Does the US Government Hedge against Fiscal Risk?*

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ABSTRACT

We develop a method for measuring the amount of insurance the portfolio of government liabilities provides against fiscal shocks, and apply it to postwar US data. We define fiscal shocks as surprises in defense spending. Our results indicate that the US federal government is partially hedged against wars and other surprise increases in defense expenditures. Seven percent of the total cost of defense spending shocks in the postwar era was absorbed by lower real returns on the federal government's outstanding liabilities. More than half of this is due to reductions in expected future, rather than contemporaneous, holding returns on government debt. This implies that changes in US government's fiscal position help predict future bond returns. Our results also have implications for active management of government debt.

JEL Classifications: C5, E4, E6, G1, H6

Keywords: Fiscal shocks, defense spending, hedging, predictability, bond returns, debt management

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1. Introduction

In the normative models of fiscal policy, public debt serves an essential role. It provides a fiscal hedge against government spending shocks. Standard models in this literature feature a benevolent government that minimizes the excess burden of taxation by varying its debt returns. The extent to which it can do this is determined by the asset market structure it faces. In complete-market models, a decline in debt returns (or equivalently, a fall in the real value of the government's liabilities) absorbs the surprise increase in spending needs, allowing the government to maintain a constant excess burden of taxation. In incomplete-market models, however, interstate hedging of fiscal shocks is constrained, hence fiscal insurance through bond markets is limited.[1] Several authors have used the implications of these normative models and the empirical behavior of tax rates and debt levels to assess the incompleteness of debt markets.[2] The empirical evidence uncovered and documented in these papers suggests that debt markets are incomplete and hence do not provide *full* insurance against fiscal shocks. However, the prior literature does not quantify how much fiscal insurance the government does achieve through bond markets in practice.

This paper develops a method for measuring the amount of fiscal insurance the government's bond portfolio provides against its spending shocks and applies it to postwar US data. To quantify the degree of fiscal insurance, we make use of the government's intertemporal budget constraint, which is present in and consistent with most dynamic fiscal models. The government's budget constraint dictates that spending shocks must be financed through either an increase in the endogenous component of current and future surpluses or a reduction in expected current and future returns on the government's portfolio of liabilities. By log-linearizing the budget constraint, we isolate the response of returns to news about government spending. We define hedging to be the fraction of the variation in the cost of fiscal shocks absorbed by variation in debt returns. We find that adjustments to returns have absorbed 7% of the cost of spending shocks in the postwar era. In this sense, the US government has made significant use of its debt portfolio to hedge fiscal shocks and hence dampen variation in its surplus growth.

To quantify the degree of fiscal hedging, we proceed in three steps. The first step involves the log-linearization of the government's budget constraint which permits a tractable decomposition

[1] See Section 2 for references and a more thorough discussion of normative models.
[2] See, inter alia, Barro (1979), Marcet and Scott (2009) and Scott (2007).

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of the policy response to fiscal shocks into news about the *present discounted value (PDV)* of future surplus growth and news about the *PDV* of current and future debt returns. We argue that fiscal shocks can be identified as news to the *PDV* of current and future defense spending growth. In the second step, we use an unstructured VAR to obtain empirical measures of these news variables. Finally, we utilize the constructed news variables to estimate hedging betas that describe the response of expected returns to fiscal shocks.

Our results suggest that in the postwar period, innovations to real returns on government debt decrease by *forty-seven* basis points when innovations to defense spending growth increases by one percent. There are two components to this. The first is standard, ex ante hedging, achieved through return variations that are contemporaneous with fiscal news and the focus of much of the normative literature on optimal fiscal policy. We find that this sort of ex ante hedging typically makes a smaller contribution to the financing of fiscal shocks. The ex ante hedging beta (of current debt returns) is *-.17* in postwar US data. Second, the innovations to *future* debt returns decrease when news about higher defense spending growth is released. The ex post hedging beta (of news about future debt returns) is *-.30*. When defense spending growth innovations increase by one percent, the average real return investors expect to earn on government debt in the future decreases by thirty basis points. This latter result does not have an analogue in the normative literature where the focus is on the contemporaneous returns, but it is a robust feature of the data. The ex ante and ex post beta add up to a total hedging beta of *-.47*. The amount of fiscal hedging depends upon both the hedging betas and the average defense to total spending ratio. Over the postwar period, the latter has been 35%, implying that 7 percent of total defense spending risk is hedged. Evaluated at the lower end-of-sample defense to total spending ratio (2007.III) of 25%, that hedging fraction is closer to 10%.

Our empirical results have implications for active management of government debt. The initial hedging calculations are based on the response of *weighted average* returns on government debt to fiscal shocks. The weights are computed as the total market value of government debt with a particular maturity, divided by the total market value of all government liabilities. The maturity composition of the government's portfolio therefore affects these weighted returns. More importantly, it has a significant impact on their volatility.

We document the higher volatility in excess returns associated with long-term debt and measure the effect of the maturity composition of debt on the amount of fiscal hedging the US government can achieve. Our results indicate that if the response of the term structure to expenditure shocks is taken as given, the Treasury can better hedge against fiscal shocks by lengthening the maturity of its liabilities. More specifically, we find that if the government were to increase the average maturity of its outstanding liabilities to an average of 20 years, the ex post beta would jump to *-.60* and the ex ante beta would more than double to *-.49*. As a result, total hedging would amount to more than 15% of total expenditure risk. Although our results rely on a partial equilibrium assumption, they do suggest that long-term debt has an important fiscal role - superior hedging performance - that has not been documented empirically before.[3]

The robustness of our hedging results relies on the precision of the identification of fiscal shocks. There are potentially two caveats associated with using a VAR model to estimate these shocks. One is the possible failure of the VAR to time them correctly. The other is its potential failure to detect changes in *expected* defense expenditures that ultimately are not reflected in aggregate data. To address both issues, we augment our benchmark VAR to include information embedded in the stock returns of companies in the defense industry. Our logic is straightforward. In so far as defense companies' profits and dividends are tied to defense spending, defense stock return variables should respond contemporaneously to news about *perceived* future defense spending growth. The results from the augmented VAR confirm our intuition. Defense spending growth is much more precisely estimated and total expenditure risk hedged remains seven percent, consistent with our earlier results.

The paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 log-linearizes and decomposes the budget constraint, and Section 4 formally defines our hedging measure and expenditure shocks. Section 5 presents our benchmark VAR model and reports the empirical results from this benchmark case. Section 6 discusses the implications of our results for active debt management. Section 7 introduces the VAR model augmented with defense stock variables and reports the associated results. Section 8 concludes.

[3]In the normative fiscal literature, couple of papers display the use of long-term debt to hedge fiscal shocks. One is Lustig, Sleet and Yeltekin (2008), who show that the long-term debt helps the government smooth distortions from costly unanticipated inflation in a dynamic model of optimal fiscal and monetary policy with nominal rigidities, and nominal non-contingent debt of various maturities. The other is Angeletos (2002), who argues that if the maturity structure of public debt is carefully chosen ex ante, the ex post variation in the market value of outstanding long-term debt may offset the contemporaneous variation in the level of fiscal expenditure.

2. Literature

Budget constraints and predictability We follow the predictability literature in finance (e.g. Lettau and Ludvigson (2001)), Campbell's (1993) asset pricing analysis and Gourinchas and Rey's (2007) work on international financial adjustment and organize our thinking around a log-linearized budget constraint; in our case, the government's budget constraint. Campbell's focus is asset pricing, whereas Gourinchas and Rey's is international adjustment to large trade or asset imbalances. The issue of hedging exogenous shocks is absent from these papers whereas it is our central focus. In particular, we are interested in quantifying the role of fiscal insurance in stabilizing the US fiscal balance following expenditure shocks. We make use of the log-linearized government budget constraint to construct a measure of fiscal hedging.

Fiscal Insurance In a related paper, Fraglia, Marcet and Scott (FMS) (2008) investigate whether or not fiscal insurance plays a role in stabilizing the fiscal balance of a handful of OECD countries between 1970 and 2000. They propose a battery of tests, some based on the normative models, some based on fiscal accounting, to assess bond market incompleteness. They conclude that incompleteness of bond markets coupled with little variation in the term structure across time provide some evidence that fiscal insurance does not play a significant role in government's finances. We, on the other hand, use the government's budget constraint to construct a measure of hedging and directly estimate it. Our results, unlike FMS, suggest that although fiscal insurance through bond markets is limited, it is certainly non-negligible. There are other significant differences between our and FMS's approach, including the definition and identification of fiscal shocks, the sample time period, and the estimation of key inputs such as holding returns and market value of debt. Our sample period starts in 1946, whereas FMS sample period starts in 1970. We define fiscal shocks as *innovations* to defense spending, whereas they identify it by using a Cholesky decomposition on a VAR with debt/GDP, primary deficit/GDP and GDP growth as state variables. Finally, FMS approximate the market value and holding returns of debt by using average coupon rates and maturity dates, whereas we extract a discount function from bond price data and value each outstanding bond to construct the aggregate numbers.

Optimal tax literature The normative theory of fiscal policy provides perspective and motivation for our primary focus: quantifying the degree of fiscal insurance. Standard models in the normative literature feature a benevolent government that minimizes the welfare losses arising from variation in marginal tax rates over time and states. If the tax system is sufficiently constrained, then the government will wish to smooth inter-state marginal tax rates and the excess burden of taxation by varying the return it pays on its debt.[4,5] The extent to which it can do this is determined by the asset market structure it faces. In complete market models, there are no restrictions on the government's ability to hedge shocks through return variations. In the simplest versions of these models, fiscal variables such as taxes are functions of shocks only and inherit their statistical properties from these shocks.[6] At the other extreme, if the government can trade only one period real non-contingent debt, then interstate hedging is proscribed and optimal policy entails intertemporal rather than interstate smoothing of taxes and the excess burden. Tax rates and debt values then evolve according to (risk-adjusted) martingales; they exhibit a unit root component and are more persistent than the underlying shocks.[7] Intermediate cases in which fiscal hedging is possible, but costly, deliver intermediate results. In these, the government optimally responds to shocks with a mixture of interstate and intertemporal smoothing of taxes and the excess burden.[8]

Several contributors, beginning with Barro (1979), have used normative models of the sort described above to assess empirical fiscal policy. Early analysis found evidence of persistence in tax rates consistent with incomplete-market models.[9] More recent work by Scott (2007) and Marcet

[4]If the government has access to lump sum taxation, then Ricardian Equivalence implies that it need make no recourse to bond markets. If it can tax private assets without inducing any contemporaneous distortion, then asset taxation can substitute for variations in debt returns. Finally, if the government can flexibly adjust both consumption and income tax rates in response to shocks, then again debt is redundant as a fiscal hedge (see Correia, Nicolini and Teles (2008)). On the other hand, if the tax system is sticky or if the government is constrained to adjust income tax rates in the aftermath of shocks, then debt's essential role as a fiscal hedge is reinstated.

[5]Scott (2007) shows that when markets are complete, the government maintains the excess burden of taxation (the shadow value of the future primary surplus stream) at a constant level. Labor tax rates still vary to the extent that the compensated labor supply elasticity varies. However, these variations are typically dampened relative to an incomplete-market setting.

[6]In more elaborate versions with capital or habit formation, fiscal variables depend on other real state variables, but they are no more persistent than these variables.

[7]See Barro (1979) and Aiyagari, Marcet, Sargent and Seppala (2002).

[8]One example is Lustig, Sleet and Yeltekin (2008). There, a government trades non-contingent nominal debt of various maturities. Costly contemporaneous or expected future inflations allow it to hedge fiscal shocks. See also Siu (2004). Another example is Sleet (2004) who requires fiscal policy to satisfy incentive compatibility restrictions.

[9]See, for example, Sahaskul (1986), Bizer and Durlauf (1990) and Hess (1993). However, as Bohn (1998) and Scott (2007) point out, the unit root tests used in this literature have low power against the alternative of optimal policy in an environment with complete markets and persistent shocks.

and Scott (2009) has obtained and empirically assessed the implications of complete and incomplete market optimal policy models. These two papers provide further evidence of persistence in debt levels and tax rates relative to allocations, suggestive of incomplete-market models and hence limited access to fiscal insurance through bond markets.

