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Shareholders Unanimity with Incomplete Markets*

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When markets are incomplete, shareholders typically disagree on the firm's optimal investment plan. This paper studies the shareholders' preferences with respect to the firm's investment in a model with aggregate risk, incomplete markets and heterogeneous households who trade in firms' shares instead of directly accumulating physical capital. If the production function exhibits constant returns to scale and borrowing limits are not binding, a firm's shareholders unanimously agree on its optimal level of investment. In contrast, with binding borrowing constraints, constrained shareholders prefer a higher level of investment than unconstrained ones.

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1. Introduction

During recent years, macroeconomists have explored the implications of dynamic general equilibrium models with heterogeneous agents, incomplete markets and aggregate risk for a variety of issues, including asset pricing (see e.g. Krusell and Smith, 1997 and Storesletten, Telmer, and Yaron, 2007), business cycles (see e.g. Krusell and Smith, 1998), and the distribution of income and wealth (see e.g. Castaneda, Diaz-Gimenez and Rios-Rull, 1998). In these models, consumers face idiosyncratic and aggregate uncertainty and accumulate physical capital which is rented to firms. The assumption that consumers, rather than firms, accumulate physical capital can either be interpreted literally, or alternatively as capturing a situation in which each consumer is also an entrepreneur. The latter directly operates a technology by employing his privately accumulated physical capital and by hiring and supplying labor in a competitive labor market.[2]

This paper considers a version of the incomplete markets models cited above in which consumers trade firms' shares rather than physical capital. In addition, firms' shareholders make decisions regarding physical capital investment. The incompleteness of the asset market implies that, in principle, this setting is not equivalent to one in which consumers accumulate physical capital directly. This is due to the fact that, when the asset market is incomplete, a firm's shareholders do not necessarily agree on what the objective of the firm should be, and, in particular, on whether the firm should maximize its stock market value or not (see e.g., Grossman and Hart, 1979). Instead, when the asset market is complete, marginal rates of substitution are equalized among shareholders, who, therefore, unanimously agree on choosing the level of investment that maximizes the stock market value of the firm. In this context it is irrelevant whether consumers or firms accumulate physical capital.

The present paper investigates the conditions under which a firms' shareholders agree on its level of investment. The paper's main result can be summarized as follows. Using a two-period version of the standard incomplete markets model used in macroeconomics, we show that, if the production function exhibits constant returns to scale in capital and labor and borrowing constraints are not binding, then a firm's shareholders will unanimously agree on the optimal level of investment in a production equilibrium.

[2] The equivalence of these two alternative interpretations is guaranteed by the standard assumption of constant returns to scale in production and the existence of an economy-wide labor market for labor. These assumptions guarantee that capital-labor ratios will be equalized across production units, independently of whether firms rent capital from consumers or the latter operate their own technology. Angeletos and Calvet (2005, 2006) consider a version of this kind of model in which there is no economy-wide labor market. Each entrepreneur employs his own capital and labor in the firm, and, as a result, capital-labor ratios are not equalized across firms.

A production equilibrium is defined as an equilibrium in which the production plan of each firm is chosen by aggregating the preferences of its shareholders, taking as given the production decisions and stock prices of other firms. The assumption of constant returns to scale is crucial because it allows a firm's shareholders to infer from the stock prices of other firms how the firm's stock price will change in response to variations in its level of investment. In fact, under constant returns to scale, each firm's dividends can be expressed as the product of its capital stock and a stochastic term that depends, among other things, on the aggregate productivity shock. Given this, the stock market value of a firm is simply scaled up or down by variations in its investment level. It follows that a firm's shareholders are unanimous in choosing a level of capital that maximizes the firm's stock market value as long as they are unconstrained in their portfolio choices.

The paper also considers two important extensions of the basic two-period model. First, when borrowing limits are binding for some initial shareholders, we show that the latter prefer a higher level of investment than unconstrained ones. This is due to the fact that a higher level of investment raises the firm's stock price. As short-sellers of the stock, constrained shareholders value relatively more a higher current stock price than the cost of higher future dividends. Therefore, they favor increasing investment above the level preferred by unconstrained shareholders. The paper discusses how this conflict between constrained and unconstrained shareholders might be resolved using majority voting.

In a second extension, we consider a multiperiod version of the economy keeping the assumptions of constant returns to scale and no binding borrowing limits. We first show how the unanimity result derived in the two-period model easily extends to this situation. Specifically, we show that there is no disagreement on the firm's investment decision among contemporaneous shareholders. Furthermore, we show that our unanimity result is even stronger in a multiperiod economy, since there is no disagreement among shareholders at different dates.

Whereas the macroeconomic literature with incomplete markets has mostly assumed away the problem of joint ownership of the firm, this issue has received much more attention in the theory literature, starting from the seminal paper of Diamond (1967). The economies considered in this literature bear some similarity with the ones usually considered by macroeconomists, but there are some important differences. First, the theory literature generally considers models in which capital is the only input in production and the production function exhibits decreasing returns to scale (see the review by Magill and Quinzii, 1996). Macroeconomists, instead, typically assume constant returns to scale production technologies that use as inputs capital and labor. Here, it is important to point out the unanimity result of Ekern and Wilson (1974), who show that shareholders will agree on the investment decision of the firm under incomplete markets if the firm's

vector of dividends is spanned by the payoffs of existing securities. In our model economy, this spanning condition is implied by the constant returns to scale assumption. However, our two-period economy differs from the one analyzed by Ekern and Wilson in that in our model shareholders can trade their stocks after the firms' investment decisions have been made. Thus, they need to anticipate the consequences of these investment choices on firms' stock market prices. Our unanimity result requires that, in this process, a firm's shareholders take as given the stock prices of other firms. This assumption of competitivity, originally introduced by Grossman and Stiglitz (1977, 1980), complements Ekern and Wilson (1974)'s original spanning assumption.

Second, as mentioned above, a firm's shareholders have to evaluate the effect of their investment decisions on the stock market value of the firm. While this information is straightforward to obtain under complete markets, it is in general much harder to gather in economies with incomplete markets. In an important paper, Grossman and Hart (1979) propose a solution to this problem by postulating that each shareholder will use his own discount factor to evaluate the effect of changes in the level of investment on the firm's market value. However, the assumption of competitive price perceptions, as it became known, is applicable only when all initial shareholders are unconstrained in their portfolio choices and it implies that shareholders will disagree about the effect of a change in the firm's investment decision on its stock price.

In contrast, we do not assume competitive price perceptions but instead use the restrictions of the model itself to derive the sensitivity of a firm's stock price to the amount of capital chosen by its shareholders. This derivation is, of course, greatly facilitated by the assumption of constant returns to scale in production, in which case it is easy to show that the competitive price perceptions condition of Grossman and Hart (1979) is also satisfied. The advantage of our approach is that it can be applied to situations in which borrowing limits are binding for some initial shareholder. While this possibility is usually ignored in the theory literature, binding borrowing constraints play a more central role in macroeconomics (see e.g. Krusell and Smith, 1998).[3] Thus, it is important to investigate their effect on the investment decision of the firm.

Third, the theory literature typically considers two-period models, while the macroeconomic literature considers infinite horizon economies. It turns out that, while the main intuitions of this paper can be presented in a two-period setting, the main results also extend to a multiperiod version of the model. The two-period model also makes it easier to relate the results of this paper with the classic contributions of Diamond (1967), Grossman and Hart (1979), and Ekern and Wilson (1974).

Finally, the assumptions of constant returns to scale and no binding borrowing limits also imply that

[3] The absence of shortseelling is one of the crucial assumptions made by Grossman and Hart (1979, page 299, footnote 5), for example.

shareholders are unanimous in choosing a level of capital that maximizes the firm's stock market value. In turn, this implies that the allocation of resources in the economy we consider is the same as in the standard setting where firms maximize period-by-period profits. A more general version of this result can be found in Carceles-Poveda and Coen-Pirani (2006), where we study an infinite horizon version of the two-period economy considered here, allowing for general borrowing constraints. In that paper we assume that firms maximize their discounted cash flows using as discount factors state prices that do not allow for arbitrage opportunities. We show that, under this objective, the model gives rise to the same allocation of resources as the standard one with static firms, regardless of the particular discount factor chosen by the firm. An important difference relative to the present paper is that this result is valid even if borrowing limits are binding. In the present paper, instead, binding borrowing limits induce disagreement between constrained and unconstrained shareholders and also among constrained shareholders, potentially leading to allocations that differ from the ones obtained in the standard setting (Krusell and Smith, 1998).

The rest of the paper is organized as follows. Section 2 introduces a standard two-period model economy with incomplete markets and idiosyncratic risk. Section 3 defines exchange and production equilibria for this economy. Section 4 characterizes the production equilibrium of the economy and presents our main unanimity result. Section 5 discusses the unanimity result and relates it to the literature on shareholder unanimity under incomplete markets. Section 6 considers two extensions of the model: the case of multiple period and the case of binding borrowing limits. Finally, section 7 concludes.

2. Model Economy

This section introduces the model economy, which is a version of the benchmark incomplete markets model used in macroeconomics (see e.g. Aiyagari, 1994 and Krusell and Smith, 1998). We start with a simplified two-period framework, since this makes the relevant intuition for our results more transparent. In a later section, we extend the analysis to a multiperiod setup.

Time is discrete and indexed by $t = 0, 1$. The economy is populated by a finite number I of consumers, indexed by $i = 1, 2, .., I$, and by a finite number J of firms, indexed by $j = 1, 2, ..J$.[4] There is both aggregate and individual uncertainty. In particular, there is no uncertainty in period 0, but there is a finite number of possible states in period 1. The state of the economy in period 1 is denoted by $s_1 \in S_1$. To simplify notation, we let $\{h_1\} \equiv (h(s_1))_{s_1 \in S_1}$ for any generic random variable h throughout the text. Further, $\pi(s_1)$ denotes

[4] Whereas all the results in the paper hold with a continuum of agents, assuming a finite number of agents considerably simplifies the notation.

the probability that state s_1 is realized.

