Real-Time Forecasting with a Large, Mixed Frequency, Bayesian VAR

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Key issues to forecasting in practice

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Yet, in addition, allows for a general framework, that enables us

- to construct forecasts conditional on "appropriate monetary policy" (i.e. condition on the path of the funds rate)
- to construct impulse response functions and density forecasts
- to explain "why" the forecasts changed since ...

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From econometric point of view

- parsimonious models usually perform better: tradeoff between over-fitting and out-of-sample performance
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Resolutions

- autoregressive models
- VARs in common low (quarterly) frequency
- factor models: Stock & Watson (2002), Giannone, Reichlin & Small (2008), Aruoba, Diebold & Scotti (2009), etc.
- MIDAS models: Ghysels, Santa Clara & Volkanov (2004), etc.
- VARs: Bańbura, Giannone & Reichlin (2010), Schorfheide & Song (in press), Foroni, Guérin & Marcellino (in press), etc.

Our Resolution

Take the VAR route

- Define a VAR that allows for monthly and quarterly data
- Monthly variables are treated at a quarterly frequency - blocking, stacking (Chen, Anderson, Deistler, Filler, 2011)
- Impose certain restrictions on the VAR consistent with the timing of data releases

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Similar to Ghysels (in press), but in out-of-sample, large VAR and used in real-time environment while relying on shrinkage

What is Blocking? An Example 1

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- Let $W_t = [x_{t-2/3}, x_{t-1/3}, x_t, y_t]'$
- Specify the VAR

$$A_0 W_t = C + \sum_{\ell=1}^{4} A_\ell W_{t-\ell} + e_t$$

  - $E(e_t | W_1, ..., W_{t-1}) = 0$
  - $E(e_t e_t' | W_1, ..., W_{t-1}) = I$
  - A_ℓ is unrestricted
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  - consider restrictions on A_0 motivated by the temporal ordering
- At time t UR is released before GDP (1st week vs. last week)

$$A_0 = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

What is Blocking? An Example 2

- Suppose 3 series: quarterly GDP growth $\{y_t\}$, monthly UR $\{x_{t-2/3}, x_{t-1/3}, x_t\}$ and monthly EMP $\{z_{t-2/3}, z_{t-1/3}, z_t\}$

What is Blocking? An Example 2

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- Let $W_t = [x_{t-2/3}, z_{t-2/3}, x_{t-1/3}, z_{t-1/3}, x_t, z_t, y_t]'$
- EMP, UR are released on the first Friday

$$A_0 = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} \end{pmatrix}$$

Real-time Data

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- Weekly and daily data aggregated to monthly - end of period, average
- Data has been transformed to stationarity

Real-time Data (Cont.)

Name	Release Date	Publ. Lag	Vintage
	Group 1		
ISM Manufacturing PMI (s.a.)	1st business day	one month	1997:03
ISM Supplier Deliveries Index (s.a.)	1st business day	one month	2009:11
ISM New Orders Index (s.a)	1st business day	one month	2009:11
Civilian Unempl. Rate, 16+ (s.a.)	1st Friday	one month	1980:01
Empl. on Nonfarm Payrolls: Total (s.a.)	1st Friday	one month	1980:01
Avg Weekly Manufacturing Hours	1st Friday	one month	1980:01
	Group 2		
Industrial Production	after 2 weeks	one month	1980:01
New Res. Constr./Housing Starts (s.a)	12th workday	one month	1980:01
Phily Fed Bus. Outlook Survey (s.a.) *	3rd Thurs	curr. month	1980:01
CPI Headline (s.a.)	varying, mid-month	one month	1980:01
CPI Core (s.a.)	varying, mid-month	one month	1996:12

Real-time Data (Cont.)

Name	Release Date	Publ. Lag	Vintage
	Group 3		
New (1-Family) Houses Sold (s.a.)	17th workday	one month	1999:07
Consumer Sentiment Index (n.s.a) *	last Friday	curr. month	1980:01
GDP Advance Estimate (s.a.)	last week of month	one month	1980:01
	Group 4		
PCE Headline (s.a.)	day after GDP	one month	2000:07
PCE Core (s.a.)	day after GDP	one month	2000:07
Personal Income (s.a)	day after GDP	one month	1980:01
	Group 5		
Initial Unempl Insurance Claims (s.a.)	last Thursday	one week	2009:06
Federal Funds (Effective) Rate *	last day	curr. month	1980:01
Term Spread (10-Year - 3-Month) *	last day	curr. month	1980:01
WTI Oil Price *	last day	curr. month	1980:01
S&P 500 Stock Index *	last day	curr. month	1980:01
Credit Spread (Baa - Aaa) *	last day	curr. month	1980:01
Trade Weighted Exch. Rate*	last day	curr. month	1980:01

Timing assumptions

- The goal is to take the timing assumptions of data releases (information flow) seriously

- Order the data based on release date not by reference date

January, 2013 GDP Advance Release	g_1^1 g_2^1 g_3^1 g_4^1 g_5^1	some associated with December, 2012 January, 2013 4th quarter, 2012

$\Downarrow$

part of the information set in January, 2013 - balanced panel

Timing assumptions

- Variables in each group are contemporaneously correlated
- Variables in each group respond contemporaneously to the group before (if exists), but not to the ones after

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- More than one would want
- Particularly for data spanning 1971:1-2013:4 $\sim$ 42 years of monthly/quarterly data
- The hope is that with enough shrinkage we can control the excessive estimation risk
  - Banbura, Giannone, and Reichlin (2010), De Mol, Giannone, and Reichlin (2008)

Estimation: Sims-Zha Shrinkage Prior

Re-write the system as

$$w'_{t+h}B = x'_t G + \epsilon'_t$$

Consider a prior of a following form:

$$b_i \sim N(0, \bar{S}_i) \quad \text{and} \quad g_i | b_i \sim N(\bar{P}_i b_i, \bar{H}_i),$$

such that $\bar{H}_{i_{jj}} = \frac{\lambda_0^2 \lambda_1^2}{\sigma_j^2 p^{2\lambda_3}}$ and $\bar{S}_{i_{jj}}$ are defined by $\frac{\lambda_0^2}{\sigma_j^2}$

- prior hierarchical in nature

λ_0	?	controls the overall tightness of the beliefs
λ_1	?	tightens the prior around the mean
λ_3	1	rate of contraction with an increase in lag length
λ_4	1	controls the tightness of the constant

Estimation: Algorithm

Waggoner and Zha (2003):

$$w'_{t+h}B = x'_t G + \epsilon'_t$$

Given the model and the restrictions

$$Q_i b_i = 0$$
$$R_i g_i = 0$$

one can find U_i and V_i such that

$$\psi_i = U_i b_i$$
$$\phi_i = V_i g_i$$

The posterior distributions take the form

$$p(\psi_1, ..., \psi_n | x_t) \propto |det[U_1\psi_1|...|U_n\psi_n]|^T exp\left(-\frac{T}{2}\sum_{i=1}^{n}\psi_i' S_i^{-1}\psi_i\right)$$

$$p(\phi_i|\psi_i, x_t) = \varphi(P_i\psi_i, H_i).$$

On the Prior

- How to pick the hyperparameters?
  - Use values from the literature - have not been optimized for the monthly/quarterly structure that we have in our setup.
  - Consider hyperparameter selection mechanism - grid search

