Bayesian Environmental Policy Decisions: Two Case Studies

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BAYESIAN ENVIRONMENTAL POLICY DECISIONS: TWO CASE STUDIES

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Abstract. Statistical decision theory can be a valuable tool for policy-making decisions. In particular, environmental problems often benefit from the application of Bayesian and decision-theoretic techniques that address the uncertain nature of problems in the environmental and ecological sciences. This paper discusses aspects of implementing statistical decision-making tools in situations where uncertainty is present, looking at issues such as elicitation of prior distributions, covariate allocation, formulation of loss functions, and minimization of expected losses subject to cooperation constraints. These ideas are illustrated through two case studies in environmental remediation.

Key words: Bayesian statistics; decision theory; elicitation; environmental remediation; prior distributions; utility functions.

INTRODUCTION

In environmental and ecological problems, the end use of a statistical analysis is often to make a decision. The decision to be made varies: sometimes it is a question of whether or not a species should be moved to the endangered list; in other cases, it may be how best to allocate resources; or it can be deciding whether or not remedial action is necessary after environmental contamination. Regardless of the specific nature of the decision problem, the common thread underlying these scenarios is the need to formally account for uncertainty in the decision-making process. Although this paper will focus primarily on two case studies in environmental remediation, the general methods outlined are applicable to a variety of ecological and environmental problems.

The goal of this paper is to illustrate how Bayesian decision-theoretic techniques can be used in environmental and ecological problems that require decision-making under uncertainty. The organization of this paper is to introduce some of the basic tenets of decision theory and some of the specific tools that are needed for the Bayesian approach, and then illustrate the use of these tools through two case studies in environmental remediation.

Whenever we make a decision, be it in our personal lives or as scientists, we have a natural desire to incorporate all the information we have available. When we think about statistical models, however, traditional statistical methods often consider the data collected for a particular study to be the only available source of information. Other sources of information, most notably, prior beliefs and utilities, are also available and can be of use in the decision-making process (Barnett 1982).

Prior beliefs generally represent some amalgamation of information that is available before data collection. This information may come from previous experience with similar types of problems, or knowledge about the underlying process from which the data are to be collected. Prior beliefs are most useful to a statistician when they are expressed in the form of a prior distribution; the prior is then combined with the sample data, in the form of the likelihood, to obtain a posterior distribution. Prior distributions can be constructed in a variety of ways, based both on subjective belief systems (Savage 1954) and on empirical data. In this paper, we give examples of both methods of constructing prior distributions. The construction of prior distributions based on subjective beliefs is a process known as elicitation, and, in particular, the elicitation of the beliefs of subject-matter experts.

Meyer and Booker (1991) and Chaloner (1996) provide several possible reasons why a decision maker may want to use the subjective opinion of an expert for his or her prior distribution. When investigating new, rare, complex or poorly understood phenomena, expert opinion may be one of the few sources of a priori information, for example, in attempting to model the environmental impact of the establishment of a new industrial facility or remediation of a contaminated site.

1 Manuscript received 14 August 1995; revised 18 December 1995; accepted 22 December 1995; final version received 15 April 1996.
2 For reprints of this group of papers, see footnote 1 on p. 1034.
Another situation is one where it may be necessary to reach a decision without collecting large amounts of data; when data alone will not provide enough information to make the decision, elicited expert opinion can be used to fill the gaps in knowledge. One may also want to use expert opinion to forecast future events or to design an experiment where collecting superfluous data is undesirable. Eliciting expert opinion can identify the segments of the design space in which information is highly uncertain. Elicitation also provides a way to document information and beliefs about planned treatments and to incorporate historical data and experiences that may not be well documented.

The utilities represent a formalization of the possible consequences of the available decisions; by constructing an exclusive and exhaustive list of possible decisions and specifying the consequences for each decision (dependent on an uncertain state of nature as represented in the posterior distribution), it then becomes possible to combine all the available sources of information to make a decision. The decision maker, when behaving as a rational Bayesian, would then choose the action under which his or her expected posterior utility is maximized, or equivalently, the expected posterior loss is minimized. In other words, the decision maker would choose the decision that, based on his or her prior beliefs, updated by collected data, is likely to have the least adversarial consequences when compared to other possible decisions. This is a personal and subjective view of Bayesian inference (Savage 1954), which seems particularly applicable to environmental problems, where there is considerable uncertainty both in the nature of forms of environmental contaminants and in the effect these contaminants may have on human health and the environment. Furthermore, in situations where there are divergent prior beliefs about the extent of damage and different utilities for the same outcome, methods for quantifying the decision-making process are necessary.

