Fundamental Disagreement

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Disagreement About Future Economic Outcomes

- Observed in every survey of financial analysts, households, professional forecasters, FOMC members...

- At odds with full information rational expectation setup.

- Key in models with info. frictions / heterogenous beliefs.
  
  
  - Finance: Scheinkman-Xiong (2003), Nimark (2009), Burnside-Eichenbaum-Rebelo (2012) ...

- Are empirical properties of disagreement informative about such models?
This Paper

- New facts related to the term structure of disagreement.
  - People disagree about fundamentals (long-horizon forecasts).

- Introduce a class of expectation models to match the facts.
  - Imperfect info. / Uncertainty about the long-run / Multivariate.

- Use macro and survey data to calibrate the model.
  - Reproduce most of the new facts.
  - Informative about perceived macro-relationships (monetary policy).
The Blue Chip Financial Forecasts Survey

• \( \sim 50 \) professional forecasters.

• We look at forecasts for RGDP growth \((g)\), CPI inflation \((\pi)\), FFR \((i)\).

• Sample period is 1986:Q1-2013:Q2.

• For 1Q, 2Q, 3Q, 4Q: observe individual forecasts.

• For 2Y, 3Y, 4Y, 5Y and long-term (6-to-11Y): observe average forecasts, top 10 average forecasts, and bottom 10 average forecasts.

• Our measure of\textit{ disagreement}: top 10 average — bot 10 average.
The Term Structure of Disagreement in the BCFF

Output
Inflation
Federal Funds Rate

Q1  Q2  Q3  Q4  Y2  Y3  Y4  Y5  Y6−11
0
0.5
1
1.5
2
2.5
3

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The Time Series of Long Run Disagreement

![Time Series of Long Run Disagreement](image-url)
• **True state** \( z = \{g, \pi, i\} \) where

\[
\begin{align*}
  z_t &= (I - \Phi) \mu_t + \Phi z_{t-1} + v_t^z, \\
  \mu_t &= \mu_{t-1} + v_t^\mu,
\end{align*}
\]

with \( v_t^z \sim iid \ N(0, \Sigma^z) \) and \( v_t^\mu \sim iid \ N(0, \Sigma^\mu) \).

• **Parameters**: \( \theta = (\Phi, \Sigma^z, \Sigma^\mu) \)
Model

Information Friction: Noisy Information

- Forecaster $j$ observes:

$$y_{jt} = z_t + \eta_{jt}$$

with $\eta_{jt} \sim iid \ N(0, \Sigma^\eta)$, $\Sigma^\eta$ diagonal.

- Individual $j$’s optimal forecast computed using the Kalman filter.

- Model parameters: $(\theta, \Sigma^\eta)$.

- Disagreement driven by variance of observation errors $\Sigma^\eta$. 
Model

Information Friction: Sticky Information

- At each date, a forecaster \( j \) observes \( k^{th} \) element of \( y_t \) with a fixed probability \( \lambda_k \); otherwise sticks to latest observation.

- Individual \( j \)'s optimal forecast computed using the Kalman filter with missing observations.

- Same number of parameters as in noisy info with \( \lambda \)'s instead of \( \Sigma^{\eta} \).

- Generate time variance of disagreement (\( \neq \) noisy information).
Calibration via Penalized MLE

Principle

• Can we find \((\theta, \Sigma^n) / (\theta, \lambda)\) consistent with the data?

• Rely on (i) realizations \(Y = \{GDP, INF, FFR\}\) and (ii) moments \(S = \{\text{avg. forecast, disag}\}\) observed in surveys.

• We minimize the Likelihood associated to true state + ...

• ... a penalty function measuring the distance between model implied moments and their survey data counterpart.
Calibration in Practice

- We target 15 moments:
  - Std-dev of consensus forecasts for Q1, Q4, Y2 and Y6-11.
  - Disagreement about Q1 forecasts only.

- Various penalty parameters $\alpha = 1, \ldots, 50$.

- Simulate $R = 100$ histories of shocks $\epsilon_t$ and observation noises $\eta^i_t$ with $T = 120$ (nb of dates) and $N = 50$ (nb of forecasters).

- Sample: realizations 1955Q1-2013Q2; survey 1986Q1-2013Q2.
Summary of Parameter Estimates

- True state parameters ($\theta$) robust to type of info. friction.

- Long-run vol. ($\Sigma^\mu$) much lower than short-run vol. ($\Sigma^z$).

