Real-Time Forecasting with a Large, Mixed Frequency, Bayesian VAR

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Problem

Key issues to forecasting in practice

• many variables released
• at varying frequencies
• with publication lags
• at different times during the quarter
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How to postulate a model that combines the information in the data in the best way given the restrictions?
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How to postulate a model that combines the information in the data in the best way given the restrictions?

Yet, in addition, allows for a general framework, that enables us

- to construct forecasts conditional on “appropriate monetary policy” (i.e. condition on the path of the funds rate)
- to construct impulse response functions and density forecasts
- to explain “why” the forecasts changed since ...
Problem: it is Hard!

From econometric point of view

- parsimonious models usually perform better: tradeoff between over-fitting and out-of-sample performance
- included variables should be “worth” the estimation risk
Problem: it is Hard!

From econometric point of view

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Resolutions

• autoregressive models
• VARs in common low (quarterly) frequency
• factor models: Stock & Watson (2002), Giannone, Reichlin & Small (2008), Aruoba, Diebold & Scotti (2009), etc.
• MIDAS models: Ghysels, Santa Clara & Volkanov (2004), etc.
• VARs: Bańbura, Giannone & Reichlin (2010), Schorfheide & Song (in press), Foroni, Guérin & Marcellino (in press), etc.
Our Resolution

Take the VAR route

- Define a VAR that allows for monthly and quarterly data

- Monthly variables are treated at a quarterly frequency - blocking, stacking (Chen, Anderson, Deistler, Filler, 2011)

- Impose certain restrictions on the VAR consistent with the timing of data releases
Our Resolution

Take the VAR route

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Similar to Ghysels (in press), but in out-of-sample, large VAR and used in real-time environment while relying on shrinkage
What is Blocking? An Example 1

- Suppose 2 series: quarterly GDP growth \( \{y_t\} \) and monthly UR \( \{x_{t-2/3}, x_{t-1/3}, x_t\} \)
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- Let \( W_t = [x_{t-2/3}, x_{t-1/3}, x_t, y_t]' \)
What is Blocking? An Example 1

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- Let \( W_t = [x_{t-2/3}, x_{t-1/3}, x_t, y_t]' \)
- Specify the VAR

\[
A_0 W_t = C + \sum_{\ell=1}^{4} A_{\ell} W_{t-\ell} + e_t
\]

- \( E(e_t|W_1, ..., W_{t-1}) = 0 \)
- \( E(e_t e'_t|W_1, ..., W_{t-1}) = I \)
- \( A_{\ell} \) is unrestricted
- consider restrictions on \( A_0 \) motivated by the temporal ordering
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• \( A_\ell \) is unrestricted
• consider restrictions on \( A_0 \) motivated by the temporal ordering

• At time \( t \) UR is released before GDP (1st week vs. last week)

\[
A_0 = \begin{pmatrix}
a_{11} & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 \\
a_{31} & a_{32} & a_{33} & 0 \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
\]
What is Blocking? An Example 2

• Suppose 3 series: quarterly GDP growth \( \{y_t\} \), monthly UR \( \{x_{t-2/3}, x_{t-1/3}, x_t\} \) and monthly EMP \( \{z_{t-2/3}, z_{t-1/3}, z_t\} \)
What is Blocking? An Example 2

- Suppose 3 series: quarterly GDP growth \( \{y_t\} \), monthly UR \( \{x_{t-2/3}, x_{t-1/3}, x_t\} \) and monthly EMP \( \{z_{t-2/3}, z_{t-1/3}, z_t\} \)
- Let \( W_t = [x_{t-2/3}, z_{t-2/3}, x_{t-1/3}, z_{t-1/3}, x_t, z_t, y_t]' \)
What is Blocking? An Example 2

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- Let \( W_t = [x_{t-2/3}, z_{t-2/3}, x_{t-1/3}, z_{t-1/3}, x_t, z_t, y_t]' \)
- EMP, UR are released on the first Friday

\[
A_0 = \begin{pmatrix}
a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0 \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77}
\end{pmatrix}
\]
Real-time Data

- Compiled from ALFRED (Archival Federal Reserve Economic Data) and Haver Analytics

- We have monthly vintages from 1980:1 to 2013:7 (using the one staring at 1985:1)
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- Some series start later, e.g., PCE headline and core in 2000:7 which implies a changing model structure
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- We have 24 data series: only one is quarterly

- Weekly and daily data aggregated to monthly - end of period, average
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• We have 24 data series: only one is quarterly

• Weekly and daily data aggregated to monthly - end of period, average

• Data has been transformed to stationarity
## Real-time Data (Cont.)