**Bayesian Decision Theory**

Statistical decision theory is concerned with making decisions in the presence of uncertainty. The uncertainty is generally formalized using statistical techniques, and the decision theory problem becomes a mathematical optimization procedure; if the statistical model is specified, and data are collected, and all possible decisions are outlined, along with the consequences of each decision, then an “optimal” decision will be one that minimizes the potential negative consequences (Berger 1985). In Appendix A, a brief mathematical outline is given of how prior distributions and likelihoods are combined to form a posterior distribution, and how the posterior distribution is combined with utilities to find a decision that is optimal. In the rest of this section, we outline some of the details of identifying prior distributions and utility functions.

**Quantifying prior information**

There are essentially two ways in which prior information can be summarized: empirically or by elicitation. An “empirical” prior is one that is constructed from other available data, typically by analyzing another data set and using the output from that analysis as the prior distribution for the current analysis. An example of how this is done is given in Case Study 2 of this paper. An “elicited” prior distribution is one that is constructed by asking questions of experts and constructing the prior distribution based on those responses, as in Case Study 1. The former is familiar, in the sense that data analysis is a familiar subject; it is the latter topic on which attention will be focused in this section.

There is a great deal of literature available on the cognitive aspects of eliciting probabilities. For many applications, the most useful prior distribution will be in the form of a conjugate prior. Conjugate priors have the property that the prior distribution and the posterior distribution have the same form. As an example, with a binomial likelihood, the conjugate prior distribution is a Beta distribution with parameters \( \alpha \) and \( \beta \). The parameters of the prior distribution are typically referred to as hyperparameters. The posterior distribution in this instance is also a Beta distribution, but with parameters \( \alpha' \), \( \beta' \), where the initial values of \( \alpha \) and \( \beta \) have been updated based on the sample data to \( \alpha' \), \( \beta' \).

In this section, rather than describing specific methods for eliciting a particular prior distribution, we review some of the characteristics of “good” probability elicitation schemes, and provide pointers to more detailed literature.

To begin with, there are several pitfalls to be aware of in elicitation (more on this topic can be found in Hogarth 1975, Wilson 1994, Wright and Ayton 1994, Chaloner 1996, and the other references cited):

1) Availability: Assessors link their probabilities to the frequency with which they can recall an event (Tversky and Kahneman 1974).
2) Adjustment and Anchoring: Judgments are anchored at some starting value and adjust outward, usually insufficiently (Winkler 1967a, b, Alpert and Raiffa 1982).
3) Overconfidence: There is difficulty in assessing the tails of a distribution (Wallsten and Budescu 1983).
4) Conjunction Fallacy: This is usually not applicable to expert elicitation; it occurs when a higher probability is assigned to an event that is a subset of an event with lower probability (Mullin 1986).
5) Hindsight Bias: If the assessor has seen the sample data, the elicited opinion may already have been updated based on the data (Morgan and Henrion 1990).

In light of these problems, the following recommendations for elicitation schemes are found in most of the statistical literature on the subject:

1) Expert opinion is the most worthwhile to elicit.
2) Experts should be asked to assess only observable quantities, conditioning only on covariates (which are also observable) or other observable quantities.

3) Experts should not be asked to estimate moments of a distribution (except possibly the first central moment, which is the mean); they should be asked to assess quantiles or probabilities of the predictive distribution.

4) Frequent feedback should be given to the expert during the elicitation process.

5) Experts should be asked to give assessments both unconditionally and conditionally on hypothetical observed data.

There are two main distinctions that are made in elicitation methods: general vs. application-specific elicitation methods, and predictive vs. structural elicitation (see Kadane and Wolfson 1996). The first distinction deals with whether an elicitation scheme is meant for a class of problems (i.e., is suitable for always eliciting the prior distribution for a linear regression model, for example), or is specific to the application at hand. In Case Study 1, an elicitation scheme is proposed that is useful only for the problem being studied, which happens to have an unusual structure.

Whenever the task of expert elicitation is undertaken, the underlying assumption is that the expert whose opinion is being elicited has some “true” underlying beliefs; the purpose of the (general or application-specific) elicitation method is to extract them. The second distinction deals with how elicitation questions are asked. Questions can be asked structurally, which means that the expert is asked to give numerical values to certain parameters of the model. An example of a question of this type is to ask what the probability of an event is, or to ask for mean and variance of the prior distribution. A method that is often preferred to structural elicitation is predictive elicitation. The reason for the preference is drawn from the list of elicitation recommendations given earlier in this paper; the experts should be asked only about observable quantities. The prior distribution is a distribution on the parameters of the likelihood, and thus is generally not a directly observable quantity. However, if the expert is asked to make assessments about future observable quantities, this is an “observable” quantity that the expert should be able to judge. Consider the following example: in attempting to construct a prior distribution about the effect of exposure to lead pollution, a doctor specializing in this area is consulted. The variable of interest is the level of lead in the bloodstream. Several different “typical patient” scenarios are constructed, identifying characteristics such as socioeconomic status, race, gender, and age. For each patient scenario, the doctor is asked to “predict” a 50th and 75th percentile of how much lead they would expect to see in a blood sample, if they considered the results of many such patients. From this information, the hyperparameters of the prior distribution can be calculated.

