{i \in H_k} Q^H_{ik}$
$\sum_{m \in S_k} F^*_m \Delta h_{mk}$
Interval k
$\sum_{j \in C_k} Q^C_{jk}$
$\sum_{n \in W_k} F^W_n \Delta h_{nk}$
R_k

Fig. 3

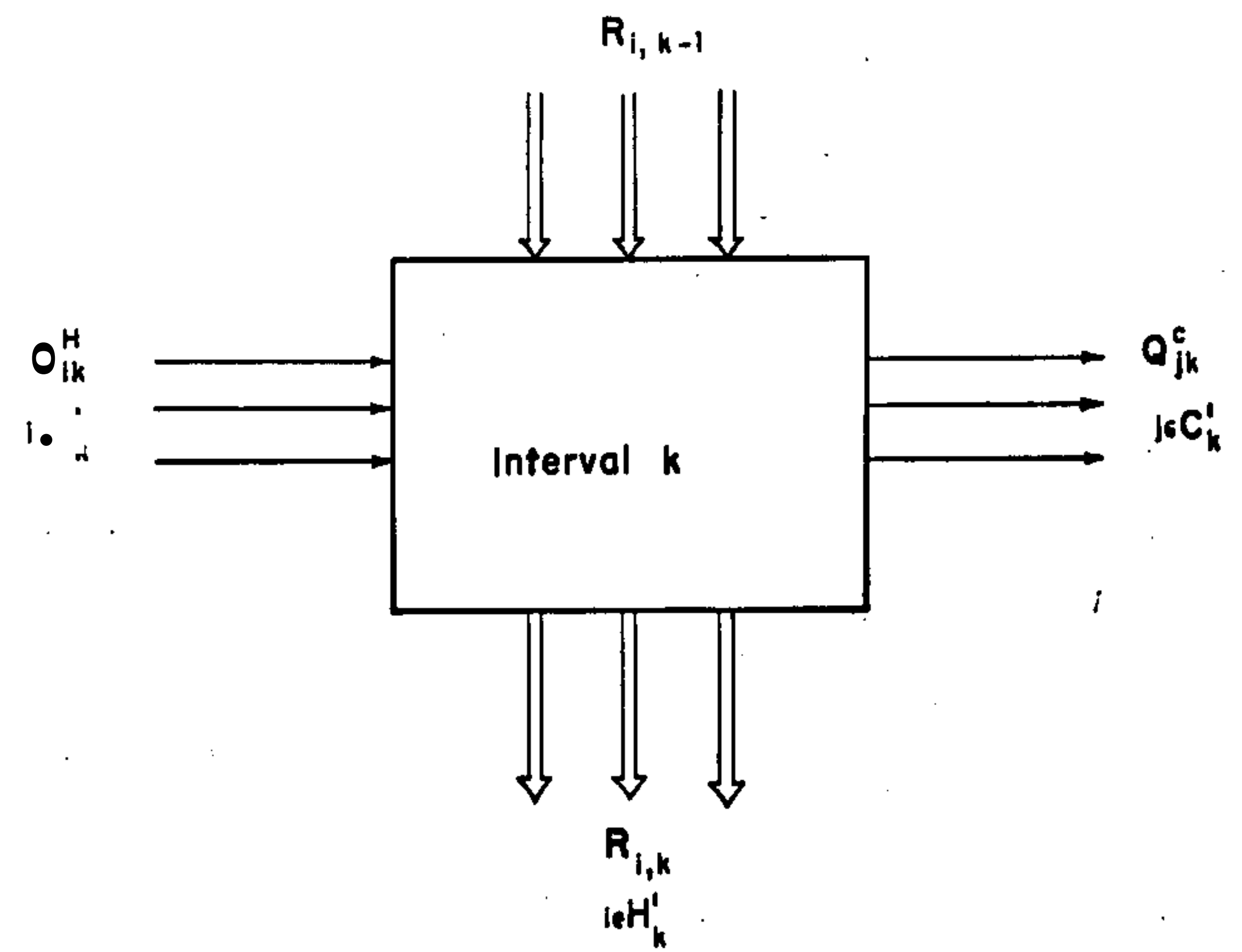


Fig. 4

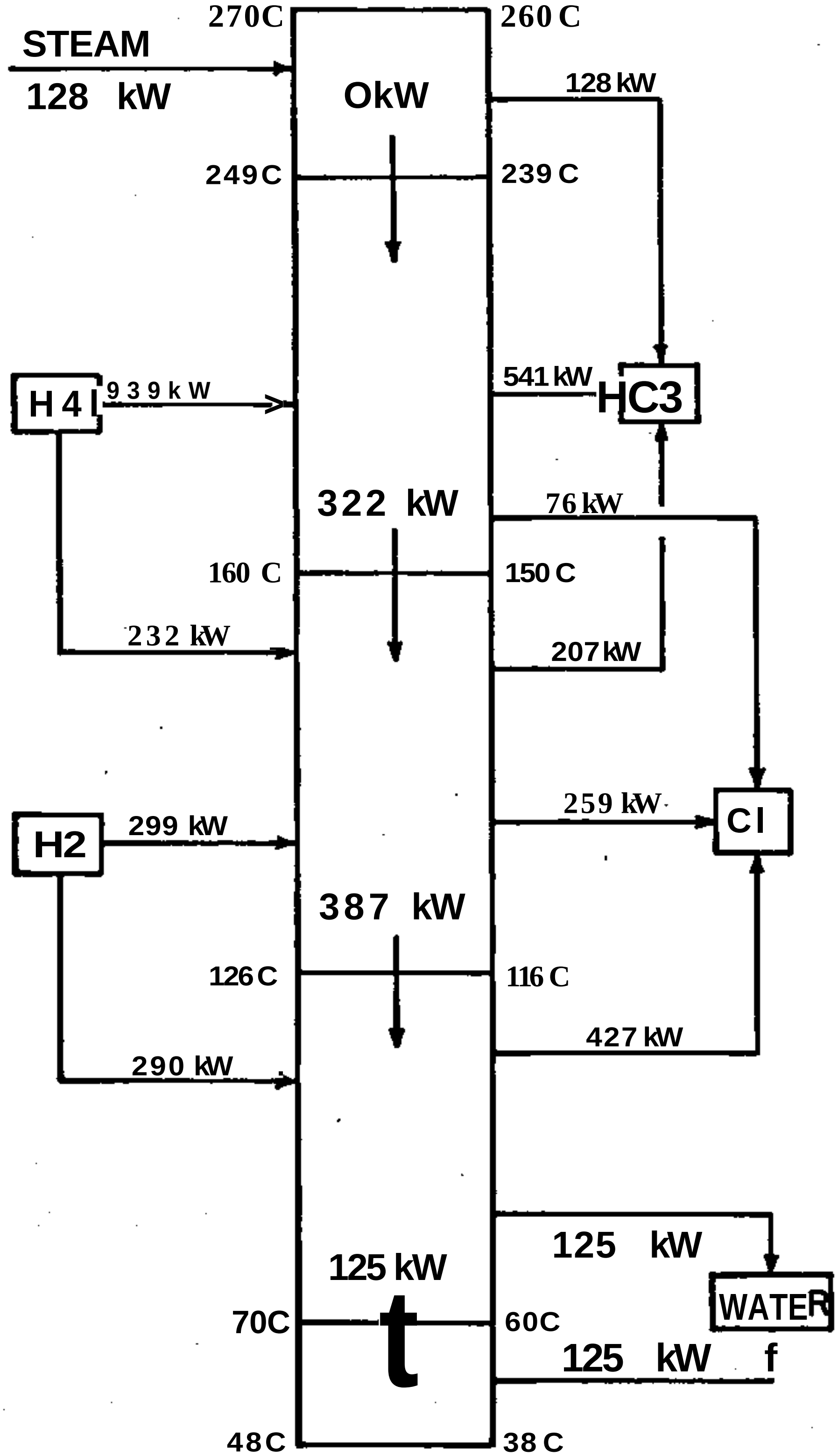

270C
260 C
STEAM
128 kW
OkW
128 kW
249 C
239 C
H4
939 kW
541 kW
HC3
322 kW
76 kW
160 C
150 C
232 kW
207 kW
259 kW
CI
H2
299 kW
387 kW
126 C
116 C
427 kW
290 kW
125 kW
125 kW
WATER
70C
60C
125 kW
t
f
48C
38 C