Relative to these papers, we are, to the best of our knowledge, the first to use the government's budget constraint to directly quantify the degree of hedging and show that fluctuations in bond prices deliver a sizable degree of hedging for the US government in the postwar era. Our work complements the existing literature by suggesting that contemporaneous hedging of shocks is limited, although we make no attempt to ascribe this to market incompleteness per se. That is, we do not distinguish between an inability or an unwillingness to engage in ex ante hedging. On the other hand, our work indicates the relative importance of ex post hedging – variations in expected future returns play a significant role in financing shocks – that is ignored in the optimal tax literature.

3. Government Budget Constraint and Hedging

To quantify the extent to which the government is hedged against expenditure shocks, we use the government's intertemporal budget constraint. The dynamic period-by-period version of the government's budget constraint is given by:

$$B_{t+1} = R^b_{t+1}\left(B_t - S_t\right),$$

where B_t denotes the real market value of outstanding government debt inclusive of cash at the start of period t, $S_t = T_t - G_t$ denotes the federal government's real primary surplus: receipts T_t less expenditures G_t. R^b_{t+1} denotes the simple gross real return paid on the government's portfolio between t and $t+1$. This equation can be re-arranged to yield the following expression for the growth rate of government debt as a function of the return on this debt and the primary surplus to debt ratio:

$$\frac{B_{t+1}}{B_t} = R^b_{t+1}\left(1 - \frac{S_t}{B_t}\right). \tag{1}$$

Our goal is to measure the impact of news about current and future spending on the budget constraint, and the extent to which this impact is offset by contemporaneous and subsequent declines in the market value of outstanding debt. To accomplish this task, we first separate the various components of the budget constraint by log-linearizing Equation (1).[10]

Log-linearizing the budget constraint Campbell's linearization of the household budget constraint treats labor income as the return on human capital and, hence, part of the return on the household's overall portfolio. The constraint is then re-expressed as a function of household wealth (inclusive of human capital) and consumption, both of which are taken to be positive. In contrast, we treat government income from taxation as a part of the surplus flow rather than as a return on a government asset. The fact that the surplus may be either positive or negative creates difficulties for the log-linearization of (1). We circumvent these issues by expanding around both the average log receipts to debt and log spending to debt ratios and then constructing a weighted log primary surplus. This procedure is valid under the following assumptions regarding spending, receipt and surplus to debt ratios.

First, we assume that for all t, the market value of outstanding government debt, B_t, is positive and larger than the primary surplus, S_t. Second, we assume that the logarithm of the receipts to debt ratio, $\log(T_t/B_t)$, and the logarithm of the spending to debt ratio, $\log(G_t/B_t)$, are stationary around their average values $\overline{\tau b}$ and $\overline{gb}$, respectively. Lastly, we suppose that $(\exp(\overline{\tau b}) - \exp(\overline{gb}))$ lies between 0 and 1.

We have verified that our assumptions are supported by the data for the sample period 1946.I to 2007.III. Figure 1 displays the time series of $\log(T_t/B_t)$ and $\log(G_t/B_t)$.[11] Optimizing the Bayesian Information Criterion proposed by Schwartz (1978), we find an optimal lag length of one for both time series.[12] The associated Augmented Dickey-Fuller test statistics reveal that the unit-root hypothesis can be rejected for both $\log(T_t/B_t)$ and for $\log(G_t/B_t)$ at the 5% level.[13]

Throughout, our notational convention is to use lower cases to denote log variables and Δ to denote a difference, so that $b_t = \log B_t$, $\Delta b_{t+1} = \log B_{t+1} - \log B_t$, and so on. Let ns_t denote the

[10]The log-linearization of the government's budget constraint follows a similar procedure to the log-linearization of the household budget constraint and the country external budget constraint of Campbell (1993) and Gourinchas and Rey (2007) respectively.

[11]Details of the fiscal data used to construct T and G can be found in Appendix C.

[12]The Akaike Information Criterion (AIC), based on Akaike (1974), penalizes the number of parameters less severely and as a result suggests that including five lags is optimal. In any case, the AIC test statistics were fairly flat for one to ten lags, for both time series.

[13]The ADF(0) test statistic is -2.9826 for $\log(T_t/B_t)$ and -3.0429 for $\log(G_t/B_t)$, each with a 5% critical value of -2.8418. See Said and Dickey (1984) for details.

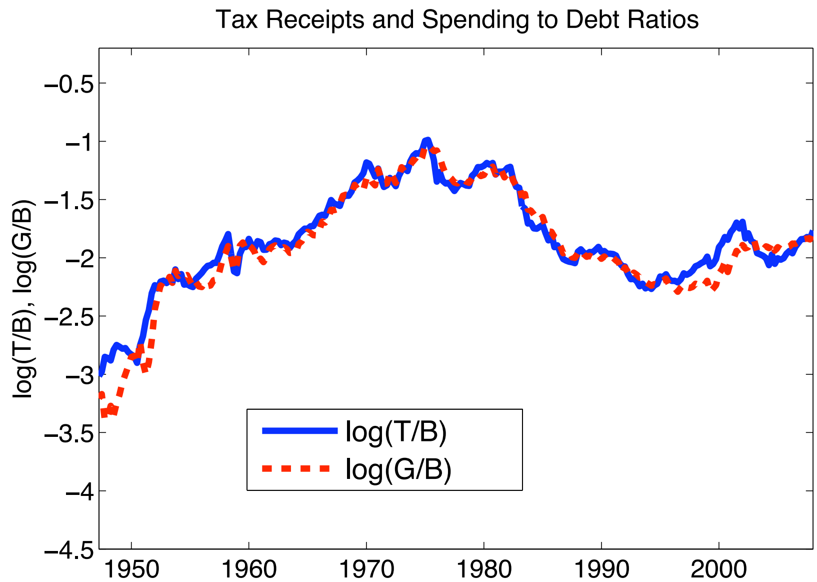

Figure 1. Government Spending and Receipts. This plot shows the log of government spending and tax receipts as a ratio of the market value of debt. The sample period is 1946.I-2007.III.

weighted log primary surplus:

$$ns_t = \mu_\tau \tau_t - \mu_g g_t. \tag{2}$$

The weights are given by

$$\mu_\tau = \frac{\mu_{\tau b}}{\mu_{\tau b} - \mu_{gb}} \qquad \text{and} \qquad \mu_g = \frac{\mu_{gb}}{\mu_{\tau b} - \mu_{gb}}, \tag{3}$$

where $\mu_{\tau b} = \exp(\overline{\tau b})$ and $\mu_{gb} = \exp(\overline{gb})$. In Appendix A we show that under the above assumptions, and ignoring unimportant constants, the log-linearization yields the following approximation for the law of motion for debt:

$$\Delta b_{t+1} = r^b_{t+1} + \left(1 - \frac{1}{\rho}\right)(ns_t - b_t), \tag{4}$$

where $\mu_{sb} = \mu_{\tau b} - \mu_{gb}$ and $\rho = (1 - \mu_{sb}) \in (0, 1)$.

Equation (4) implies the first-order difference equation:

$$b_t - ns_t = \rho r^b_{t+1} + \rho \Delta ns_{t+1} + \rho(b_{t+1} - ns_{t+1}). \tag{5}$$

Solving (5) forward and imposing the tail condition $\lim_{j\to\infty}\rho^j(ns_{t+j} - b_{t+j}) = 0$, we obtain the following expression for the *weighted log surplus to debt ratio*, $ns_t - b_t$:

$$ns_t - b_t = E_t \sum_{j=1}^{\infty} \rho^j \left(r^b_{t+j} - \Delta ns_{t+j} \right). \tag{6}$$

The expression in (6) implies that, if the log surplus to debt ratio fluctuates, it has to be due to either a change in expected future returns on outstanding debt, or a change in expected surplus growth. The log surplus to debt ratio reveals deviations from the long-run relationship between surpluses and debt. If it is negative, the surplus is small relative to the market value of debt. In this case, we expect low future returns on government debt or high future surplus growth. If the log surplus to debt ratio is positive, we anticipate high future returns on debt or low future surplus growth.

4. Government Hedging

Through further manipulation of the linearized government budget constraint, we gain a better understanding of the different ways in which the government can hedge against shocks to its expenditures. We first re-arrange the expression for the log of the surplus to debt ratio in Equation (6) to decompose the news about the weighted log surplus into two parts:[14]

$$ns_{t+1} - E_t ns_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^b_{t+j+1} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta ns_{t+j+1}. \tag{7}$$

This equation states that a positive shock to the (weighted log) surplus today (period $t+1$) must correspond to either to a positive shock to expected returns on government debt or to a negative shock to expected surplus growth. As a corollary, we can infer news about surplus growth from news about returns on government debt.[15]

As stated before, our goal is to quantify the impact of government expenditure risk on the contemporaneous and subsequent market value of outstanding debt. Therefore, we decompose Equation (7) further to isolate the component of the government's budget that we identify with

[14] See Appendix A for its derivation.

[15] For the remainder of the paper, we refer to $(E_{t+1} - E_t)X_{t+1}$ as innovations/news/shocks to X_{t+1} to avoid more cumbersome verbal descriptions of a change in the information set expectations are conditioned on.

exogenous expenditure shocks.

Exogenous shocks to government expenditures The presence of active fiscal policy and its associated implementation lags complicate the timing and extraction of news to government spending, i.e, expenditure shocks, from aggregate government spending data. Ramey (2008) advocates using defense spending data to identify fiscal shocks. She argues that fluctuations in defense spending account for almost all of the fluctuations in total government spending relative to its trend and that non-defense spending accounts for most of the trend in government spending. Ramey also shows evidence that suggests most non-defense spending is done by state and local governments rather than the federal government, undermining the ability of empirical estimations relying on aggregate expenditure data to capture exogenous shocks to government spending.

We define exogenous shocks to government spending, i.e. fiscal shocks, as innovations to defense spending growth. To identify these fiscal shocks, we first separate government spending into defense and non-defense components. We denote growth in defense spending between periods t and $t+1$ by Δg_{t+1}^{def} and growth in surplus excluding defense spending by Δns_{t+1}^{endo}. The weight μ_{def} denotes the average fraction of government spending that is defense spending and we use it to replace Δns by the weighted average of Δg^{def} and Δns^{endo}. Then re-arranging Equation (7) produces the following relation between news about defense spending growth, news about government debt returns and news about non-defense surplus growth:

$$
\begin{aligned}
(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta g_{t+j+1}^{def} &= -\frac{1}{\mu_g} \frac{1}{\mu_{def}} \left((E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{t+j+1}^b \right) \\
&\quad + \frac{1}{\mu_g} \frac{1}{\mu_{def}} \left((E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta ns_{t+j+1}^{endo} \right), \qquad (8)
\end{aligned}
$$

where μ_g is the weight of spending in the government's budget, as defined in Equation (3). The above equation implies that a positive shock to expected defense expenditure growth has to coincide with one of two things: a negative shock to expected returns on debt, or a positive shock to expected endogenous (i.e. non-defense) surplus growth. We will refer to the first of these adjustments as government hedging, broadly defined. When the government is fully hedged, the negative shock to expected returns completely offsets the surprise increase in expected defense spending growth. The second effect absorbs the slack: if innovations to debt returns do not fully offset the effect of news about an increase in exogenous expenditure growth, the government will have to run larger surpluses now or in the future. If there are shocks to government expenditures that are not captured by innovations to defense spending (growth), then the precision of our subsequent empirical estimates of hedging will be reduced. If, on the other hand, innovations to defense spending contain non-exogenous components, then our hedging estimates will be biased downwards, not overstated.[16]

Ex ante versus ex post hedging Government hedging broadly defined occurs either through a contemporaneous decline in the returns on the government's debt portfolio when the news about higher defense spending growth is revealed, or a decline in expected future returns. We distinguish between these two channels and label them *ex ante hedging* and *ex post hedging*, respectively. The normative fiscal theory literature emphasizes response of current returns to fiscal shocks as a device for smoothing the excess burden of taxation. The role of ex post hedging and quantitative measures of either hedging channel have not been explored in the normative literature.

