A Multivariate Analysis of Forecast Disagreement: Confronting Models of Disagreement with Survey Data

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Motivation

Disagreement about Inflation Outlook (EA) as Measured from SPF Data (2015q2)
Motivation

- Disagreement about future values of macroeconomic variables is high
- So far, only univariate disagreement has been analyzed empirically
- Models of heterogeneous expectation formation are calibrated/evaluated based on information about univariate/marginal disagreement

What properties does multivariate forecast disagreement have?
Which implications does this have for theoretical models of heterogeneous expectation formation?
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A Stylized Scatter of Forecasts
Contribution of Paper

- Documentation of multivariate disagreement in macroeconomic expectations
- In particular, two new stylized facts:
  - Forecasters disagree about the relationship between different macroeconomic variables
  - Different forecasters tend to have different levels of disagreement
- Discussion of implications for models of heterogeneous expectation formation:
  - Along which dimensions do models fail?
  - Modifications of models to fit observed multivariate disagreement better
  - Suggestion how multivariate aspects can be taken into account during estimation of models

What I do not:

- Abstracting from behavioral issues (e.g., incentives or interaction between forecasters)
- Not yet considering option of model uncertainty
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Related Literature

- **Empirical analyses of forecast disagreement** (Mankiw et al., 2003; Lahiri and Sheng, 2008; Patton and Timmermann, 2010; Rich and Tracy, 2010; Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013; Andrade et al., 2014; Dovern et al., 2012)

- **Empirical studies with multivariate aspects** (Banterngansa and McCracken, 2009; Dräger and Lamla, 2015)

- **Reduced-form models of heterogeneous expectations:**
  - **Noisy information models** (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013; Andrade et al., 2014)
  - **Models with agent-specific priors** (Lahiri and Sheng, 2008; Patton and Timmermann, 2010)
  - **Sticky information models** (Mankiw et al., 2003; Coibion and Gorodnichenko, 2012)

- **DSGE models with heterogeneous expectation formation:**
  - **Models with constant levels of disagreement** (Nimark, 2008; Maćkowiak and Wiederholt, 2009; Lorenzoni, 2009)
  - **Sticky information DSGE models** (Mankiw and Reis, 2002)
  - **“Man-bites-dog” model** (Nimark, 2014)
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Agenda

1 Motivation
2 Contribution of Paper
3 Related Literature
4 Measurement of Disagreement
5 Data and Stylized Facts
6 Implications for Models of Disagreement
7 Conclusion
How to Measure Multivariate Disagreement

- Measures proposed by Banterghansa and McCracken (2009)
- Basic inputs: vector-valued forecasts \((y_{i,t+h|t})\) and their (cross-sectional) covariance

\[
S_{t+h|t} = N_{t+h|t}^{-1} \sum_{i=1}^{N_{t+h|t}} (y_{i,t+h|t} - \bar{y}_{t+h|t})(y_{i,t+h|t} - \bar{y}_{t+h|t})'
\]

- Overall/Multivariate disagreement (absolute):

\[
D_{t+h|t} = \sqrt{\text{det}(S_{t+h|t})}
\]

- Individual disagreement (relative):

\[
d_{i,t+h|t} = \sqrt{(y_{i,t+h|t} - \bar{y}_{t+h|t})' S_{t+h|t}^{-1} (y_{i,t+h|t} - \bar{y}_{t+h|t})}
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Data

- **Survey of Professional Forecasters for Euro area:**
  - Forecasts from research institutes, banks, and large corporations
  - Sample: 1999q1–2015q1
  - 1-year-ahead forecasts \((h = 4)\) and long-term forecasts \((h = 20)\)
  - Forecasts for growth, inflation and the unemployment rate

- **Survey of Professional Forecasters for United States:**
  - Longer sample: 1981q3–2014q4
  - 1-quarter-ahead forecasts \((h = 1)\) and 1-year-ahead forecasts \((h = 4)\)
  - Robustness check with additional forecasts for short- and long-term interest rates and industrial production

- **Consensus Economics survey data for twelve advanced economies:**
  - Fixed-event forecasts (annual averages)
  - Concentrate on forecasts with \(h = 13\)
  - Sample: 1990m1–2015m4
Multivariate Disagreement ($D_{t+h|t}$) over Time

Figure 3: Multivariate Disagreement among SPF Participants

(a) US-SPF

(b) EA-SPF

Notes:

Estimates are computed using equation (2).

The higher disagreement about the correlations between variables is reflected in lower cross-sectional correlations between pairs of long-run forecast relative to correlations based on short-run forecasts.