- FFR is perfectly observed:
  - *Noisy*: observation error ($\Sigma_\eta$) for FFR is zero.
  - *Sticky*: probability of observing FFR ($\lambda_i$) is one.

- Quantifying information frictions:
  - *Noisy*: observation errors on GDP roughly twice as for CPI.
  - *Sticky*: avg. proba. of updating $g$ or $\pi$ is $\simeq 4Q$ ($\lambda = 0.26$).
This figure displays the model-implied (time) average of disagreement across different horizons for the generalized noisy information model (dark blue) and the generalized sticky information model (light blue) calibrated with $\alpha = 50$ along with the Blue Chip Financial Forecasts survey (red). Open circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, \ldots, S_T)$.
Disagreement and Consensus Volatility
Noisy

The first column displays the model-implied disagreement for the generalized noisy information model calibrated with $\alpha = 50$ (blue) and the noisy information model without shifting endpoints calibrated with $\alpha = 50$ (green) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open white circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, \ldots, S_T)$ for the model without shifting endpoints. Open white and light blue circles designate survey moments used to form the penalization term for the generalized noisy information model. Model-implied 95% confidence intervals for the model with and without shifting endpoints are designated by shaded regions and dotted lines, respectively.
The first column displays the model-implied (time) variance of disagreement for the generalized noisy information model calibrated with $\alpha = 50$ (blue) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding correlation of disagreement between variables. Model-implied 95% confidence intervals are designated by shaded regions.
The first column displays the model-implied (time) variance of disagreement for the generalized sticky information model calibrated with $\alpha = 50$ (blue) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding correlation of disagreement between variables. Model-implied 95% confidence intervals are designated by shaded regions. Results are based on 2,500 simulations.

**Figure 8: Second Moments of Disagreement**

**Sticky Information Model**

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Y2</th>
<th>Y3</th>
<th>Y4</th>
<th>Y5</th>
<th>Y6−11</th>
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<tbody>
<tr>
<td><strong>Real Output Growth</strong></td>
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<td><strong>Real Output Growth &amp; CPI Inflation</strong></td>
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<tr>
<td><strong>Real Output Growth &amp; Federal Funds Rate</strong></td>
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<td><strong>Federal Funds Rate</strong></td>
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</table>
Role of Key Ingredients

- Imperfect information + permanent and transitory components:
  - Generate fundamental disagreement.
  - Don’t need asymmetric agents with different models / immutable priors / signal-to-noise ratios.
    \[ \Rightarrow \text{Appealing since hard to find “super forecaster” in the data.} \]

- Multivariate model:
  - Explain disagreement about future FFR even though perfectly observed.
  - Univariate version of our model cannot generate upward-sloping disagreement unless \( \sigma_\mu > \sigma_z \).
Disagreement about FFR and the Taylor Rule

- Generate individual FFR forecasts from a Taylor rule

\[ i_t = \rho \cdot i_{t-1} + (1 - \rho) \cdot i^*_t + \epsilon_t \]

\[ i^*_t = \bar{i}_t + \varphi_\pi \cdot (\pi_t - \bar{\pi}_t) + \varphi_g \cdot (g_t - \bar{g}_t) \]

- Find Taylor rule parameters giving best fit of reduced form model disagreement for FFR.

- Compare with various parametric restrictions.

  - Std Taylor rule parameters: \( \tilde{\rho} = 0.9, \tilde{\varphi}_\pi = 2, \tilde{\varphi}_g = 0.50. \)
‘Standard’ Taylor Rule

Data Model-implied Rule
Standard Rule
Standard Rule with $\rho = 0$
Role of Uncertainty about the Long-Run

Figure 6: Monetary Policy Rules
Noisy Information Model

This figure shows the results of the analysis discussed in Section 4.1.1. The top chart displays model-implied disagreement for different values of $(\rho, \phi^\pi, \phi^g)$ along with the Blue Chip Financial Forecasts survey (red). The "standard rule" is given by $(\rho, \phi^\pi, \phi^g) = (0.90, 2.0, 0.5)$. The bottom chart shows model-implied disagreement for different specifications of $\bar{i}_t$. Open circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, ... , S_T)$. 
Conclusion

• Present new facts about forecaster disagreement.
  • May help discriminate between models of expectation formation.

• Show that imperfect info models combined with permanent/transitory decomposition explains most of the facts for sound parameter values.
  • Minimal departure from REH: agents know and agree on true model/params.