- Derive the marginal data density for our VAR (similar to Giannone, Lenza and Primiceri, 2015)

Unconditional Forecasting

- Three states of our world: end of first, second, and third month of quarter
- In the second (third) month of the quarter we have one complete set of month one (and two) variables
- At the end of the first month of each quarter - January, April, July, October vintages - our quarterly information set is complete
- We estimate the VAR only when we have a full set of data, i.e. once a quarter
- We evaluate the forecasts using quarterly vintages, i.e. forecasts produced in January, February and March are all evaluated against the April vintage

Unconditional Forecasting

The system is

$$w'_{t+h}B = x'_t G + \epsilon'_t$$

Forecasts and forecast errors are

- At the end of the first month of quarter

$$\hat{w}'_{t+1|t} = x'_t G B^{-1}$$

- At the end of the second month of each quarter

$$\hat{\epsilon}_{t+1/3} = B' i_{1/3}(w_{t+1/3} - i'_{1/3}\hat{w}_{t+1})$$
$$\hat{w}'_{t+1|t+1/3} = \hat{w}'_{t+1|t} + [\hat{\epsilon}_{t+1/3}; 0]' B^{-1}$$

- At the end of the third month of each quarter

$$\hat{\epsilon}_{t+2/3} = B' i_{2/3}(w_{t+2/3} - i'_{2/3}\hat{w}_{t+1|t+1/3})$$
$$\hat{w}'_{t+1|t+2/3} = \hat{w}'_{t+1|t+1/3} + [0; \hat{\epsilon}_{t+2/3}; 0]' B^{-1}$$

Performance: the Alternatives

Quarterly models:

- "AR-Quarterly"
  - Estimate monthly and quarterly ARs with January, April, July, October vintages
  - For monthly variables construct up to three-step-ahead forecasts (with no new information)
  - Compare the average monthly forecast to the average monthly realization
- "VAR-Quarterly"
  - Average the monthly variables to a quarterly frequency
- "BVAR-Quarterly" - similar to "AR-Quarterly"
  - No need for multi-step-ahead forecasting since the set-up generates it by construction

Performance: Results

Mixed models:

- "AR-Mixed"
  - Estimate monthly and quarterly ARs with January, April, July, October vintages
  - For monthly variables construct three one-step-ahead forecasts with inter-quarter vintages
  - Compare forecasts to the realizations in next quarterly vintage

- "BVAR-Mixed"
  - Estimate the VARs with January, April, July, October vintages
  - Construct forecasts with inter-quarter vintages
  - Compare forecasts to the realizations in next quarterly vintage

- SPF (Survey of Professional Forecasters)

Performance: Results

Evaluation is based on the mean of the forecast distribution which is consistent with quadratic loss (Gneiting, 2012)

Quarterly Models

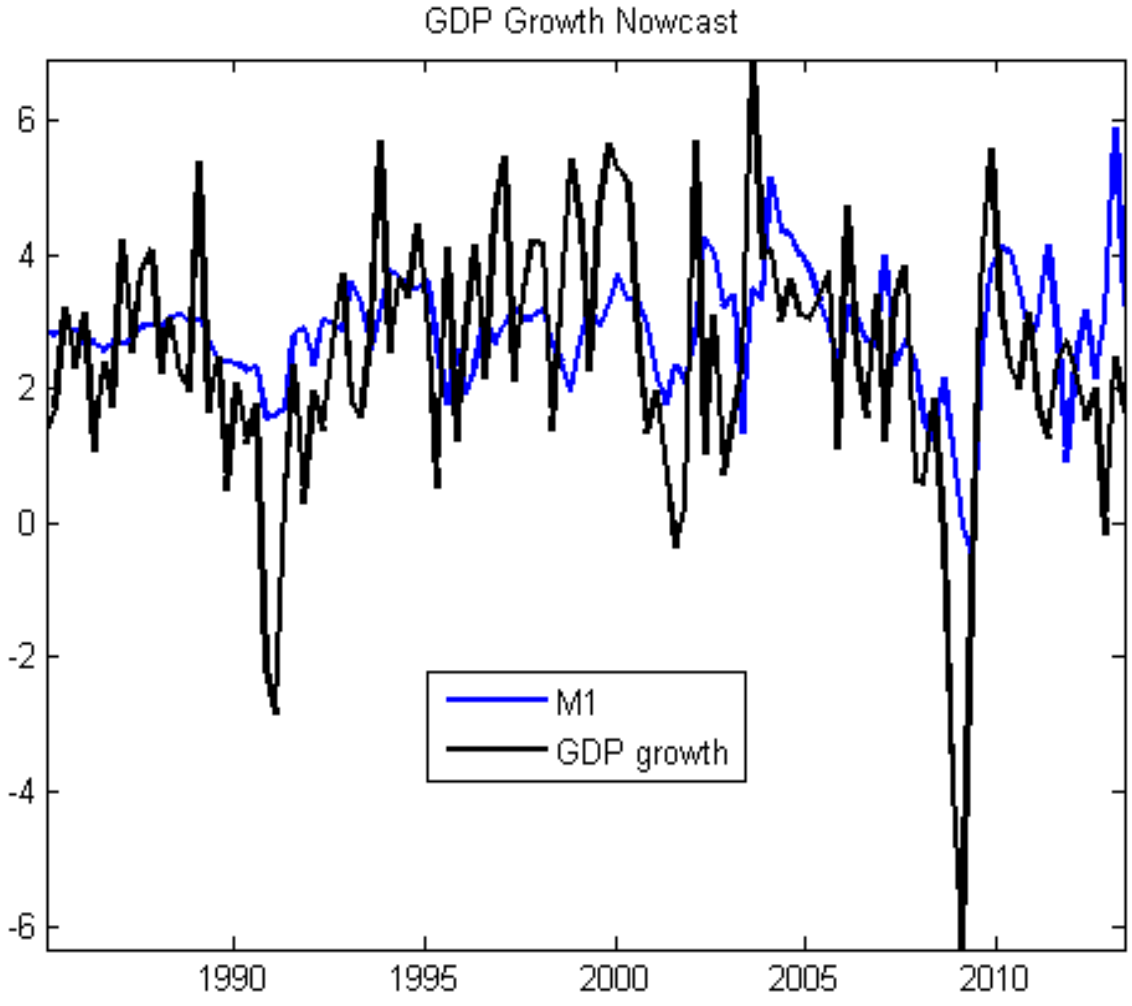
	EMP	CPI	FFR	GDP
AR-Quarterly	0.24	0.70	0.58	1.78
VAR-Quarterly	1.71	1.50	1.46	1.23
BVAR-Quarterly	1.91	1.53	1.26	0.99
SPF	-	-	-	0.74

