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Jonathan Cagan
Carnegie Mellon University

Thomas R. Kurfess

Carnegie Mellon University.Engineering Design Research Center.

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J. Cagan, T. Kurfess

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Jonathan Cagan
and
Thomas R. Kurfess

Department of Mechanical Engineering
Carnegie Mellon University
Pittsburgh, PA 15213

0. Abstract

We introduce a methodology for concurrent design that considers the allocation of tolerances and manufacturing processes for minimum cost. Cost is approximated as a hyperbolic function over tolerance, and worst-case stack-up tolerance is assumed. Two simulated annealing techniques are introduced to solve the optimization problem. The first assumes independent, unordered, manufacturing processes and uses a Monte-Carlo simulation; the second assumes well known individual process cost functions which can be manipulated to create a single continuous function of cost versus tolerance with discontinuous derivatives solved with a continuous simulated annealing algorithm. An example utilizing a system of friction wheels over the manufacturing processes of turning, grinding, and saw cutting bar stock demonstrates excellent results.

1. Introduction

Techniques for concurrent design promise to impact the way design is done in **industry. Too often** design conceptualization, detailed design, and manufacturing design are **done** independently causing inferior product quality and excessive cost. In this paper we propose a technique to bring manufacturing concerns into the design process. Our emphasis is on the design of and manufacturing influences on tolerance stack-up. Product tolerancing is often the least emphasized part of the design process, yet the tolerance on individual components can have a major effect on the performance of a system. This paper assumes the worse-case stack-up of tolerances which occurs when each part is manufactured consistently on the upper or lower end of the tolerance range. Although each part is in tolerance, the worst case sum of part tolerances (the stack-up) may violate a performance constraint.

There has been significant work on tolerance selection. Speckhart (1972), Michael and Siddall (1981), Spotts (1973), and Sutherland and Roth (1975) have applied optimization techniques for tolerance selection to minimize cost given a fixed manufacturing process. Dong and Soom (1990) have examined optimal tolerance allocation in multiple dimension chains. Lee and Woo (1990) employ probabilistic techniques to analyze tolerance tradeoffs with performance. Each of these efforts assume a fixed manufacturing process.

We will show that selection of tolerances alone is not enough; the tolerances must be selected along with the manufacturing process if costs are to be minimized. Manufacturing a part to tight tolerances can be an expensive process; thus parts are usually designed for as large a tolerance range as possible. However, components can be manufactured with different processes at different costs, and each process is best suited to hold different tolerance ranges. Once the form of the design is determined, the part must be designed and manufactured such that no constraints are violated and the cost is kept to a **minimum. Our thesis is that *consideration of geometric tolerances should be a major influence in selecting a manufacturing process while minimizing cost for a given performance.***

In this paper we introduce two optimization-based approaches to determine the geometric tolerance on each component, as well as its manufacturing process, in a system of components. In both techniques, the combinatoric problem is solved using the stochastic pptimization technique of simulated annealing (Kirkpatrick, *et. al,* 1983). In the first technique, during each iteration of the simulated annealing algorithm a set of manufacturing processes is assumed and the tolerance of each component is determined to minimize the cost of the system and maintain a pre-defined output tolerance. This technique is useful with purely disjoint sets of manufacturing processes. Here the simulated annealing is used only to control a Monte-Carlo analysis and can only find the actual global minimum if the correct rr vufacturing processes are at some time selected. Although statistically this is likely happen, it is not guaranteed

In the second technique, we examine the inverted problem. Instead of randomly generating a set of manufacturing processes and then determining the tolerances, the tolerance versus cost curve is viewed as a continuum with discontinuous first derivatives; at each discontinuity, a different manufacturing process becomes dominant. Gradient techniques are inappropriate because of the discontinuous derivatives; however, simulated annealing is now used to solve the continuous problem. This technique is guaranteed, in theory, to converge on the global minimum.