Firms produce an homogeneous good that can be either consumed or invested. Each firm operates the production function $F(k, l; z)$, where k denotes the physical capital input, l is the labor input, and z is a productivity shock, which is common across firms. The capital stock depreciates at the rate δ. We assume that the production function F is twice continuously differentiable with respect to k and l and that it displays constant returns to scale, that is, $F(\mu k, \mu l; z) = \mu F(k, l; z)$ for all $\mu > 0$. As we will see later, the constant returns to scale assumption plays a key role in the analysis.

The timing of the firms' decisions is as follows. Each firm j is initially endowed with a level of capital k_0^j. This is exogenously given to each firm and can be different across firms. At time zero, the level of aggregate productivity z_0 is known and the firm decides on the labor input l_0^j and on next period's capital stock k_1^j. At time $t = 1$, the aggregate uncertainty in the economy is captured by the non-negative random variable $z_1(s_1)$ and the firm decides on the labor input $l_1^j(s_1)$ after state s_1 is realized.

Consumer $i \in I$ has the following preferences:

$$U(c^i) = u\left(c_0^i\right) + \beta \sum_{s_1} \pi(s_1)\, u\left(c_1^i(s_1)\right), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, $c^i \equiv \left(c_0^i, \left\{c_1^i\right\}\right)$, c_0^i denotes consumption at time zero, and $\left\{c_1^i\right\}$ denotes consumption at time one. The period utility function u is assumed to be concave and twice continuously differentiable.

Consumer $i \in I$ is initially endowed with x_0^i units of labor. This is non-random and can differ across consumers. The aggregate labor endowment at time zero is denoted by L_0:

$$L_0 = \sum_{i \in I} x_0^i. \tag{2}$$

Individual uncertainty is realized in period 1, since a consumer's labor endowment $\left\{x_1^i\right\}$ is random. The aggregate units of labor available for production at date one and state s_1 are then equal to:

$$L_1(s_1) = \sum_{i \in I} x_1^i(s_1). \tag{3}$$

At time zero, consumer $i \in I$ is also endowed with θ_{j0}^i shares of firm $j \in J$. In what follows, we let $\theta_0^i \equiv (\theta_{j0}^i)_{j \in J}$ represent consumer i's portfolio of initial shares.

The measure of each firm's shares outstanding is normalized to one and the initial distribution of shares satisfies $\sum_{i \in I} \theta_{j0}^i = 1$. In addition, we assume that the supply of a firm's stock remains constant over time. For convenience, we denote by J_{j0} the set of shareholders with strictly positive initial shares of firm j:

$$J_{j0} = \left\{i \in I : \theta_{j0}^i > 0\right\}.$$

The timing of the household decisions is as follows. At the beginning of time zero, firms pay a wage of w_0 per unit of labor endowment and a dividend per share of d_0^j to their shareholders. When the stock market opens, consumers can rebalance their portfolios by buying or selling their initial shares at the (ex-dividend) price p_0^j. We let θ_{j1}^i denote the final shares of firm j held by consumer i after trading in the stock market and $\theta_1^i \equiv (\theta_{j1}^i)_{j\in J}$ denote consumer i's portfolio of shares. We assume that there is a lower bound on the holdings of each firm's shares:

$$\theta_{j1}^i \geq -\underline{\theta}, \tag{4}$$

where $\underline{\theta} \geq 0$ must be such that the consumer is always able to pay back his debt in period 1, while enjoying non-negative consumption.

At the beginning of period 1, the state s_1 is realized and aggregate and individual uncertainties are revealed. Each firm j pays a wage of $w_1(s_1)$ per unit of labor endowment and a dividend per share of $d_1^j(s_1)$ to its shareholders. Further, each consumer observes his labor shock and consumes $c_1^i(s_1)$.

Consumers maximize utility, as defined in equation (1), subject to the portfolio constraint in (4) and to the following budget constraints:

$$c_0^i + \sum_{j\in J} p_0^j \theta_{j1}^i = \sum_{j\in J} \theta_{j0}^i \left(d_0^j + p_0^j\right) + w_0 x_0^i, \tag{5}$$

$$c_1^i(s_1) = \sum_{j\in J} \theta_{j1}^i d_1^j(s_1) + w_1(s_1)\, x_1^i(s_1), \text{ for all } s_1. \tag{6}$$

The dividends per share are determined by the budget constraints of the firm:

$$d_0^j = F\left(k_0^j, l_0^j; z_0\right) - w_0 l_0^j - \left(k_1^j - (1-\delta)\, k_0^j\right), \tag{7}$$

$$d_1^j(s_1) = F\left(k_1^j, l_1^j(s_1); z_1(s_1)\right) - w_1(s_1)\, l_1^j(s_1) + (1-\delta)\, k_1^j, \text{ for all } s_1. \tag{8}$$

Note that we have assumed that firms finance their investment solely with retained earnings. Given this, the dividends paid by each firm are the residual of its profits (output net of wages) and investment, whereas investment is equal to the new capital stock net of the undepreciated capital.[5]

[5] It is straightforward to add bonds to this economy without altering the main unanimity results of the paper. On the other hand, as long as the initial shareholders are unconstrained with respect to their choice of stocks and bonds, the Modigliani-Miller theorem holds and the financial structure of the firm is indeterminate.

3. Equilibrium

3.1. Exchange Equilibrium

This section defines an exchange equilibrium for the economy introduced in the previous section. To do this, we let y^j denote firm j's production plan:

$$y^j = \left[l_0^j, \left\{l_1^j\right\}, k_1^j\right]. \tag{9}$$

The previous equation reflects the fact that a production plan comprises choices of labor in the two periods and a choice of physical capital for period 1. In addition, equations (7) and (8) imply that a plan y^j, together with the realization of the productivity shock $z_1(s_1)$ and the wages w_0 and $w_1(s_1)$, determines the firm's dividends d_0^j and $d_1^j(s_1)$. Following the literature, our definition of an exchange equilibrium assumes that both the wages and the production plans (together with the associated dividends) are taken as given. In other words, our equilibrium concept abstracts from the decisions of the firms.

Definition 1: Exchange Equilibrium. Given the wages $w = (w_0, \{w_1\})$ and some initial distribution of shares $\left(\theta_0^i\right)_{i\in I}$ and capital stocks $k_0 = \left(k_0^j\right)_{j\in J}$, an exchange equilibrium relative to the production plans $y = \left(y^j\right)_{j\in J}$ and the associated dividends $d = \left(d_0^j, \left\{d_1^j\right\}\right)_{j\in J}$ is a vector of stock prices $p = (p_0^j)_{j\in J}$, a consumption allocation $\left(c^i\right)_{i\in I}$ and portfolio choices $(\theta_1^i)_{i\in I}$ such that:

(i) Given p, the choices of consumer $i \in I$, c^i and θ_1^i, are optimal, that is, they maximize (1) subject to (4), (5) and (6).

(ii) p is such that the market for the shares of each firm in period 1 clears:

$$\sum_{i\in I} \theta_{j1}^i = 1 \text{ for all } j \in J. \tag{10}$$

(iii) The goods market in periods 0 and 1 clears:

$$\sum_{i\in I} c_0^i = \sum_{j\in J} d_0^j + w_0 L_0, \tag{11}$$

$$\sum_{i\in I} c_1^i(s_1) = \sum_{j\in J} d_1^j(s_1) + w_1(s_1)L_1(s_1), \text{ for all } s_1. \tag{12}$$

In the next section, we define a production equilibrium in which both the wages and the production plans of all firms are determined endogenously.

3.2. Production Equilibrium

In general, it is not straightforward to define a production equilibrium when markets are incomplete, since each firm is owned by many consumers-shareholders who might disagree on its optimal production plan. In

this section, we analyze the shareholders' preferences with respect to this plan. To do this, we assume that initial, rather than final, shareholders make the investment decision.[6] In addition, we assume that only the shareholders with a positive interest in the firm can decide over its production plan.

In order to determine a shareholder i's preferred production plan y^j for firm j, we can replace his optimal consumption choices into the objective function (1). The resulting indirect utility function $V^i(y)$ will, among other things, depend on the plans $y = (y^j)_{j \in J}$ undertaken by all firms through the consumption choices c^i. In other words, each consumer will have indirect preferences over these plans. The preferred choice for shareholder i can then be obtained by maximizing his indirect utility function $V^i(y)$ with respect to y^j subject to equations (7)-(8) and taking as given the wages, investment decisions and stock prices of the other firms.

Here, it is important to note that a firm's choice of labor, l_0^j and $\left\{l_1^j\right\}$ is a static one. In particular, this problem is equivalent to maximizing (7) and (8) with respect to l_0^j and $\left\{l_1^j\right\}$ respectively. In contrast, the choice of a preferred level of period 1 capital k_1^j by a shareholder involves an intertemporal trade-off. Therefore, disagreement among shareholders cannot be ruled out a priori. In sum, all shareholders will agree on the firm's labor choice and therefore, in what follows, we focus on the potential disagreement regarding k_1^j.

Let k_1^{ij} be the preferred level of capital of firm j for shareholder i. Given that k_1^{ij} might in principle differ across firm j's shareholders, it is necessary to specify a mechanism by which potential disagreement results in a choice k_1^j for the firm as a whole. In this section, we represent this mechanism in a general way by means of the following function:

$$\Psi\left(\left(k_1^{ij}\right)_{i \in J_{j0}}, \left(\theta_{j0}^i\right)_{i \in J_{j0}}\right) : \mathbb{R}_+^{J_{j0}} \times \mathbb{R}_+^{J_{j0}} \to \mathbb{R}_+ \tag{13}$$

This function maps the initial distribution of firm's shares and the preferred levels of capital of the firm's shareholders into a choice for the firm's capital in period 1, k_1^j. The only assumption we impose on the aggregator Ψ is that it is responsive to shareholders when there is unanimity. In other words, if all the shareholders in J_{j0} prefer the same level of capital k_1^j for period 1, this level will be chosen by the firm regardless of the initial distribution of shares $\left(\theta_{j0}^i\right)_{i \in J_{j0}}$:

$$k_1^j = \Psi\left(\left(k_1^j\right)_{i \in J_{j0}}, \left(\theta_{j0}^i\right)_{i \in J_{j0}}\right).$$

[6] The results of this section easily apply to the case in which final shareholders decide on the investment. In essence, our assumption implies that initial shareholders have to form expectations about the effect of a change in investment on the firm's stock price. Note also that this distinction becomes less relevant in a multiperiod setting, where shareholders have to form expectations about the changes in future prices, regardless of whether initial or final shareholders make the investment decision.