Performance: the Alternatives

Mixed Models

	AR-Mixed	BVAR - M1	BVAR - M2	BVAR - M3
EMP-M1	0.46	1.18		
CPI-M1	1.08	1.09		
FFR-M1	0.86	1.03		
EMP-M2	0.49	1.26	1.14	
CPI-M2	1.15	1.21	1.02	
FFR-M2	0.84	1.17	1.61	
EMP-M3	0.45	1.28	1.15	1.13
CPI-M3	1.04	1.27	1.22	1.04
FFR-M3	0.71	1.22	1.34	1.33
GDP	1.78	0.99	0.85	0.87

Performance: Forecasts



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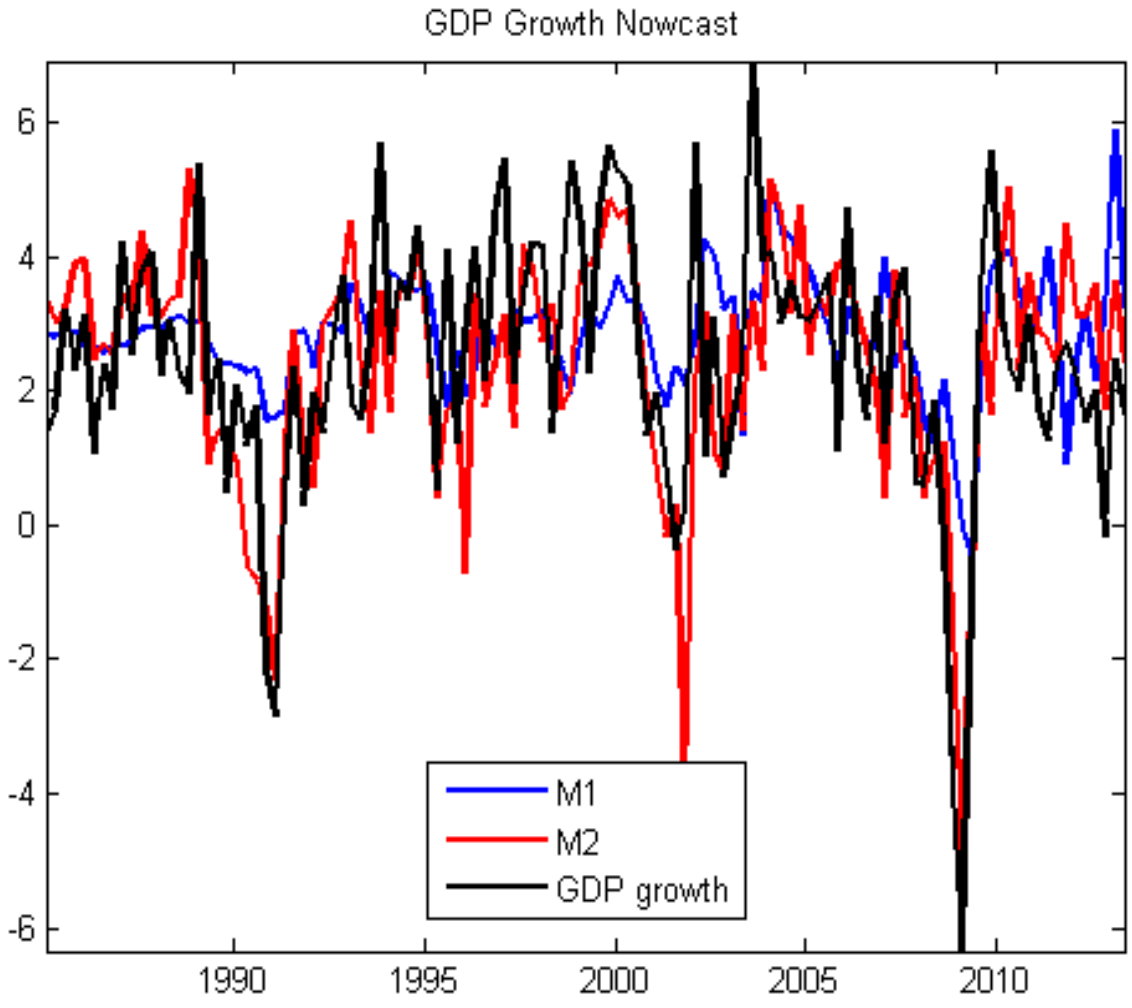

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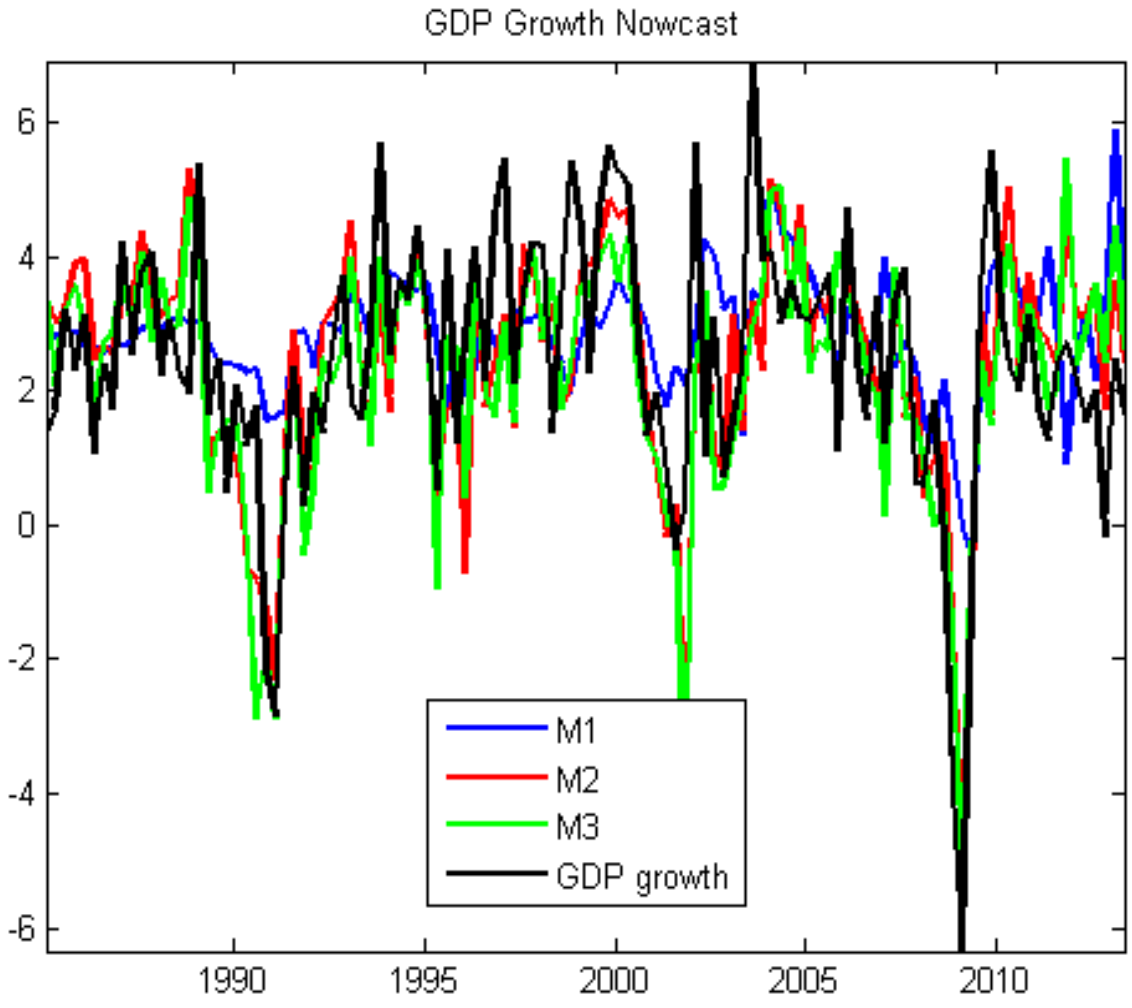