To simplify matters, we introduce some additional notation. We denote news about current and future defense spending growth by:

$$\widetilde{h}_{t+1}^{g,def} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta g_{t+j+1}^{def},$$

news about current returns on government debt by:

$$\widetilde{r}_{t+1}^{b} = r_{t+1}^{b} - E_t r_{t+1}^{b},$$

and news about future returns on government debt by:

$$\widetilde{h}_{t+1}^{r^b} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+j+1}^{b}.$$

The linearized budget constraint (7) then implies that news about the weighted log surplus growth $\widetilde{h}_{t+1}^{ns} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta ns_{t+j+1}$ is given by $\widetilde{r}_{t+1}^{b} + \widetilde{h}_{t+1}^{r^b}$.

With these pieces of notation in place, we may formally define ex ante hedging to be a negative

[16] For completeness, we also estimate quantity of hedging with total government spending replacing defense spending.

covariance between news to current returns and news about current and future defense spending growth in the current period and in the future:

$$cov\left(\widetilde{h}^{g,def}_{t+1},\widetilde{r}^{b}_{t+1}\right)<0.$$

Analogously, ex post hedging is defined to be a negative covariance between news about future returns on government debt and news about current and future defense expenditure growth:

$$cov\left(\widetilde{h}^{g,def}_{t+1},\widetilde{h}^{r^b}_{t+1}\right)<0.$$

Measuring the government portfolio's g-beta To assess how well the government is hedged, we compute the government portfolio betas of news about current and future returns, one for ex post hedging, β^p, and one for ex ante hedging, β^a, and a last one for the total amount of hedging, β^f:

$$\begin{aligned}\widetilde{h}^{r^b}_{t+1}&=\beta^p_0+\beta^p_1\widetilde{h}^{g,def}_{t+1}+\varepsilon^p_{t+1},\\ \widetilde{r}^{b}_{t+1}&=\beta^a_0+\beta^a_1\widetilde{h}^{g,def}_{t+1}+\varepsilon^a_{t+1},\\ \widetilde{r}^{b}_{t+1}+\widetilde{h}^{r^b}_{t+1}&=\beta^f_0+\beta^f_1\widetilde{h}^{g,def}_{t+1}+\varepsilon^f_{t+1}.\end{aligned}\tag{9}$$

If the total g-beta, β^f_1, is minus one, the total decline in innovations to current and future debt returns is one percent when the innovations to current and future defense expenditure growth rises by one percent.

According to Equation (8), these betas map directly into fractions of total exogenous expenditure risk hedged by the government. If $\beta^p_1\frac{1}{\mu_g\mu_{def}}$ is minus one, the government is obviously fully hedged in the ex post sense. The government budget constraint does not require any additional increase in future non-defense surplus growth. Similarly, if $\beta^a_1\frac{1}{\mu_g\mu_{def}}$ is minus one, the government is obviously fully hedged in the ex ante sense. No additional adjustment is required.

5. Empirical Results

This section presents our estimation results. We start by setting up a benchmark VAR model that is used to construct innovations to current and future defense spending growth and innovations to current and future government debt returns. These news variables are then used to estimate the fraction of total expenditure risk that is hedged by the government.

5.1. Estimating the News Variables

A benchmark VAR We use unrestricted VARs to forecast future government debt returns and defense spending growth. From these forecasts, we construct estimates of return and defense spending news. The state vector, z_t, includes the log real (holding) returns on government debt, r_t^b, the weighted surplus to debt ratio, $ns_t - b_t$, and the growth rate of defense spending, Δg_t^{def}. We now describe each of these inputs in turn.

Log real returns, r_t^b, are constructed in several steps. Using CRSP Treasury bill and coupon-bond price data, we first employ the Nelson and Siegel (1987) technique to extract the time-t nominal zero-coupon yield curve. This enables us to compute nominal discount rates, which are converted to real terms using the, Consumer Price Index (CPI). Let P_t^k denote the real price of a synthetic zero-coupon government bond that matures at time $t+k$, for $k = 1, \ldots, 120$, where time steps are measured in quarters. The time-t real holding return on government debt maturing at $t+k$ can then be computed as $r_t^k = log(P_t^k) - log(P_{t-1}^{k+1})$. Lastly we obtain r_t^b by forming the weighted average of the quarterly real holding returns r_t^k, across all maturities k. The weights are determined based on the quantity and maturity data on government debt as obtained from CRSP and Treasury bulletins. Further details are provided in Appendix B.

Note that the inclusion of $ns_t - b_t$ is motivated by our linearized budget constraint (6), according to which $ns_t - b_t$ is likely to contain useful information about future returns and future defense spending growth. We compute the market value of outstanding government debt, B_t, by aggregating the time-t price of all future coupon and principal payments promised by the government. The current price of future payments is computed relative to the zero-coupon yield curve constructed from CRSP data, as described above. In what follows, we abstract from other federal government liabilities that may change in value when shocks to expenditures arise.

To compute $ns_t - b_t$, we use two different methods. In the first, we estimate the weights μ_τ and μ_g in Equation (3) from sample averages for the log of the receipts to debt ratio and the log of the spending to debt ratio, respectively. We then use the following definition to obtain the weighted log surplus to debt ratio, nsb_t:

$$nsb_t = \mu_\tau \tau_t - \mu_g g_t - b_t. \tag{10}$$

In the second method, we construct $ns_t - b_t$ from the residuals of the co-integration relation between τ_t, g_t and b_t. If $\{\tau_t\}$, $\{g_t\}$ and $\{b_t\}$ are co-integrated, the residuals from the co-integrating vector give the deviations from the long-run relationship between weighted log surplus and the log of the market value of government debt. We estimate the co-integrating vector by simple OLS regression of g_t on a constant, τ_t and b_t (see Appendix D for details). The residual (inclusive of constant) from this relation is labeled $\widetilde{nsb}_t$. It is obtained as:

$$\widetilde{nsb}_t = \widetilde{\mu}_\tau \tau_t - \widetilde{\mu}_g g_t - b_t, \tag{11}$$

where $\widetilde{\mu}_\tau$ and $\widetilde{\mu}_g$ are the weights constructed from the OLS coefficients. Figure 2 displays the weighted log surplus to debt ratio series associated with each method. It shows that the two series are almost identical, up to a constant. When we document our empirical results, we explicitly specify which series are used in the estimations.

The quarterly growth rate of defense spending, Δg^{def}, is constructed from data on national defense expenditures. Details on the source of the data can be found in Appendix C. Since our focus ultimately will be on *innovations* to Δg^{def}, we eliminate the trend component of defense spending growth using a two-sided HP filter.[17]

The state vector z_t for the benchmark VAR also includes quarterly inflation, π_t, and the slope of the yield curve, sl_t, as additional forecasting variables. The former is computed as the quarterly rate of change of the CPI, whereas the latter is defined as the difference between the ten-year and

[17]Stock and Watson (1999) argue for a one-sided HP filter. We prefer the two-sided filter for reasons similar to Gourinchas and Rey (2007). First, dropping observations leads to a less accurate estimate of the trend. Second, the one-sided filter keeps more high frequency components inside the trend in the beginning of the sample relative to the end of the sample. This is a concern in our case, since the largest fluctuations in defense spending growth occur at the beginning of our sample. We have re-estimated our benchmark VAR and hedging betas with the one-sided filter. As we anticipated, the total amount of hedging is slightly dampened due to the second point raised here.

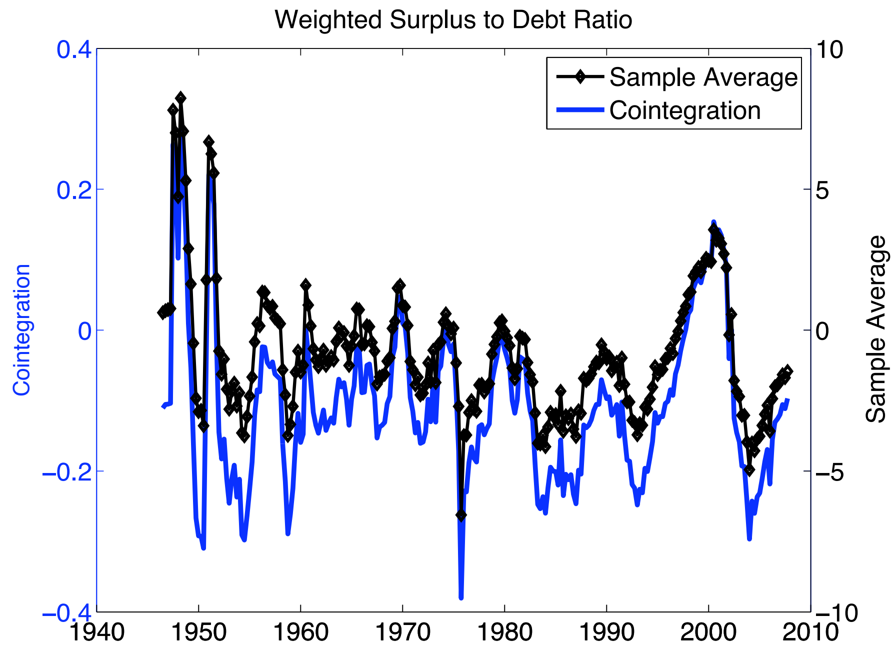

Figure 2. Weighted Log Surplus to Debt Ratio. This figure plots nsb_t constructed from sample averages (right axes) and $\widetilde{nsb}_t$ constructed from the residuals of the co-integrating relation between τ_t, g_t and b_t (left axes). The estimated weights $(\mu_g, \mu_\tau, \widetilde{\mu}_g, \widetilde{\mu}_\tau)$ are equal to $(19.6, 20.5, 11.9, 13.3)$. The sample period is 1946.I-2007.III.

the one-year yield on zero-coupon Treasury bonds.[18] Thus,

$$z_t = \begin{pmatrix} r_t^b & \pi_t & nsb_t & sl_t & \Delta g_t^{def} \end{pmatrix}.$$

All variables, except inflation and the slope of the yield curve, are deflated using the CPI. We demean all the variables and impose a first-order structure on the VAR:

$$z_{t+1} = Az_t + \varepsilon_{t+1}.$$

Table 1 reports the GMM estimates with their t-statistics. Our results show that this simple specification does reasonably well in predicting the returns on government debt. In particular, we find that nsb_t helps to forecast the returns on government debt at $t+1$: the coefficient on the log surplus to debt ratio is statistically significant. The negative sign of this coefficient can be reconciled with equation (6) as follows. An increase in nsb_t can be either due to an increase in ns_t

[18] In Appendix F, we provide results from VARs with alternative state variables, including $\widetilde{nsb}_t$.

Table 1
Benchmark VAR Estimates

This table reports the results of the benchmark VAR estimation. The benchmark VAR includes five variables, one lag and uses quarterly data. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. T-statistics for the GMM estimates are reported in brackets. We use the Newey-West variance-covariance matrix with four lags as the weighting matrix. The last column reports the R-squared. The sample period is 1946.I-2007.III.

	r_t^b	π_t	nsb_t	sl_t	Δg_t^{def}	R^2
r_{t+1}^b	0.0454	-0.1432	-0.0017	0.0920	0.0094	0.0522
	[0.3942]	[-0.6869]	[-2.1606]	[0.3883]	[0.6835]	
π_{t+1}	0.0092	0.4434	0.0002	-0.2116	-0.0036	0.2873
	[0.2874]	[6.1365]	[0.6873]	[-2.5553]	[-0.6523]	
nsb_{t+1}	-2.4001	10.3007	0.8680	-3.8203	-1.3727	0.7970
	[-0.7030]	[0.7504]	[27.0602]	[-0.3901]	[-1.0524]	
sl_{t+1}	0.0137	0.0166	-0.0002	0.7940	0.0008	0.6651
	[0.4209]	[0.3640]	[-1.0059]	[12.7208]	[0.6748]	
Δg_{t+1}^{def}	0.0032	0.2043	0.0053	0.3220	0.0009	0.1008
	[0.0331]	[0.6016]	[2.4351]	[1.2538]	[0.0140]	

or a decrease in b_t. If the latter case is true, it implies a decline in real bond prices in period t. If this decline is persistent, then an increase in nsb_t can be associated with a lower r_{t+1}^b.