Note that our aggregator function encompasses a wide variety of mechanisms, such as majority voting. A competitive production equilibrium can then be defined as follows:

Definition 2: Production Equilibrium. Given some initial distribution of shares $\left(\theta_0^i\right)_{i\in I}$ and capital stocks $k_0 = (k_0^j)_{j\in J}$, a production equilibrium is a vector of stock prices $p = (p_0^j)_{j\in J}$, production plans $y = \left(y^j\right)_{j\in J}$, dividends $d = \left(d_0^j, \left\{d_1^j\right\}\right)_{j\in J}$, wages $w = (w_0, \{w_1\})$, consumption allocations $\left(c^i\right)_{i\in I}$, portfolio choices $\left(\theta_1^i\right)_{i\in I}$ and preferred levels of investment $\left(\left(k_1^{ij}\right)_{j\in J}\right)_{i\in J_{j0}}$ for each shareholder such that:

i) p, θ_1^i, and c^i constitute an exchange equilibrium relative to the production plans y and the associated dividends d.

ii) A shareholder $i \in J_{j0}$'s preferred level of investment in firm j is such that k_1^{ij} maximizes $V(y^j)$ subject to equations (7) and (8), and taking as given wages, the dividends paid by all the other firms and their stock prices.

iii) The level of capital k_1^j for each firm $j \in J$ is such that:

$$k_1^j = \Psi\left(\left(k_1^{ij}\right)_{i\in J_{j0}}, \left(\theta_{j0}^i\right)_{i\in J_{j0}}\right).$$

iv) For each firm j, the labor demands l_0^j and $\left\{l_1^j\right\}$ maximize the current and future dividends d_0^j and $\left\{d_1^j\right\}$. This implies that the wages w satisfy the standard marginal product conditions:

$$w_0 = F_l\left(k_0^j, l_0^j; z_0\right), \tag{14}$$

$$w_1(s_1) = F_l\left(k_1^j, l_1^j(s_1); z_1(s_1)\right), \text{ for all } s_1. \tag{15}$$

v) The dividends d are given by equations (7) and (8).

4. Characterization of the Production Equilibrium

4.1. Labor, Dividends, and Stock Prices

This section derives some important results that will help us to characterize the aggregator function Ψ that maps the preferences of shareholders with respect to different investment alternatives into a production plan for the firm. To characterize these preferences, it is important to note that a shareholder has to know how a firm's stock price will be affected by variations in the investment decision to determine his preferred level

of capital. Our first result shows that the assumption of constant returns to scale in production allows the shareholder to infer this information from the stock prices and choices of capital of other firms.

To see this, we first consider firm j's decision regarding the labor input in period 1. As mentioned in the previous section, all shareholders will agree on selecting $\left\{l_1^j\right\}$ in order to maximize period 1 dividends. This implies that the optimal choice for $l_1^j(s_1)$ satisfies the standard marginal product condition in equation (15). Further, since under constant returns to scale the derivative $F_l(k,l;z)$ is homogeneous of degree zero in k and l, it follows that all firms have the same capital-labor ratio at $t=1$. In particular, the optimal choice of labor by firm j at s_1 can be expressed as:

$$l_1^j(s_1) = k_1^j g_l(w_1(s_1), z_1(s_1)), \tag{16}$$

where g_l is a function of the aggregate variables $w_1(s_1)$ and $z_1(s_1)$ only. Replacing equation (16) into equation (8), and using the constant returns to scale assumption to collect k_1^j, the dividends in period 1 can then be rewritten as:

$$d_1^j(s_1) = k g_d(w_1(s_1), z_1(s_1)), \text{ for all } s_1. \tag{17}$$

Notice that, according to equation (17), the first period dividend payments $d_1^j(s_1)$ are proportional to the level of investment k_1^j, with a factor of proportionality that is common across firms and equal to:

$$g_d(w_1(s_1), z_1(s_1)) \equiv F(1, g_l(w_1(s_1), z_1(s_1)); z_1(s_1)) - w_1(s_1) g_l(w_1(s_1), z_1(s_1)) + 1 - \delta, \text{ for all } s_1.$$

A key implication of equation (17) is summarized in the following proposition.

Proposition 1. In a production equilibrium, the rates of return on the stocks of all firms are equalized for all possible realizations of the state s_1 in period 1. Formally:

$$\frac{d_1^{j'}(s_1)}{p_0^{j'}} = \frac{d_1^j(s_1)}{p_0^j} \text{ for all } j, j' \in J \text{ and } s_1. \tag{18}$$

Proof of Proposition 1. To prove the proposition, suppose that there exist a couple of firms j and j' and a state of the world s_1 such that equation (18) does not hold. Then, using equation (17), it must be the case that, say:

$$\frac{k_1^j g_d(w_1(s_1), z_1(s_1))}{p_0^j} > \frac{k_1^{j'} g_d(w_1(s_1), z_1(s_1))}{p_0^{j'}},$$

so that the equality condition is in fact violated for all possible realizations of s_1. As the previous inequality reflects, the rate of return on firm j's stock will always be higher than the rate of return on firm j''s stock, for all possible realizations of s_1. But this clearly implies that there would be an

incentive for a consumer holding firm j''s stock to sell it and buy instead firm j's stock. In turn, this implies that the economy is not in a production equilibrium. Therefore, equation (18) must hold.■

The result of Proposition 1 provides a way to compute the effects of a variation in firm j's investment on its stock price. If we use equation (17) to replace $d_1^j(s_1)$ in (18), we obtain:

$$p_0^j = k_1^j \frac{p_0^{j'}}{k_1^{j'}} \text{ for all } j, j' \in J. \tag{19}$$

Further, since firm j's shareholders are assumed to take as given the stock price and the investment decision made by all other firms when deciding on k_1^j, equation (19) provides them with information about the effect of different investment levels on their firm's stock price. Taking the derivative of p_0^j with respect to k_1^j, and using again equation (19), yields:

$$\frac{\partial p_0^j}{\partial k_1^j} = \frac{p_0^j}{k_1^j}. \tag{20}$$

Equation (20) provides a way to evaluate the change in a firm's stock price when it changes the investment level. In particular, it implies that the elasticity of a firm's stock price to its period 1 capital stock is equal to one.

4.2. Unanimity in the Absence of Binding Borrowing Constraints

In what follows, we use the results above to study the preferred choice of period 1 capital for firm j by shareholder $i \in J_{j0}$ for the case in which the borrowing constraint (4) is not binding. This would occur, for example, if the constraint was set equal to the natural borrowing limit. In such a situation, we show that shareholders will be unanimous in selecting a level of investment k_1^j that maximizes the stock market value of the firm.

As mentioned in section 3.2, the preferred level of capital from the perspective of a shareholder is obtained by maximizing his indirect utility function $V^i(y)$ with respect to k_1^j, taking as given the stock prices and production decisions of all other firms. Further, when choosing his preferred level of k_1^j, a shareholder has to take into account how changes in k_1^j give rise to changes in current and future dividends d_0^j and $\left\{d_1^j\right\}$ and in the stock market value of the firm p_0^j. The change in utility experienced by shareholder i when firm j changes its level of investment is therefore given by:[7]

$$\begin{aligned} \frac{\partial V^i(y)}{\partial k_1^j} &= \theta_{j0}^i \frac{\partial u\left(c_0^i\right)}{\partial c_0^i}\left(\frac{\partial p_0^j}{\partial k_1^j} + \frac{\partial d_0^j}{\partial k_1^j}\right) \\ &\quad -\theta_{j1}^i\left(\frac{\partial u\left(c_0^i\right)}{\partial c_0^i}\frac{\partial p_0^j}{\partial k_1^j} - \sum_{s_1}\beta\pi\left(s_1\right)\frac{\partial u\left(c_1^i\left(s_1\right)\right)}{\partial c_1^i\left(s_1\right)}\frac{\partial d_1^j\left(s_1\right)}{\partial k_1^j}\right). \end{aligned} \tag{21}$$

[7] See appendix A for a derivation of this expression.

Equation (21) is the key to understanding the effect of incomplete markets on shareholders' investment decisions. As initial owners of the firm, shareholders care about the way investment in physical capital affects the net stock market value of the firm $p_0^j + d_0^j$. This effect is captured by the term multiplying θ_{j0}^i in (21). As final owners of the firm, shareholders care about the way the firm's investment affects what Magill and Quinzii (1996) refer to as the "spanning services of the firm's equity contract", which is equal to its stock price in period 0 and the dividends it pays out in period 1. This effect is captured by the term multiplying θ_{j1}^i in (21). To simplify notation, we let the marginal rate of substitution of shareholder i between consumption in the initial period and consumption in state s_1 be given by:

$$m_0^i(s_1) \equiv \frac{\beta\pi(s_1)\frac{\partial u\left(c_1^i(s_1)\right)}{\partial c_1^i(s_1)}}{\frac{\partial u\left(c_0^i\right)}{\partial c_0^i}}.$$

Further, we follow Grossman and Hart (1979) and define b_{j0}^i as the the maximum amount of date 0 income (possibly negative) that a shareholder is willing to give up for a marginal change in the initial investment of firm j. Using this notation, equation (21) implies that:

$$b_{j0}^i = \theta_{j0}^i\left(\frac{\partial p_0^j}{\partial k_1^j} + \frac{\partial d_0^j}{\partial k_1^j}\right) - \theta_{j1}^i\left(\frac{\partial p_0^j}{\partial k_1^j} - \sum_{s_1} m_0^i(s_1)\frac{\partial d_1^j(s_1)}{\partial k_1^j}\right) \tag{22}$$

Notice that shareholders will unanimously agree on increasing or decreasing the firm's level of investment as long as b_{j0}^i has the same sign for all of them. Moreover, a shareholder's preferred level of capital for period 1, k_1^{ij}, must be such that the marginal benefits and cost of a change in the firm's capital stock exactly offset each other, implying that, at an optimal choice, $b_{j0}^i = 0$. We now show that this is the case in the present setting. In particular, a firm's shareholders are unanimous in their choice of capital, i.e., $k_1^{ij} = k_1^j$ for all $i \in J_{j0}$.