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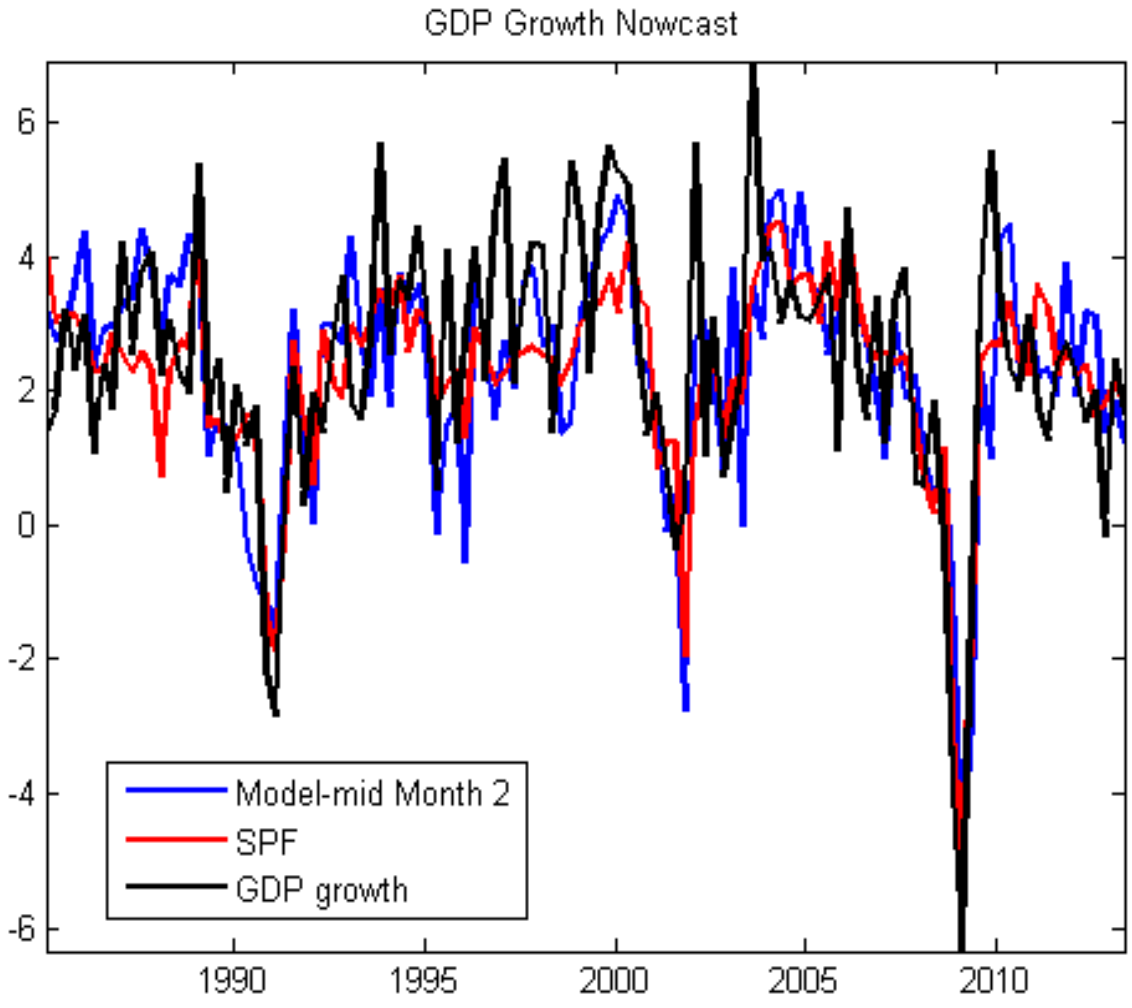
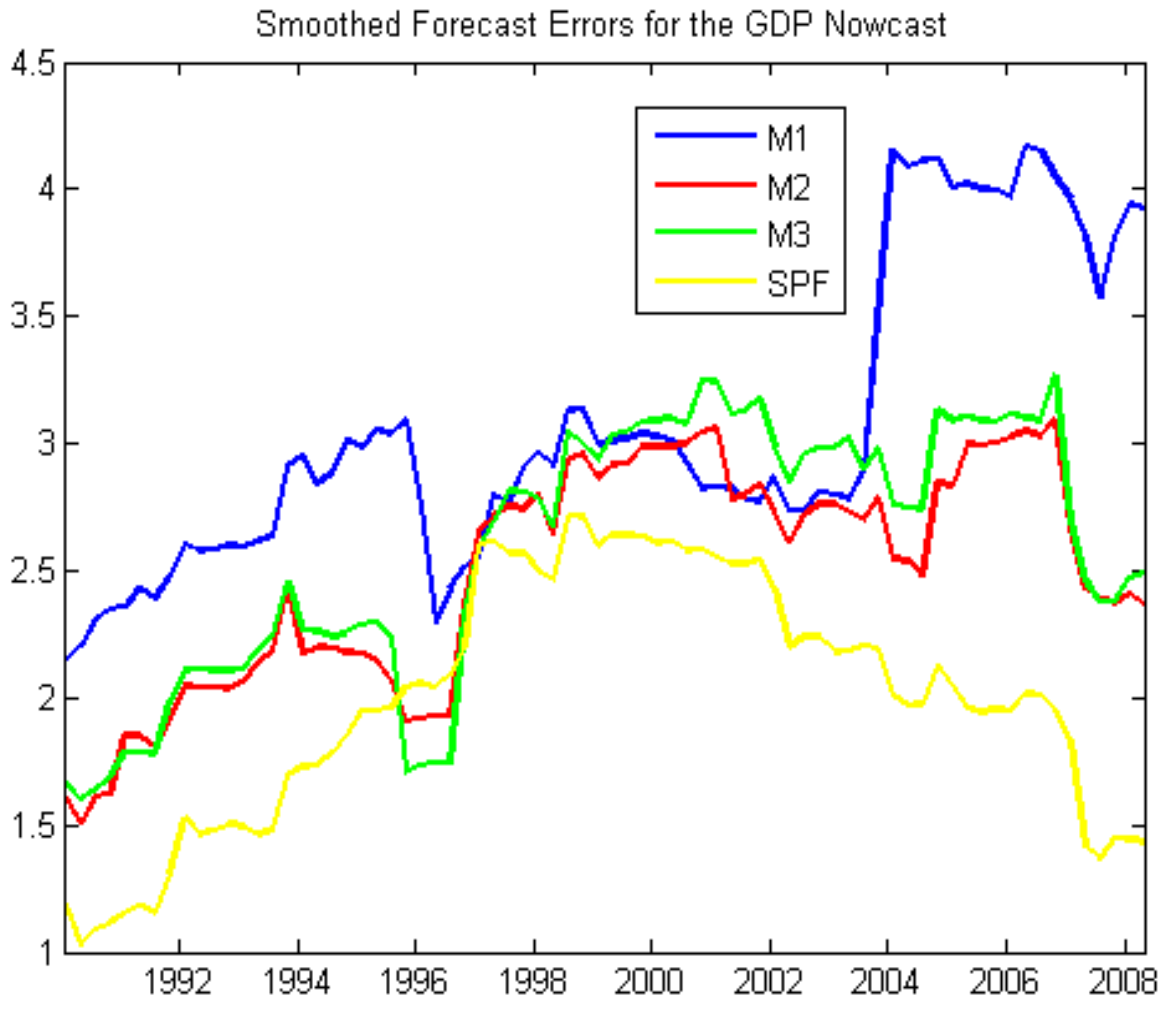