Table 1 reveals that nsb_t also has some predictive power for defense spending growth. This may be due to the fact that nsb_t contains defense spending at time t and hence affects Δg_{t+1}^{def}. It may also indicate that some of the defense spending growth is endogenous. If this is the case and nsb does not capture all of the predictable components of Δg^{def}, our hedging results (which make use of the residuals) will be biased downwards, not overstated, as discussed in Section 4.

Calculating the news variables We set $\rho = 1 - \mu_{sb}$ equal to its postwar sample value of *.9924*. This allows us to easily back out news about current and future defense spending growth from the benchmark VAR estimates as:

$$\widetilde{h}_{t+1}^{g,def} = e_5(I - \rho A)^{-1}\varepsilon_{t+1},$$

where e_i represents a row vector of dimension five, with one in the i'th position and zero everywhere else and $\{\varepsilon\}$ represent the VAR residuals. We obtain news about current government debt returns via:

$$\widetilde{r}_{t+1}^b = e_1\varepsilon_{t+1},$$

Table 2
Correlation between Innovations

This table reports the standard deviations (diagonals) and the correlations (off-diagonals) of the news variables constructed from the benchmark VAR. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. The sample period is 1946.I-2007.III.

	$\widetilde{r}^b_{t+1}$	$\widetilde{h}^{r^b}_{t+1}$	$\widetilde{h}^{g,def}_{t+1}$	$\widetilde{\pi}_{t+1}$	$\widetilde{h}^{\pi}_{t+1}$
$\widetilde{r}^b_{t+1}$	0.02				
$\widetilde{h}^{r^b}_{t+1}$	0.44	0.02			
$\widetilde{h}^{g,def}_{t+1}$	-0.32	-0.67	0.04		
$\widetilde{\pi}_{t+1}$	-0.53	-0.34	0.22	0.01	
$\widetilde{h}^{\pi}_{t+1}$	-0.51	-0.83	0.46	0.60	0.02

and news about future government debt returns via:

$$\widetilde{h}^{r^b}_{t+1} = e_1 \rho A (I - \rho A)^{-1} \varepsilon_{t+1}.$$

In a similar fashion, we also compute innovations to current inflation, $\widetilde{\pi}_{t+1}$, and innovations to future inflation, $\widetilde{h}^{\pi}_{t+1}$.

5.2. Hedging results

Empirical correlations Table 2 reports the correlations between the news variables on its off diagonals; the diagonals contain the standard deviations of these variables. The main results are as follows. First, news about current and future debt returns are positively correlated, implying that there is no mean-reversion in debt returns in response to a fiscal shock. Second, news about current and future defense spending growth are negatively correlated with news about current returns on government debt (*-.32*), and are even stronger negatively correlated with news about future returns on government debt (*-.67*), providing evidence for both ex-ante and ex-post hedging. Third, innovations to current and future defense spending growth have twice the volatility of government debt returns.

Figure 3 plots $\widetilde{h}^{g,def}_{t+1}$ and $\widetilde{h}^{r^b}_{t+1}$. There are three large positive expenditure shocks in the earlier part of our sample: one in 1946, one in 1947 and one in 1950, at the start of the Korean war. As is apparent from Figure 3, the latter two are accompanied by negative shocks to expected future returns on government debt of roughly one-fourth the size of the g-shocks (both objects are in the

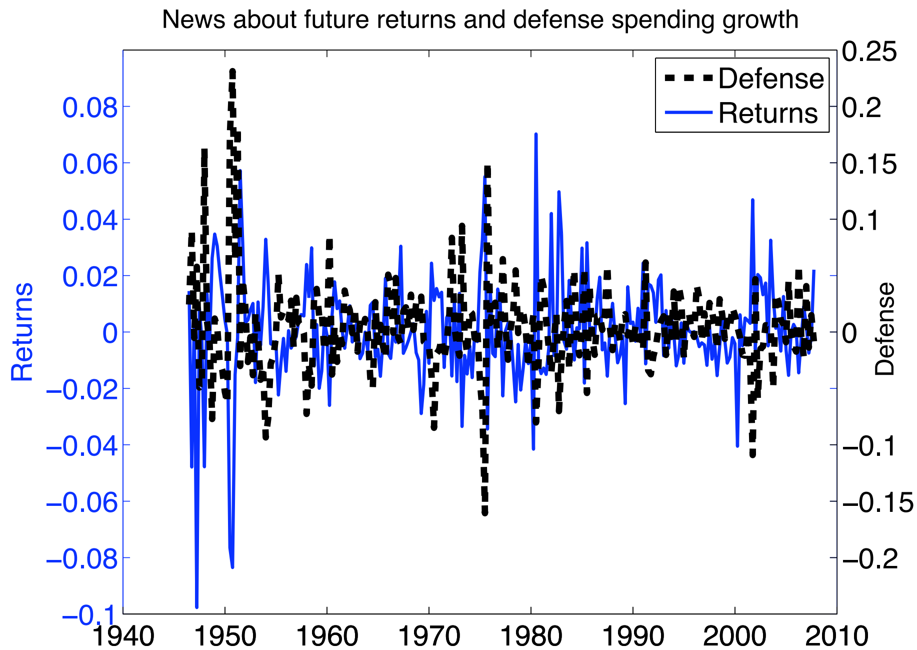

Figure 3. News about Future Debt Returns and Defense Spending Growth. This plot shows the innovations to future government debt returns, $\widetilde{h}^{r^b}$ (solid line), and the innovations to current and future defense spending growth, $\widetilde{h}^{g,def}$ (dotted line). Innovations are computed from the benchmark VAR. The sample period is 1946.I-2007.III.

same units). The first fiscal shock in the postwar period is accompanied by an equal-sized negative shock to expected returns (roughly 10%). The defense shocks in the remainder of the sample are smaller, but the negative correlation between innovations to future government debt returns and innovations to current and future defense spending growth is still apparent.

Figure 4 plots the innovations to current government debt returns against innovations to current and future defense spending growth. We detect a contemporaneous response of government debt returns to news about defense spending growth, but this response is smaller than the adjustment in future government debt returns.

***g*-Betas** Table 3 reports the g-betas for the government portfolio. The ex post g-beta is *-.30*, its ex ante counterpart is *-.17*. Both are significantly different from zero with the ex post beta being more precisely estimated. The two estimates add up to a total g-beta of *-.47*, implying that a one-percent shock to expected defense spending growth induces, on average, a *forty-seven* basis points unexpected drop in returns on outstanding public debt. This implies that a sizable degree

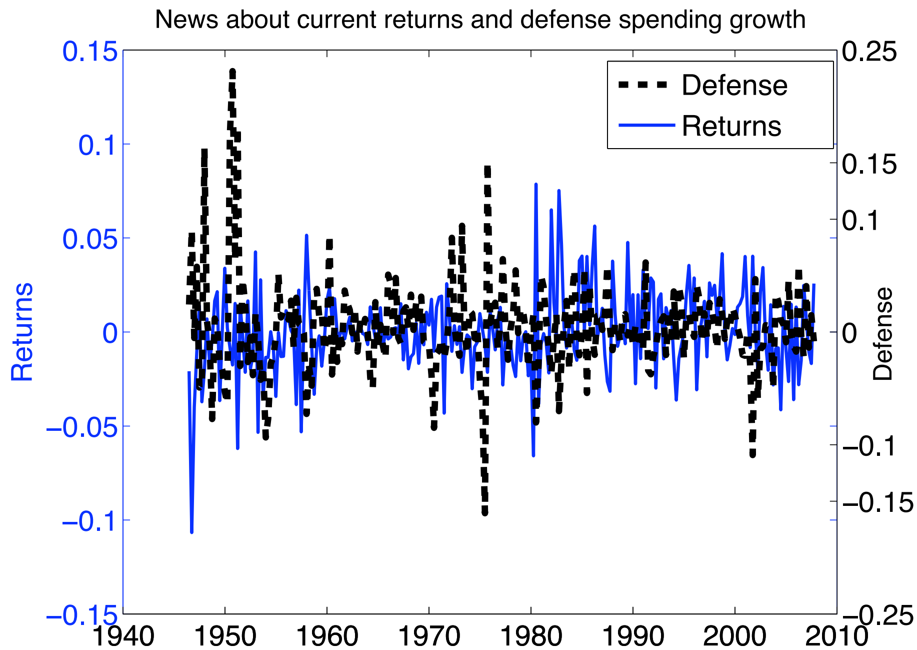

Figure 4. News about Current Debt Returns and Defense Spending Growth. This plot shows the innovations to current government debt returns, $\widetilde{r}^b$ (solid line), and the innovations to current and future defense spending growth, $\widetilde{h}^{g,def}$ (dotted line). Innovations are computed from the benchmark VAR. The sample period is 1946.I-2007.III.

of government spending risk was born by bond-holders in the postwar era. Over this period, innovations to current and future defense spending growth can account for almost *thirty-two percent* of the total variation in innovations to current and future holding returns on the federal government's outstanding debt.

Quantifying fiscal hedging To compute the fraction of fiscal risk hedged by the government, we need to obtain estimates for μ_g, the weight on expenditures in the weighted log primary surplus defined in (2), and for μ_{def}, the value of defense spending as a fraction of total government spending (see Section 4). Over the 1946.I-2007.III sample period, the defense spending to total spending ratio, μ_{def}, is *.35*, and the weight on expenditures, μ_g, is *19.6*. Together with the results reported in the third column of Table 3, this implies that the government hedges on average about 4.3 percent of its expenditure risk through ex post hedging, and a smaller fraction, about 2.4 percent, through ex ante hedging.[19] This adds up to a total of almost *7 percent* for the postwar period (last column of

[19]As discussed in Section 4, we identify fiscal shocks as innovations to defense spending.

Table 3
Hedging Betas

This table reports the results from regressing $\widetilde{h}^{r^b}_{t+1}$, $\widetilde{r}^b_{t+1}$, etc. on $\widetilde{h}^{g,def}_{t+1}$, as described in (9). The first two columns show the intercept and the hedging beta, with their t-statistics in brackets. The third column reports the R-squared, and the final column shows the fraction of expenditure risk hedged. Innovations are computed from the benchmark VAR. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. The sample period is 1946.I-2007.III.

	β_0	β_1	R^2	*fraction*
$\widetilde{h}^{r^b}$	0.0015	-0.3027	0.4532	0.0431
	[1.4423]	[-11.8297]		
$\widetilde{r}^b$	0.0007	-0.1715	0.0993	0.0244
	[0.4364]	[-4.0137]		
$\widetilde{r}^b + \widetilde{h}^{r^b}$	0.0022	-0.4742	0.3161	0.0675
	[1.0105]	[-8.7516]		

Table 3).

Obviously, these hedging fractions are sensitive to the weights, μ_g and μ_{def}. The sample average of thirty-five percent for the defense spending to total spending ratio seems rather high. At the end of our sample, in 2007.III, the ratio amounted to only twenty-five percent. Evaluated at that end-of-sample figure, the government is hedged against *ten percent* of expenditure risk, rather than seven percent.[20]

5.3. Additional results

Expected returns The correlation between innovations to government spending and innovations to returns on government debt adds up to a striking pattern between expected defense expenditure growth and expected returns. Instead of considering innovations, we now focus on the expected PDV of current and future defense spending growth as measured by $h^{g,def}_{t+1} = E_{t+1} \sum_{j=0}^{\infty} \rho^j \Delta g^{def}_{t+j+1}$. Figure 5 plots this time series against the expected PDV of future returns on government debt, $h^{r^b}_{t+1} = E_{t+1} \sum_{j=1}^{\infty} \rho^j r^b_{t+j+1}$. Both series are in deviations from their respective sample means. Here,

[20]In Appendix F, we provide hedging results from the first-order VAR with $\widetilde{nsb}$, where the weights are obtained from the cointegrating relationship between government receipts, spending and debt, instead of sample averages. Results are reported in Table 8. The government g-betas stay roughly the same, but the total amount of expenditure risk hedged increases to 13%, mostly due to a lower estimated weight on spending in the government's budget.