First, as shown in the previous section, the effect of a change in the investment plan on the stock price of the firm can be inferred from equation (20). Similarly, taking the derivative of $d_1^j(s_1)$ with respect to k_1^j in equation (17) yields:

$$\frac{\partial d_1^j(s_1)}{\partial k_1^j} = \frac{d_1^j(s_1)}{k_1^j}, \tag{23}$$

whereas the period zero budget constraint of the firm in equation (7) implies that:

$$\frac{\partial d_0^j}{\partial k_1^j} = -1. \tag{24}$$

Substituting these expressions in equation (22) yields:

$$b_{j0}^i = \theta_{j0}^i\left(\frac{p_0^j}{k_1^j} - 1\right) - \theta_{j1}^i\left(\frac{p_0^j}{k_1^j} - \sum_{s_1} m_0^i(s_1)\frac{d_1^j(s_1)}{k_1^j}\right). \tag{25}$$

Second, notice that the second term in parenthesis on the right hand side of equation (25) is equal to zero for all the shareholders $i \in J_{j0}$ for whom the borrowing constraint (4) is not binding.[8] In this case, the first order condition associated with the optimal portfolio choice of shareholder i is given by:

$$p_0^j = \sum_{s_1} m_0^i(s_1)\, d_1^j(s_1) \text{ for all } j \in J. \tag{26}$$

Because of equation (18), if condition (26) holds with equality for firm j, then it must hold with equality for all other firms $j' \neq j$. Thus, the expression in parenthesis multiplying θ_{j1}^i in equation (25) is equal to zero. In other words, stock market trading equalizes the final shareholders' valuations of future dividends. These observations lead us to the main result of the paper.

Proposition 2. If the borrowing constraint (4) is not binding for any shareholder $i \in J_{j0}$, then the choice of period 1 capital for each firm $j \in J$ in a production equilibrium is characterized by:

1. **Unanimity.** The shareholders $i \in J_{j0}$ of a firm $j \in J$ are unanimous in their preferred level of period 1 capital for firm j, that is,

$$k_1^{ij} = k_1^j \text{ for all } j \in J \text{ and all } i \in J_{j0}.$$

2. **Value maximization.** The level of capital k_1^j chosen by a firm j's shareholders is equal to:

$$k_1^j = p_0^j \text{ for all } j \in J.$$

The previous proposition states that all shareholders are unanimous about the investment level of firm j provided that the borrowing limits are not binding. Moreover, they agree on setting the level of investment equal to the stock price of the firm. Note that this level of investment is actually the one that maximizes the net market value of the firm $p_0^j + d_0^j$.

4.3. Equivalent Economy

In the next section, we discuss the relationship between our unanimity result and the findings in the literature. Before doing this, it is important to note that one of the relevant implications of the previous results is that

[8] Sufficient conditions for this to be the case are: i) $\underline{\theta} = 0$, (ii) the minimum possible labor endowment realization in period 1 is zero, i.e., $\min_{s_1} x_1^i(s_1) = 0$, and iii) the marginal utility of consumption at zero is infinite. These assumptions imply that a consumer will never choose $\theta_{j1}^i = 0$ for any j. Suppose, in fact, that for some firm j, $\theta_{j1}^i = 0$ and that the first order condition with respect to stocks holds as an inequality. Then, equation (18) implies that the first order condition holds as an inequality for all firms $j \in J$. Thus, $\theta_{j1}^i = 0$ for all $j \in J$. In this case a consumer will experience zero consumption with positive probability. The assumption that the marginal utility at zero is infinite is sufficient to rule this case out. Thus, the borrowing constraint will never bind if (i)-(iii) are satisfied.

unanimity among initial shareholders implies that the equilibrium allocation of consumption and capital in this economy is the same as in an economy where consumers, instead of firms, accumulate physical capital directly. To see this, consider such a framework and abstract for simplicity from the firm sub-index j.[9] The consumers' budget constraint is given by:

$$c_0^i + k_1^i = (r_0 + 1 - \delta) k_0^i + w_0 x_0^i, \tag{27}$$

$$c_1^i(s_1) = (r_1(s_1) + 1 - \delta) k_1^i + w_1(s_1) x^i(s_1) \text{ for all } s_1, \tag{28}$$

where r_0 and $r_1(s_1)$ are the rental rates of capital. These are equal to the marginal products of capital in the two periods:

$$\begin{aligned} r_0 &= F_k(k_0, L_0; z_0) \\ r_1(s_1) &= F_k(k_1, L_1(s_1); z_1(s_1)). \end{aligned}$$

Since consumers make the investment decisions, market clearing implies that the aggregate capital stock is equal to:

$$k_0 = \sum_{i \in I} k_0^i, \text{ and } k_1 = \sum_{i \in I} k_1^i.$$

Finally, in this setting each consumer accumulates physical capital according to the following condition:

$$1 = \sum_{s_1} m_0^i(s_1)(r_1(s_1) + 1 - \delta). \tag{29}$$

It is easy to see that the equations that characterize the equilibrium of this economy are the same as the conditions that characterize a production equilibrium of our economy. Clearly, the first order condition (29) is the same as (26), since $k_1 = p_0$ and

$$r_1(s_1) + 1 - \delta = \frac{d_1(s_1)}{k_1}.$$

In addition, the budget constraints in (27) and (28) are equivalent to the ones in the economy of section 2 (equations (5) and (6)) after setting $r_0 = \frac{d_0}{k_0} - (1 - \delta)$, $k_0^i = k_0 \theta_0^i$ and $k_1^i = p_0 \theta_1^i$. Given this, the equilibrium is the same in the two economies.[10]

[9] Note that this is without loss of generality, since all firms will make the same production decisions in equilibrium.

[10] The relationship between the two economies is discussed in detail in Carceles-Poveda and Coen-Pirani (2006), where the authors consider an infinite horizon economy in which firms discount future cash flows with present value processes that do not allow for arbitrage opportunities. In a setup with general borrowing constraints, they show that this objective generates the same allocations as in a framework where consumers accumulate capital directly, regardless of the particular present value process used by the firm to discount profits. Under this objective, however, not every shareholder agrees with the investment plan of the firm.

5. Discussion and Relationship with the Literature

As we have seen, the key assumptions to obtain unanimity are that the production function displays constant returns to scale and that the borrowing constraint is not binding. These assumptions jointly guarantee that the term in parenthesis multiplying θ_{j1}^{i} in equation (21) is equal to zero. First, the constant returns to scale assumption guarantees that the latter term can be written as in equation (25); second, the fact that the portfolio decision is interior guarantees that the first order condition for agent i's portfolio choice can be used to set this term to zero.

It is easy to show that this unanimity result is robust to extending the model to consider: 1) preferences that are characterized by non-expected utility; 2) preference heterogeneity among consumers; 3) different "opinions" among consumers about the likelihood of a given state of the world; 4) adjustment costs in the installation of new capital, as long as the adjustment cost function is homogeneous of degree one in current capital and investment (as in Hayashi, 1982). In this case, homogeneity guarantees that period 1 dividends can still be written as k_1^j times a term that depends only on aggregate variables.

In this section, we discuss how the unanimity result obtained here relates to some of the main contributions to the theoretical literature on shareholder disagreement in economies characterized by incomplete markets. Before considering the literature on incomplete markets, it is instructive to summarize why unanimity obtains under complete financial markets. In the latter case, the individual state prices $m_0^i(s_1)$ are equalized among shareholders. Since the state price $m_0(s_1)$ is independent of an individual firm's choice of capital, equation (26) implies that

$$\frac{\partial p_0^j}{\partial k_1^j} = \sum_{s_1} m_0(s_1) \frac{\partial d_1^j(s_1)}{\partial k_1^j}.$$

Clearly, this condition implies that the second term in (21) is equal to zero for all shareholders, independently of whether the production function displays constant returns to scale or not. When the asset market is incomplete, however, the state prices are not necessarily equal among shareholders. In this circumstance, the assumption of constant returns to scale in production guarantees that trading in the stock market is sufficient to make shareholders agree on the expected discounted value of an extra unit of investment, as long as borrowing constraints are not binding.