Fig. 5

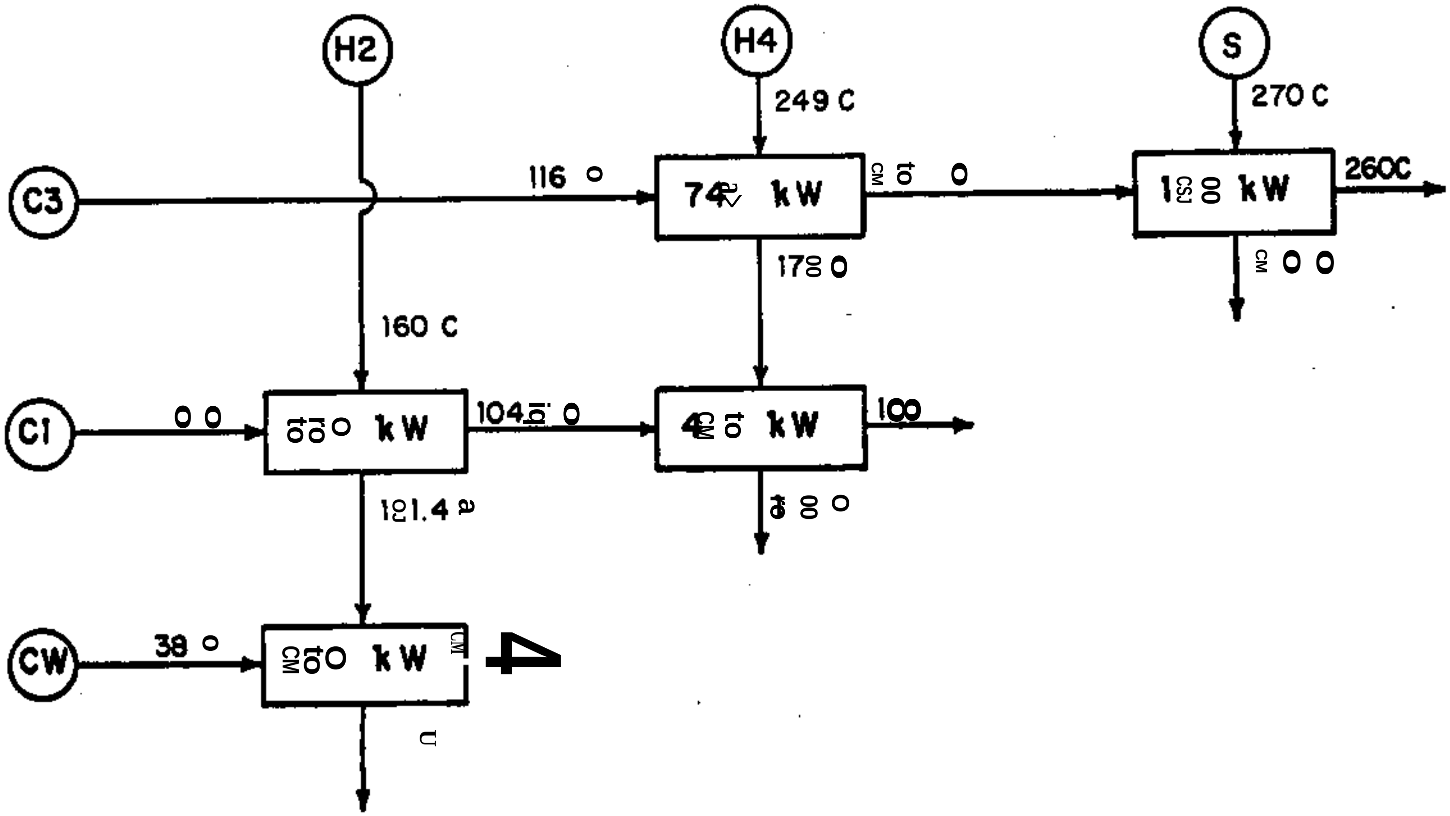