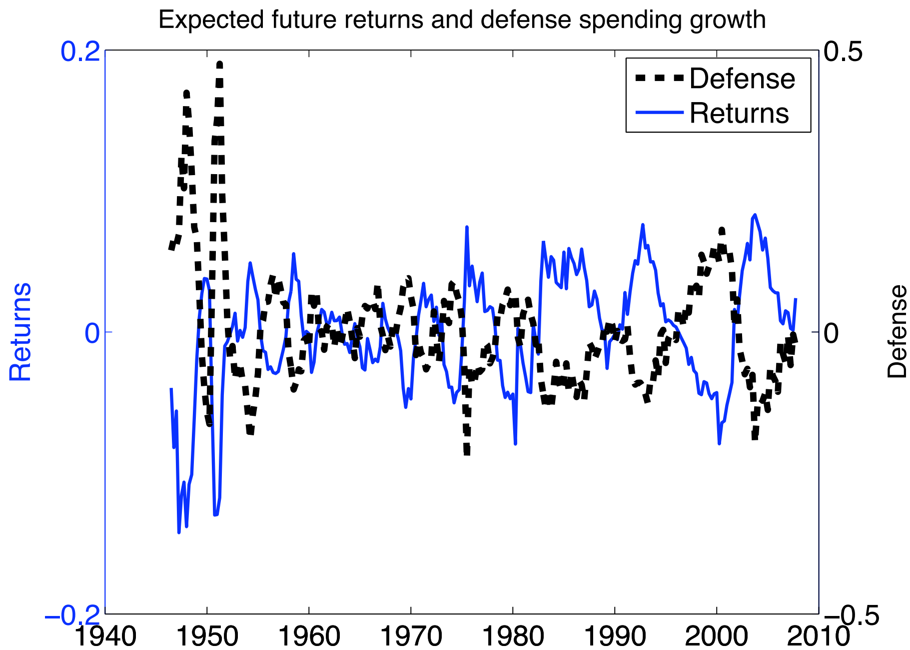

Figure 5. Expected Future Debt Returns and Expected Defense Spending Growth. This plot shows the expected PDV of future government debt returns, h^{r^b} (solid line), and the expected PDV of current and future defense spending growth, $h^{g,def}$ (dotted line), both in deviations from their respective sample means. Expected values are computed from the benchmark VAR. The sample period is 1946.I-2007.III.

$h^{g,def}_{t+1}$ and $h^{r^b}_{t+1}$ are calculated as:

$$
\begin{aligned}
h^{r^b}_{t+1} &= e_1 \rho A (I - \rho A)^{-1} z_{t+1}, \\
h^{g,def}_{t+1} &= e_5 (I - \rho A)^{-1} z_{t+1}.
\end{aligned}
$$

Over the entire sample, the correlation between these two objects is *-.91*. For example, at the start of the Korean war, expected defense spending growth increases in PDV to roughly *fifty percent* above its sample mean, while the expected future returns on government debt decrease to *fifteen percent* below their sample mean. Similarly, in the late nineties, expected defense spending is *twenty percent* above its mean and expected future returns on bonds are *nine percent* below their mean. This strong negative correlation result is remarkably robust to changes in the VAR specification, including adding additional lags and forecasting variables.

Fiscal shocks and inflation The results in Table 3 suggest that the US government has achieved a significant level of fiscal insurance in the postwar period. Adjustments in the real value of its outstanding liabilities have helped stabilize fiscal imbalances following expenditure shocks. These results, however, do not offer an explanation as to how this state contingency is achieved given that the US government issues nominal, non-contingent debt of several maturities only. Several theoretical papers in the optimal tax literature, including Siu (2004) and Lustig, Sleet and Yeltekin (2008), have shown that when a government trades non-contingent nominal debt only, costly contemporaneous or expected future inflations can allow it to provide insurance against fiscal shocks. Our benchmark VAR model allows us to construct innovations to current and future inflation and relate them to innovations to current and future defense spending growth, our proxy for fiscal shocks.

Figure 6 plots news about future inflation against news about defense spending growth. The correlation between the two time series is *.46*; and it is *.22* for news about current inflation. There are, however, three large surprises to defense spending growth in the early part of the sample: one in 1946, one in 1947, and one in 1950. Each of these events is accompanied by a substantial surprise in current and future inflation. They are also accompanied by increases in *expected* inflation (see Figure 7). Immediately after the Second World War, expected inflation rises to almost *ten percent* above its mean, to *five percent* above its mean in 1947, and to *nine percent* above its mean at the outset of the Korean war. This pattern breaks down in the seventies and the eighties (with the exception of 1973-4).

To quantify the relationship between the innovations to inflation and fiscal shocks more precisely, we regress $\widetilde{\pi}_{t+1} + \widetilde{h}^{\pi}_{t+1}$, on $\widetilde{h}^{g,def}_{t+1}$. The regression coefficient is .22 and highly significant, implying that in postwar US, a one percent increase in innovations to defense spending growth leds to 22 basis points increase in innovations to inflation. These results suggest that at quarterly frequencies, surprise inflation accounts for a non-negligible part of the surprise decrease in real returns after a spending shock. They also deliver some supporting evidence for the aforementioned theoretical models.

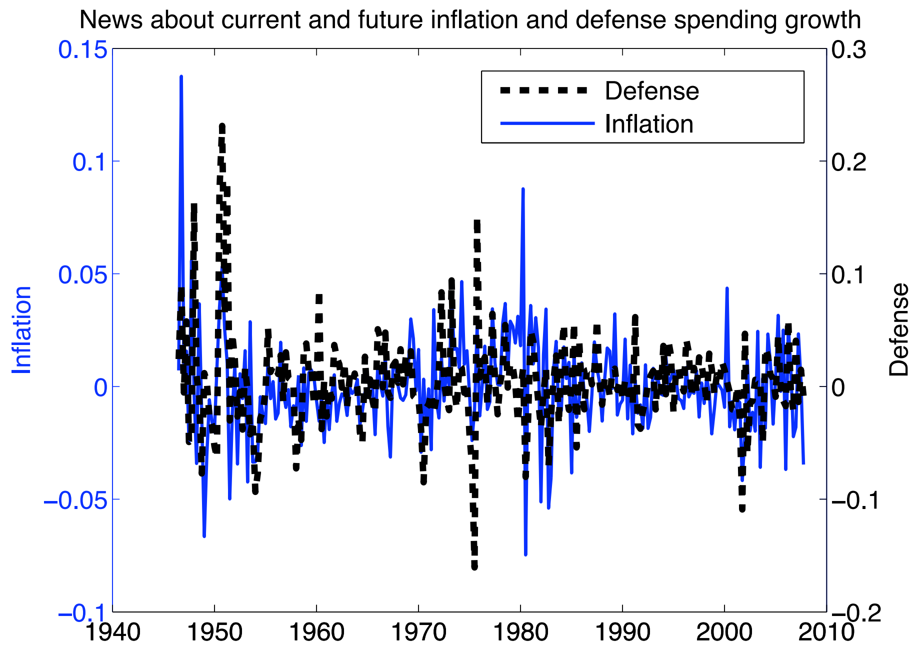

Figure 6. News about Inflation and Defense Spending Growth. This plot shows the innovations to current and future inflation, $\widetilde{\pi} + \widetilde{h}^{\pi}$ (solid line), and the innovations to current and future defense spending growth, $\widetilde{h}^{g,def}$ (dotted line). Innovations are computed from the benchmark VAR. The sample period is 1946.I-2007.III.

6. Implications for Active Debt Management

We now explore the implications of our hedging results for active management of government debt. The log holding returns $\{r_t^b\}$ are the weighted average of returns on the portfolio of outstanding government bonds. The maturity composition of this portfolio affects the level, but more importantly, the volatility of the $\{r_t^b\}$ series. Table 4 displays the average excess (4-quarter holding) return and its standard deviation for bonds of different maturities. It shows that excess returns on long-term debt have a higher mean than short-term debt and they are significantly more volatile. In particular, the average one-year excess return on a 10-year zero coupon bond was fifty-four basis points higher over the postwar period (compared to a one-year zero-coupon bond), while the average yield spread was about seventy-three basis points higher.

The higher volatility of returns to long-term debt has led to arguments for shortening the maturity structure, both in the normative tax literature and in other related work. Campbell (1995) argues that a cost-minimizing government should respond to a steeply sloped nominal yield curve by shortening the maturity structure since high yield spreads tend to predict high expected

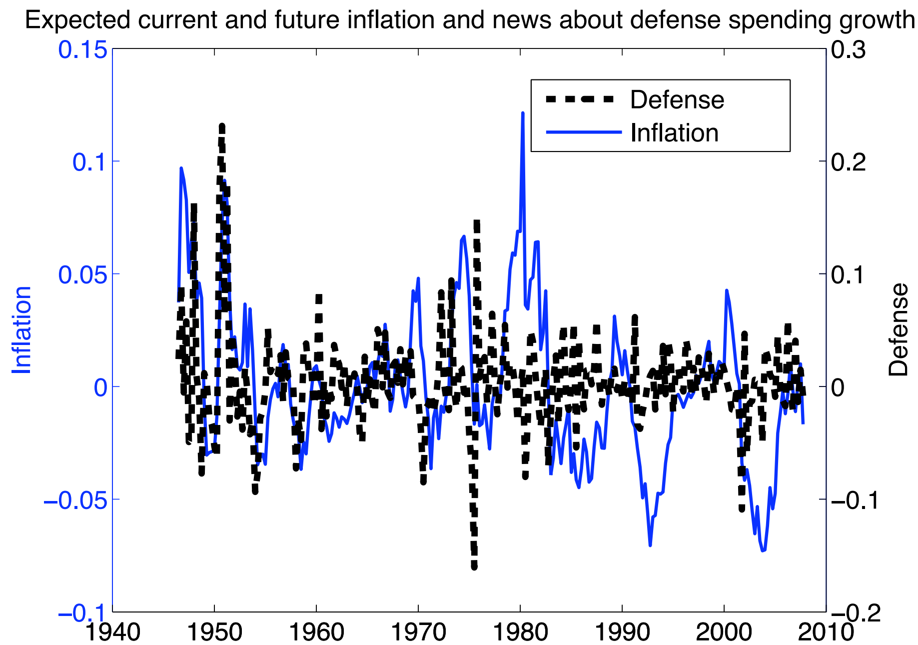

Figure 7. Expected Inflation and News about Defense Spending Growth. This plot shows the expected current and future inflation in deviations from its sample mean (solid line), and the innovations to current and future defense spending growth, $\widetilde{h}^{g,def}$ (dotted line). Both series are computed from the benchmark VAR. The sample period is 1946.I-2007.III.

bond returns in the future. Barro (1997) emphasizes tax smoothing considerations. He argues that governments can reduce their risk exposure and better smooth taxes by shortening the maturity structure when the inflation process becomes more volatile and persistent. These arguments, however, ignore the potential hedging benefits of long-term debt.

To quantitatively assess the potential hedging benefits of different maturities of debt, we assume the federal government has only one of the following securities outstanding: 1-year, 5-year, 10-year, 15-year and 20-year zero-coupon bonds. We include the returns $r_t^{b,k}$ on each of these securities (in addition to all the previous variables) in a separate, first-order VAR:

$$z_{t+1}^k = A^k z_t^k + \varepsilon_{t+1}^k \quad for\ k = 1, 5, 10, 15, 20.$$

The state vector z_t^k includes six variables:

$$z_t^k = \begin{pmatrix} r_t^b & \pi_t & nsb_t & sl_t & \Delta g_t^{def} & r_t^{b,k} \end{pmatrix}.$$

Table 4
Yield Spreads and Excess Returns

This table reports the average yield spread (relative to a 3-month T-bill) and the average 4-quarter log holding return (in excess of the return on a 3-month T-bill) on government debt of different maturities. Standard deviations are reported in parentheses. Zero-coupon yield curves are constructed from CRSP data (see Appendix B). The sample period is 1946.I-2007.III.

