The Demand for Liquid Assets: Evidence from Stochastic Volatility Demand Systems

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Research Question

- What is the price elasticity of demand for liquid assets in the United States?

- What is the degree of substitutability among monetary assets?
Motivation

- Why study demand for money?
  - Plays a major role in macroeconomic analysis;
  - Selects appropriate monetary policy action.

- Why study the substitutability?
  - Simple-sum approach to monetary aggregates:
    \[ M_t = \sum_{i=1}^{n} x_{it} \]
  - Views all assets as perfect substitutes;
  - Not suitable given complex structures of modern financial markets;
  - Defective monetary data misleads macroeconomic policy making.
Related Literature

**Time Series Approach**

- Cointegration Analysis
  - Johansen cointegration test or Engle-Granger procedure;
  - Variables: Money demand (M1, M2, Currency), real GDP, T-bill rate, inflation rate, investments.
  - Sriram (2001) survey paper;
- Limitation: Ad hoc and atheoretical.
Related Literature

Flexible Functional Forms Approach

- Estimate elasticities based on a demand system
  - Derived from neoclassical microeconomic theory.

However,

- Derived from theory ≠ Consistent with the theory

  - Theoretical regularity: **Positivity, Monotonicity, Curvature**.

- Typical assumption of homoscedasticity.
A summary of flexible functional forms estimation of monetary assets demand:

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Model used</th>
<th>Curvature imposed</th>
<th>Homoscedasticity assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barnett (1983)</td>
<td>Minflex Laurent</td>
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<td>Ewis and Fisher (1985)</td>
<td>Fourier</td>
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<td>Serletis and Robb (1986)</td>
<td>Translog</td>
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<td>Translog</td>
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<td>Fleissig (1997)</td>
<td>Minflex, GL, Translog</td>
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<td>Fleissig and Swofford (1997)</td>
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<td>Drake, Fleissig, and Mullineux (1999)</td>
<td>AIM</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Fleissig and Serletis (2002)</td>
<td>Fourier</td>
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<td>Drake, Fleissig, and Swofford (2003)</td>
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<td>✓</td>
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<tr>
<td>Drake and Fleissig (2004)</td>
<td>Fourier</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Serletis and Shahmoradi (2005)</td>
<td>AIM and Fourier</td>
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<td>Serletis and Shahmoradi (2007)</td>
<td>GL, BTL, AIDS, Minflex, NQ</td>
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<td>Drake and Fleissig (2010)</td>
<td>Fourier</td>
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<td>✓</td>
</tr>
<tr>
<td>Serletis and Feng (2010)</td>
<td>AIM</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Our Approach

- Apply four flexible functional forms, the basic Translog (BTL), AIDS, QUAIDS, and Minflex Laurent model;

- Pay attention to theoretical regularity conditions: check and impose the conditions if they are not satisfied;

- Model the covariance matrix of the errors to be a BEKK GARCH (1,1);

- Consider three monetary assets: M1, Savings Deposits, Time Deposits;

Summary Results

- Only the curvature constrained Minflex Laurent model satisfies all three theoretical regularity conditions at all data points;
- Evidence shows heteroscedasticity and significant ARCH effects;
- Only the curvature constrained Minflex Laurent with the BEKK specification is able to generate results consistent with both economic and econometric regularity;
- The demand for money in the United States is inelastic and the monetary assets are Morishima substitutes.
Contributions

- Provide empirical inferences about money demand consistent with economic theory;

- Incorporate heteroscedastic error structures in the demand analysis of monetary assets.
Outline

1. The AIDS, QUAIDS, BTL, and Minflex Laurent model;
2. Regularity conditions & Ways to check;
3. A method to impose curvature conditions;
4. A method to impose the BEKK GARCH(1,1) specification;
5. Data;
6. Results;
7. Conclusion.
The Almost Ideal Demand System (AIDS)
Deaton and Muellbauer (1980)

■ The AIDS Indirect Utility Function:

\[ h(p, y) = \left[ \ln y - \ln a(p) \right] \beta_0^{-1} \prod_{i=1}^{n} (p_i)^{-\beta_i} \]

where,

\[ \ln a(p) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j \]

\[ \ln b(p) = \ln a(p) + \beta_0 \prod_{i=1}^{n} (p_i)^{\beta_i} \]