<table>
<thead>
<tr>
<th>Name</th>
<th>Release Date</th>
<th>Publ. Lag</th>
<th>Vintage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISM Manufacturing PMI (s.a.)</td>
<td>1st business day</td>
<td>one month</td>
<td>1997:03</td>
</tr>
<tr>
<td>ISM Supplier Deliveries Index (s.a.)</td>
<td>1st business day</td>
<td>one month</td>
<td>2009:11</td>
</tr>
<tr>
<td>ISM New Orders Index (s.a)</td>
<td>1st business day</td>
<td>one month</td>
<td>2009:11</td>
</tr>
<tr>
<td>Civilian Unempl. Rate, 16+ (s.a.)</td>
<td>1st Friday</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td>Empl. on Nonfarm Payrolls: Total (s.a.)</td>
<td>1st Friday</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td>Avg Weekly Manufacturing Hours</td>
<td>1st Friday</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td><strong>Group 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial Production</td>
<td>after 2 weeks</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td>New Res. Constr./Housing Starts (s.a)</td>
<td>12th workday</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td>Phily Fed Bus. Outlook Survey (s.a.) *</td>
<td>3rd Thurs</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>CPI Headline (s.a.)</td>
<td>varying, mid-month</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td>CPI Core (s.a.)</td>
<td>varying, mid-month</td>
<td>one month</td>
<td>1996:12</td>
</tr>
</tbody>
</table>
### Real-time Data (Cont.)

<table>
<thead>
<tr>
<th>Name</th>
<th>Release Date</th>
<th>Publ. Lag</th>
<th>Vintage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group 3</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New (1-Family) Houses Sold (s.a.)</td>
<td>17th workday</td>
<td>one month</td>
<td>1999:07</td>
</tr>
<tr>
<td>Consumer Sentiment Index (n.s.a) *</td>
<td>last Friday</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>GDP Advance Estimate (s.a.)</td>
<td>last week of month</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td><strong>Group 4</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE Headline (s.a.)</td>
<td>day after GDP</td>
<td>one month</td>
<td>2000:07</td>
</tr>
<tr>
<td>PCE Core (s.a.)</td>
<td>day after GDP</td>
<td>one month</td>
<td>2000:07</td>
</tr>
<tr>
<td>Personal Income (s.a)</td>
<td>day after GDP</td>
<td>one month</td>
<td>1980:01</td>
</tr>
<tr>
<td><strong>Group 5</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Unempl Insurance Claims (s.a.)</td>
<td>last Thursday</td>
<td>one week</td>
<td>2009:06</td>
</tr>
<tr>
<td>Federal Funds (Effective) Rate *</td>
<td>last day</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>Term Spread (10-Year - 3-Month) *</td>
<td>last day</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>WTI Oil Price *</td>
<td>last day</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>S&amp;P 500 Stock Index *</td>
<td>last day</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>Credit Spread (Baa - Aaa) *</td>
<td>last day</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
<tr>
<td>Trade Weighted Exch. Rate*</td>
<td>last day</td>
<td>curr. month</td>
<td>1980:01</td>
</tr>
</tbody>
</table>
Timing assumptions

- The goal is to take the timing assumptions of data releases (information flow) seriously

- Order the data based on release date not by reference date

<table>
<thead>
<tr>
<th>January, 2013 GDP Advance Release</th>
<th>$g_1^1$ $g_2^1$ $g_3^1$ $g_4^1$ $g_5^1$</th>
</tr>
</thead>
</table>

$v$

part of the information set in January, 2013 - balanced panel
Timing assumptions

- Variables in each group are contemporaneously correlated.
- Variables in each group respond contemporaneously to the group before (if exists), but not to the ones after.
Good luck! How Many Parameters are there to Estimate?