Maturity	1	2	5	10
		1939.1-2006.12		
Spread	0.45	0.55	0.81	1.14
	(0.59)	(0.67)	(0.94)	(1.24)
Excess Return	0.08	0.20	0.49	0.76
	(0.62)	(1.72)	(4.91)	(9.13)
		1947.1-2006.12		
Spread	0.46	0.56	0.83	1.19
	(0.62)	(0.71)	(1.01)	(1.31)
Excess Return	0.11	0.16	0.44	0.65
	(0.64)	(1.82)	(5.22)	(9.69)
		1970.1-2006.12		
Spread	0.60	0.75	1.11	1.51
	(0.69)	(0.80)	(1.13)	(1.48)
Excess Return	0.07	0.41	1.23	2.09
	(0.75)	(2.10)	(6.41)	(11.39)

For each maturity k, we re-estimate the VAR and compute the news about current and future government returns, $\widetilde{r}_{t+1}^{b,k}$ and $\widetilde{h}_{t+1}^{r^{b,k}}$, by:

$$\begin{aligned} \widetilde{r}_{t+1}^{b,k} &= e_6 \varepsilon_{t+1}^k, \\ \widetilde{h}_{t+1}^{r^{b,k}} &= e_6 \rho A^k (I - \rho A^k)^{-1} \varepsilon_{t+1}^k. \end{aligned}$$

As before, we then regress these news variables on innovations to the *PDV* of defense spending growth, $\widetilde{h}_{t+1}^{g,def}$. The resulting hedging beta estimates are reported in Table 5.[21] All of the estimated betas are significantly negative at the five percent level. The total g-beta of government debt returns more than doubles from *-.34* for one-year debt to *-.72* for 15-year debt. Correspondingly, the fraction of total risk hedged increases from *five to ten percent*, evaluated at the sample average weights. When evaluated at the end-of-sample defense spending ratio of twenty-five percent, the fraction increases from *nineteen percent* to *thirty-six percent*.

[21]The VAR estimates are not included for space considerations, but are available from the authors upon request.

As the maturity of bonds increases, total amount of hedging rises for two reasons. First, the ex ante beta decreases from *-.07* for the one-year bond to *-.32* for the 15-year bond. Second, the ex post beta decreases from *-.26* to *-.40*. The more negative ex ante beta just reflects the fact that the price of longer maturity bonds experience a much larger (in fact eight times larger) price drop when the news about higher future defense spending growth is revealed. This is the main the source of the higher hedging fractions at longer maturities.

The results in Table 5 document the superior hedging power of longer term debt. To the best of our knowledge, this is the first empirical documentation of the role of long-term debt as an effective hedging instrument for the government.[22] This suggests that active debt management recommendations may benefit from considering the additional role of long-term debt. More specifically, our results indicate that if the response of the term structure to defense expenditure shocks is taken as given, the Treasury can lengthen the maturity of its liabilities to take better advantage of the degrees of state-contingency built into them.[23]

Actual maturity structure of government debt Our benchmark hedging results in Section 5.2 suggest that innovations in bond prices, and hence holding returns, can help finance about 7% of defense expenditure risk. These holding returns were computed as the weighted average of returns on the portfolio of outstanding government bonds. The average maturity of government debt over the postwar sample period 1946-2007 is 5.5 years. The last panel of Table 5 shows that if the government had 5-year bonds in its portfolio only, it could hedge 7.5% of the its defense expenditure risk, in line of our initial 7% hedging results.

Figure 8 displays the actual maturity structure of US government debt. The maturity series does fluctuate substantially at low frequencies. At the end of the Second World War, the average maturity was around 10 years, and it was about 3 years in the mid-seventies. The average maturity started to increase again in the eighties and stayed above 6 years until early 21st century. The results in Table 5 coupled with the evolution of the average maturity of debt indicates that the US government was able to hedge defense expenditure shocks in the mid 40s to early 50s and again from

[22] Lustig, Sleet and Yeltekin (2008) analyze the structure of optimal debt management in an environment with non-contingent nominal debt of various maturities. They show that when costly contemporaneous or expected future inflations allow the government to hedge fiscal shocks, optimal debt management calls for issuing long term debt only, due to its superior hedging performance.

[23] This exercise is more in the partial equilibrium spirit of Campbell (1995), not in the general equilibrium spirit of the optimal tax literature.

Table 5
Hedging Betas for Each Maturity

This table reports the results of the hedging beta regressions in (9), maturity by maturity. The first two columns show the intercept and the hedging beta, with their t-statistics in brackets. The third column reports the R-squared, wheras the final column shows the hedging fractions. The VAR includes six variables, one lag and uses quarterly data. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. The sample period is 1946.I-2007.III.

	β_0	β_1	R^2	*fraction*
Maturity		Hedging Betas $\widetilde{h}^{r^{b,k}}$		
1	0.0012	**-0.2673**	0.4553	0.0380
	[1.3058]	[-10.3389]		
5	0.0015	**-0.2954**	0.4658	0.0420
	[1.4598]	[-12.2557]		
10	0.0020	**-0.3857**	0.2758	0.0549
	[1.1507]	[-10.1340]		
15	0.0021	**-0.4030**	0.1268	0.0573
	[0.7649]	[-7.6523]		
20	0.0031	**-0.6044**	0.0895	0.0860
	[0.6195]	[-5.5591]		
		Hedging Betas $\widetilde{r}^{b,k}$		
1	0.0003	**-0.0745**	0.0533	0.0106
	[0.3673]	[-2.8309]		
5	0.0009	**-0.2306**	0.0794	0.0328
	[0.3961]	[-3.6782]		
10	0.0011	**-0.2777**	0.0454	0.0395
	[0.2966]	[-3.2163]		
15	0.0012	**-0.3220**	0.0199	0.0458
	[0.1976]	[-2.6730]		
20	0.0018	**-0.4931**	0.0090	0.0701
	[0.1252]	[-2.2481]		
		Hedging Betas $\widetilde{h}^{r^{b,k}} + \widetilde{r}^{b,k}$		
1	0.0016	**-0.3417**	0.3412	0.0486
	[1.0534]	[-7.7875]		
5	0.0024	**-0.5259**	0.2276	0.0748
	[0.8264]	[-7.4190]		
10	0.0031	**-0.6634**	0.1633	0.0944
	[0.7088]	[-6.1800]		
15	0.0034	**-0.7251**	0.0894	0.1031
	[0.5281]	[-5.2174]		
20	0.0049	**-1.0975**	0.0469	0.1561
	[0.3636]	[-4.0056]		

mid-80s and to the end of the 90s more effectively. It also suggests that any defense expenditure shock arriving in the 70s could have been possibly better insured against had the government not

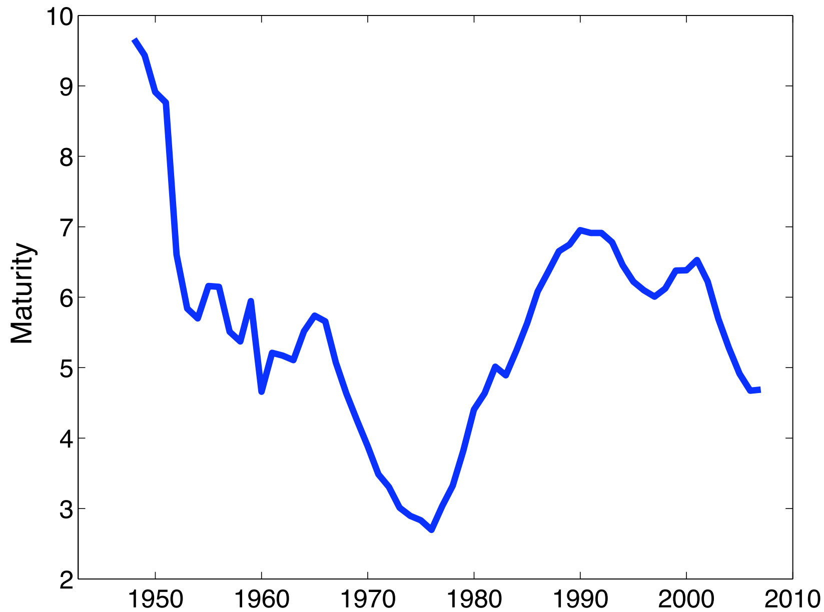

Figure 8. Maturity Structure of Publicly Held Debt. This figure plots the average maturity of publicly held debt. The sample period is 1947-2006.

decreased the maturity level to 3 years during this time.

In a related paper, Faraglia, Marcet and Scott (2008) investigate the role of debt management in providing insurance against budget shocks. They propose a battery of tests to assess the quality of debt management in OECD countries for the sample period 1970 to 2000. Their findings suggest that there is little link between debt structure (maturity and indexation) and fiscal insurance. They conclude that because holding returns show little variation, better fiscal insurance can only be achieved through extreme portfolio positions or alternative forms of contingent securities. In contrast, by conditioning on maturities, we are able to show that increasing the maturity structure from an average of 5 years to 15 years can increase fiscal insurance by 50%. Admittedly, our calculations ignore the impact of the maturity structure on prices of bonds, but they do suggest that a substantial increase in hedging can be achieved without taking extreme portfolio positions.

7. Defense Shocks and Defense Stocks

Throughout, we have defined fiscal hedging as the use of government debt returns to absorb variations in the expected PDV of defense spending growth. To assess the extent of fiscal hedging, it is clearly essential to forecast future defense spending well. Two factors potentially complicate the extraction of such forecasts from macroeconomic data. First, agents may learn about political and/or military events driving future defense spending growth in advance of this growth occurring or affecting other aggregate variables. Thus, VARs relying exclusively on such aggregate data may fail to identify the true date of the shock.[24] Second, innovations to the expected PDV of defense spending growth may not result in *realized* increases in defense spending. For example, an international dispute may raise expectations of future defense spending, but if the dispute is resolved through negotiation, this spending may not occur. In this case, a VAR specification relying on defense spending and other macroeconomic data would completely fail to detect this change in expectations.

Previous papers on identifying fiscal shocks, although divided in their approach, have mainly concentrated on resolving the timing issue.[25] These existing approaches, however, do not address the detection issue. We propose a new VAR specification that augments our benchmark one and addresses both the timing and the detection issues simultaneously. More specifically, the augmented VAR includes information embedded in the stock returns of companies in the defense industry.[26] Our logic is straightforward. In so far as defense companies' profits and dividends are tied to defense spending, defense stock return variables should respond contemporaneously to news about *perceived* future defense spending growth. If our intuition is correct, then this immediate response of defense stock return variables to news about defense spending growth will help address both the timing and detection concerns.

The augmented VAR includes the excess returns on defense stocks, r_t^{def}, relative to the market return, r_t^m, and the difference between the log price to dividend ratio of defense stocks, pd_t^{def}, and the market return, as additional forecasting variables.[27] The inclusion of these additional variables is motivated by the Campbell and Schiller (1998) expression for the price to dividend ratio:

$$d_t - p_t = E_t \sum_{j=1}^{\infty} \rho^j \left(r_{t+j}^s - \Delta d_{t+j} \right),$$

[24] See Ramey (2008) for a discussion of the causes and implications of mis-timing shocks when using the VAR approach.

[25] See, Ramey and Shapiro (1998) and Ramey (2008) for the narrative approach and Blanchard and Perotti (2002) and Perotti (2005) amongst others, for the restricted VAR approach to identifying fiscal shocks.

[26] For the definition of the defense industry, see Appendix E.