Fig. 6

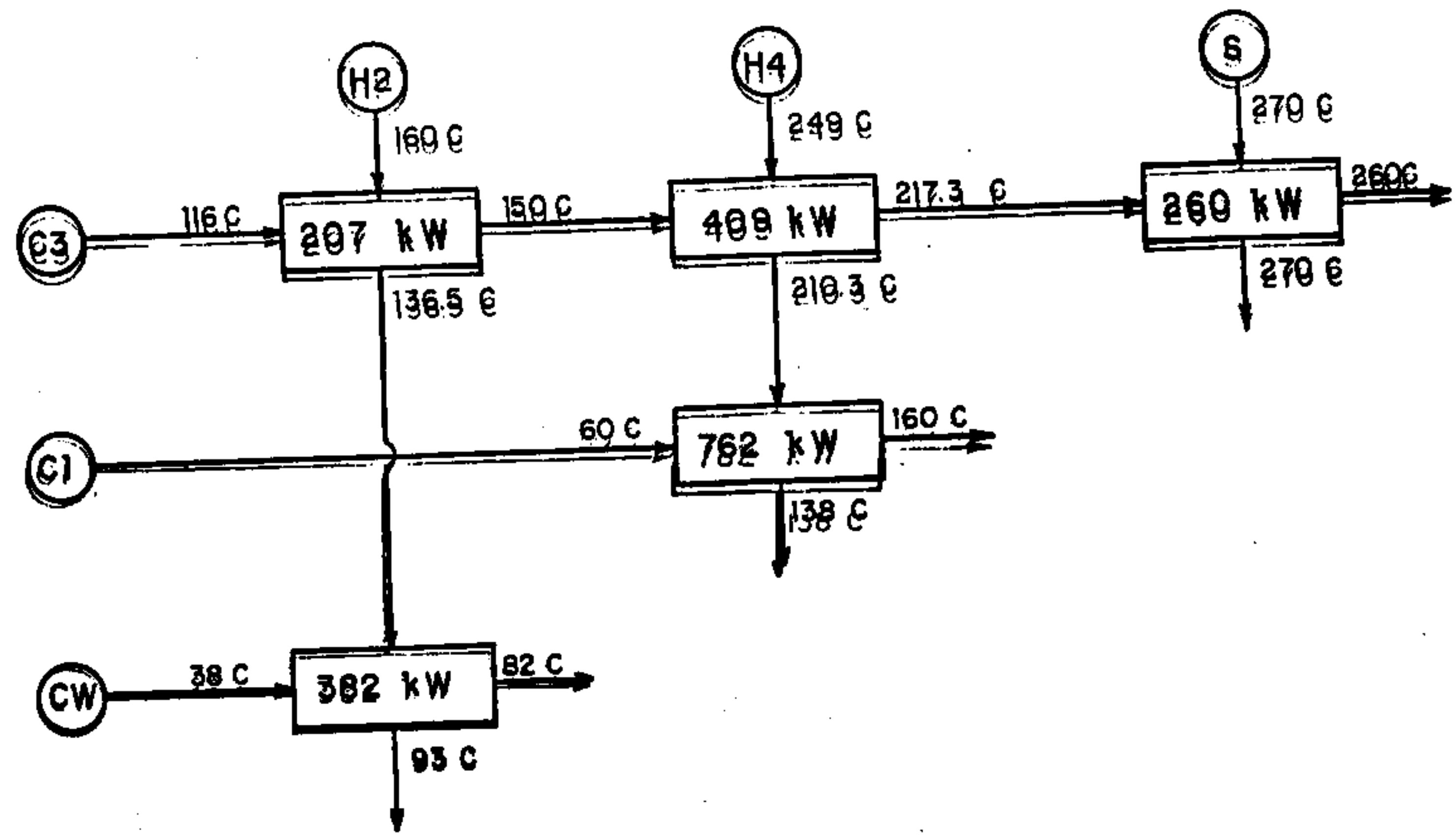