Performance: Rolling RMSE



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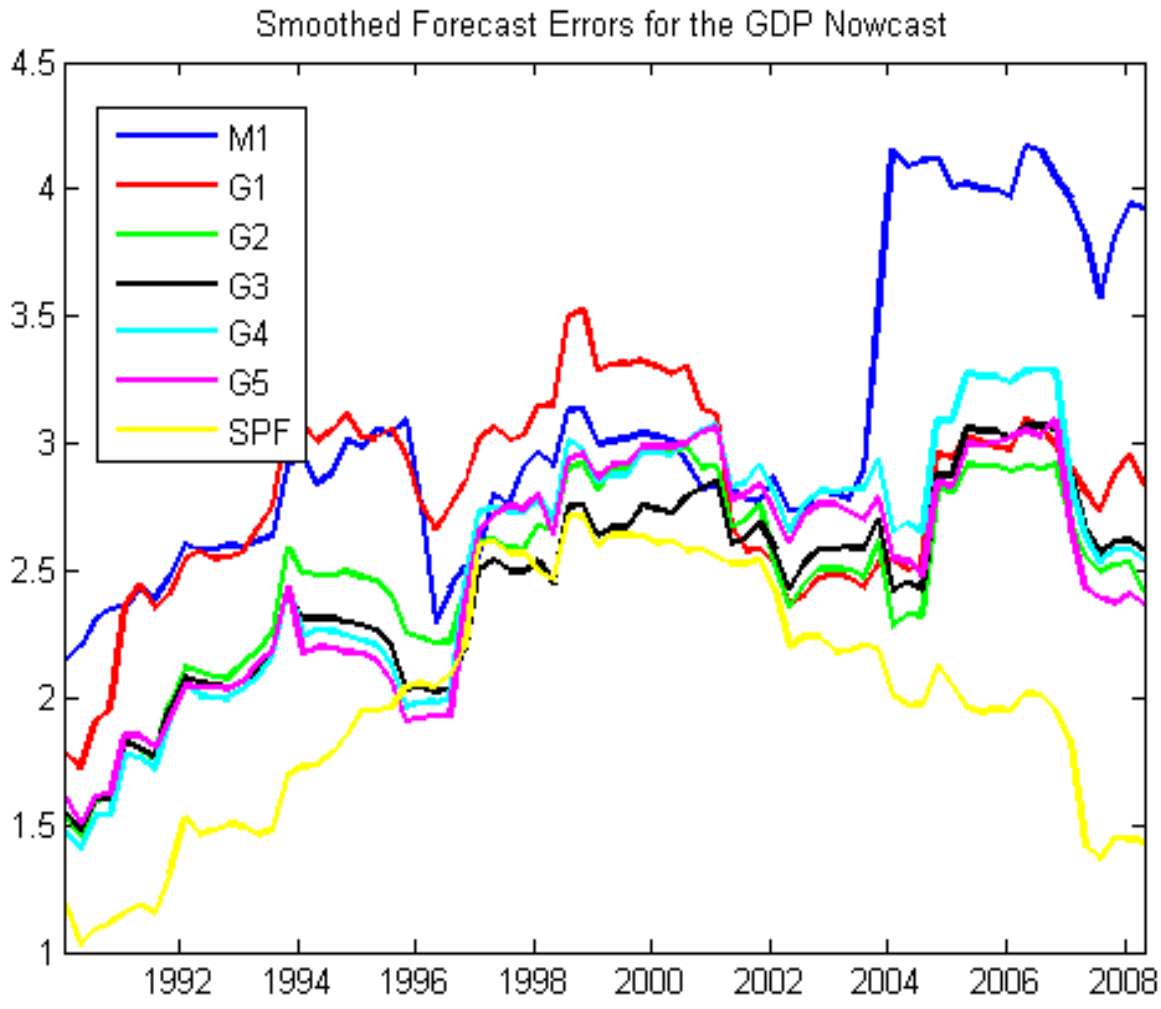