One of the first papers discussing the firm's choice of capital under incomplete financial markets is Diamond (1967). Diamond focused his attention on the case where: 1) uncertainty faced by firms takes a multiplicative form; 2) capital is the only input used in production; 3) production occurs under decreasing returns to scale according to a production function of the form $zG(k)$, where $G' > 0$ and $G'' < 0$; and 4)

capital fully depreciates within a period, i.e., $\delta = 1$. Under these assumptions a firm's period 1 dividend is equal to $d_1^j(s_1) = z_1(s_1)\,G(k_1^j)$. Given that aggregate uncertainty affects dividends in a multiplicative way, an argument analogous to the one developed in section 4 implies the equalization of ex-post rates of return among firms (equation 18). Thus, the equivalent of equation (19) is:

$$p_0^j = p_0^{j'} \frac{G(k_1^j)}{G(k_1^{j'})} \text{ for all } j, j' \in J.$$

It then follows that

$$\frac{\partial p_0^j}{\partial k_1^j} = G'(k_1^j) \frac{p_0^j}{G(k_1^j)}. \tag{30}$$

Replacing (30) and the first order condition (26) in (21) yields the equation that determines the optimal choice of capital for shareholder i:

$$\theta_{j0}^i \left(G'(k_1^j) \frac{p_0^j}{G(k_1^j)} - 1 \right).$$

This implies that shareholders will be unanimous in their choice of k_1^j. To see the relationship between this result and the one obtained in the previous section, consider a possible extension of Diamond's result to an economy where the good is produced using capital and labor, while assumptions 1) and 3)-4) are preserved. Denote the production function by $zG(k, l)$, where G does not necessarily to exhibit constant returns to scale. The dividend in period 1 is equal to

$$d_1^j(s_1) = z_1(s_1)\,G(k_1^j, l_1^j(s_1)) - w_1(s_1)\,l_1^j(s_1). \tag{31}$$

In order to extend Diamond's result to such an economy one has to show that, once the optimal $l_1^j(s_1)$ has been replaced into $d_1^j(s_1)$, the latter can be written as:

$$d_1^j(s_1) = h(k_1^j)\widetilde{g}_d(z_1(s_1), w_1(s_1)), \tag{32}$$

where h and $\widetilde{g}_d$ are two functions that are common across firms. If this were the case, then we could apply Diamond's argument summarized above. The problem with this argument is that, even if uncertainty enters multiplicative in the *original* production function, there is no guarantee that the decomposition (32) applies. To see this, consider, for example the decreasing returns to scale production function:

$$zG(k, l) = z \log\left(1 + k + l^{\frac{1}{2}}\right).$$

Solving for the labor demand problem and plugging the optimal $l_1^j(s_1)$ back into (31) yields:

$$d_1^j(s_1) = z_1(s_1) \log \frac{1}{2} \left\{ 1 + k_1^j + \left[(1 + k_1^j)^2 + \frac{2z_1(s_1)}{w_1(s_1)} \right]^{\frac{1}{2}} \right\},$$

which clearly does not take the form (32). However, when the production function takes the commonly used Cobb-Douglas form, this problem does not arise. For example, if $G(k,l) = k^{\alpha} l^{\sigma}$, with $\alpha + \sigma < 1$, then:

$$d_1^j\left(s^1\right) = \left(k_1^j\right)^{\frac{\alpha}{1-\sigma}} \left(\frac{\sigma z_1\left(s_1\right)}{w_1\left(z_1\left(s_1\right)\right)}\right)^{\frac{\sigma}{1-\sigma}} z_1\left(s_1\right)\left(1-\sigma\right),$$

which satisfies (32). In summary, in the model considered here, Diamond's assumption of *multiplicative uncertainty* does not, in general, give rise to unanimity among shareholders if the production function exhibits decreasing returns to scale in capital and labor. The assumption of constant returns to scale in production, however, is sufficient to obtain unanimity.[11] Finally, as the previous example with a Cobb-Douglas production function indicates, the assumption of constant returns to scale is not necessary in order to obtain the unanimity result.

After Diamond (1967), the theoretical literature concerning the objectives of the firm under incomplete markets has mostly made the assumption of *competitive price perceptions* (from now on CPP). This was originally introduced by Grossman and Hart (1979) in their seminal paper (see also Magill and Quinzii (1996) for a discussion of this approach). It is important to note that Grossman and Hart do not assume that the production function F displays constant returns to scale. Instead, they postulate that consumers have CPP, that is, that they use their own state prices $m_0^i\left(s_1\right)$ to evaluate the effect of a change in k_0^j on the firm's stock price:

$$\left.\frac{\partial p_0^j}{\partial k_1^j}\right|_i = \sum_{s_1} m_0^i\left(s_1\right) \frac{\partial d_1^j\left(s_1\right)}{\partial k_1^j}, \tag{33}$$

where the subindex i on the left hand side indicates that the perception of this derivative varies across shareholders. Notice that this amounts to setting the second term in equation (21) equal to zero by assumption. Further, this implies that shareholders will be unanimous in their desire to maximize the firm's net stock market value, by setting

$$\left.\frac{\partial p_0^j}{\partial k_1^j}\right|_i = 1,$$

but they will, in general, disagree on the choice of k_1^j. Grossman and Hart resolve this conflict by allowing for income transfers among a firm j's initial shareholders at time zero. In their approach, an optimal investment plan for the firm is such that there is no other investment plan and a set of income transfers among shareholders such that all shareholders are better off.

In our model, even if shareholders used their own state prices to evaluate $\partial p_0^j / \partial k_1^j$, they would still agree on the magnitude of this derivative. This is because the assumption of constant returns to scale guarantees

[11] Notice also that, if the production function displays constant returns to scale, the aggregate shock z must affect this function multiplicatively.

that the right-hand side of equation (33) is the same among all shareholders, so that Grossman and Hart's CPP condition holds.[12] Moreover, as discussed below, if one considers a multiperiod setting, the constant returns to scale assumption not only rules out disagreement among contemporaneous shareholders, but also among shareholders in different periods. An advantage of not postulating CPP is that we can extend our approach to the case where borrowing constraints are binding, as in section 6.2. In such a case, the CPP approach is not applicable because the right-hand side of (33) does not measure the marginal benefit of a higher investment by the firm for a constrained shareholder.[13] Moreover, the CPP approach implies that shareholders have different opinions about the sensitivity of stock prices to the level of investment in an incomplete markets equilibrium.

Finally, it is important to note that the unanimity result obtained here can also be interpreted as a special case of the unanimity result under *spanning* originally derived by Ekern and Wilson (1974) and extended to a multiperiod setting under the additional assumption of *competitivity* by Grossman and Stiglitz (1977, 1980). First, Ekern and Wilson show that shareholders will be unanimous in approving investment plans generating vectors of dividends that are spanned by the payoffs of existing securities, even if markets are incomplete. On the other hand, Grossman and Stiglitz show that "spanning" per se does not imply unanimity among shareholders in settings, like ours, where a firm's shares are traded after the investment decision has been taken. In particular, they emphasize that unanimity requires also competitivity in addition to spanning, in the sense that "each consumer believes that if the output of any firm increases by $b\%$ in each and every state of nature, then the value of the firm increases by $b\%$" (see Grossman and Stiglitz, 1977). Under these two conditions, the authors show that shareholders will agree on a production plan that maximizes the net market value of the firm.

Our constant returns to scale assumption implies that the spanning condition initially postulated by Ekern and Wilson (1974) is satisfied due to the fact that each firm j's period 1 dividend vector $\left\{d_1^j\right\}$ is just a multiple of the dividend vector of any other firm j', since

$$d_1^j(s_1) = \frac{k_1^j}{k_1^{j'}} d_1^{j'}(s_1), \text{ for all } s_1.$$

However, note that the reason why spanning holds here is non-trivial, in the sense that it is not an assumption, but rather an implication of constant returns to scale in production. The key to spanning lies

[12] Grossman and Hart (1979, page 300) indirectly refer to this case by mentioning that when the spanning assumption of Ekern and Wilson (1974) holds, shareholders will be unanimous in their investment choice.

[13] See footnote 5 in Grossman and Hart (1979) for a discussion of this point. As the authors point out, if an initial shareholder faces binding short-sale restrictions, his state prices do not contain sufficient information on the change in the firm's stock market price following a change in its investment.

in the fact that, from the point of view of period 0, the production function in period 1 effectively displays constant returns to scale in physical capital only. This is because the labor input in period 1 is chosen after the capital stock is already in place, and, by constant returns to scale, the optimal labor input in period 1 is then a linear function of capital (see equation 16).

The competitivity condition postulated by Grossman and Stiglitz allows shareholders to infer the effects of different levels of investment on a firm's stock price from the information contained in other firms' stock prices. This is because each firm is assumed to take as given the stock prices and investment decisions of other firms when considering changing its own investment decision. Consequently, as equation (20) implies, an increase in a firm's investment level leads to the same percentage increase in its stock price.

6. Extensions

In this section, we consider two important extensions of our benchmark model economy. Specifically, the first subsection shows how the unanimity result derived in the two-period model of section 4 carries over to a multiperiod economy. In the second subsection, we consider the case of binding borrowing limits.

6.1. Multiple Periods

Two new features arise in a multiperiod version of the economy of section 2. First, stock returns depend not only on future dividends but also on future stock prices. Second, as noted by Grossman and Hart (1979), disagreement may arise not only among shareholders in the same period but also among shareholders at different dates. This implies that, when characterizing the equilibrium, one has to make an assumption about which period's shareholders decide on a firm's capital stock in a given period.

We start by assuming that the decision about the firm's capital in a given period is made by its shareholders in that period. Using a three-period version of the economy of section 2, we first sketch how to extend our unanimity result to a multiperiod setting under the assumptions of constant returns to scale and no binding borrowing limits. Moreover, we show that, under the same assumptions, there is no disagreement among shareholders in different periods. In particular, all the shareholders of a firm at different dates agree on setting its capital stock equal to the stock market value of the firm.

6.1.1. Unanimity in a Multiperiod Economy

For simplicity, we consider a three period economy. Appendix B shows that all the results of this section carry over to an economy with T periods, where T can be infinity. The three periods are indexed by $t = 0, 1, 2$.

As before, we assume that there is no uncertainty at period $t = 0$ but there are a finite number of states in periods $t = 1, 2$. We denote the state of the economy in period $t = 1$ by $s_1 \in S_1$ and the history of the economy at $t = 2$ by $s^2 = (s_1, s_2)$, where $s_2 \in S_2$ is the particular realization of uncertainty at $t = 2$.

At $t = 0$, consumer $i \in I$ maximizes the following utility function:

$$U\left(c^i\right) = u\left(c_0^i\right) + \beta \sum_{s_1} \pi\left(s_1\right) u\left(c^i\left(s_1\right)\right) + \beta^2 \sum_{s^2} \pi\left(s^2\right) u\left(c^i\left(s^2\right)\right).$$

The consumer is subject to trading constraints at $t = 0, 1$:

$$\theta_{jt+1}^i\left(s_t\right) \geq -\underline{\theta}(s_t),$$

where $\underline{\theta}(s_t)$ is a possibly state-dependent borrowing limit. In what follows, we assume that this borrowing constraint is never binding for any shareholder at any period. As in the two-period model, a sufficient condition for this to be the case is that the limit is set to its natural level.