The technical details of how this type of predictive elicitation can be done have been worked out as general elicitation methods, and are summarized in detail in Wolfson (1995). Many of the elicitation methods have been developed as computer software programs available on StatLib through the World Wide Web. A brief summary of available methods is given here. Chaloner and Duncan (1983) and Gavaskar (1988) have developed elicitation methods for the hyperparameters of the beta prior distribution for a binomial model; Kadane et al. (1980), Garthwaite and Dickey (1988), and Garthwaite and Dickey (1992) have developed methods for eliciting the hyperparameters of the conjugate prior for the normal linear model; elicitation for the Cox proportional hazards model was written by Chaloner et al. (1993); for the AR(1) time series model by Kadane et al. (1996); for the gamma prior distribution for an exponential model by Wolfson (1995); and for ANOVA models by Laskey and Black (1989).

Quantifying utilities

Making decisions based on maximizing expected utility is synonymous with making decisions that minimize expected loss. Determining expected loss means constructing loss functions that consist of outcomes based on a quantity of interest, Q, and the loss associated with each outcome. The loss can be fixed or random. In general, there is a set A of possible actions that can be taken, and for each element of A, there are losses that may or may not depend on Q.

Lindley (1985) points out that the set A must be exhaustive; in other words, all possible actions must be enumerated and each action must be exclusive of every other action.

Consider the situation where there are two parties involved in the decision-making process. Both parties should be involved in the elicitation of the loss functions, since the ideal solution would be that the loss functions of the two parties coincide. These utilities may be useful in negotiating a compromise and/or providing information about why two parties disagree.

The first stage of the elicitation process should be to choose the action set A and the quantity of interest Q. The set of actions should be well defined and must contain at least two elements. These actions are usually dependent on some quantity of interest. For example, if the action set consists of assigning patients to two different treatments in a clinical trial, the quantity of interest might be some quantification of the state of the patients’ health. In choosing the quantity of interest, it is important to make sure that Q is a quantity that incorporates the uncertainty inherent in the problem being addressed. Generally, Q will be a random quantity defined by a statistical model, and, as such, it can have several dimensions. Attention in this paper is restricted to the case in which Q is one-dimensional.

Once the action set A and the quantity of interest Q have been determined, the second stage of the eliciti-
tation is to construct loss functions based on them. This is the most difficult stage of the elicitation process. Each party will construct these separately and must give careful thought to the consequences resulting from each action in $A$ for any value of $Q$.

In general, there will be losses associated with any action. These losses can be monetary costs, or they can represent the relative benefits, at a particular value of $Q$, of one course of action versus another. Specifying the losses can be tricky; it is often advisable to construct several loss functions for each party. Each loss function represents the values being placed by an individual (or group) on various costs and benefits.

Some examples of the elements of a loss function, particularly those related to health, environmental, and ecological issues, are: monetary costs associated with actions, loss or gain of goodwill, increase in life expectancy, medical problems associated with an action or outcome, quality of life, the value of biodiversity, the value of future resources, and potential threat of litigation. This is not an exhaustive list, but merely an indication of the type of elements to consider.

The loss function can be specified parametrically, rather than by assigning a single specific value at the outset to each of the elements of the loss functions. It is often the case, as demonstrated in this paper, that the optimal decision will depend on the relationship between elements of the loss function rather than the absolute value of the elements. A key point is that loss functions specified in this manner allow the investigation of the relative values of the parameters of the loss function that will give rise to different decisions.

**Case Study 1: Radioactive Groundwater Contamination**

In the area surrounding a landfill site, there has been some concern on the part of various stakeholders (in this case, concerned residents as well as some public interest groups) that radioactive waste had been dumped in the landfill when it was operative, and as a result, the groundwater was contaminated. The Environmental Protection Agency (EPA) did not authorize dumping of radioactive waste in the landfill, but some illicit dumping may have occurred, and residents of the area claim to have seen trucks with radioactive symbols on them dumping waste during the night on several occasions.

To determine if there has indeed been radioactive contamination of the groundwater supply, the EPA authorizes testing that reports whether or not a sample is "above" or "below" a radioactive threshold, but not the amount above or below.

Contamination is considered to have occurred if the proportion of samples above threshold, $p$, is $>20\%$. Otherwise, the proportion of samples above threshold is deemed to be consistent with "ordinary" contamination levels that one might expect to see in a suburban area.