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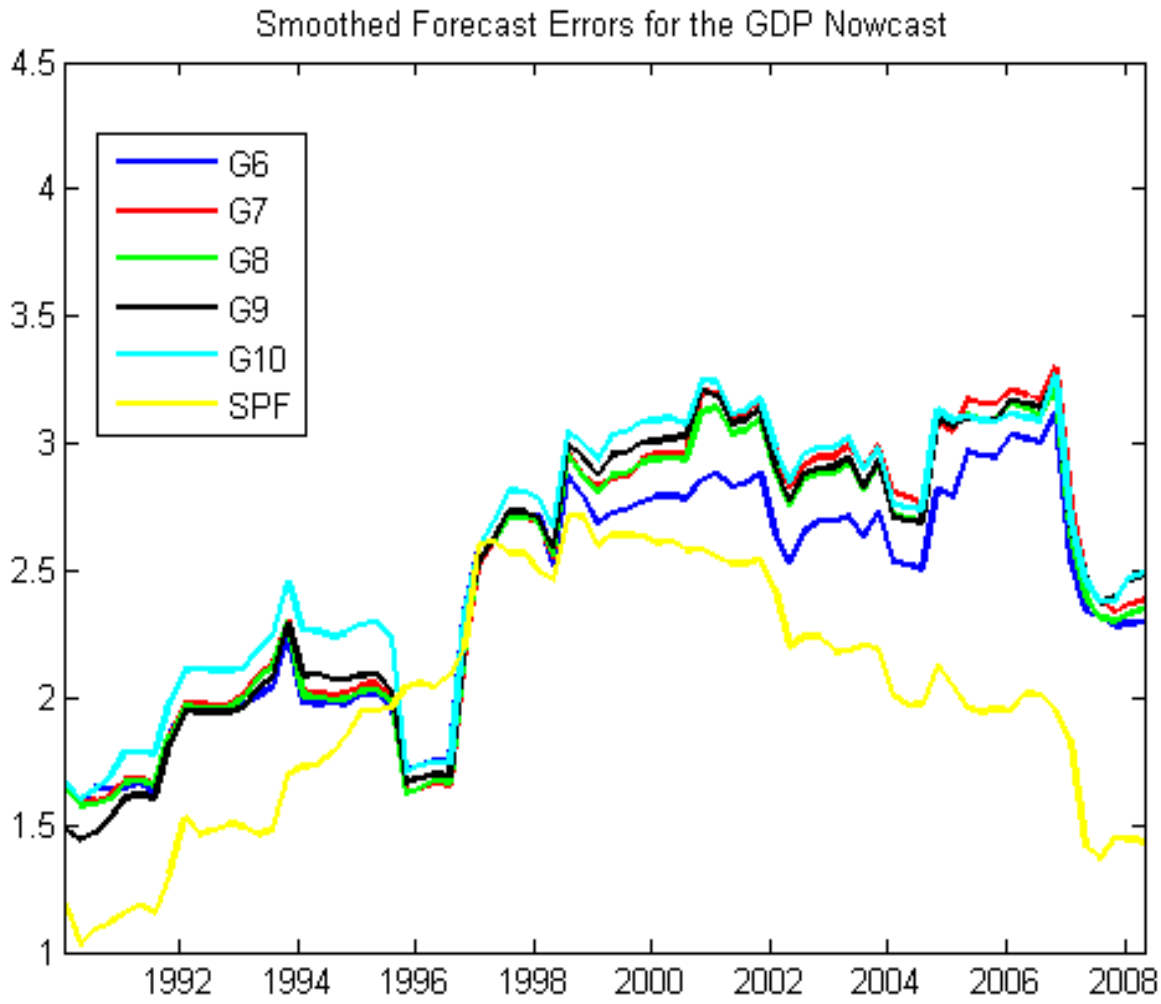
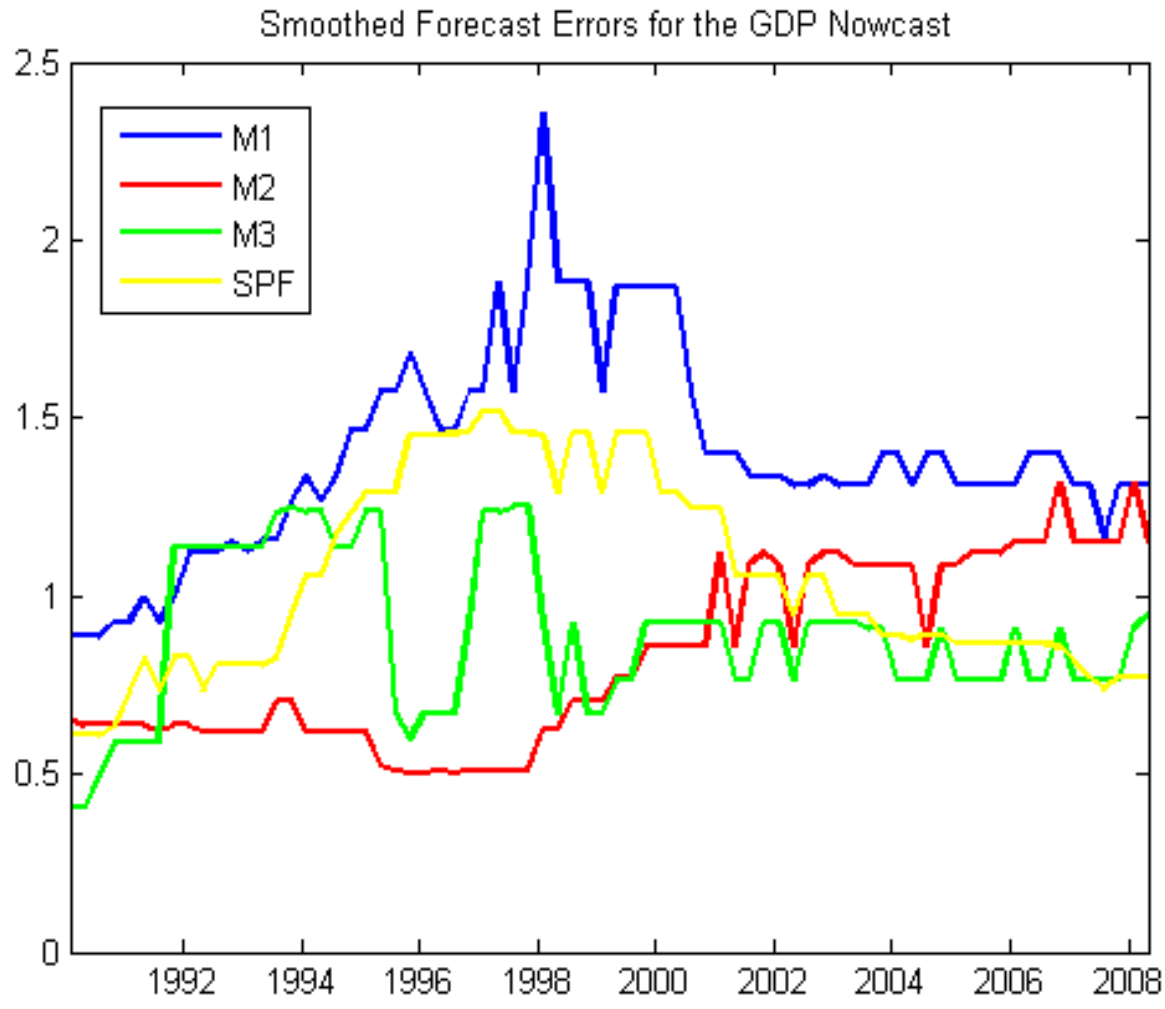