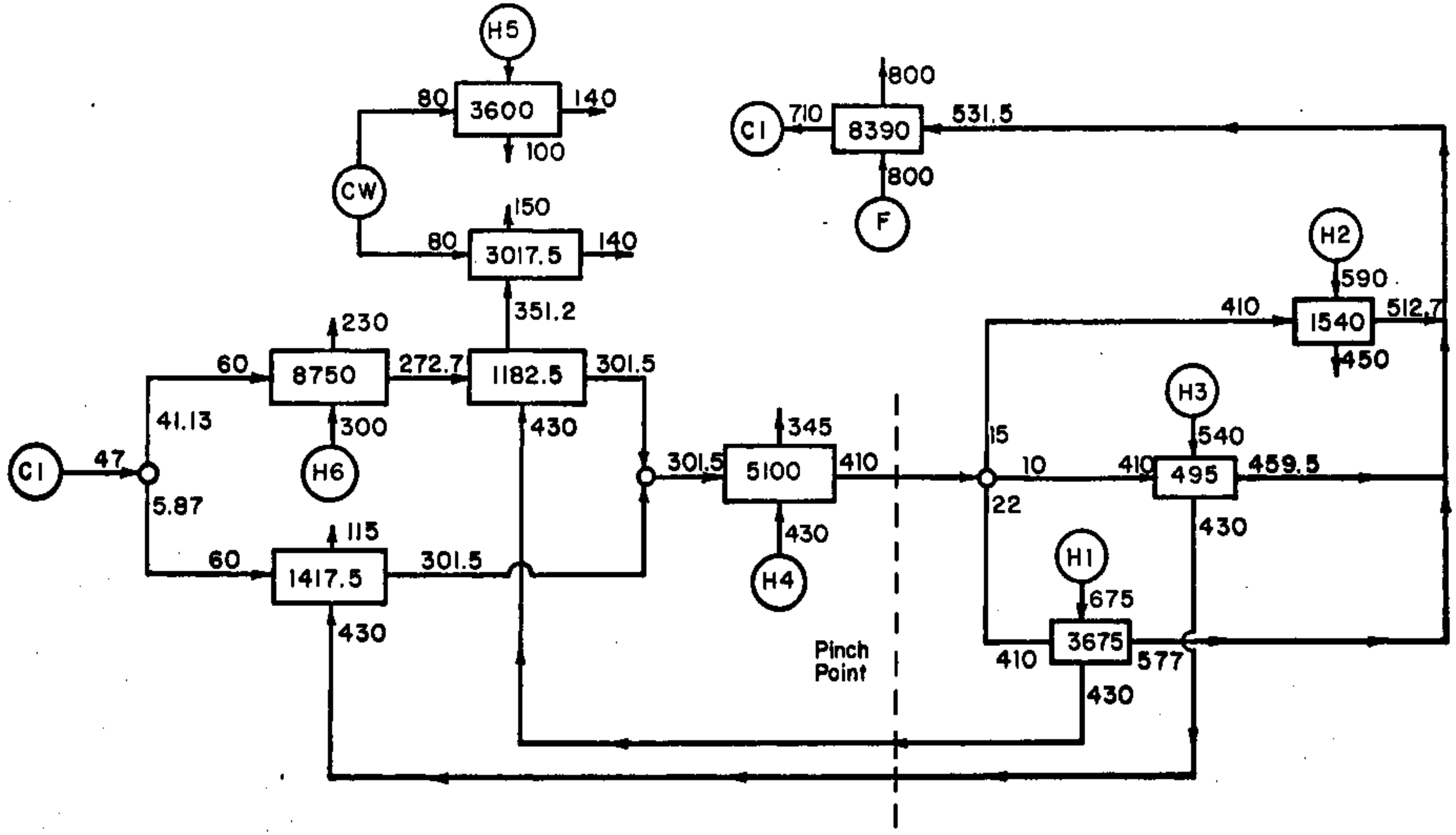