16 Disagreement about the long-run inflation rate and the long-run growth rate is, on average, lower than that about the nearer term movements of those variables. Finally, there is not much difference in terms of the level of disagreement between the nowcasts from the US-SPF (plot for US-SPF, red line) and disagreement about the one-year-ahead outlook (red line).

What elements of $S_{t+h|t}$ are driving most of the fluctuations in multivariate disagreements over time? Is it variations in the level of disagreement about future business-cycle scenarios (elements on the main diagonal)? Or is it time-variation in the level of disagreement about the appropriate model of the economy (correlations on the off-diagonal elements)? Figure 4 shows—for each survey and forecast horizon—the time series of each of the elements in $S_{t+h|t}$. It is evident that medium-to-low-frequency movements in multivariate disagreement are driven primarily by movements in the cross-sectional standard deviation for each of the three variables while fluctuations of the implied bivariate correlations are of higher frequency. The impression is confirmed by a regression of $D_{t+h|t}$ on the three standard deviations in one case and on the three correlations in the other case. With respect to the EA-SPF, at the short-term horizon the univariate standard deviations alone explain 85% of the variation of $D_{t+4|t}$ while the correlations...
... a Closer Look at Individual Components

United States: Short-Term ($h = 4$)
... a Closer Look at Individual Components

Unconditional Averages

<table>
<thead>
<tr>
<th></th>
<th>Euro area</th>
<th></th>
<th>US</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 4$</td>
<td>$h = 20$</td>
<td>$h = 1$</td>
<td>$h = 4$</td>
</tr>
<tr>
<td>$sd(g)$</td>
<td>0.36</td>
<td>0.27</td>
<td>1.07</td>
<td>0.75</td>
</tr>
<tr>
<td>$sd(\pi)$</td>
<td>0.27</td>
<td>0.22</td>
<td>0.88</td>
<td>0.62</td>
</tr>
<tr>
<td>$sd(u)$</td>
<td>0.31</td>
<td>0.65</td>
<td>0.12</td>
<td>0.31</td>
</tr>
<tr>
<td>$Corr(g, \pi)$</td>
<td>0.16</td>
<td>0.10</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>$Corr(g, u)$</td>
<td>$-0.25$</td>
<td>$-0.17$</td>
<td>$-0.17$</td>
<td>$-0.42$</td>
</tr>
<tr>
<td>$Corr(\pi, u)$</td>
<td>$-0.10$</td>
<td>$-0.02$</td>
<td>$-0.07$</td>
<td>$-0.14$</td>
</tr>
</tbody>
</table>

**Notes:** The values shown are based on the averages for each element of $S_{t+h|t}$ over the full sample. These averages are computed as $\bar{s}_{ij,h} = \frac{1}{T} \sum_{t=1}^{T} s_{t+h|t}^{ij}$, where $s_{t+h|t}^{ij}$ denotes the element of $S_{t+h|t}$ corresponding to the $i^{th}$ row and the $j^{th}$ column. $sd(\bullet)$ is computed as the square root of the main diagonal elements. $Corr(\bullet, \bullet)$ is computed as the correlations that correspond to the off-diagonal elements.
Assessment of Rank Persistence (based on $d_{i,t+h|t}$)

Visualizing Persistence ...
Assessment of Rank Persistence

Measurement

- Kendall’s coefficient of concordance: Formal assessment of rank persistence
- \( W=0 \): No persistence
- \( W=1 \): Perfectly stable ranking
Assessment of Rank Persistence

**EA-SPF/US-SPF**

<table>
<thead>
<tr>
<th></th>
<th>Euro area</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 4$</td>
<td>$h = 20$</td>
</tr>
<tr>
<td>Since 1981q3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Since 1999q1</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>1999q1–2008q2</td>
<td>0.16</td>
<td>0.31</td>
</tr>
<tr>
<td>Since 2008q3</td>
<td>0.22</td>
<td>0.37</td>
</tr>
</tbody>
</table>

For comparison (since 1999q1):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Point forecasts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>0.13</td>
<td>0.30</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.19</td>
<td>0.44</td>
</tr>
<tr>
<td>$u$</td>
<td>0.14</td>
<td>0.27</td>
</tr>
</tbody>
</table>

|                  |          |            |
| Forecast uncertainty |      |            |
| $g$              | 0.56     | 0.65       | -         | -         |
| $\pi$            | 0.32     | 0.62       | -         | -         |
| $u$              | 0.54     | 0.56       | -         | -         |

**Notes:** The table shows values of Kendall’s coefficient of concordance. The statistics have been calculated by accounting for missing values as proposed in Boero et al. (2014).
Assessment of Rank Persistence
Consensus Forecast Data