Lee and Woo (1989) solve a similar problem with integer programing by assuming discrete tolerances. We explore tmiated annealing because, in the general case, a machine could have from several thousand to several million parts (Kalpakjian, 1991) produced by many different manufacturing processes. Thus the combinatorics are unwieldy and application of integer programing becomes impractical. Note that for smaller problems such as the one presented in this paper, the integer programming solution is more efficient than the Monte-Carlo approach; we utilize the simple example only to demonstrate the methodology of our theory. However, the continuous solution is still the most efficient approach.

A system of friction wheels is utilized to illustrate the theory. We choose friction wheels because they are simple enough to analyze in closed form, and yet complex enough to illustrate the difficulties in actual applications. In the next section we review the technique of simulated annealing. The cost versus tolerance function for the wheels is then developed for any given ma: process and the two techniques are formalized. Finally, the methods are applied to ^ wheel system where each component can be manufactured by three different manuiacturing processes (giving $3^4 = 81$ possible combinations).

2. Simulated Annealing

Simulated annealing is a stochastic optimization technique which has been shown able to solve both ordered combinatoric problems and non-linear continuous problems even with objectives of discontinuous slope. Traditional gradient-based optimization techniques (e.g., Papalambros and Wilde, 1988) are not readily able to solve such problems. Kirkpatrick, *et al.*, (1983) developed the simulating annealing algorithm based on Metropolis[1] Monte-Carlo technique (1953). The idea is analogous to the annealing of metals. A cooling schedule is defined giving a temperature reduction over the number of iterations. Temperature, T, is a gradient variable with no relation to physical temperature. At high temperature, selection of a solution point is quite random while at lower temperatures the solution is more stable; the metal annealing analogy is that at high temperatures the molecules are at a highly random state while at lower temperatures they reach a stable minimum energy state.

The approach to simulated annealing is to randomly pick a feasible state, *sj*> and evaluate the energy at that state, E_{Sj}. A different feasible state, 52, is then selected by randomly picking a new state within a given range of the available design space (which we call the *mutation space)*. State *S2* is then evaluated to £y2- ^ *&s2* < ^*si** ^ e n S*2* becomes the new solution state. If $E_{s2} > E_{sl}$, then there is a probability based on the temperature that the new state will be accepted anyhow. Acceptance is determined by the probability calculation:

$$\Pr\{E_{s_2}\} = \quad , \tag{1}$$

where Z(T) is a normalization factor. A random number, r, uniformly distributed between 0 and 1 is generated and compared with Pr{£y2)- ^r < JM ^ J * e n *c new state *s accepted anyhow; otherwise the old state is retained. The temperature is reduced and the process continues until convergence is reached or the temperature reaches 0. Also, the size of the mutation space is also reduced so that it asymptotes to 0. It can be proven that if the system reaches *equilibrium* at each temperature, then as the temperature approaches zero the algorithm will asymptotically converge on the global optimum (Lundy and Mees, 1986). Because we cannot guarantee sufficient time to reach equilibrium or a slow enough decrease in temperature, we search only for a good solution and do not require the absolute best solution. However, in the example problem of section 4, the global optimum is attained.

Simulated annealing has been applied to various mechanical engineering problems. Jain and Agogino applied simulated annealing to the continuous problem of mechanism design (1988) and the integer problem of selecting teeth for the gears in a multispeed gearbox (1990). Jain, *et ai*, (1990) applied simulated annealing to the nesting of blanks for metal stamping with excellent results for scrap minimization of the continuous problem. Other references of simulated annealing are given in van Laarhoven and Aarts (1987).

3 Relating Feeds, Speeds, Tolerances, and Costs

This section presents the methods employed in determining the cost functional used in the optimization algorithm. It is divided into several subsections that describe the computation of the three different machining costs per part for saw cutting, turning, and cylindrical grinding. The theory is valid for a variety of processes, the ones discussed are chosen for practical demonstration. The first subsection describes the generic computation of the machine costs on ah hourly basis, and the assumptions made. The second sub section describes the computation of the machining times for each process. We conclude this section with the determination of hyperbolic cost functional which are quite general and differ from those found in the literature.

Determination of Hourly Machine Costs

To compute the costs of any machine on an hourly basis, three figures are required: the annual payments on the machine, A, the annual maintenance for the machine, A_m, and the cost of operator and maintenance labor, Q. Although there are a large number of costs involved with operating a machine tool, these are the major ones and, therefore,we address them in our economic analysis .