In the three period economy, the consumer's budget constraints are given by:

$$\begin{aligned}
c_0^i + \sum_{j \in J} p_0^j \theta_{j1}^i &= \sum_{j \in J} \theta_{j0}^i \left(d_0^j + p_0^j\right) + w_0 x_0^i, \\
c_1^i\left(s_1\right) + \sum_{j \in J} p_1^j\left(s_1\right) \theta_{j2}^i\left(s_1\right) &= \sum_{j \in J} \theta_{j1}^i \left(d_1^j\left(s_1\right) + p_1^j\left(s_1\right)\right) + w_1\left(s_1\right) x_1^i(s_1), \text{ for all } s_1, \\
c_2^i\left(s^2\right) &= \sum_{j \in J} \theta_{j2}^i\left(s_1\right) d_2^j\left(s^2\right) + w_2\left(s^2\right) x_2^i(s^2), \text{ for all } s^2.
\end{aligned}$$

while the dividends paid by the firm are equal to:

$$d_0^j = F\left(k_0^j, l_0^j; z_0\right) - w_0 l_0^j - \left(k_1^j - (1-\delta) k_0^j\right), \tag{34}$$

$$d_1^j\left(s_1\right) = F\left(k_1^j, l_1^j\left(s_1\right); z_1\left(s_1\right)\right) - w_1\left(s_1\right) l_1^j\left(s_1\right) - \left(k_2^j\left(s_1\right) - (1-\delta) k_1^j\right) \text{ for all } s_1, \tag{35}$$

$$d_2^j\left(s^2\right) = F\left(k_2^j\left(s_1\right), l_2^j\left(s^2\right); z_2\left(s^2\right)\right) - w_2\left(s^2\right) l_2^j\left(s^2\right) + (1-\delta) k_2^j\left(s_1\right) \text{ for all } s^2. \tag{36}$$

In the present setting, unanimity with respect to $k_2^j\left(s_1\right)$ among the period 1 shareholders can be proved exactly as in section 4, so we do not repeat the proof here. In particular, it is easy to see that they will be unanimous in choosing $k_2^j\left(s_1\right) = p_1^j\left(s_1\right)$ (see proposition 2).

Consider now the decision of the period 0 shareholders regarding k_1^j. Differentiating their indirect utility function with respect to k_1^j subject to the constraints (34)-(36), it is easy to show that their willingness to sacrifice period 0 consumption in exchange for a marginal increase in k_1^j is equal to:

$$\begin{aligned}
b_{j0}^i = {} & \theta_{j0}^i \left[\frac{\partial p_0^j}{\partial k_1^j} + \frac{\partial d_0^j}{\partial k_1^j}\right] - \theta_{j1}^i \left[\frac{\partial p_0^j}{\partial k_1^j} - \sum_{s_1} m_0^i(s_1) \left(\frac{\partial p_1^j(s_1)}{\partial k_1^j} + \frac{\partial d_1^j(s_1)}{\partial k_1^j}\right)\right] + \\
& - \sum_{s_1} \theta_{j2}^i(s_1) m_0^i(s_1) \frac{\partial p_1^j(s_1)}{\partial k_1^j} \left[1 - \frac{\partial k_2^j(s_1)}{\partial p_1^j(s_1)} \sum_{s^2|s_1} \frac{m_0^i(s^2)}{m_0^i(s_1)} \frac{\partial d_2^j(s^2)}{\partial k_2^j\left(s_1\right)}\right].
\end{aligned} \tag{37}$$

where $m_0^i(s_2)$ is the marginal rate of substitution between time zero consumption and consumption in state s^2. Note that, similarly to the two-period economy, when choosing k_1^j, the period 0 shareholders have to compute the effect of their decision on p_0^j, d_0^j and $d_1^j(s_1)$. As stated earlier, there are two additional features that do not arise in the two-period economy. First, shareholders also need to consider the effect of their decision on the firm's stock price in period 1, $p_1^j(s_1)$. Second, they need to anticipate how the change in $p_1^j(s_1)$ will affect the investment decision undertaken by the period 1 shareholders in period 2, $k_2^j(s_1)$, and, through that channel, the dividends paid by the firm in period 2, $d_2^j(s^2)$.

We now show that the term in square brackets on the second line of equation (37) is equal to zero. This follows from three observations. First, as observed above, shareholders at $t = 2$ choose $k_2^j(s_1) = p_1^j(s_1)$. We therefore have that $\partial k_2^j(s_1)/\partial p_1^j(s_1) = 1$. Second, our constant returns to scale assumption implies that $\partial d_2^j(s^2)/\partial k_2^j(s_1) = d_2^j(s^2)/k_2^j(s_1)$. Third, the period 0 shareholder's first order condition for stock holding in period 1 implies that

$$k_2^j(s_1) = p_1^j(s_1) = \sum_{s^2|s_1} \frac{m_0^i(s^2)}{m_0^i(s_1)} d_2^j(s^2).$$

Given this, the term in the square brackets cancels out. In turn, this implies that the marginal valuation $b_{j0}^i(s_0)$ of an increase in k_1^j for a period 0 shareholder takes the same form as in the two-period economy (see equation (22)), with the important difference that a marginal change in k_1^j leads to a variation in the return $d_1^j(s_1) + p_1^j(s_1)$ and not only in the period 1 dividend $d_1^j(s_1)$. Since $k_2^j(s_1) = p_1^j(s_1)$, one can use equation (35) to show that the return on holding firm j's stock between 0 and 1 can be rewritten as:

$$\frac{d_1^j(s_1) + p_1^j(s_1)}{p_0^j} = \frac{k_1^j g_d(w_1(s_1), z_1(s_1))}{p_0^j}. \tag{38}$$

The numerator of this equation is the counterpart of equation (17) in the two-period economy. Following the same steps as in section 4, it is then easy to show that also the second term in equation (37) is also equal to zero.[14] The shareholder's marginal valuation of an increase in k_1^j then simplifies to:

$$b_{j0}^i = \theta_{j0}^i \left[\frac{\partial p_0^j}{\partial k_1^j} + \frac{\partial d_0^j}{\partial k_1^j} \right].$$

Since $\partial d_0^j/\partial k_1^j = -1$ and $\partial p_0^j/\partial k_1^j = p_0^j/k_1^j$, this implies that the period 0 shareholders are unanimous in selecting a level of capital $k_1^j = p_0^j$.

Finally, it is easy to see that these arguments can be generalized to a general time horizon and investment choice. In such a setting, we can denote the history of the environment up to period t by s^t and we let $s^{t+1}|s^t$

[14] It is worthwhile noticing that under competitive price perceptions (Grossman and Hart, 1979), both the second and third terms in equation (37) would be set equal to zero by assumption. The fact that in our approach these two terms disappear is, instead, a result.

be the set of immediate successor nodes of event s^t. The following proposition, whose proof is relegated to appendix B, extends our main result to a general multiperiod setting.

Proposition 4. In the multiperiod economy with no binding trading constraints, all the initial shareholders of a firm j at node s^t agree to set its capital stock $k_{t+1}^j\left(s^t\right)$ according to the condition:

$$k_{t+1}^j\left(s^t\right) = p_t^j\left(s^t\right). \tag{39}$$

6.1.2. Disagreement Among Shareholders At Different Dates

The previous section has shown that, in a general multiperiod setting, all shareholders of a given firm at period t will agree on setting the capital stock equal to the stock market value of the firm in that period. We now ask whether shareholders of the firm in the periods preceding t will also agree with the period t's shareholders choice, which is summarized in equation (39). This is an important question, since disagreement among shareholders at different dates provides incentives for early shareholders of the firm to manipulate the current investment decision to try to affect future decisions. In principle, this possibility considerably complicates the solution of the model. Given this, some authors like Grossman and Hart (1979) assume that shareholders at the initial period can make a legally binding decision for all future levels of investment, allowing for no possibility of revisions for future shareholders at a later date.

In what follows, we show that, under no binding borrowing constraints, the assumption of constant returns to scale rules out disagreement among shareholders at different periods. To illustrate this, we employ again the three period economy, and consider a firm j's level of investment at date $t = 1$ and event $\overline{s}_1$. We now compare the investment level preferred by a period 0 shareholder with the one preferred by a shareholder in period 1.

Let's first analyze a period 1 shareholder's preferred choice of capital. From the perspective of period 1 the economy is the same as the two-period version of the model. Using the same arguments as in section 4, it follows easily that shareholder i's propensity to sacrifice period 1 consumption in state $\overline{s}_1$ for a marginal increase in $k_2^j\left(\overline{s}_1\right)$ is given by:

$$b_{j1}^i(\overline{s}_1) = \theta_{j1}^i\left[\frac{p_1^j(\overline{s}_1)}{k_2^j\left(\overline{s}_1\right)} - 1\right].$$

Thus, shareholders at period $t = 1$ will unanimously want to set $k_2^j\left(\overline{s}_1\right) = p_1^j(\overline{s}_1)$. Second, we ask whether an initial shareholder in period $t = 0$ would want to deviate from the choice $k_2^j\left(\overline{s}_1\right) = p_1^j(\overline{s}_1)$, taking into account that the period 1 capital is determined according to equation (39), that is, $k_1^j = p_0^j$. The shareholder's

willingness to sacrifice period 0 consumption in exchange for a marginal increase in $k_2^j(\overline{s}_1)$ is given by:

$$(40) \quad b_{j0}^i = \theta_{j0}^i \left[\frac{\partial p_0^j}{\partial k_2^j(\overline{s}_1)} + \frac{\partial d_0^j}{\partial k_2^j(\overline{s}_1)} \right] - \theta_{j1}^i \frac{\partial p_0^j}{\partial k_2^j(\overline{s}_1)} \left[1 - \frac{\partial k_1^j}{\partial p_0^j} \sum_{s_1} m_0^i(s_1) \left(\frac{\partial p_1^j(s_1)}{\partial k_1^j} + \frac{\partial d_1^j(s_1)}{\partial k_1^j} \right) \right]$$
$$- \sum_{s_1} \theta_{j2}^i(s_1) m_0^i(s_1) \frac{\partial p_1^j(s_1)}{\partial k_2^j(\overline{s}_1)} \left[1 - \frac{\partial k_2^j(s_1)}{\partial p_1^j(s_1)} \sum_{s^2|s_1} \frac{m_0^i(s^2)}{m_0^i(s_1)} \frac{\partial d_2^j(s^2)}{\partial k_2^j(s_1)} \right],$$

A marginal change in $k_2^j(\overline{s}_1)$ produces an effect on the firm's stock price and dividend at time 0 (first term in equation (40)). Further, since $k_1^j = p^j$, the variation in p_0^j leads to an equal variation in k_1^j, and through the latter to a change in the firm's stock price and dividend in period 1 (second term in in equation (40)). Finally, a marginal change in $k_2^j(\overline{s}_1)$ produces an effect on the firm's stock price in period 1. Since $k_2^j(s_1) = p_1^j(s_1)$, the latter gives rise to an equal variation in $k_2^j(s_1)$, and this changes the dividends paid out by the firm in period 2.