[27] The market return is measured as the return on the value-weighted CRSP market portfolio.

where d is the log dividend, p is the log price and Δd is the dividend growth rate of a stock. Campbell and Shiller argue that a high log dividend to price ratio implies high expected future holding returns or low expected future dividend growth. For our case, it implies that both the price to dividend ratio and excess returns on defense stocks contain information about the PDV of future dividend growth in the defense industry. The state space now includes 7 variables:

$$z_t = \begin{pmatrix} r_t^b & \pi_t & nsb_t & sl_t & \Delta g_t^{def} & pd_t^{def-m} & r_t^{def-m} \end{pmatrix}, \tag{12}$$

where $r^{def-m} = r_t^{def} - r_t^m$ and $pd_t^{def-m} = pd_t^{def} - r_t^m$.

Results Table 6 reports our estimation results using the new augmented VAR specification (12). Our results indicate that the excess returns on defense stocks help predict future defense spending growth, providing empirical evidence that defense stock returns do, indeed, contain new information about future defense spending growth. The variation in defense spending growth is explained better compared to our benchmark VAR: the R^2 improves to 14.6 percent. Additionally, (12) proves to be a slightly better specification for explaining the variation in quarterly returns on government debt (the R^2 is six percent).

Hedging We construct the news variables from the augmented VAR and confirm that the correlations between news to defense expenditure growth, our proxy for fiscal shocks, do covary negatively with news to current holding returns (-0.32) and future holding returns (-0.64), providing, once more, evidence for ex ante and ex post hedging. Table 7 reports the g-betas and hedging fractions from our augmented model. Our hedging results remain virtually the same. *Six and a half* percent of total defense spending shocks are hedged; four percent is hedged ex post and two and a half percent is hedged ex ante. All of the estimated beta coefficients are significant at the five percent level. The ex ante beta is *-.17*, the ex post beta is *-.29*; so, in total, one percent increase in innovations to defense spending growth leads to forty-six basis point decrease in innovations to returns.

Maturity composition Finally, we check the robustness of our earlier results about the links between the maturity composition of debt and hedging performance. We re-estimate the augmented VAR after replacing the average return on government debt with the 1-year, 5-year, 10-year, 15-

Table 6
Augmented VAR Estimates

This table reports the results of the augmented VAR estimation. The augmented VAR includes seven variables, one lag and uses quarterly data. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. T-statistics for the GMM estimates are reported in brackets. We use the Newey-West variance-covariance matrix with four lags as the weighting matrix. The last column reports the R-squared. The sample period is 1946.I-2007.III.

	r_t^b	π_t	nsb_t	sl_t	Δg_t^{def}	pd_t^{def-m}	r_t^{def-m}	R^2
r_{t+1}^b	0.0330	-0.1527	-0.0016	0.0983	0.0111	0.0005	0.0218	0.0601
	[0.2859]	[-0.6806]	[-2.0146]	[0.3971]	[0.7485]	[0.0821]	[1.2557]	
π_{t+1}	0.0133	0.4467	0.0002	-0.2135	-0.0041	-0.0001	-0.0073	0.2911
	[0.4159]	[5.9914]	[0.5879]	[-2.4198]	[-0.7393]	[-0.0624]	[-0.9085]	
nsb_{t+1}	-2.0465	10.2379	0.8627	-3.9030	-1.3999	-0.1224	-0.2094	0.7962
	[-0.5664]	[0.6900]	[26.1683]	[-0.3837]	[-1.0317]	[-0.5212]	[-0.2483]	
sl_{t+1}	0.0125	0.0153	-0.0002	0.7941	0.0010	-0.0001	0.0022	0.6656
	[0.3684]	[0.3215]	[-0.9945]	[12.0556]	[0.7715]	[-0.0970]	[0.5919]	
Δg_{t+1}^{def}	-0.0391	0.2003	0.0059	0.3433	0.0044	0.0128	0.0588	0.1462
	[-0.3969]	[0.5386]	[2.4936]	[1.2591]	[0.0641]	[1.3405]	[2.0306]	
pd_{t+1}^{def-m}	-0.0992	-0.7993	-0.0102	-0.8159	0.2966	0.6701	0.0224	0.4900
	[-0.1521]	[-0.6123]	[-1.2121]	[-0.5427]	[4.9761]	[8.2946]	[0.1169]	
r_{t+1}^{def-m}	-0.1271	-0.4664	-0.0042	-0.4510	0.1628	-0.0420	0.1222	0.0465
	[-0.3614]	[-0.6439]	[-1.0448]	[-0.5510]	[4.4041]	[-1.3379]	[1.3894]	

Table 7
Hedging Betas: Augmented VAR

This table reports the results from regressing $\widetilde{h}_{t+1}^{r^b}$, $\widetilde{r}_{t+1}^{b}$, etc. on $\widetilde{h}_{t+1}^{g,def}$, as described in (9). The first two columns show the intercept and the hedging beta, with their t-statistics in brackets. The third column reports the R-squared, and the final column shows the fraction of expenditure risk hedged. Innovations are computed from the augmented VAR. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. The sample period is 1946.I-2007.III.

	β_0	β_1	R^2	*fraction*
$\widetilde{h}^{r^b}$	0.0015	-0.2880	0.4126	0.0410
	1.4142	[-10.6806]		
$\widetilde{r}^b$	0.0007	-0.1710	0.1021	0.0243
	0.4494	[-4.1125]		
$\widetilde{r}^b + \widetilde{h}^{r^b}$	0.0022	-0.4591	0.3059	0.0653
	[1.0368]	[-8.6221]		

year and 20-year zero-coupon bond real holding returns.[28] The ex post beta more than doubles from *-.25* at the one-year maturity to *-.60* at the twenty-year maturity. At the same time, the ex

[28] The estimates are not reported, but are available from the authors upon request.

ante beta decreases dramatically from -*.09* to -*.55*, a six-fold change. The total g-betas are -*.34*, and -*1.15* and significant at the 1 and 20-year maturity respectively, confirming our earlier results on the superior hedging performance of long-term bonds. The total fraction of expenditure risk that is hedged increases with maturity and ranges from *five* percent to *sixteen* percent. At the longer maturities, the difference between ex ante and ex post hedging fractions become smaller, with each one providing roughly half of the total risk hedged. This suggests that the longer the average maturity of the government's portfolio, the bigger the contemporaneous drop in the real value of its outstanding liabilities in the aftermath of a fiscal shock, and hence the larger the effect of ex ante hedging.

8. Conclusion

In normative fiscal theory, government debt plays an essential hedging role. Variations in debt returns absorb shocks to spending needs, enabling governments to minimize variations in the excess burden of taxation. The amount of hedging debt can provide depends on the asset market structure assumed. In complete markets, spending shocks are fully hedged; the contemporaneous devaluation in the market value of government liabilities finances the surprise spending needs completely. However, empirical evidence on the persistence of tax rates and debt levels suggests that bond markets are incomplete, hence the fiscal insurance they provide is limited. The size of the actual fiscal insurance has not been quantified before however.

In this paper, we develop a method for quantifying fiscal hedging that is derived from the government's log-linearized intertemporal budget constraint, and hence is not wedded to any particular fiscal model. We show the government's fiscal balance helps predict future bond returns and that 7% of fiscal shocks has been hedged on average in the postwar US. Some of this is due to fall in contemporaneous real returns, which is the focus of the normative literature, but most of it comes from adjustments to future real returns, which has so far not been explored in the fiscal literature.

Additionally, we show that our results on the superior hedging performance of long-term bonds have implications for active management of government's liabilities. In particular, they suggest that the US government can hedge fiscal shocks better by lengthening its average debt maturity.

The method we develop for quantifying fiscal hedging is based on a VAR model. The VAR estimations are used to extract changes in bond-holders' expectations regarding future government expenditures. It's been noted in the previous empirical fiscal literature that the VAR approach may not correctly time these changes in expectations. In addition to the mis-timing problem, there is a possible detection problem. If changes in expected spending growth do not necessarily translate into realized changes in spending growth, the VAR can fail to detect the innovations to the bond investors' information set. To address both issues, we include defense industry stock return variables as additional forecasting variables in our VAR model. The predictability literature in finance suggests that dividend price ratios and excess returns contain information about future dividend growth of stocks. If the dividend growth of defense firms are tied to defense spending growth, then these defense return variables should help us predict future defense spending growth. The results from the our augmented VAR show that they indeed do. Our paper is the first to include defense return variables to help detect fiscal shocks and hence in addition to providing a measure of fiscal hedging, it contributes to the empirical literature on identifying fiscal shocks.

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Appendices

A. Linearization of the Government Budget Constraint

We start with the dynamic budget constraint of the government. All variables are expressed in real terms. Let B_t denote the market value of outstanding government liabilities, inclusive of cash, at the beginning of period t. The government budget constraint is given by:

$$B_{t+1} = R^b_{t+1}\left(B_t - S_t\right).$$

where R^b_{t+1} is the gross real return on government debt between t and $t+1$. The government's real primary surplus, $S_t = T_t - G_t$, is computed as the difference between receipts T_t and expenditures G_t. The growth rate of government debt can be stated simply as the gross return times one minus the primary surplus to debt ratio:

$$\frac{B_{t+1}}{B_t} \quad = \quad R^b_{t+1}\left(1 - \frac{S_t}{B_t}\right) \tag{A.1}$$

We assume that for all t, $B_t > 0$ and $B_t > S_t$. Additionally, we assume that the log receipts to debt ratio, $\log(T_t/B_t)$, and the log spending to debt ratio, $\log(G_t/B_t)$, are stationary around their respective average values $\overline{\tau b}$ and $\overline{gb}$.[29] Finally, we assume that $\exp(\overline{\tau b}) - \exp(\overline{gb})$ is between 0 and 1. Using lower case letters to denote logs, (A.1) may be rewritten as:

$$\begin{aligned}
\Delta b_{t+1} &= r^b_{t+1} + \log(1 - \exp(s_t - b_t)) \quad \text{ if } S_t > 0 \\
&= r^b_{t+1} + \log(1 + \exp(d_t - b_t)) \quad \text{ if } D_t = -S_t > 0,
\end{aligned}$$

where we distinguish between the case in which the government is running surpluses and the case in which it is running deficits. If the government only ran surpluses, then we could expand the right-hand side of the log budget constraint as a function of $s_t - b_t$ around $\overline{sb} = \overline{\log SB}$:

$$\log(1 - \exp(s_t - b_t)) \quad \approx \quad \log(1 - \exp(\overline{sb})) - \frac{\exp(\overline{sb})}{1 - \exp(\overline{sb})}\left[(s_t - b_t) - \overline{sb}\right].$$

[29] See Section 3 for supporting evidence on these assumptions.

First-order expansion Since governments run deficits, an alternative expansion is required. We rewrite $\log(1-S_t/B_t)$ as $\log(1-\exp(\tau_t - b_t)+\exp(g_t - b_t))$ and expand around $(\overline{\tau b}, \overline{gb})$. We obtain:

$$\begin{aligned}
\log\left(1-\frac{S_t}{B_t}\right) &\approx \log(1-\exp(\overline{\tau b})+\exp(\overline{gb})) - \frac{\mu_{sb}}{1-\mu_{sb}}\left(\frac{\mu_{\tau b}\left(\tau_t - b_t - \overline{\tau b}\right) - \mu_{gb}\left(g_t - b_t - \overline{gb}\right)}{\mu_{sb}}\right) \\
&= K - \frac{\mu_{sb}}{1-\mu_{sb}}\left(\frac{\mu_{\tau b}}{\mu_{\tau b}-\mu_{gb}}\tau_t - \frac{\mu_{gb}}{\mu_{\tau b}-\mu_{gb}}g_t - b_t\right), \qquad \text{(A.2)}
\end{aligned}$$

where K absorbs unimportant constants. The weights are defined as $\mu_{sb} = \mu_{\tau b} - \mu_{gb}$, with $\mu_{\tau b} = \exp(\overline{\tau b})$ and $\mu_{gb} = \exp(\overline{gb})$.