### Table 1: Prior distributions for $p$, conditional on contamination levels, where $p$ is the proportion of samples above a radioactive threshold.

| Contamination level ($\mathcal{X}$) | $\pi(p | \mathcal{X})$ | $E(p | \mathcal{X})$ |
|-------------------------------------|-------------------------|---------------------|
| No contamination (NC)               | $(1 - p)^2 I(0 \leq p < 0.2)$ | 0.09                |
| Contamination (C)                   | $(1 - p)^2 I(0.2 \leq p \leq 1)$ | 0.4                 |

In this study, the EPA and the stakeholders have different prior beliefs about whether or not contamination has occurred, as well as having different utilities for the consequences of the decision to remediate or not remediate.

The question of interest in this example is not so much what the remediation decision should be, but how many samples should be collected in order to make the decision. See Wolfson et al. (1996) for a detailed exposition of this case study.

### Eliciting the prior distribution

The sampling model for the data is the binomial ($n, p$) distribution, since what will be observed is a sequence of Bernoulli trials. The usual conjugate prior for the binomial distribution is the Beta distribution (see DeGroot 1970 for more details on how conjugate priors are chosen; the essential property that needs to be known here is that a conjugate prior is one such that the posterior and prior distributions have the same form). In this case, the prior distribution is conditional on $\mathcal{X}$, the state of nature; $\mathcal{X} = C$ indicates that contamination has occurred, defined as $0.2 \leq p \leq 1.0$, where the parameter $p$ represents the probability that a given sample is above threshold. If contamination has not occurred, then $\mathcal{X} = NC$, and $0 \leq p < 0.2$.

One method for constructing a Beta prior distribution is to combine a prior mean with a prior sample size, and to use numerical methods to find the combination of the parameters $\alpha, \beta$ in solving the equation

$$E_p[X] = \int_0^1 \frac{\Gamma(\alpha, \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \, dx,$$  

subject to the constraint that $\alpha + \beta$ is equal to the prior sample size.

Since the prior distribution is divided into two regions, $\mathcal{X} = NC: 0 \leq p < 0.2$, and $\mathcal{X} = C: 0.2 \leq p \leq 1.0$, the prior distribution on each piece is proportional to a truncated Beta distribution with parameters $\alpha$ and $\beta$ obtained by solving Eq. 1. The expert is asked to assess $E(p | \mathcal{X})$, the prior expectation of what proportion of the sample will show contamination, given the state of nature, as well as a "weight," $\alpha + \beta$, which is their indication of a "prior sample size." The numerical results are shown in Table 1.

To make the prior distribution over the entire range of $p$ integrate to one, weights must be put on each of the two pieces, reflecting the subjective opinion of which state of nature was most likely a priori. $\mathcal{X} = C$ or $\mathcal{X} = NC$. Because $\pi(p | C)$ and $\pi(p | NC)$ differ only
by a constant, then if and only if the weights on the two pieces are \( P(\text{NC}) = \frac{p^*}{2} \) and \( P(C) = \frac{1}{2} \). If the sample is above the detection threshold, then the converse is true for the EPA, resulting in the prior predictive distributions shown in Table 2.

The method just described for constructing the prior distribution is an application-specific method, where structural elicitation (the probabilities on contamination and no contamination) is combined with predictive elicitation (where the expected predictive mean and sample sizes are specified).

The binomial sampling model dictates that a sample of size \( n \) will be collected, summarized in the vector \( X = (X_1, \ldots, X_n) \), where \( X_i = 1 \) if the sample is above the detection threshold, and 0 if the sample is below the detection threshold. Given this sampling model and the prior distributions on contamination of both the stakeholders and the regulatory agency, Bayes’ formula can be used to compute the posterior probability of contamination. The necessary calculations are shown in Appendix B.

Fig. 1 shows that the prior predictive probability distribution of \( X/n = \sum_{i=1}^{n} X_i/n \) will be concentrated in the region \( X/n < 0.2 \) for the EPA, and in the region \( 0.2 \leq X/n \) for the stakeholders.

Eliciting utilities

Suppose that the utilities of the stakeholders (S) are expressed by the following loss function:

\[
L_S(p, \text{not remediating}) = \nu_S, \\
L_S(p, \text{remediating}) = \nu_S(1 - p).
\]

Thus, \( \nu_S \) is the value placed by the stakeholders on the effect of contamination, and \( \nu_S \) is the value they place on public spending by the EPA when it may not be necessary. Thus, the stakeholders would choose to remediate when \( \nu_S > \nu_S \), where \( \nu_S \) is the posterior expectation of \( p \) for the stakeholders, given the collected evidence \( X \).