In order to show that the time 0 shareholder does not want to deviate from $k_2^j(\overline{s}_1) = p_1^j(\overline{s}_1)$, we need to show that at this choice, b_{j0}^i is equal to zero. Following the same steps as in the previous section, it is easy to show that the second and last terms in equation (40) are equal to zero. To show that the first term is also equal to zero, notice first that, since $k_1^j(s_0) = p_0^j$, the definition of d_0^j in equation (34) implies that:

$$p_0^j + d_0^j = F\left(k_0^j, l_0^j; z_0\right) - w_0 l_0^j + (1-\delta) k_0^j.$$

Since k_0^j is fixed and the choice of l_0^j is static, it then follows that:

$$\frac{\partial p_0^j}{\partial k_2^j(\overline{s}_1)} + \frac{\partial d_0^j}{\partial k_2^j(\overline{s}_1)} = 0.$$

This implies that $b_{j0}^i = 0$. Therefore, an initial shareholder at period $t = 0$ will agree with setting $k_2^j(\overline{s}_1) = p_1^j(\overline{s}_1)$. In other words, under constant returns to scale in production and no binding constraints, there is no disagreement among shareholders at different periods about the value of $k_2^j(s_1)$ for any state s_1. As stated before, this potential source of disagreement does not disappear under the assumption of CPP originally postulated by Grossman and Hart (1979). Given this, the authors assume that the firm commits itself to a production plan in the initial period, allowing for no possibility of revisions by future shareholders at a later date. In the present setting, however, unanimity obtains without such a restrictive assumption.

6.2. Binding Short-Sales Constraints

This section analyzes the two-period economy introduced in section 2 assuming that the borrowing constraint (4) binds for some initial shareholders of a firm. In such a case, the term multiplying θ_{j1}^i in equation (22) for these initial shareholders is not equal to zero. Except when the limit is $\underline{\theta} = 0$, in which case the second

term in (22) disappears, this situation introduces the possibility of disagreement among initial shareholders about the optimal size of investment to be undertaken by the firm.

To see this, consider an initial shareholder i for whom this borrowing constraint (4) binds. As noticed above, if this constraint binds for one firm j, then it must also bind for all firms. Since a constrained shareholder j's first order condition with respect to θ^i_{j1} (equation 26) holds as an inequality, there exists a $\xi^i_{j0} \in (0,1)$ such that:

$$p^j_0 \xi^i_{j0} = \sum_{s_1} m^i_0(s_1)\, d^j_1(s_1), \text{ for all } j \in J.$$

Replacing this into the first order condition for shareholder i's preferred investment level (equation 22), taking into account equation (20) and the fact that $\theta^i_{j1} = -\underline{\theta}$, yields:

$$\theta^i_{j1}\left(\frac{p^j_0}{k^j_1} - 1\right) + \underline{\theta}\frac{p^j_0}{k^j_1}\left(1 - \xi^i_{j0}\right) = 0.$$

The previous equation has the following implication.

Proposition 5. The constrained initial shareholders of firm $j \in J$ prefer a higher level of period 1 capital than the unconstrained shareholders. In particular, constrained shareholder i's preferred capital is given by:

$$k^{ij}_1 = p^j_0\left[1 + \frac{\underline{\theta}}{\theta^i_{j1}}\left(1 - \xi^i_{j0}\right)\right] \text{ for all } j \in J, \tag{41}$$

while an unconstrained shareholder i' would want to choose $k^{i'j}_1 = p^j_0$.

The previous proposition implies that a constrained initial shareholder would like to set the firm's investment to the point where the derivative $\partial p^j_0/\partial k^j_1$ is smaller than one, that is, for each dollar of additional investment in the firm, the firm's stock market price increases by less than a dollar. Despite the fact that the increase in the firm's price is smaller than the cost of increasing the firm's capital, a constrained shareholder benefits from this choice. In fact, the higher stock price translates into greater proceedings from short-selling the firm's stock. Given that the shareholder is constrained, each extra dollar obtained from short-selling can be consumed today, while the discounted marginal cost of repaying this debt back tomorrow is only $\xi^i_{j0} < 1$.

It is also important to note that, while the investment level preferred by constrained shareholders in equation (41) depends on the agent-specific multiplier ξ^i_{j0}, the unconstrained shareholders agree among themselves on the level of investment to be undertaken by the firm. This property suggests that, as long as the unconstrained shareholders initially hold more than fifty percent of the firm's shares, their preferred level of investment will prevail if the latter was determined by majority voting. In this case, the price of one

share of firm j will coincide with the price of a unit of physical capital, and the equilibrium will again be the same as in the analogous economy where consumers accumulate physical capital directly.

When the initially constrained shareholders own more than fifty percent of the firm's shares and $\underline{\theta} \neq 0$, however, the analysis becomes more complex. In a majority voting equilibrium, provided that it exists, the period 1 capital stock would tend to be higher than a firm's stock price, since $k_1^j > p_0^j$ is preferred by the constrained shareholders.

Finally, note that these arguments also extend to the general multiperiod economy studied in the previous setting. In particular, the unanimity result extends to the case of binding borrowing constraints only if this constraint is equal to $\underline{\theta} = 0$. On the other hand, disagreement will arise if some shortselling is allowed and the associated limit is binding for some shareholders. Also in this circumstance, differences among shareholders may be resolved by majority voting. However, the analysis would become much more complicated, since a shareholder that is unconstrained in one period would have to anticipate the possibility of becoming constrained in a later period. In turn, this implies that unconstrained shareholders in one period would in general not be unanimous on the level of investment to be undertaken by the firm, since they would face different chances of becoming constrained later on. For this reason, the analysis of a multiperiod economy with binding borrowing constraints and majority voting by shareholders is a difficult task that we therefore leave for future research.

7. Summary and Conclusions

This paper studies versions of the standard incomplete markets economy considered by Aiyagari (1994) and Krusell and Smith (1998) under the assumption that firms accumulate physical capital and that investment decisions explicitly reflect the underlying preferences of firms' shareholders. We show that, if borrowing constraints are not binding and production occurs under constant returns to scale, a firm's shareholders will unanimously agree on its optimal level of investment. In particular, shareholders will choose next period's capital stock to equal the stock market value of the firm in the current period. Further, under this level of investment, the equilibrium allocation of this economy is shown to coincide with the one that characterizes the standard model.

Our result contributes to two different strands of literature. The first is the theory literature on shareholder unanimity that developed after Diamond (1967) and the second is the macroeconomic literature that originated after Aiyagari (1994). Relative to the former one, we show how the assumption of constant returns to scale in production in the Aiyagari-Krusell-Smith model implies Ekern and Wilson (1974)'s spanning

condition in a non-trivial way. We also relate our approach and results to the CPP approach of Grossman and Hart (1979). In particular, our assumptions of constant returns to scale and no binding borrowing constraints also imply competitive price perceptions. In addition, these conditions rule out disagreement among shareholders at different dates, a result that in general cannot be obtained under Grossman and Hart's assumption. Last, we explicitly consider the case in which borrowing constraints are binding.

Our paper also contributes to the macroeconomic literature that emphasizes the importance of incomplete asset markets by clarifying the conditions under which a firm's shareholders are unanimous with respect to its investment decision. While in the Aiyagari-Krusell-Smith model it is common practice to assume constant returns to scale in production, this assumption alone is not sufficient to guarantee unanimity. Binding borrowing constraints will, in general, lead to disagreement among shareholders, even with constant returns to scale. The only exception to this rule, in our version of the model, is the case in which the borrowing constraint completely prevents short-selling of a firm's shares.[15] Our analysis therefore suggests that the practical importance of assuming that firms, rather than consumers, accumulate capital in the Aiyagari-Krusell-Smith model depends on the extent to which borrowing constraints are binding. This is ultimately a quantitative issue that we leave for future research.

[15] Notice that, even if no-shortselling is the typical borrowing constraint in the standard macroeconomic model with incomplete markets (see Krusell and Smith, 1998), the unanimity result that ensues in this case is not robust to the introduction of a second asset (such as a bond) in the economy.

A. Derivation of Equation (21)

To derive equation (21), we can define $V^i(y)$ as follows:

$$\begin{aligned} V^i(y) &= \max_{c^i,\theta_1^i} \left[u\left(c_0^i\right) + \beta \sum_{s_1} \pi\left(s_1\right) u\left(c_1^i\left(s_1\right)\right) \right] \\ & \quad \text{s.t.} \\ c_0^i + \sum_{j\in J} p_0^j \theta_{j1}^i &= \sum_{j\in J} \theta_{j0}^i \left(d_0^j + p_0^j\right) + w_0 x_0^i, \\ c_1^i\left(s_1\right) &= \sum_{j\in J} \theta_{j1}^i d_1^j\left(s_1\right) + w_1\left(s_1\right) x_1^i(s_1), \text{ for all } s_1. \end{aligned}$$

Equation (21) is then an implication of the envelope theorem. Specifically, if we differentiate $V^i(y)$ with respect to k_1^j considering that only variations of p_0^j, d_0^j and $d_1^j\left(s_1\right)$ in response to a marginal change in k_1^j have to be taken into account, the equation immediately follows. Notice that d_0^j, $d_1^j\left(s_1\right)$ and p_0^j depend on k_1^j, as evident from equations (7), (17) and (19), respectively.