Law of motion for debt The approximation in (A.2) implies the following law of motion for debt:

$$\Delta b_{t+1} = r^b_{t+1} + \left(1-\frac{1}{\rho}\right)(ns_t - b_t),$$

where $\rho = 1-\mu_{sb}$. Rearranging terms produces:

$$(ns_t - b_t) = \rho r_{t+1} - \rho \Delta ns_{t+1} + \rho\left(ns_{t+1} - b_{t+1}\right).$$

This is a first-order difference equation that can be solved by repeated substitution for the weighted log surplus to debt ratio. Imposing the tail condition $\lim_{j\to\infty}\rho^j(ns_{t+j} - b_{t+j}) = 0$ and taking expectations, we obtain:

$$(ns_t - b_t) = E_t \sum_{j=1}^{\infty} \rho^j \left(r^b_{t+j} - \Delta ns_{t+j}\right). \qquad \text{(A.3)}$$

Equation (A.3) implies that the news about current and future returns on government debt equals the news about current and future surplus growth:

$$(E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j r^b_{t+j+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta ns_{t+j+1}.$$

This follows because:

$$(ns_{t+1} - b_{t+1}) - E_t (ns_{t+1} - b_{t+1}) = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^b_{t+j+1} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta ns_{t+j+1}.$$

And so using $b_{t+1} - E_t b_{t+1} = r^b_{t+1} - E_t r^b_{t+1}$, we have:

$$ns_{t+1} - E_t ns_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^b_{t+j+1} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta ns_{t+j+1}.$$

B. Computing the Value of Total Government Liabilities and Returns on Government Debt

The Treasury reports the interest cost of total government debt, calculated by summing up all the principal and coupon payments the government has promised to deliver at $t + k$ as of time t. The Treasury's methodology makes no distinction between coupon payments and principal payments, and hence mismeasures the cost of funds. We follow Hall and Sargent's (1997) accounting technique for computing the government's cost of funds, holding returns (i.e. capital gains and losses) on government debt, as well as the change in the market value of government debt. To accomplish this task, we first convert nominal yields to maturity on government debt into prices of claims on future dollars in terms of current prices. In other words, we unbundle a coupon bond into its constituent pure discount bonds and value these components. We then add up the values of the each of the components to attain the value of the bundle.

Let s^k_t be the number of time-$(t+k)$ dollars the government has promised to deliver as of time t. Let P^k_t be the number of time-t goods it takes to buy a dollar delivered at time $t+k$. Hence, P^k_t is the real (inflation-adjusted) price at time t of a zero-coupon bond maturing k periods ahead. Let D_t be the government's real net-of-interest budget deficit, measured in units of time-t goods. Finally, let m_t be the nominal value of the monetary base. Then the government's budget constraint can be written as:

$$\underbrace{P_t^0\, m_t}_{\substack{\text{real value of mon.}\\ \text{base at } t}} + \underbrace{\sum_{k=1}^{K} P_t^k\, s_t^k}_{\substack{\text{real value of debt}\\ \text{at } t}} = \underbrace{P_t^0\, m_{t-1}}_{\substack{\text{real value of}\\ (t-1)\text{-mon. base at } t}} + \underbrace{\sum_{k=1}^{K} P_t^{k-1}\, s_{t-1}^k}_{\substack{\text{real value of}\\ (t-1)\text{-debt at } t}} + D_t. \tag{B.1}$$

Here, P_t^0 is the inverse of the aggregate price level at time t. Monetary base is viewed as matured government bonds, hence bond-holders and money-holders are treated symmetrically. We can re-arrange Equation (B.1) to get

$$P_t^0\, m_t + \sum_{k=1}^{K} P_t^k\, s_t^k = \underbrace{\left(P_t^0 - P_{t-1}^0\right)\, m_{t-1}}_{\substack{\text{- seignorage}\\ \text{borrowing cost of "money"}}} + \underbrace{\sum_{k=1}^{K} \left(P_t^{k-1} - P_{t-1}^k\right)\, s_{t-1}^k}_{\substack{\text{borrowing cost of debt}\\ \text{"cost of funds"}}}$$

$$+ \underbrace{P_{t-1}^0\, m_{t-1}}_{\substack{\text{real value of mon.}\\ \text{base at } t-1}} + \underbrace{\sum_{k=1}^{K} P_{t-1}^k\, s_{t-1}^k}_{\substack{\text{real value of debt}\\ \text{at } t-1}} + D_t. \tag{B.2}$$

To decompose the government's budget constraint in this manner, we need to calculate the quantities s_t^k and the prices P_t^k.

The quantity data We compute the series s_t^k from the CRSP government bonds files on monthly Treasuries going back to 1960 and from the monthly Treasury Bulletins, the Wall Street Journal and the New York Times for the years preceding 1960. These files contain monthly data on the maturity and face value of outstanding publicly held debt, plus coupon-rate data on virtually all negotiable direct obligations of the United States Treasury, from 1946 to the present. We construct the series s_t^k by adding up all the dollar principal amounts plus the coupon payments that the government has promised, as of date t, to deliver to the public at date $t+k$. Note that CRSP does not report the face value of Treasury bills held by the public, and that these data are obtained from table FD-5 of the monthly Treasury Bulletin.

The price data To compute the series P_t^k, we employ the Nelson and Siegel (1987) approach to extract the time-t zero-coupon yield curve from the CRSP Treasury bill and coupon-bond price data. To facilitate the yield-curve extraction, we clean the price data so that it contains only straight bonds with a maturity of at least one year plus T-bills with 30-days or longer until maturity. We also remove all bonds with 1.5% coupon rates, as they have been documented to contain large spurious errors.[30] Once the zero curves are constructed, we compute nominal discount rates, which are converted to real terms P_t^k by dividing by the CPI.[31]

Returns on the government's debt portfolio We compute the average real holding return on the government's outstanding debt between $t-1$ and t, r_t^b, as

$$r_t^b \quad = \quad \sum_{k=1}^{K-1} \frac{s_{t-1}^{k+1} P_{t-1}^{k+1}}{\sum_{l=1}^{K-1} s_{t-1}^{l+1} P_{t-1}^{l+1}} \, r_t^k, \tag{B.3}$$

where $r_t^k = \log(P^(k)_t) - \log(P_{t-1}^{k+1})$.

C. Fiscal Data

The source of our fiscal budget data is NIPA Table 3.2, Government Current Receipts and Expenditures, seasonally adjusted and measured in billions of dollars. Government receipts T are current receipts (Line 1) which include current tax receipts, contributions for social insurance, income receipts on other assets and current transfer assets. Government expenditures G include current expenditures (Line 40), gross government investment (Line 41), and capital transfer payments (Line 42). We subtract consumption of fixed capital (Line 44) and debt interest payments (Line 28) from

[30]For details, see pg. 27 of the CRSP Monthly Treasury U.S. Database Guide.
[31]The value of the currency P_t^0 is set equal to the inverse of the consumer price index at t.

current expenditures. National defense spending data are from NIPA Table 3.9.5., Line 11 (national defense expenditures). They are seasonally adjusted at annual rates and measured in billions of dollars.

D. Cointegrating Relation Between Spending, Receipts and Market Value of Debt

We deflate the market value of debt, government expenditures and government receipts using the CPI. We test for a cointegrating relationship between τ_t, g_t and b_t using the Johansen (1988) procedure and we find evidence in favor a cointegrating relation between these three variables on the 1946.I-2007.III sample. The estimated co-integrating vector is: $[1, 1.1233, -0.0843]$. We can back out the implied weights from these estimates.

E. Defense Industry Return Variables

Defense stocks are identified as firms with SIC codes between 3760-3769 (Guided missiles and space vehicles), 3795-3795 (Tanks and tank components) and 3480-3489 (Ordnance & accessories). This is identical to the Fama-French definition of the "Guns" industry in the 48 industry portfolios.[32] We use CRSP cum-dividend returns for all defense stocks to compute value-weighted quarterly portfolio returns for the defense industry, from 1946.I to 2007.III. In addition, we also compute price dividend ratios at the portfolio level, using CRSP data on dividend cash amount (data item DIVAMT).

F. Robustness Checks and Additional Results

In this section, we report hedging results from alternative specifications of our benchmark VAR state vector. First, we show the results after replacing the weighted surplus to debt ratio nsb with $\widetilde{nsb}$, where the weights are obtained from the co-integrating vector. Table 8 shows that in this case, the government hedging betas are -0.4053 and -0.1536 for future returns and current returns,

[32]For details, see Kenneth French's data library at
$http : //mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.$

Table 8
Hedging Betas: Cointegration Weights

This table reports the results from regressing $\widetilde{h}^{r^b}_{t+1}$, $\widetilde{r}^b_{t+1}$, etc. on $\widetilde{h}^{g,def}_{t+1}$, as described in (9). The first two columns show the intercept and the hedging beta, with their t-statistics in brackets. The third column reports the R-squared, and the final column shows the fraction of expenditure risk hedged. Innovations are computed from a VAR with five variables. We use $\widetilde{nsb}_t$ for the weighted log surplus to debt ratio, with the weights obtained from the co-integrating relation between τ_t, g_t and b_t. The sample period is 1946.I-2007.III.

	β_0	β_1	R^2	*fraction*
$\widetilde{h}^{r^b}$	0.0017	-0.4053	0.3434	0.0971
	[0.9420]	[-4.0180]		
$\widetilde{r}^b$	0.0004	-0.1536	0.1707	0.0368
	[0.2893]	[-4.2904]		
$\widetilde{r}^b + \widetilde{h}^{r^b}$	0.0021	-0.5589	0.4816	0.1338
	[1.1012]	[-4.9327]		
$\widetilde{r}^\pi + \widetilde{h}^\pi$	-0.0006	0.2401	0.4214	0.0575
	[-0.5140]	[6.8926]		

respectively. These g-betas are larger than in our benchmark case. Additionally, the weight of spending in the government's budget, μ_g, is smaller when these weights are obtained from the co-integrating relationship between spending, taxes and debt (11.9 versus 19.6). The combination of larger g-betas and a smaller spending weight deliver a higher fraction of fiscal risk hedged: 13.4 % versus 6.7 % in the benchmark case.

Next, we construct innovations to *total* government spending growth by replacing defense spending with total spending in our benchmark VAR. We then obtain hedging results by regressing innovations to future returns and to current returns on innovations to growth of total spending. Table 9 shows that the government hedging betas for total spending are larger than in the case with defense spending. Most significantly, the beta in the current returns equation increases from -0.17 to -0.26. In postwar US data, innovations to real returns on government debt decreases by *sixty-four* basis points when innovations to government spending growth increases by one percent. This implies that over the postwar period, 3.1% total spending risk has been hedged by returns (total hedging beta divided by μ_g). We also regress $\widetilde{h}^g$ on $\widetilde{\pi}$ and $\widetilde{h}^\pi$. The regression coefficient is .3228 and highly significant, implying that in postwar US data, a one percent increase in innovations to total spending growth leads to a 32 basis point increase in innovations to inflation.

Table 9
Hedging Betas: Total Government Spending

This table reports the results from regressing $\widetilde{h}^{r^b}_{t+1}$, $\widetilde{r}^b_{t+1}$, etc. on $\widetilde{h}^g_{t+1}$, innovations to total government spending. The first two columns show the intercept and the hedging beta, with their t-statistics in brackets. The third column reports the R-squared, and the final column shows the fraction of expenditure risk hedged. Innovations are computed from a VAR with five variables, where defense spending growth is replaced by total government spending growth. We use nsb_t for the weighted log surplus to debt ratio, with the weights obtained from sample averages. The sample period is 1946.I-2007.III.

	β_0	β_1	R^2	*fraction*
$\widetilde{h}^{r^b}$	0.0009	-0.3815	0.4645	0.0186
	[0.9296]	[-11.8582]		
$\widetilde{r}^b$	0.0006	-0.2606	0.1724	0.0127
	[0.3738]	[-5.2731]		
$\widetilde{r}^b + \widetilde{h}^{r^b}$	0.0015	-0.6420	0.4076	0.0313
	[0.7626]	[-14.0582]		
$\widetilde{r}^\pi + \widetilde{h}^\pi$	-0.0008	0.3228	0.2633	
	[-0.5985]	[7.0683]		