The EPA's utility can be expressed by a loss function that says there is a fixed loss, \( \nu_E \), when remediation occurs (the cost of the remediation), and there will be a loss proportional to \( p \) when remediation does not occur. The value \( \nu_E \) represents the value placed on underremediation by the EPA:

\[
L_E(p, \text{not remediating}) = \nu_E, \\
L_E(p, \text{remediating}) = \nu_E(p).
\]

The EPA would choose to remediate when \( \nu_E > \nu_E \), which is the EPA's posterior expectation of \( p \) given the data \( X \). Because of the different prior opinions, there is no relationship between \( \nu_S, \nu_S, \nu_E, \nu_E \) that results in the decision made by both parties being the same for all values of \( X/n \).

Making an optimal decision

Lindley and Singpurwalla (1991), Etzioni and Kadane (1993), and Lodh (1993) consider the problem of selecting the optimal sample size in this type of situation, where the two parties have differing prior opinions and different utility functions. Lindley and Singpurwalla (1991) refer to this as an “adversarial” relationship. In the problem just described, the EPA will choose the sample size. It can be shown that the sample size that the EPA would choose, if the decision were being made under \( L_E \), would be substantially less than the one they would choose if the decision were made based on \( L_S \). To illustrate, if the EPA were the only party involved in making the decision, and the cost of sampling were \( \nu_E \) per unit, then the sample size \( n \) would be chosen to minimize the EPA's expected loss.
FIG. 2. Posterior expected value of \( p \), represented by the vertical axis. The horizontal axis represents possible outcomes (x/n). Sample size \( n = 30 \) in the left-hand panel, and \( n = 96 \) in the right-hand panel. If the data collected are highly informative (results in region B or D) then the EPA and Stakeholders will agree on a course of action. When data fall in region C, there will be disagreement. As the sample size increases, the probability of observing data in region C will decrease.

(note that the operators \( P_{\text{EPA}}, E_{\text{EPA}}, P_{\text{S}}, E_{\text{S}} \) denote the posterior probability and expectations of the EPA and the stakeholders respectively):

\[
v_3 n + v_3 P_{\text{EPA}} \left[ \frac{v_1}{v_4} < E_{\text{EPA}}[p \mid X] \right] + v_4 E_{\text{EPA}} \left[ p \cdot P_{\text{EPA}} \left[ \frac{v_2}{v_4} \geq E_{\text{EPA}}[p \mid X] \right] \right].
\]

This "pre-posterior" analysis can also be done from the perspective that the decision about whether or not to remediate after the sample is drawn will be made by the stakeholders instead of the EPA. In that situation, the sample size would be chosen by the EPA to minimize

\[
v_3 n + v_3 P_{\text{EPA}} \left[ \frac{v_1}{v_1 + v_2} < E_{\text{S}}[p \mid X] \right] + v_4 E_{\text{EPA}} \left[ p \cdot P_{\text{EPA}} \left[ \frac{v_2}{v_1 + v_2} \geq E_{\text{S}}[p \mid X] \right] \right].
\]

The sample sizes in Eqs. 4 and 5 can be determined numerically for particular values of \( v_1 \ldots v_5 \). As an example, suppose that the stakeholders choose \( v_2/(v_1 + v_2) = 0.1 \), implying that the loss for underremediation is nine times that of overremediation. Further, suppose that the EPA values the penalty for underremediation at four times the cost of remediation, so that \( v_2/v_4 = 0.25 \), and the cost of a single sample is \( v_5 = 1/4000 \). Then the sample size chosen by the EPA under Eq. 4 would be \( n = 30 \), and the sample size chosen by the EPA under Eq. 5 would be \( n = 96 \). If \( v_2/v_4 = 0.33 \) and \( v_1, v_2, \) and \( v_3 \) remain the same, then under Eq. 4 only one sample would be taken, since, as Fig. 1 shows, the EPA's prior predictive probability that \( x/n > 0.33 \) is very small. Under Eq. 5, the optimal sample size would be \( n = 45 \). If \( v_2/v_4 \) were decreased to 0.2, keeping all other values the same, then the EPA would choose a sample size of \( n = 40 \) under \( L_{\text{EPA}} \), and a sample size of \( n = 116 \) under \( L_{\text{S}} \).

The diagonal curves in Fig. 2 show the posterior expected value of \( p \) for the stakeholders and the EPA for samples of size \( n = 30 \) and \( n = 96 \), over the range of possible values that could be obtained for \( x \). For the stakeholders, this "curve" is actually a straight line, whereas it is a curve for the EPA. The reason is that, given the stakeholders prior opinion, their posterior opinion about \( p \) is that \( p \) increases linearly as a function of the number of samples observed above threshold. For the EPA, the number of samples above threshold does not have as large an influence on their posterior opinion when the data are somewhat uninformative. In the plots, four regions are defined. In region "A," under \( L_{\text{EPA}} \) the stakeholders would conclude that remediation is mandated, but the EPA would not; in region "C," under \( L_{\text{S}} \) both parties would agree that remediation is mandated. In regions "B" and "D," both parties would reach the same decision regardless of whose utility is being used. As the sample size increases, the proportion of samples above threshold that would lead to disagreement by putting a decision maker in the indeterminate region decreases. As the sample size increases, the proportion of samples above threshold that would lead to disagreement by putting a decision maker in the indeterminate region decreases. Table 3 gives exact numbers, which, summarized, imply that with a sample of size \( n = 30 \), 27% of the possible outcomes will put the decision maker in the indeterminate region; with a sample of size \( n = 96 \), only 19% of the possible outcomes will do this. As the sample size gets larger, the area of regions "A" and "C" decreases.