B. Proof of Proposition 4

To prove the proposition, note that the first order condition of household i for holding stocks of firm j at s^t is given by:

$$p_t^j\left(s^t\right) = \sum_{s^{t+1}|s^t} m_t^i\left(s^{t+1}\right)\left(d_{t+1}^j\left(s^{t+1}\right) + p_{t+1}^j\left(s^{t+1}\right)\right), \text{ for all } j \in J \tag{42}$$

where

$$m_t^i\left(s^{t+1}\right) \equiv \beta \frac{\pi\left(s^{t+1}\right)}{\pi\left(s^t\right)} \frac{U\left(c_{t+1}^i\left(s^{t+1}\right)\right)}{U\left(c_t^i\left(s^t\right)\right)}.$$

and the condition holds with equality due to the fact that the shareholder is unconstrained. In addition, the firm's dividends at node s^t are equal to:

$$d_t^j\left(s^t\right) = F\left(k_t^j\left(s^{t-1}\right), l_t^j\left(s^t\right); z_t\left(s^t\right)\right) - w_t\left(s^t\right) l_t^j\left(s^t\right) + (1-\delta) k_t^j\left(s^{t-1}\right) - k_{t+1}^j\left(s^t\right). \tag{43}$$

We first assume that the horizon is finite. To show that our unanimity results extend to a general multiperiod setup, consider period T. Since shareholders will unanimously choose $k_{T+1}^j\left(s^T\right) = 0$, by constant returns to scale in production, $d_T^j\left(s^T\right)$ can be written as:

$$d_T^j\left(s^T\right) = k_T^j\left(s^{T-1}\right) g_d\left(w_T\left(s^T\right), z_T\left(s^T\right)\right). \tag{44}$$

This implies that period $T-1$ is the same as period 0 in the two-period economy of section 4. Given this, the initial shareholders at $T-1$ will unanimously agree on setting:

$$k_T^j\left(s^{T-1}\right) = p_{T-1}^j\left(s^{T-1}\right). \tag{45}$$

In turn, this equation, together with (43), implies that:

$$\begin{aligned} & d_{T-1}^{j}\left(s^{T-1}\right)+p_{T-1}^{j}\left(s^{T-1}\right) \\ = & F\left(k_{T-1}^{j}\left(s^{T-2}\right), l_{T-1}^{j}\left(s^{T-1}\right); z_{T-1}\left(s^{T-1}\right)\right)-w_{T-1}\left(s^{T-1}\right) l_{T-1}^{j}\left(s^{T-1}\right)+(1-\delta) k_{T-1}^{j}\left(s^{T-2}\right) \\ = & k_{T-1}^{j}\left(s^{T-2}\right) g_{d}\left(w_{T-1}\left(s^{T-1}\right), z_{T-1}\left(s^{T-1}\right)\right), \end{aligned} \tag{46}$$

where the second equality follows from the constant returns to scale assumption. At the beginning of period $T-2$, a shareholder of firm j would want to choose $k_{T-1}^{j}\left(s^{T-2}\right)$ in order to maximize his value function. This gives rise to the first order condition:

$$\begin{aligned} & \theta_{jT-2}^{i}\left(s^{T-3}\right)\left(\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}+\frac{\partial d_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}\right) \\ & -\theta_{jT-1}^{i}\left(s^{T-2}\right)\left(\frac{\partial p_{T-2}^{j}\left(s^{T-2}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}-\sum_{s^{T-1} \mid s^{T-2}} m_{T-2}^{i}\left(s^{T-1}\right) \frac{\partial\left(d_{T-1}^{j}\left(s^{T-1}\right)+p_{T-1}^{j}\left(s^{T-1}\right)\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}\right) \\ & -\sum_{s^{T-1} \mid s^{T-2}} \theta_{jT}^{i}\left(s^{T-1}\right) m_{T-2}^{i}\left(s^{T-1}\right) \frac{\partial p_{T-1}^{j}\left(s^{T-1}\right)}{\partial k_{T-1}^{j}\left(s^{T-2}\right)}\left(1-\frac{\partial k_{T}^{j}\left(s^{T-1}\right)}{\partial p_{T-1}^{j}\left(s^{T-1}\right)} \sum_{s^{T} \mid s^{T-1}} \frac{m_{T-2}^{i}\left(s^{T}\right)}{m_{T-2}^{i}\left(s^{T-1}\right)} \frac{\partial d_{T}^{j}\left(s^{T}\right)}{\partial k_{T}^{j}\left(s^{T-1}\right)}\right)=0. \end{aligned} \tag{47}$$

The first line of this equation captures the effect of a higher $k_{T-1}^{j}\left(s^{T-2}\right)$ on the net stock market value of the firm for an initial shareholder i at time $T-2$. The second line reflects the effect of a higher $k_{T-1}^{j}\left(s^{T-2}\right)$ on the expected discounted value, as perceived by agent i, of the firm's dividend and stock value at $T-1$. The third line captures the effect of a higher $k_{T-1}^{j}\left(s^{T-2}\right)$ on the expected market value of the firm at $T-1$ and dividends at T.

Using the arguments already developed before, it is possible to simplify equation (47) considerably. First, by equation (44):

$$\frac{\partial d_{T}^{j}\left(s^{T}\right)}{\partial k_{T}^{j}\left(s^{T-1}\right)}=\frac{d_{T}^{j}\left(s^{T}\right)}{k_{T}^{j}\left(s^{T-1}\right)}$$

In addition, equation (45) implies that:

$$\frac{\partial p_{T-1}^{j}\left(s^{T-1}\right)}{\partial k_{T}^{j}\left(s^{T-1}\right)}=\frac{p_{T-1}^{j}\left(s^{T-1}\right)}{k_{T}^{j}\left(s^{T-1}\right)}.$$

Given this, the first order condition (42) evaluated at $T-1$ implies that:

$$\frac{\partial p_{T-1}^{j}\left(s^{T-1}\right)}{\partial k_{T}^{j}\left(s^{T-1}\right)}-\sum_{s^{T} \mid s^{T-1}} m_{T-1}^{i}\left(s^{T}\right) \frac{\partial d_{T}^{j}\left(s^{T}\right)}{\partial k_{T}^{j}\left(s^{T-1}\right)}=0. \tag{48}$$

Using the fact that $m_{T-1}^{i}\left(s^{T}\right)=\frac{m_{T-2}^{i}\left(s^{T}\right)}{m_{T-2}^{i}\left(s^{T-1}\right)}$, equation (48) implies that the last term in (47) cancels out,

and we can therefore rewrite it as follows:

$$\theta^i_{jT-2}\left(s^{T-3}\right)\left(\frac{\partial p^j_{T-2}\left(s^{T-2}\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}+\frac{\partial d^j_{T-2}\left(s^{T-2}\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}\right) \tag{49}$$

$$-\theta^i_{jT-1}\left(s^{T-2}\right)\left(\frac{\partial p^j_{T-2}\left(s^{T-2}\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}-\sum_{s^{T-1}|s^{T-2}} m^i_{T-2}\left(s^{T-1}\right)\frac{\partial\left(d^j_{T-1}\left(s^{T-1}\right)+p^j_{T-1}\left(s^{T-1}\right)\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}\right)=0.$$

In addition, using the same arguments as the ones in section 3.1, notice that (46) implies the no-arbitrage condition:

$$\frac{d^j_{T-1}\left(s^{T-1}\right)+p^j_{T-1}\left(s^{T-1}\right)}{p^j_{T-2}\left(s^{T-2}\right)}=\frac{d^{j'}_{T-1}\left(s^{T-1}\right)+p^{j'}_{T-1}\left(s^{T-1}\right)}{p^{j'}_{T-2}\left(s^{T-2}\right)} \text{ for all } j,j',s^{T-2},s^{T-1}.$$

It therefore follows that:

$$\frac{\partial p^j_{T-2}\left(s^{T-2}\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}=\frac{p^j_{T-2}\left(s^{T-2}\right)}{k^j_{T-1}\left(s^{T-2}\right)}.$$

Replacing this into (49) and using the first order condition (42) for holding stocks evaluated at $T-2$ yields:

$$\frac{\partial p^j_{T-2}\left(s^{T-2}\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}-\sum_{s^{T-1}|s^{T-2}} m^i_{T-2}\left(s^{T-1}\right)\frac{\partial\left(d^j_{T-1}\left(s^{T-1}\right)+p^j_{T-1}\left(s^{T-1}\right)\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}=0.$$

Finally, since $\frac{\partial d^j_{T-2}\left(s^{T-2}\right)}{\partial k^j_{T-1}\left(s^{T-2}\right)}=-1$ any shareholder i of firm j in period $T-2$ will want to set:

$$k^j_{T-1}\left(s^{T-2}\right)=p^j_{T-2}\left(s^{T-2}\right).$$

In other words, all initial shareholders of firm j in period $T-2$ will also unanimously agree to set the level of capital to be used in the production at $T-1$ equal to the stock price of the firm in $T-2$. Using the same logic, this unanimity result can be generalized to any period $t<T$ until the initial period $t=0$.

Consider now an infinite horizon economy. When $T\to\infty$, there is no final period that can be used as a starting point for the backward induction type of argument illustrated above. On the other hand, the definition of dividends in (43) and the assumption of constant returns to scale in production still imply that, for all s^t:

$$d^j_t\left(s^t\right)+k^j_{t+1}\left(s^t\right)=k^j_t\left(s^{t-1}\right)g_d\left(w_t\left(s^t\right),z_t\left(s^t\right)\right),$$

where the lack of a terminal point prevents us from directly replacing $p^j_t\left(s^t\right)$ instead of $k^j_{t+1}\left(s^t\right)$ on the left hand side of this equation. On the other hand, we can guess that $p^j_t\left(s^t\right)=k^j_{t+1}\left(s^t\right)$ and then verify that this guess is valid. Note that showing that the guess is valid amounts to showing that there is unanimity among initial shareholders regarding the investment decision of every firm. ■

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