■ The AIDS Budget Share Form:

\[ s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[ \frac{y}{a(p)} \right] \]
The AIDS Model
Deaton and Muellbauer (1980)

- Restrictions:
  - Symmetry Condition: $\gamma_{ij} = \gamma_{ji}$
  - Adding-up Condition: $\sum_{i=1}^{n} \alpha_{i} = 1$
  - Homogeneity Condition: $\sum_{i=1}^{n} \beta_{i} = \sum_{i=1}^{n} \gamma_{ij} = \sum_{j=1}^{n} \gamma_{ij} = 0$.
  - $(n^2 + 3n - 2)/2$ free parameters.
The Quadratic AIDS Model (QUAIDS)
Banks et al. (1997)

- The QUAIDS Indirect Utility Function:

\[ \ln h(p, y) = \left\{ \left[ \frac{\ln y - \ln a(p)}{b(p)} \right]^{-1} + \lambda(p) \right\}^{-1} \]

where,

\[ \lambda(p) = \sum_{i=1}^{n} \lambda_i \ln p_i, \text{ where } \sum_{i=1}^{n} \lambda_i = 0 \]

- The QUAIDS Budget Share Form:

\[ s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln \left[ \frac{y}{a(p)} \right] + \frac{\lambda_i}{b(p)} \left\{ \ln \left[ \frac{y}{a(p)} \right] \right\}^2 \]

\( (n^2 + 5n - 4)/2 \) free parameters.
The Basic Translog Model (BTL)
Christensen *et al*. (1975)

- **The BTL Reciprocal Indirect Utility Function:**

\[
\log h(\mathbf{v}) = a_0 + \sum_{i=1}^{n} a_i \log v_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \log v_i v_j
\]

where, \( v_i = p_i / y \).

- **The BTL Budget Share Form:**

\[
s_{i} = \frac{a_{i} + \sum_{j=1}^{n} \beta_{ij} \log v_{j}}{\sum_{j=1}^{n} a_{j} + \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{jk} \log v_{k}}
\]

\((n^2 + 3n - 2)/2\) free parameters.
The Minflex Laurent Model
Barnett (1983)

- The Minflex Laurent Reciprocal Indirect Utility Function:

\[ h(v) = a_0 + 2 \sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n} a_{ii} v_i + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} v_i^{1/2} v_j^{1/2} - \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} v_i^{-1/2} v_j^{-1/2}. \]

where, \( v_i = p_i / y \).

- The Minflex Laurent Budget Share Form:

\[ s_i = \frac{a_i v_i^{1/2} + a_{ii} v_i + \sum_{j=1}^{n} a_{ij} v_i^{1/2} v_j^{1/2} + \sum_{j=1}^{n} b_{ij} v_i^{-1/2} v_j^{-1/2}}{\sum_{i=1}^{n} a_i v_i^{1/2} + \sum_{i=1}^{n} a_{ii} v_i + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} v_i^{1/2} v_j^{1/2} + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} v_i^{-1/2} v_j^{-1/2}}. \]
The Minflex Laurent Model
Barnett (1983)

Restrictions:

\[ a_{ij}b_{ij} = 0 \quad \forall i, j ; \quad \sum_{i=1}^{n} a_{ii} + 2 \sum_{i=1}^{n} a_i + \sum_{j=1}^{n} a_{ij}^2 - \sum_{j=1}^{n} b_{ij}^2 = 1. \]

\((n^2 + 3n)/2\) free parameters;
Theoretical Regularity Conditions

- **Positivity** is satisfied, if predicted budget shares are nonnegative;

- **Monotonicity** is satisfied, if estimated indirect utility is negatively related to the prices and positively related to income;

- **Curvature** is satisfied, if the estimated Slutsky matrix is negative semidefinite.
Impose Curvature Conditions  
Ryan and Wales (1998)

At a reference point \((x^*)\), the \(n \times n\) Slutsky matrix \(S\) of \(f(x)\):

\[
S = B + C
\]

\(B:\) \(n \times n\) symmetric matrix of the parameters associated with the independent variables (prices);

\(C:\) \(n \times n\) symmetric matrix of functions of all other parameters in the model.