Performance: Rolling Root Median Squared Error



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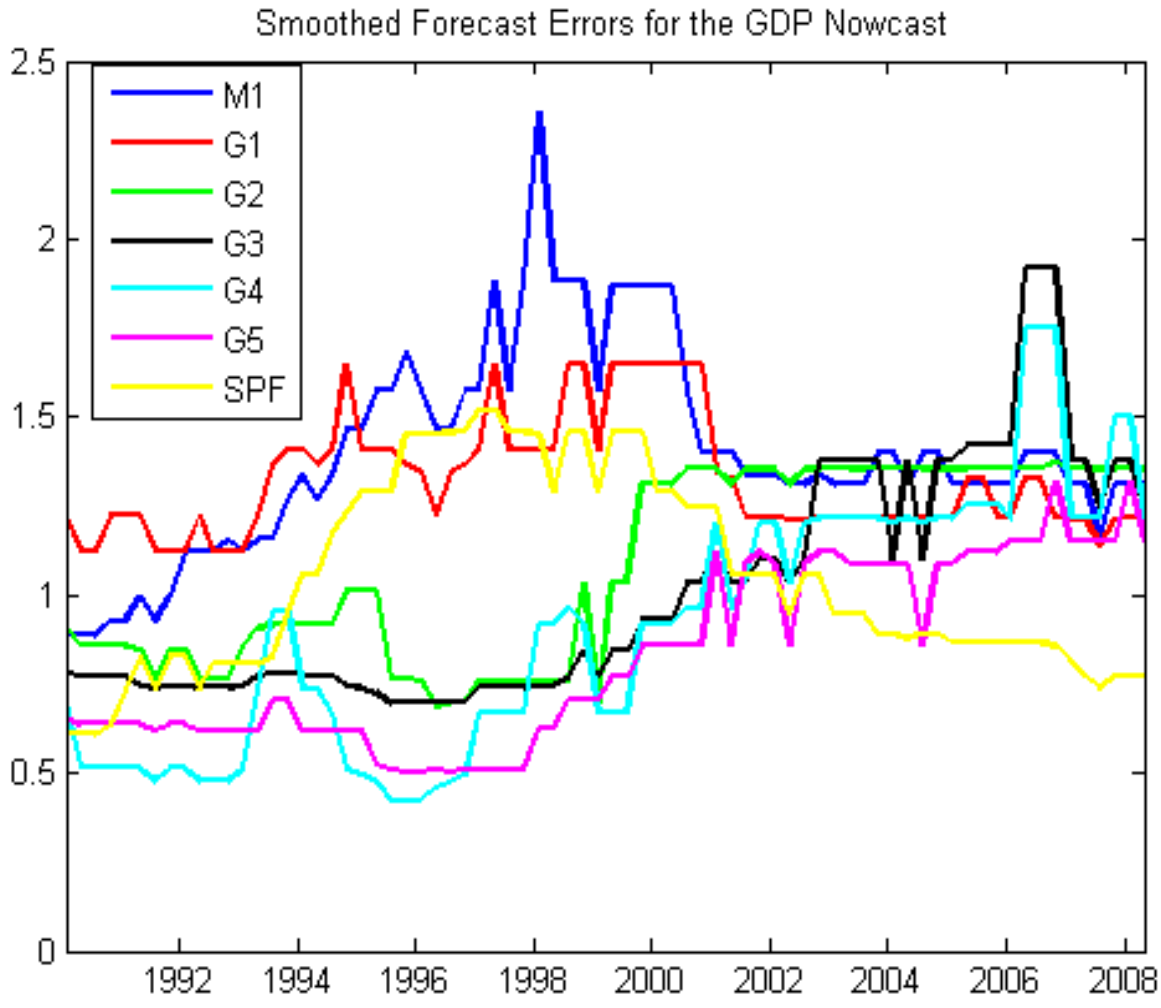

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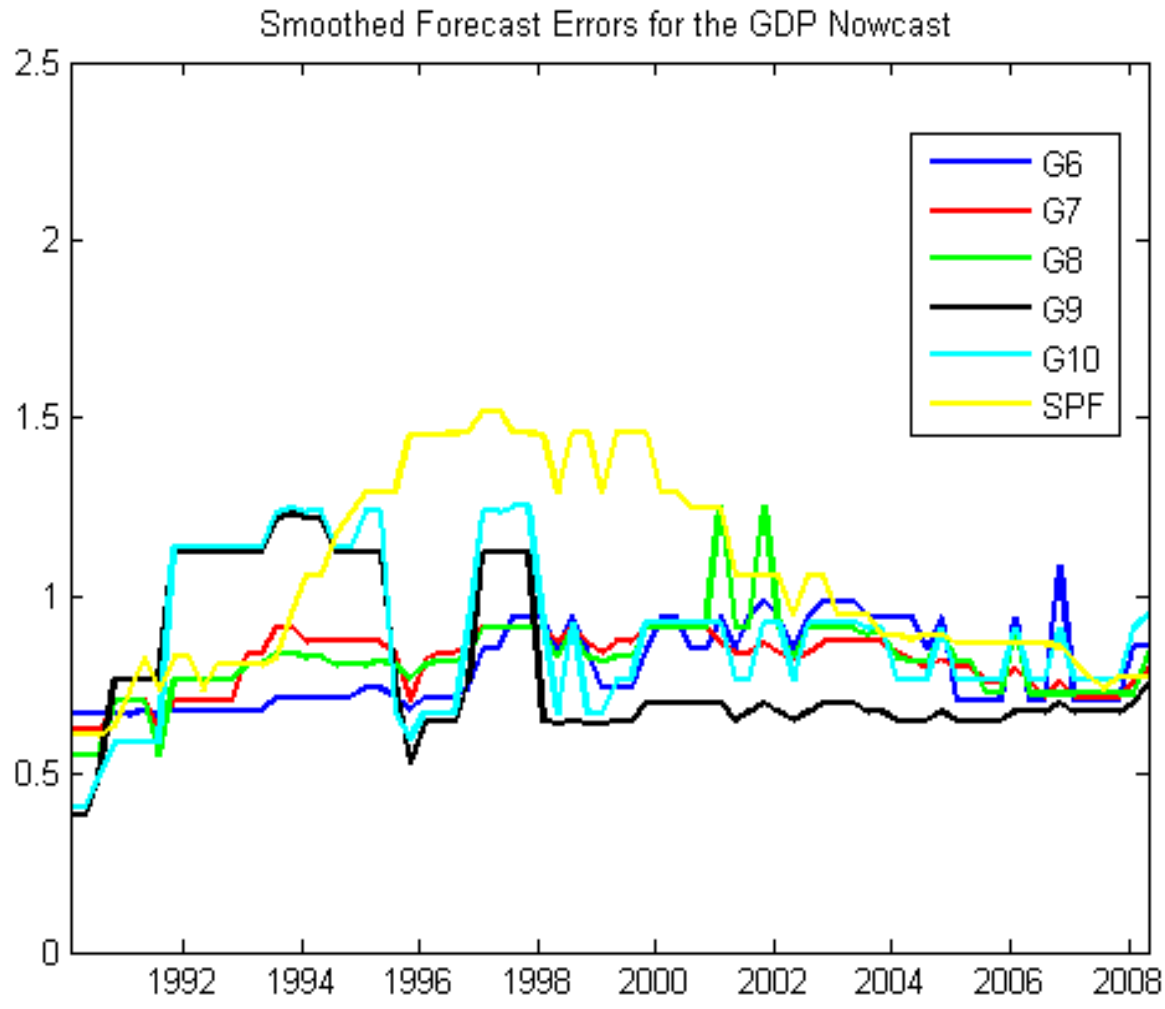
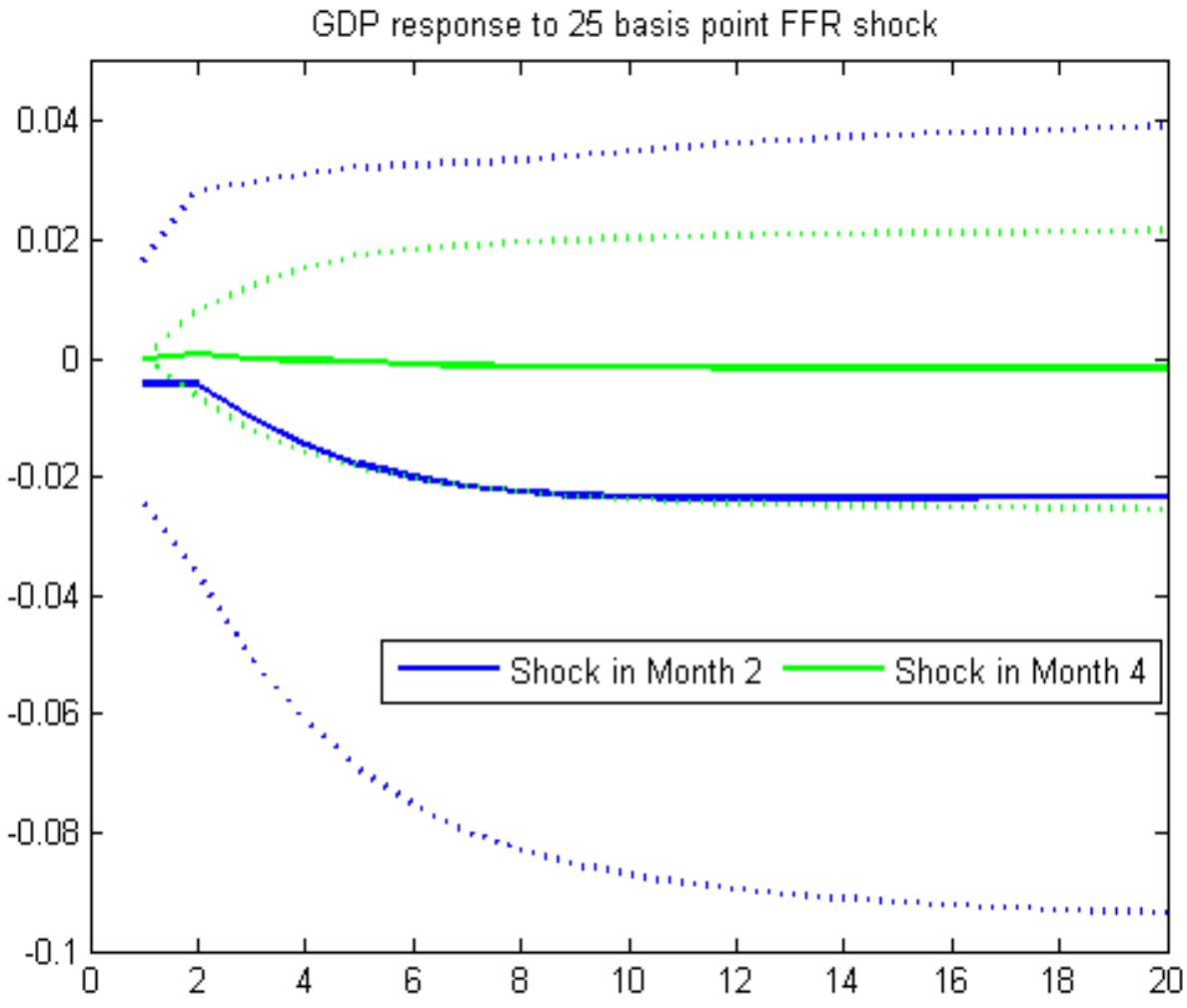