<table>
<thead>
<tr>
<th>Country</th>
<th>Full Sample</th>
<th>Since 1999</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 1$</td>
<td>$h = 13$</td>
</tr>
<tr>
<td>USA</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>Japan</td>
<td>0.14</td>
<td>0.26</td>
</tr>
<tr>
<td>Germany</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>France</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>UK</td>
<td>0.30</td>
<td>0.36</td>
</tr>
<tr>
<td>Italy</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
<td>Canada</td>
<td>0.26</td>
<td>0.22</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>Norway</td>
<td>0.21</td>
<td>0.22</td>
</tr>
<tr>
<td>Spain</td>
<td>0.38</td>
<td>0.30</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.26</td>
<td>0.20</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.24</strong></td>
<td><strong>0.24</strong></td>
</tr>
</tbody>
</table>

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Model structure:

\begin{align*}
y_t &= (I_k - \Phi)\mu_t + \Phi y_{t-1} + \nu_t^y \\
\mu_t &= \mu_{t-1} + \nu_t^\mu \\
y_{it} &= y_t + \eta_{it} \tag{3}
\end{align*}

with $\nu_t^y \sim \mathcal{N}(0, \Sigma^y)$, $\nu_t^\mu \sim \mathcal{N}(0, \Sigma^\mu)$, and $\eta_{it} \sim \mathcal{N}(0, \Sigma^\eta)$

- Agents form optimal forecasts based on Kalman filtering
- Model features not in line with new stylized facts:
  - No persistence of disagreement ranking
  - Relatively high cross-sectional correlation of forecasts
Implications for Models of Disagreement

Andrade et al. (2014): Effects of Introducing True Heterogeneity

- New feature: **Heterogenous signal-to-noise ratios:**

\[ \eta_{it} \sim \mathcal{N}(0, \delta_i \Sigma^\eta) \quad \text{with} \quad \delta_i \sim \mathcal{G}(1, \sigma^2_\delta) \]  (4)
Implications for Models of Disagreement

Nimark (2014): Structure of DSGE Model

- Model structure:
  - Island economy as in Lorenzoni (2009)
  - Agents observe island-specific productivity \((a_{j,t} = a_t + \varepsilon_{j,t})\), island-specific demand, and a sub-set of goods prices
  - Additional “man-bites-dog” signal \((z_t^a = a_t + \eta_t)\) about common productivity shock
    \(\Rightarrow\) Probability of this signal rises with (absolute) size of productivity shock
  - Fig. 1 of Nimark (2014)

- Main mechanism to generate variation in disagreement:
  - Different interpretation of “man-bites-dog” signals due to different priors across agents
  - Level of disagreement depends on frequency of occurrence of these signals (and dispersion of priors)

- Model is estimated and evaluated using information about univariate disagreement only
Implications for Models of Disagreement

Nimark (2014): Implied Cross-Sectional Correlation between Different Forecasts

Fig. 6 of Nimark (2014)
Summary

- Interesting features of multivariate disagreement that have previously been ignored in the literature
- They suggest that agents are truly heterogeneous (beyond being subject to different information flows)
- Models of heterogeneous expectation formation have neglected this dimension so far
- How can models be adapted?
  - Heterogeneous signal-to-noise ratios
  - Learning models? Models with different types of forecasters?
  - At least, multivariate information should be used for estimation
- What can we learn in a broader context?
  - Observed correlations contain information about (perceived) relevance of different types of macroeconomic shocks
  - Heterogeneity of agents might not only be important in terms of the current state-vector (e.g., credit constraints) but also in terms of expectations
Background on Multivariate Measure

$D_{t+h,t}$ as a Function of One Variance
Background on Multivariate Measure

$D_{t+h,t}$ as a Function of Variances

![Graph showing the relationship between $\frac{A(1,1)}{A(i,i)}$ and $\det(S)$.

- Black line: Varying $A(1,1)$
- Blue line: Varying all $A(i,i)$

The graph illustrates how the determinant of $S$, $\det(S)$, changes as $\frac{A(1,1)}{A(i,i)}$ varies, with different scenarios for varying $A(1,1)$ or all $A(i,i)$.}

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Multivariate Disagreement
SEM: Juli 22–24, 2015
Background on Multivariate Measure

$D_{t+h,t}$ as a Function of Correlations

$\rho(1,2)/\rho(i,j)$

$\text{det}(S)$

0.00 0.25 0.50 0.75 1.00

0.00 0.25 0.50 0.75 1.00

Varying $\rho(1,2)$
Varying all $\rho(i,j)$
Occurrence of “Man-Bites-Dog” Signal

In the introduction, the Movers segment on Bloomberg Television was given as an example of man-bites-dog news reporting. It seems unlikely that individual events, perhaps with the exception of the 1987 stock market crash, directly cause what is identified as man-bites-dog episodes by the model. An alternative interpretation of these episodes at the macro-level is that, at certain times, the economy for various reasons becomes one of the main sources of news stories, dominating network news and newspaper front pages. According to the man-bites-dog dictum, this should be more likely to happen when macroeconomic developments are in some sense unusual.