- They show \(S = (-KK')\), where \(K\) is an \(n \times n\) lower triangular matrix so that \((-KK')\) by construction a negative semidefinite matrix;

- Then, \(B = -KK' - C\);

- Reparameterize the model by estimating the parameters in \(K\) and \(C\).
Impose Curvature Conditions

- Impose local curvature condition on AIDS (Ryan and Wales 1998):

  - The $ij$th element of the AIDS Slutsky matrix is:
    \[
    S_{ij} = \gamma_{ij} - (\alpha_i - \beta_i \alpha_0)\delta_{ij} + (\alpha_j - \beta_j \alpha_0)(\alpha_i - \beta_i \alpha_0) - \beta_i \beta_j \alpha_0
    \]

  - The $ij$th element of $B$ is:
    \[
    \gamma_{ij} = -(KK')_{ij} + (\alpha_i - \beta_i \alpha_0)\delta_{ij} - (\alpha_j - \beta_j \alpha_0)(\alpha_i - \beta_i \alpha_0) + \beta_i \beta_j \alpha_0
    \]

- Imposing global curvature condition (Barnett 1983):
  Replace each unsquared parameter by a squared parameter.
Impose Heteroscedasticity
Serletis and Isakin (2014)

Estimating the models of the form:

\[ s_t = g(p_t, y_t, \vartheta) + u_t \]

where \( s = (s_1, s_2, s_3)' \);

\[ g(p_t, y_t, \vartheta) = (g_1(p_t, y_t, \vartheta), g_2(p_t, y_t, \vartheta), g_3(p_t, y_t, \vartheta))' \];

\( g_i(p_t, y_t, \vartheta) \) is given by the RHS of each model;

\( \vartheta \) is the parameter vector.

- Shares sum to 1 $\Rightarrow$ Disturbance covariance matrix is singular
  $\Rightarrow$ Drop the last share equation in each model;

- Typically assume: \( u_t \sim N(0, \Omega) \) and \( u_t = (u_{1t}, u_{2t}, u_{3t})' \) $\Rightarrow$
Impose Heteroscedasticity
Serletis and Isakin (2014)

Assume heteroscedasticity:

\[ u_t | I_{t-1} \sim N(0, \Omega_t) \]

- To avoid singularity, consider \((n - 1) \times (n - 1)\) covariance matrix \(\Phi_t\) with a Baba, Engle, Kraft, and Kroner (BEKK) GARCH(1,1) with \(K=1\) representation:

\[
\Phi_t = C' C + B' \Phi_{t-1} B + A' u_{t-1} u'_{t-1} A
\]

- \((3n^2-n)/2\) free parameters.

Example
Data

- Monthly data on monetary asset quantities and their user costs from the Center of Financial Stability from 1967:1 to 2015:3 (579 obs);

- Three monetary subaggregates:
  - M1 (A): Currency, Traveler’s Checks, Demand Deposits, and Other Checkable Deposits
  - Savings Deposits (B)
  - Time Deposits (C)

- Real per capita asset quantities: U.S. CPI and total population;

- Divisia user cost indices for the associated subaggregates.
Empirical Results

I. Estimate all four models:

<table>
<thead>
<tr>
<th></th>
<th>BTL</th>
<th>AIDS</th>
<th>QUAIDS</th>
<th>Minflex Laurent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positivity violations</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Monotonicity violations</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Curvature violations</td>
<td>576</td>
<td>551</td>
<td>579</td>
<td>579</td>
</tr>
<tr>
<td>Log $L$</td>
<td>2169.537</td>
<td>2152.652</td>
<td>2217.764</td>
<td>2205.130</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LM Test ($\chi^2$)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\widehat{u}_1$</td>
<td>143.696</td>
<td>160.615</td>
<td>117.382</td>
<td>140.229</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\widehat{u}_2$</td>
<td>111.736</td>
<td>114.595</td>
<td>96.332</td>
<td>120.528</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>
Empirical Results