To compute A, we employ a standard annual payment analysis assuming an annual interest rate compounded annually, i, an initial machine cost of C_m, and n years in which to pay the machine off. This analysis assumes that the payments are made at the end of each of the n years, and ignores inflation. The annual cost of the machine is given by:

$$A = C_m \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) + A \tag{2}$$

The hourly cost of the equipment is computed as

$$Ah = \frac{A}{N \wedge T} \tag{3}$$

where Nd is the number of working days per year, and Nh is the number of working hours per day. Finally, the hourly cost of machine operation is

$$H_m = A_h + Q, \tag{4}$$

where Q the hourly cost of labor.

Computing Production Times for the Processes

Saw Cutting

The production procedure for a saw cut operation is to simply take bar stock from the steel mill and saw a predetermined disk of width, W, from the part with a power reciprocating saw. After the sawing operation, a high speed face grinder is employed to flatten the face of the disk. The time required to face grind is insignificant in comparison to the sawing process, and it is assumed that the two processes function in parallel, thus only the sawing time is required to determine the production time per wheel.

For a wheel of diameter, D, the production time in seconds is

$$T \quad \frac{60D}{\text{^>saw}\ r_{sa} W} \tag{5}$$

where Ssaw is the recommended number of strokes/minute, Fsaw is the recommended feed speed (in/stroke), and Tsaw is the load/unload time for the saw.

Turning

The turning operation requires that a saw cut blank be supplied as the initial geometry of the part. Since we are only concerned about the diameter of the part, we analyze the single point turning of the outside diameter for a single pass. In many operations, multiple passes may be employed including roughing and finishing passes; however, we do not incorporate these into our analysis. The time in seconds required to turn a part of diameter, D, and width, W, is

$$Ti_{at}he = \frac{(5)\pi DW}{S\ lathe\ Flathe}, \tag{6}$$

where Sjathe is the recommended surface speed (fpm) and Fjathe is the recommended feed speed (ipr) for single point turning.

Grinding

The grinding operation requires that a blank be sawed and then rough turned to near finished shape. Again, we assume that the blank is close to finished size and, thus, requires only a single grinding pass. In general, this analysis may be extended to multiple grinding passes; however, for the purposes of this research a single pass analysis is sufficient. The time in seconds required to grind the wheel of diameter, D, and width, W, with a grinding wheel of width, W_w, is

$$T_g = \frac{(5)\pi D\, W + W_W)}{S_{grinder}F_{grinder}} - \frac{10\,\pi D)\, W(+ W\ )}{S_{grind}\hat{}W_w}, \quad (7)$$

where Sgrind is the recommended work speed (fpm) and the recommended traverse feed, Fgrind* is one half the grinding wheel width per work-piece revolution.

Nominal Part Production Costs

The nominal part production cost may now be determined from the above nominal machining time parameters as

$$C_{nom} = J\hat{}Q\, \overline{H}\; T_i + M_i,\ i = \{saw, lathe, ginder\}, \quad \hat{}$$

where Mj is the cost of the blank material. It is important to realize that the blank costs may differ for various processes, since different processes may require different blanks. For example, the sawing operation requires bar stock as its blank; however, the turning operation requires saw cut blanks (wheels pre-cut from bar stock). Thus, the cost of sawing must be incorporated into the turning blank cost.

Computing the Hyperbolic Cost Parameters

We assume a hyperbolic cost function as depicted in Figure 1. The form of the cost function is given as

$$C = \frac{K}{A - a} + b, \quad (9)$$

where C is the cost, a is the tightest tolerance that the machine can hold, b is the cost of producing a part when no specific tolerance is specified *(i.e.,* when the machine is running at its maximum capacity without regard to holding a specific tolerance), and K is a process-dependent constant

Two common cost functions found in the literature are the inverse quadratic (Spotts, 1973; Sutherland and Roth, 1975) and exponential (Speckhart, 1972; Dong and Soom, 1990) forms. Although all of the cost functional are of the same general shape, both our hyperbolic form and the inverse quadratic form provide the limiting non-zero tolerance case as found in practice, whereas the exponential form permits impractical tolerances such as negative and unachievable values. The hyperbola differs from the inverse quadratic in that for tolerances tighter than the machine's minimal tolerance it yields negative costs providing a flag to the algorithm.