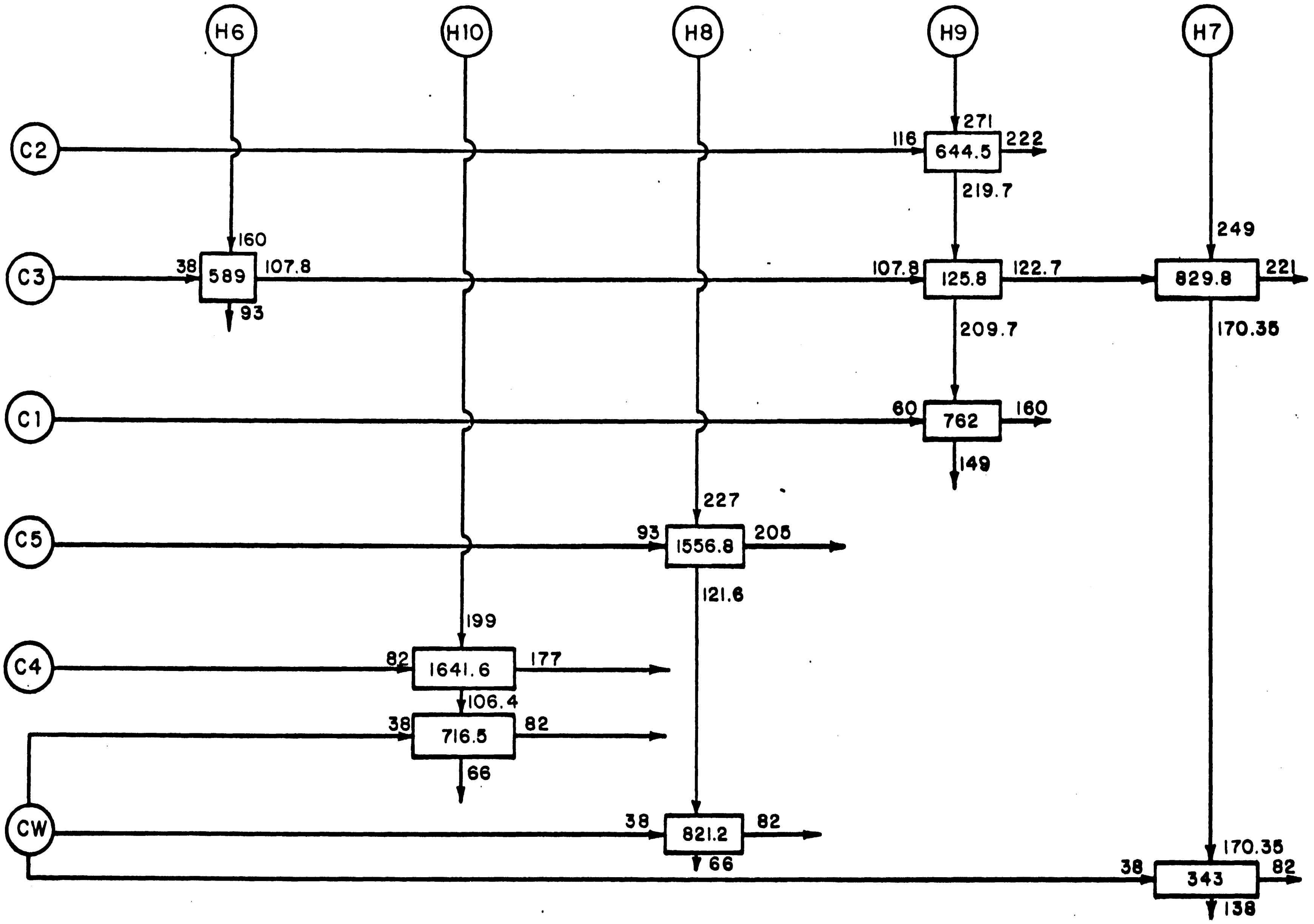