II. Estimate the curvature-constrained models:

<table>
<thead>
<tr>
<th>Models</th>
<th>Positivity violations</th>
<th>Monotonicity violations</th>
<th>Curvature violations</th>
<th>Log $L$</th>
<th>LM Test ($\chi^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BTL</td>
<td>0</td>
<td>0</td>
<td>$576 \rightarrow 523$</td>
<td>2147.473</td>
<td>$\hat{u}_1$ = 136.237 (0.000)</td>
</tr>
<tr>
<td>AIDS</td>
<td>0</td>
<td>0</td>
<td>$551 \rightarrow 495$</td>
<td>2061.007</td>
<td>$\hat{u}_1$ = 138.757 (0.000)</td>
</tr>
<tr>
<td>QUAIDS</td>
<td>0</td>
<td>0</td>
<td>$579 \rightarrow 321$</td>
<td>2199.800</td>
<td>$\hat{u}_1$ = 158.345 (0.000)</td>
</tr>
<tr>
<td>Minflex Laurent</td>
<td>0</td>
<td>0</td>
<td>$579 \rightarrow 0$</td>
<td>1996.38</td>
<td>$\hat{u}_1$ = 150.920 (0.000)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>Positivity violations</th>
<th>Monotonicity violations</th>
<th>Curvature violations</th>
<th>Log $L$</th>
<th>LM Test ($\chi^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BTL</td>
<td>115.223</td>
<td>126.130</td>
<td>$579 \rightarrow 321$</td>
<td>154.473</td>
<td>$\hat{u}_2$ = 154.73 (0.000)</td>
</tr>
<tr>
<td>AIDS</td>
<td>115.223</td>
<td>126.130</td>
<td>$579 \rightarrow 321$</td>
<td>154.473</td>
<td>$\hat{u}_2$ = 154.73 (0.000)</td>
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<td>$579 \rightarrow 321$</td>
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<td>$\hat{u}_2$ = 154.73 (0.000)</td>
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Empirical Results

III. Estimate the curvature-constrained models with the BEKK specification:

<table>
<thead>
<tr>
<th></th>
<th>BTL</th>
<th>AIDS</th>
<th>QUAIDS</th>
<th>Minflex Laurent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positivity violations</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Monotonicity violations</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Curvature violations</td>
<td>537</td>
<td>437</td>
<td>403</td>
<td>0</td>
</tr>
</tbody>
</table>

Log $L$                     | 2939.615  | 2986.993  | 2977.930  | 2852.363
Empirical Results

IV. Get the elasticities at the mean of the data based on the curvature-constrained Minflex Laurent model with the BEKK specification:

<table>
<thead>
<tr>
<th>Subaggregate</th>
<th>Income $\eta_i$</th>
<th>Own- and cross-price $\eta_iA$</th>
<th>$\eta_iB$</th>
<th>$\eta_iC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (A)</td>
<td>1.171 (0.000)</td>
<td>-0.988 (0.000)</td>
<td>-0.154 (0.000)</td>
<td>-0.028 (0.000)</td>
</tr>
<tr>
<td>Savings Deposits (B)</td>
<td>0.837 (0.000)</td>
<td>-0.177 (0.000)</td>
<td>-0.639 (0.000)</td>
<td>-0.020 (0.000)</td>
</tr>
<tr>
<td>Time Deposits (C)</td>
<td>1.304 (0.000)</td>
<td>-0.800 (0.000)</td>
<td>-0.488 (0.000)</td>
<td>-1.015 (0.000)</td>
</tr>
</tbody>
</table>

Notes: Numbers in the parentheses are $p$-values.

Standard errors are calculated using the Delta method.
V. Get Allen elasticities of substitution based on the curvature-constrained Minflex Laurent model with the BEKK specification:

<table>
<thead>
<tr>
<th>Subaggregate ( i )</th>
<th>( \sigma_{iA}^a )</th>
<th>( \sigma_{iB}^a )</th>
<th>( \sigma_{iC}^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (A)</td>
<td>-1.944 (0.000)</td>
<td>-0.409 (0.000)</td>
<td>-0.247 (0.000)</td>
</tr>
<tr>
<td>Savings Deposits (B)</td>
<td>-1.697 (0.000)</td>
<td>-0.173 (0.000)</td>
<td></td>
</tr>
<tr>
<td>Time Deposits (C)</td>
<td></td>
<td>-8.840 (0.000)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Numbers in the parentheses are \( p \)-values.
Empirical Results

VI. Get Morishima elasticities of substitution based on the curvature-constrained Minflex Laurent model with the BEKK specification:

<table>
<thead>
<tr>
<th>Subaggregate</th>
<th>( \sigma_{iA}^m )</th>
<th>( \sigma_{iB}^m )</th>
<th>( \sigma_{iC}^m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (A)</td>
<td>0.811 (0.000)</td>
<td>0.188 (0.000)</td>
<td></td>
</tr>
<tr>
<td>Savings Deposits (B)</td>
<td>0.485 (0.000)</td>
<td></td>
<td>0.152 (0.000)</td>
</tr>
<tr>
<td>Time Deposits (C)</td>
<td>0.987 (0.000)</td>
<td>0.995 (0.000)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Numbers in the parentheses are \( p \)-values.
Conclusion

- Only the curvature constrained Minflex Laurent model satisfies economic regularity;

- Evidence shows heteroscedasticity and significant ARCH effects in all four models;

- Only the curvature constrained Minflex Laurent with the BEKK specification is able to generate results consistent with both economic and econometric regularity;

- The demand for money in the United States is inelastic (except for time deposits) and the monetary assets are Morishima substitutes.

- It suggests:
  
  - Heteroscedasticity is a superior assumption that needs to be used in future demand studies;
  
  - The simple-sum approach to monetary aggregation is inappropriate and a better aggregation method is needed.
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July 23, 2015
For 2 goods, the restrictions on the $\gamma_{ij}$ terms:

$$\gamma_{11} = -k_{11}^2 + \alpha_1 - \beta_1 \alpha_0 - (\alpha_1 - \beta_1 \alpha_0)^2 + \beta_1^2 \alpha_0;$$

$$\gamma_{22} = -k_{12}^2 - k_{22}^2 + \alpha_2 - \beta_2 \alpha_0 - (\alpha_2 - \beta_2 \alpha_0)^2 + \beta_2^2 \alpha_0;$$

$$\gamma_{12} = \gamma_{21} = -k_{12} k_{12} - (\alpha_1 - \beta_1 \alpha_0) (\alpha_2 - \beta_2 \alpha_0)^2 + \beta_1 \beta_2 \alpha_0 .$$

where the $k_{ij}$ terms are the elements of the $K$ matrix.
In our case of 3 goods:

\[ h_{11,t} = c_{11}^2 + b_{11}^2 h_{11,t-1} + 2b_{11}b_{21} h_{12,t-1} + b_{21}^2 h_{22,t-1} \\
+ a_{11}^2 u_{1,t-1}^2 + 2a_{11}a_{21} u_{1,t-1} u_{2,t-1} + a_{21}^2 u_{2,t-1}^2 \]

\[ h_{12,t} = c_{11} c_{12} + b_{11} b_{21} h_{11,t-1} + (b_{11} b_{22} + b_{12} b_{21}) h_{12,t-1} + b_{21} b_{22} h_{22,t-1} \\
+ a_{11} a_{12} u_{1,t-1}^2 + (a_{11} a_{22} + a_{12} a_{21}) u_{1,t-1} u_{2,t-1} + a_{21} a_{22} u_{2,t-1}^2 \]

\[ h_{22,t} = c_{12}^2 + c_{22}^2 + b_{12}^2 h_{11,t-1} + 2b_{12} b_{22} h_{12,t-1} + b_{22}^2 h_{22,t-1} \\
+ a_{12}^2 u_{1,t-1}^2 + 2a_{12} a_{22} u_{1,t-1} u_{2,t-1} + a_{22}^2 u_{2,t-1}^2 \]
Flexible Functional Forms

Let \( f(x) \) be a \( n \)-argument, twice continuously differentiable function

- e.g. an indirect utility or expenditure function

\( f(x) \) is a flexible functional form if it has enough free parameters to satisfy:

- \( f(x^*) = f^*(x^*) \)
- \( \nabla f(x^*) = \nabla f^*(x^*) \)
- \( \nabla^2 f(x^*) = \nabla^2 f^*(x^*) \)

where \( x^* \) is the value of \( x \) at an arbitrary point;

\( f^* \) is the value of \( f \) at the arbitrary point.
Flexible Functional Forms

If $f(x)$ is an indirect utility or expenditure function and also a flexible functional form:

- Apply Roy’s identity (or Shepherd’s lemma) to obtain demand system in terms of budget shares;
- Estimate demand system and use estimated parameters to calculate elasticities.
- Usefulness: few restrictions on the model & based on underlying theories.