Some Structural Analysis: Monetary Policy Shocks



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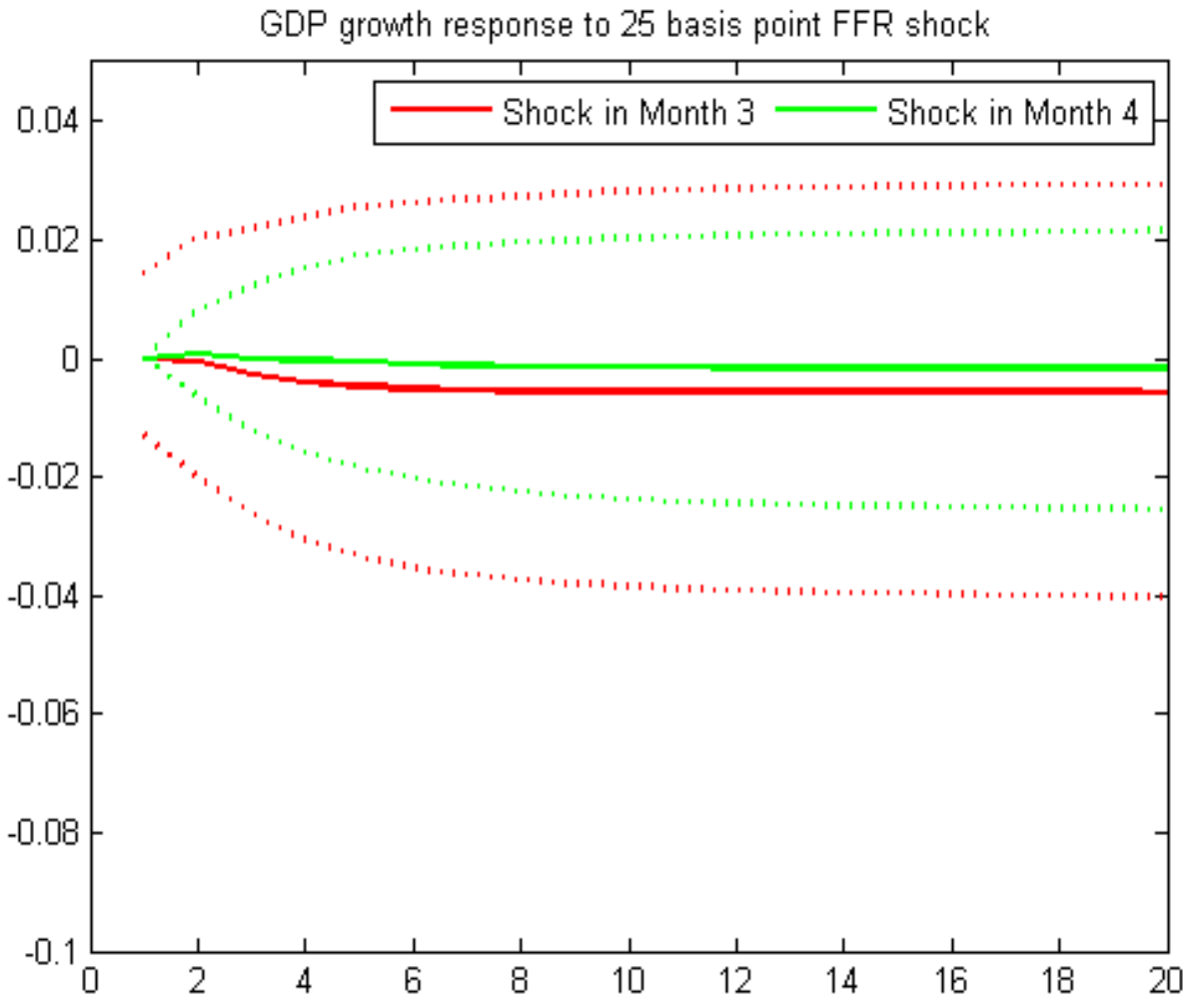

Conclusions

- We have an easy way to mix monthly and quarterly variables into a meaningful forecasting framework.
- Can be a viable alternative for providing high frequency updates.
- Can consider for "structural" analysis with interesting interpretations.
- Comments? Thank you!