H2
160 C
C3
116 C
207 kW
150 C
136.5 C
H4
249 C
409 kW
217.3 C
210.3 C
S
270 C
260 kW
260C
270 C
C1
60 C
762 kW
160 C
138 C
CW
38 C
382 kW
82 C
93 C

Fig. 7



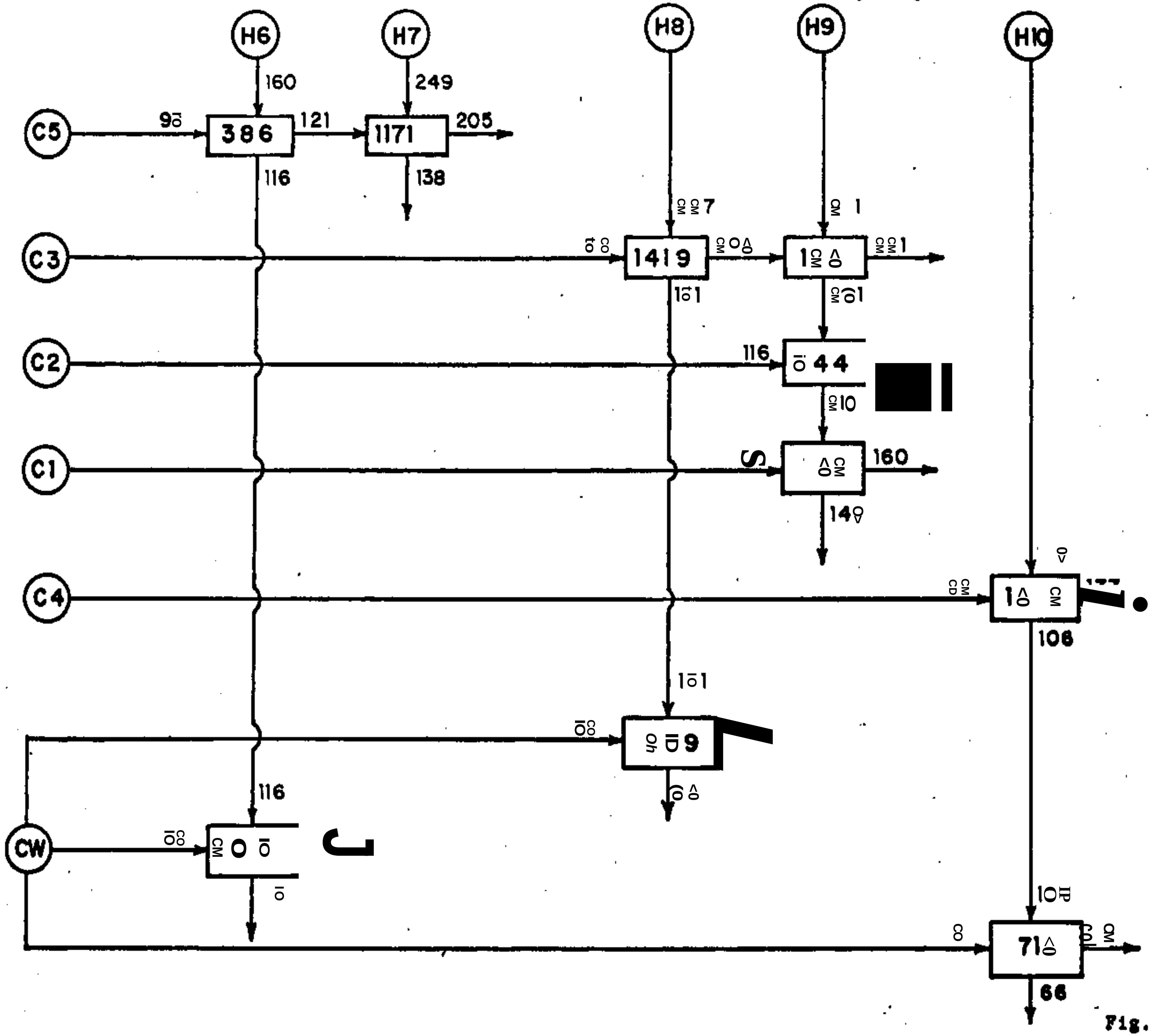
H5
80
3600
140
100
CW
150
80
3017.5
140
351.2
C1
710
8390
800
531.5
800
F
H2
590
410
1540
512.7
450
230
60
8750
272.7
1182.5
301.5
41.13
300
430
345
H3
540
C1
47
H6
301.5
5100
410
15
10
410
495
459.5
5.87
22
115
430
430
H4
H1
60
1417.5
301.5
675
430
3675
410
577
430
Pinch
Point

Fig. 8



H6
H10
H8
H9
H7
C2
C3
C1
C5
C4
CW
271
116
644.5
222
219.7
160
38
589
107.8
93
249
107.8
125.8
122.7
829.8
221
209.7
170.35
60
762
160
149
227
93
1556.8
205
121.6
199
82
1641.6
177
106.4
38
716.5
82
66
38
821.2
82
66
170.35
38
343
82
138

Fig. 9



H6
H7
H8
H9
H10
C5
C3
C2
C1
C4
CW
160
249
386
121
1171
205
116
138
1419
160
106
116
66

Fig. 10