• Disagreement informative about both degree of imperfect info and underlying DGPs.
  • Help identifying parameters driving unobserved components.
  • Informative about perceived structural relationships.
Calibration via Penalized MLE
Details (1/2)

• Consider realizations as signals about $z_t$: $\mathcal{Y}_t = z_t + \tilde{\eta}_t$ with $\tilde{\eta}_t \sim iid \ N(0, \tilde{\Sigma}^\eta)$.

• $-\mathcal{L} \left( \mathcal{Y}_1, \ldots, \mathcal{Y}_T; \theta, \tilde{\Sigma}^\eta \right) =$ likelihood obtained with Kalman filter.
• Given \((\theta, \Sigma^\eta)\) we generate individual forecasts \(f_{it}^h\) and compare some associated moments with their survey data counterparts \(S_t\).

• \(P(S_1, \cdots, S_T; \theta, \Sigma^\eta) = \text{distance between model implied expectation moments and their survey data counterpart.}\)

• We minimize the penalized likelihood:

\[
C\left(\theta, \Sigma^\eta, \tilde{\Sigma}^\eta\right) = \mathcal{L}\left(Y_1, \cdots, Y_T; \theta, \tilde{\Sigma}^\eta\right) + \alpha P\left(S_1, \cdots, S_T; \theta, \Sigma^\eta\right).
\]
## Noisy Information Model

### Table 1: Results of Calibration for $\alpha = 50$

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\Sigma^z$</th>
<th>$\sqrt{\text{diag}(\Sigma^\eta)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 0.378 &amp; -0.503 &amp; -0.153 \ 0.125 &amp; 0.974 &amp; -0.033 \ 0.147 &amp; 0.104 &amp; 0.924 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 3.419 &amp; -0.019 &amp; 0.561 \ -0.019 &amp; 0.645 &amp; 0.365 \ 0.561 &amp; 0.365 &amp; 0.632 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 2.592 \ 1.429 \ 0.000 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

| $|\text{eig}(\Phi)|$ | $\Sigma^\mu$ | $\sqrt{\text{diag}(\Sigma^\eta)}$ |
|-------------------------|--------------|------------------------------------|
| $\begin{bmatrix} 0.920 \end{bmatrix}$ | $\begin{bmatrix} 0.008 & 0.014 & 0.026 \end{bmatrix}$ | $\begin{bmatrix} 4.317 \end{bmatrix}$ |
| $0.711$ | $\begin{bmatrix} 0.014 & 0.024 & 0.045 \end{bmatrix}$ | $\begin{bmatrix} 2.731 \end{bmatrix}$ |
| $0.646$ | $\begin{bmatrix} 0.026 & 0.045 & 0.085 \end{bmatrix}$ | $\begin{bmatrix} 0.000 \end{bmatrix}$ |
### Table 1: Results of Calibration for $\alpha = 50$

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<tr>
<th>$\Phi$</th>
<th>$\Sigma^z$</th>
<th>$\text{sqrt(diag}(\tilde{\Sigma}^\eta))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 0.392 &amp; -0.478 &amp; -0.142 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 3.736 &amp; -0.065 &amp; 0.564 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 2.586 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\begin{bmatrix} 0.122 &amp; 0.939 &amp; -0.024 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -0.065 &amp; 0.911 &amp; 0.347 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1.355 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\begin{bmatrix} 0.146 &amp; 0.087 &amp; 0.931 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.564 &amp; 0.347 &amp; 0.635 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.000 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

| $|\text{eig}(\Phi)|$ | $\Sigma^\mu$ | $\lambda$ |
|-----------------|---------|--------|
| $\begin{bmatrix} 0.920 \end{bmatrix}$ | $\begin{bmatrix} 0.007 & 0.012 & 0.022 \end{bmatrix}$ | $\begin{bmatrix} 0.260 \end{bmatrix}$ |
| $\begin{bmatrix} 0.674 \end{bmatrix}$ | $\begin{bmatrix} 0.012 & 0.021 & 0.039 \end{bmatrix}$ | $\begin{bmatrix} 0.260 \end{bmatrix}$ |
| $\begin{bmatrix} 0.674 \end{bmatrix}$ | $\begin{bmatrix} 0.022 & 0.039 & 0.073 \end{bmatrix}$ | $\begin{bmatrix} 1.000 \end{bmatrix}$ |