This analysis could be carried out for other values of \( v_1 \ldots v_5 \), but the essential result is that as long as

\[
\frac{v_1}{v_4} \geq \frac{v_2}{v_1 + v_2},
\]

be
TABLE 3. Number of samples required to be above threshold. Each cell represents the minimum number of samples that would have to be above threshold for the decision to be made to remediate under given loss function and prior.

<table>
<thead>
<tr>
<th>Loss</th>
<th>Prior</th>
<th>EPA</th>
<th>Stakeholders</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{EPA}} )</td>
<td>11</td>
<td>8</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>24</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>( L_{\text{S}} )</td>
<td>3</td>
<td>3</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>10</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

then the sample size that the EPA would choose under its own loss functions would be less than the sample size chosen under the stakeholder’s utilities. Unless the samples are highly informative, in other words, they fall in region “B” or “D,” there is no decision that will satisfy the utilities of both the EPA and the stakeholders, given their different prior opinions.

CASE STUDY 2: SOIL LEAD CONTAMINATION

Sites designated to be “Superfund” sites in the United States are required to have a phased program of investigation, design, and remedial action. There are a number of former battery-recycling facilities that have been designated as Superfund sites due to the excessive soil lead concentrations detected in the areas immediately surrounding them. This case study is a former battery recycling facility that has been closed for a number of years. The full details are reported in Small et al. (1995) and L. J. Wolfson, J. B. Kadane, and M. J. Small (unpublished manuscript).

At issue here is the question of how to allocate responsibility for the high soil lead concentrations as a function of distance from the plant. Soil lead concentrations in residential urban and suburban areas usually range from 50 to 1000 mg/kg soil (Lovering 1976, Nriagu 1978), but the EPA policy at the time of the study was that soil lead concentrations >500 mg/kg are dangerous and in need of remediation (EPA 1991). As one gets farther away from the plant, the current owners of the plant (hereafter referred to as the “principally responsible parties” or PRP), argue that their responsibility for remediation decreases.

Fig. 3 shows the data collected for this study. Data set I is all sample sites, including those where the PRP had agreed to take responsibility for remediation prior to this study; data set II is the subset of data set I containing the soil lead concentrations where responsibility for remediation is questionable.

The problem in modeling the lead concentrations is what is known in statistics as a latent variable problem, because total lead (L) is the sum of lead concentrations contributed by diffuse and localized “background” (B) sources, such as lead-based paint on homes, historic vehicle emissions, and fuel and trash burning and disposal, as well as the “plant” (P) from when the battery-recycling facility is in operation. Some additional covariate information available for data set II helps to distinguish between background and plant contributions. L. J. Wolfson, J. B. Kadane, and M. J. Small (unpublished manuscript) provide a full description of the covariates. Kadane (1974) shows that the latent variable problem can often be addressed by an informative prior distribution. In this case study, the ap-
The approach taken is to combine the prior distribution with covariate allocation.

Aside from the question of how to model the soil lead concentrations, there is the issue of how to use that model, along with decision theoretic tools, to negotiate a determination of when the PRP is responsible for remediation.

**Incorporating prior information**

The model used for soil lead concentrations (L. J. Wolfson, J. B. Kadane, and M. J. Small, unpublished manuscript) is a gamma regression model with parameters \( \mu_L = \mu_B + \mu_P \) and \( \beta_L \). Some of the technical details of the model specification are given in Appendix C. The parameters \( \mu_B \) and \( \mu_P \) represent the mean lead concentrations contributed by background sources and the former battery-recycling plant, respectively. The available covariates (distance from the plant, and indicator variables for the presence of lead-based paint) are allocated as best describing either background or plant contributions.

In this instance, part of the prior is constructed from other available data. This is often referred to as an empirical prior (not to be confused with Empirical Bayes methods; see O'Hagan 1994). A reference prior is placed on \( \mu_P \), and a joint prior on \( (\mu_B, \beta_L) \) is constructed from data set III, which is a data set of soil lead concentrations contributed by background sources and the former battery-recycling plant, respectively. The available covariates (distance from the plant, and indicator variables for the presence of lead-based paint) are allocated as best describing either background or plant contributions.