- More than one would want

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- Particularly for data spanning 1971:1-2013:4 ~ 42 years of monthly/quarterly data
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- Particularly for data spanning 1971:1-2013:4 \(\sim 42\) years of monthly/quarterly data

- The hope is that with enough shrinkage we can control the excessive estimation risk
Estimation: Sims-Zha Shrinkage Prior

Re-write the system as

\[ w'_{t+h}B = x'_t G + \epsilon'_t \]

Consider a prior of a following form:

\[ b_i \sim N(0, \bar{S}_i) \quad \text{and} \quad g_i|b_i \sim N(\bar{P}_i b_i, \bar{H}_i), \]

such that \( \bar{H}_{ij} = \frac{\lambda_0^2 \lambda_1^2}{\sigma_j^2 p^2 \lambda_3} \) and \( \bar{S}_{ij} \) are defined by \( \frac{\lambda_0^2}{\sigma_j^2} \)

- prior hierarchical in nature

| \( \lambda_0 \) | ? | controls the overall tightness of the beliefs |
| \( \lambda_1 \) | ? | tightens the prior around the mean |
| \( \lambda_3 \) | 1 | rate of contraction with an increase in lag length |
| \( \lambda_4 \) | 1 | controls the tightness of the constant |
Estimation: Algorithm

Waggoner and Zha (2003):

\[ w'_{t+h} B = x'_t G + \epsilon'_t \]

Given the model and the restrictions

\[ Q_i b_i = 0 \]
\[ R_i g_i = 0 \]

one can find \( U_i \) and \( V_i \) such that

\[ \psi_i = U_i b_i \]
\[ \phi_i = V_i g_i \]

The posterior distributions take the form

\[
p(\psi_1, \ldots, \psi_n | x_t) \propto |\text{det}[U_1 \psi_1 | \ldots | U_n \psi_n]|^T \exp \left( -\frac{T}{2} \sum_{i=1}^{n} \psi'_i S_i^{-1} \psi_i \right)
\]

\[
p(\phi_i | \psi_i, x_t) = \varphi(P_i \psi_i, H_i).
\]
On the Prior

• How to pick the hyperparameters?
  • Use values from the literature - have not been optimized for the monthly/quarterly structure that we have in our setup.
  • Consider hyperparameter selection mechanism - grid search

• Derive the marginal data density for our VAR (similar to Giannone, Lenza and Primiceri, 2015)
Unconditional Forecasting

- Three states of our world: end of first, second, and third month of quarter

- In the second (third) month of the quarter we have one complete set of month one (and two) variables

- At the end of the first month of each quarter - January, April, July, October vintages - our quarterly information set is complete

- We estimate the VAR only when we have a full set of data, i.e. once a quarter

- We evaluate the forecasts using quarterly vintages, i.e. forecasts produced in January, February and March are all evaluated against the April vintage
Unconditional Forecasting

The system is

\[ w'_{t+h} B = x'_t G + \epsilon'_t \]

Forecasts and forecast errors are

- At the end of the first month of quarter

\[ \hat{w}'_{t+1} | t = x'_t GB^{-1} \]

- At the end of the second month of each quarter

\[ \hat{\epsilon}_{t+1/3} = B'i_{1/3}(w_{t+1/3} - i'_{1/3} \hat{w}_{t+1}) \]
\[ \hat{w}'_{t+1} | t+1/3 = \hat{w}'_{t+1} | t + [\hat{\epsilon}_{t+1/3}; 0]'B^{-1} \]

- At the end of the third month of each quarter

\[ \hat{\epsilon}_{t+2/3} = B'i_{2/3}(w_{t+2/3} - i'_{2/3} \hat{w}_{t+1}|t+1/3) \]
\[ \hat{w}'_{t+1} | t+2/3 = \hat{w}'_{t+1} | t+1/3 + [0; \hat{\epsilon}_{t+2/3}; 0]'B^{-1} \]
Performance: the Alternatives

Quarterly models:

• “AR-Quarterly”
  • Estimate monthly and quarterly ARs with January, April, July, October vintages
  • For monthly variables construct up to three-step-ahead forecasts (with no new information)
  • Compare the average monthly forecast to the average monthly realization

• “VAR-Quarterly”
  • Average the monthly variables to a quarterly frequency

• “BVAR-Quarterly” - similar to “AR-Quarterly”
  • No need for multi-step-ahead forecasting since the set-up generates it by construction
Performance: Results

Mixed models:

- “AR-Mixed”
  - Estimate monthly and quarterly ARs with January, April, July, October vintages
  - For monthly variables construct three one-step-ahead forecasts with inter-quarter vintages
  - Compare forecasts to the realizations in next quarterly vintage

- “BVAR-Mixed”
  - Estimate the VARs with January, April, July, October vintages
  - Construct forecasts with inter-quarter vintages
  - Compare forecasts to the realizations in next quarterly vintage

- SPF (Survey of Professional Forecasters)
Evaluation is based on the mean of the forecast distribution which is consistent with quadratic loss (Gneiting, 2012)

### Quarterly Models

<table>
<thead>
<tr>
<th>Model</th>
<th>EMP</th>
<th>CPI</th>
<th>FFR</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR-Quarterly</td>
<td>0.24</td>
<td>0.70</td>
<td>0.58</td>
<td>1.78</td>
</tr>
<tr>
<td>VAR-Quarterly</td>
<td>1.71</td>
<td>1.50</td>
<td>1.46</td>
<td>1.23</td>
</tr>
<tr>
<td>BVAR-Quarterly</td>
<td>1.91</td>
<td>1.53</td>
<td>1.26</td>
<td>0.99</td>
</tr>
<tr>
<td>SPF</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.74</td>
</tr>
</tbody>
</table>
## Performance: the Alternatives

### Mixed Models

<table>
<thead>
<tr>
<th></th>
<th>AR-Mixed</th>
<th>BVAR - M1</th>
<th>BVAR - M2</th>
<th>BVAR - M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMP-M1</td>
<td>0.46</td>
<td>1.18</td>
<td></td>
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<tr>
<td>CPI-M1</td>
<td>1.08</td>
<td>1.09</td>
<td></td>
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<tr>
<td>FFR-M1</td>
<td>0.86</td>
<td>1.03</td>
<td></td>
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<tr>
<td>EMP-M2</td>
<td>0.49</td>
<td>1.26</td>
<td>1.14</td>
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<tr>
<td>CPI-M2</td>
<td>1.15</td>
<td>1.21</td>
<td>1.02</td>
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<tr>
<td>FFR-M2</td>
<td>0.84</td>
<td>1.17</td>
<td>1.61</td>
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<tr>
<td>EMP-M3</td>
<td>0.45</td>
<td>1.28</td>
<td>1.15</td>
<td>1.13</td>
</tr>
<tr>
<td>CPI-M3</td>
<td>1.04</td>
<td>1.27</td>
<td>1.22</td>
<td>1.04</td>
</tr>
<tr>
<td>FFR-M3</td>
<td>0.71</td>
<td>1.22</td>
<td>1.34</td>
<td>1.33</td>
</tr>
<tr>
<td>GDP</td>
<td>1.78</td>
<td>0.99</td>
<td>0.85</td>
<td>0.87</td>
</tr>
</tbody>
</table>
Performance: Forecasts
Performance: Forecasts

GDP Growth Nowcast

M1
M2
GDP growth
Performance: Forecasts

GDP Growth Nowcast

- M1
- M2
- M3
- GDP growth
Performance: Forecasts

GDP Growth Nowcast

- Model-mid Month 2
- SPF
- GDP growth

Performance: Rolling RMSE

Smoothed Forecast Errors for the GDP Nowcast

- M1
- M2
- M3
- SPF

Time Period: 1992 - 2008
Performance: Rolling RMSE
Performance: Rolling RMSE
Performance: Rolling Root Median Squared Error

Smoothed Forecast Errors for the GDP Nowcast

- M1
- M2
- M3
- SPF

Years: 1992 to 2008
Performance: Rolling Root Median Squared Error

Smoothed Forecast Errors for the GDP Nowcast
Performance: Rolling Root Median Squared Error

Smoothed Forecast Errors for the GDP Nowcast
Some Structural Analysis: Monetary Policy Shocks

GDP response to 25 basis point FFR shock

- Shock in Month 2
- Shock in Month 4
Some Structural Analysis: Monetary Policy Shocks

GDP growth response to 25 basis point FFR shock

- Shock in Month 3
- Shock in Month 4
Conclusions

• We have an easy way to mix monthly and quarterly variables into a meaningful forecasting framework.

• Can be a viable alternative for providing high frequency updates.

• Can consider for “structural” analysis with interesting interpretations.

• Comments? Thank you!