One way to check more directly whether what the model interprets as man-bites-dog events are indeed related to the intensity of news coverage is to compare the posterior probabilities that $s_t = 1$ with the fraction of respondents in the Michigan Survey that have heard either unfavorable or favorable news “during the last months.” This data was not used in estimation and thus provides an independent check on how reasonable the estimates produced by the model are. The bottom panel of Figure 4 contains an index of the number of respondents that have heard any news about the economy (solid line). The bottom panel also includes indices for the fraction of respondents that have heard any unfavorable (dashed line) or any favorable (dotted line) news about the economy. 7

7 The indices are constructed by computing the fraction of survey respondents that have heard either favorable or unfavorable news. The fractions are re-normalized to have a minimum of 0 and a maximum of 1 to make them comparable.

![Graph](image-url)
Intuition behind “Man-Bites-Dog” Signal

The distribution of $x$ conditional on $y$ being available can be backed out from $p(x)$ and the conditional probability of observing $y$ by using Bayes' rule.

$\frac{p(S=1|x)}{p(S=1)} = p(x | S=1) p(x)$

This is illustrated by the dotted line in Figure 1.

Bayes' rule can also be used to plot the conditional distribution of $x$ when $y$ is not available. From the inequality, we know that it must have less probability mass in the tails than $p(x)$ (not shown). A man-bites-dog information structure thus introduces a form of conditional heteroscedasticity in the distribution of $x$. From the agents' perspective, it is as if the latent variable $x$ is drawn from a more dispersed distribution when the signal $y$ is available compared to when it is not. It is important to note that this is true even when the signal content of $y$ is not specifically about the variance, or second moment, of $x$. It is also important to keep in mind that the indicator variable $S$ is a modeling device that we use to describe the event that the signal $y$ is available and not a separate signal that agents can observe directly and independently of $y$.

Figure 1. Unconditional Distribution of $x$, Conditional Probability of Observing the Signal $y$, and the Implied Conditional Distribution of $x$
Implications for Models of Disagreement

Nimark (2014): Suggestion to Take Multivariate Aspects into Account

- Modification of model to take multivariate dimension of disagreement into account
- Nimark (2014) assumes that forecasts are drawn from univariate distributions:

\[
\begin{align*}
  f_{t,\pi}^j &\sim N \left( \int E[\pi_{t+1} | \Omega_j, t] \, dj, \sigma^2_{f,\pi}(s^t) \right) \\
  f_{t,\Delta y}^j &\sim N \left( \int E[\Delta y_{t+1} | \Omega_j, t] \, dj, \sigma^2_{\Delta y}(s^t) \right)
\end{align*}
\]

- I suggest to use a bivariate distribution with covariance matrix:

\[
\Sigma_f(s^t) = \begin{bmatrix}
  \sigma^2_{f,\pi}(s^t) & \sigma_{f,\pi,f\Delta y}(s^t) \\
  \sigma_{f,\pi,f\Delta y}(s^t) & \sigma^2_{\Delta y}(s^t)
\end{bmatrix}
\]

- Reflect this also in the model’s LLF ⇒ more effective use of sample information:
  - Keep track of forecast pairs
  - Need not even throw away univariate forecasts
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\]

\[
f^j_{t,\Delta y} \sim \mathcal{N}\left(\int E[\Delta y_{t+1}|\Omega_j,t] \, dj, \sigma^2_{\Delta y}(s^t)\right)
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\[ f_{j,\Delta y}^t \sim \mathcal{N} \left( \int E[\Delta y_{t+1} | \Omega_j, t] \, dj, \sigma_{\Delta y}^2(s^t) \right) \]

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\[ \Sigma_f(s^t) = \begin{bmatrix} \sigma_{f,\pi}^2(s^t) & \sigma_{f,\pi,\Delta y}(s^t) \\ \sigma_{f,\pi,\Delta y}(s^t) & \sigma_{\Delta y}^2(s^t) \end{bmatrix} \]

- Reflect this also in the model’s LLF \( \Rightarrow \) more effective use of sample information:
  - Keep track of forecast pairs
  - Need not even throw away univariate forecasts