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L. J. Wolfson, J. B. Kadane, and M. J. Small (unpublished manuscript) use the Laplace approximation (Tierney and Kadane 1986, Kass et al. 1989, Tierney et al. 1989) to find the posterior distribution of the model, given the prior distribution and the likelihood. Other covariates were also available for data set II and Bayesian model selection was performed; the reader is referred to the paper for further details.

**Remediation decisions**

Let the quantity of interest be

\[ R = \frac{B}{B + P}. \]

so that \( R \) represents the proportion of background lead that, in some sense, indicates the degree to which the plant is not responsible for exceeding the level at which remediation is mandated. Consider the situation in which, from the perspective of the PRP, there are three components in the loss function: dollars, goodwill, and lawsuit risk. If the PRP remediates, there is a fixed monetary cost; call it \( m \). If it does not remEDIATE when responsible, it loses goodwill in the community and opens itself to potential lawsuits. Call this loss \( g \). If it does not remEDIATE when it is not responsible, it loses nothing. Defining \( q \) as a threshold value for \( R \) that indicates whether or not the plant is responsible, the corresponding loss function \( L'_{\text{PRP}} \) is obtained as

\[
L'_{\text{PRP}}(R, \text{no remedia} \text{tion}) = \begin{cases} 0 & \text{if } R \geq q \\ g & \text{if } R < q \end{cases}
\]

\[
L'_{\text{PRP}}(R, \text{remedia} \text{tion}) = m.
\]  

(7)

\( R \) is different for each home, but \( m \), \( g \), and \( q \) are set to be the same for all homes. The action that minimizes expected loss is to remEDIATE when

\[ \frac{m}{g} < P(R \leq q). \]

A loss function from the perspective of the EPA looks somewhat different. Suppose that the EPA's belief is that there is no loss to the public if remediation does not occur when the proportion of background lead is high, but there is a loss when remediation does not occur and the proportion of background lead is low. This loss, \( h \), can be viewed as the costs associated with underremediation (i.e., when homes are not remediated, but should be), such as increased plant-imposed health risks in the surrounding community, since the EPA is charged with protecting the public interests. When the action chosen is remEDIATE, then the EPA experiences no losses if the PRP remediates and the proportion of background lead to total lead is below the cutoff point \( q \), since the PRP is then remediating when it is indeed responsible. If, however, the PRP is remediating when the proportion of background lead to total lead is high, and, thus, is above the cutoff point \( q \), then the EPA's relationship with the PRP is damaged, and the cost associated with forcing the PRP to overremediate is \( k \).

Since the EPA is not going to bear a direct financial burden, a plausible loss function might be

\[
L'_{\text{EPA}}(R, \text{not remedia} \text{tion}) = \begin{cases} 0 & \text{if } R \geq q \\ h & \text{otherwise} \end{cases}
\]

\[
L'_{\text{EPA}}(R, \text{remedia} \text{tion}) = \begin{cases} 0 & \text{if } R < q \\ k & \text{otherwise} \end{cases}
\]

(8)

which implies that the action that minimizes expected loss is to remEDIATE when

\[ \frac{k}{h} \leq \frac{P(R \leq q)}{1 - P(R \leq q)}. \]

Once again, the values for \( k \) and \( h \) need not be assessed directly, since it is their relative values that are of import. The loss functions \( L'_{\text{EPA}} \) and \( L'_{\text{PRP}} \) provide simple, yet plausible, loss representations for both the tangible and intangible policy factors of the decision-making process under uncertainty.
When Eq. 9 is not satisfied, disagreements arise on the appropriate extent of remediation. Typically, $mg$ exceeds $k/(k + h)$, with the result that the EPA wishes to extend the area of remediation farther from the site than is desired by the PRP.

**DISCUSSION**

The two case studies in this paper illustrate the implications and the use of combining information for environmental (or ecological) decision making. The use and quantification of both subjective and empirical prior information are demonstrated, as are some of the different ways in which loss functions can be utilized in decision making. In the soil lead contamination example, the PRP and the EPA agreed to a single prior distribution, based on empirical data. Despite having different utilities, it was possible for them to find a compromise. In the radioactive groundwater contamination example, the different prior distributions, combined with the different utilities, leave no room for an a priori compromise. If the data collected are highly informative, then an a posteriori compromise may be reached. It is useful that the loss functions for the two competing parties can be directly compared in the initial formulation.

Two parts to the decision-making process are presented in this paper. The first part is to settle on relative values of some intangible quantities, such as perceived risk, goodwill, etc. The second part of the decision-making process is to use those relative values to make a decision. For the first part, it is convenient for the loss functions of the two parties to be of a form that allows one to express a relationship between the utility quantities of each party in the negotiation process, such as in Eq 9. The decision-making process can be an iterative process. Once a relationship between the intangible quantities has been developed and relative values have been agreed upon, a more representative loss function can be employed.

The advantage of taking an approach of this type is twofold. First, it forces analysts to be explicit about their preconceptions. This can be a very valuable process, since quantifying prior beliefs and utilities may point out misconceptions. This can also be an iterative process, in which assessments are refined. This approach can provide a way to do some analysis prior to collecting data and may, in fact, guide the collection of data, as in the first case study.

When presenting the results of a policy decision to the public, such as when to remediate, it may facilitate the explanation to have quantified one’s prior probabilities and utilities. This type of method is recommended for any type of decision-making problem.

Analysis of the data for the two case studies presented herein used many methods, technical details of which have been omitted from this paper. The reader who is interested in the latest tools for Bayesian data analysis is referred to Gelman et al. (1995).
LITERATURE CITED


Appendix A
Bayesian Calculations

Suppose that the prior distribution is \( \pi(\theta \mid \psi) \), where \( \psi \) represents the parameters of the prior distribution, known as the hyperparameters. Then the likelihood is given by \( p(y \mid \theta) \), the range of the data \( y \) is given by \( \Omega \), and the posterior distribution is obtained by updating the prior with the likelihood as shown:

\[
p(\theta \mid y) \quad \text{Prior distribution} \\
p(y \mid \theta) \quad \text{Likelihood} \\
p(\theta \mid y, \psi) = \frac{p(\theta \mid \psi)p(y \mid \theta)}{\int_{\Theta} p(\theta \mid \psi)p(y \mid \theta) \, d\theta} \\
\]

Posterior distribution. \( \text{(A3)} \)

Now, suppose that a utility structure has been specified in the following fashion: for each possible action \( a \), a loss function \( L(\theta, a) \) is specified, reflecting the utility of the possible actions. Then the "optimal" decision, in terms of minimizing the expected posterior loss, is to choose the action \( a \) that minimizes

\[
\int_{\Theta} L(\theta, a) \pi(\theta \mid y, \psi) \, d\theta. \quad \text{(A4)}
\]

Appendix B
Posterior Calculations for Case Study 1

To find the posterior probability of viewing a specific value of the evidence, \( X = x/n = \Sigma_{i=1}^{n} X/n \), the following calculations are needed:

\[
P(C \mid X) = P(0.2 \leq p \leq 1 \mid X) = \frac{P(X \mid C)P(C)}{P(X)}
\]

where for \( \mathcal{X} \in \{ \text{C, NC} \} \), \( \alpha = 1 \), \( \beta = 3 \),

\[
P(X \mid p, \mathcal{X}) \sim \text{Binomial}(n, p)
\]

\[
P(p \mid \mathcal{X}) = \frac{p^{n+\alpha-1}(1-p)^{n+\beta-1}}{\int_{\mathcal{X}} p^{n+\alpha-1}(1-p)^{n+\beta-1} \, dp}
\]

and \( x \) is the number of the \( n \) samples that are above threshold.

Appendix C
Model Specification for Case Study 2

The model for lead concentrations is a gamma regression model with parameters \( \mu_\alpha, \beta_\alpha \), where

\[
\mu_\alpha = \mu_\gamma + \beta_\gamma
\]

where

\[
\mu_\gamma = \exp(\theta_1 + \theta_2 X' + \theta_3 Y' + \theta_4 X^2 + \theta_5 Y^2) + \theta_6 E
\]

\[
\beta_\gamma = \theta_8 + \theta_9 CO
\]

where \( X', Y' \) are the locations of the sample sites, centered at the plant, \( CO \) is an indicator for historical lead-based paint, and \( E \) is an indicator for enhanced depositions near structures.

The probability density function (pdf) is then

\[
f_{\alpha}(l) = \frac{1}{\Gamma(\mu_{\gamma}/\beta)} e^{-\mu_{\gamma}/\beta} \exp \left[ -l/\beta \right]. \quad \text{(A6)}
\]

To obtain the prior on \( \mu_\gamma, \beta_\gamma \), or equivalently on \( \theta_\mu, \theta_\gamma \), and \( \beta_\gamma \), a gamma regression model with parameters \( \mu = \phi_1 + \phi_2 CO, \phi_\beta \), and a reference prior is fit to Data set III. Estimates of the posterior modes of these parameters can be obtained, which, in this case, are exactly maximum likelihood estimates. These modes are used as prior means, and the inverse observed Fisher information is used as an estimate of the variance–covariance matrix for a multivariate normal prior distribution:

\[
f(\beta, \theta_\mu, \theta_\gamma \mid \mu, \Sigma)
\]

where

\[
\Sigma = \begin{pmatrix} 45^2 & 783 & 161 \\ 783 & 28^2 & -436 \\ 161 & -436 & 110^2 \end{pmatrix}
\]

\( (A7) \)