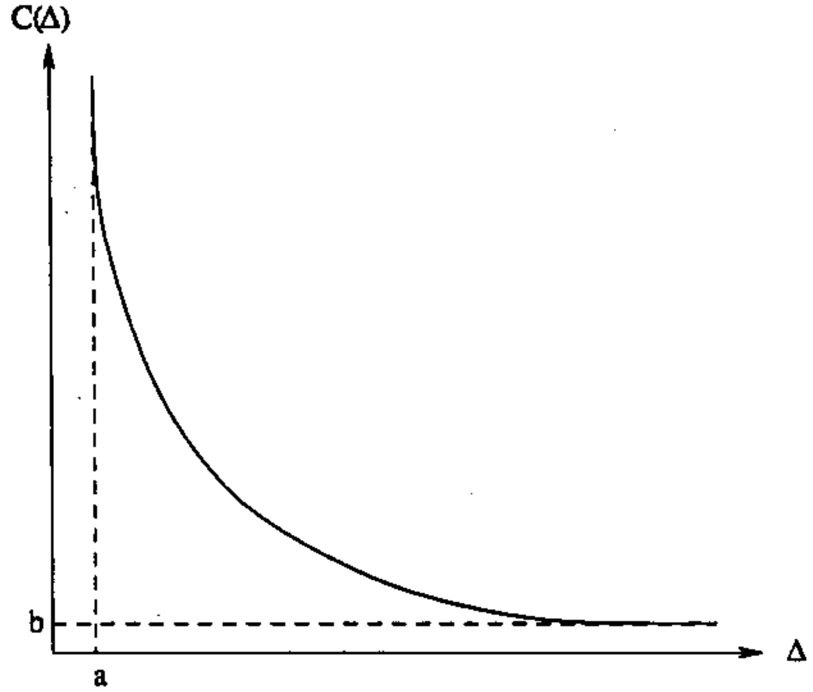

Figure 1. Typical Cost Functional

The hyperbolic shape of the cost functional is based on the standard manufacturing practice of reducing feeds, speeds and cut depths to increase accuracy. Thus, tighter tolerances may be achieved by reducing the production rate of a machine until the ultimate machine accuracy, a, is attained. The lower limit on the cost, b, is the realization that there are certain fixed costs involved in producing a part. That is, no matter how loose the tolerances are, the cost of manufacturing a part can never be zero (e.g., material costs demand this).

There are, in theory, some limitations to the hyperbolic representation, the most significant being the lower cost bound. The physical meaning of this bound is that production rates may be increased *ad infinitwn* when the tolerances become infinitely loose. Clearly, however, there are physical constraints to production rates which prevent this. Thus, in practice the infinite production rate scenario is not a significant limitation to the algorithm.

To determine the parameters for the curve in equation (9) a point on the hyperbola is required. We use a nominal tolerance

$$A_{nom} = 3a \tag{10}$$

as the tolerance generated when the recommended feeds, speeds and cut depths are employed. Thus, operating at A_{nom} results in a nominal part cost, C_{nom}. The cost functional may be written in terms of known parameters as

$$C = \frac{\quad}{\Delta - a} + b \tag{11}$$

4. Monte-Carlo Simulation

We are interested in specifying tolerance and manufacturing process for an assembly of parts. In our first approach to solve this problem, a possible manufacturing process is randomly selected for each system component. That set of processes becomes the current design state. The state is then checked to verify constraint satisfaction and then the optimization problem is solved by symbolic or numerical techniques to evaluate the optimal objective solution for the given set of manufacturing processes. That objective of manufacturing cost is the energy quantity which is evaluated by the simulated annealing algorithm to determine the optimal configuration.

The general algorithm is shown in Figure 2 where the temperature is multiplied by reduction_factor on each iteration until it reaches 0. In practice the algorithm is run until there is no improvement at an iteration (convergence is reached). Also the solution should reach equilibrium at each temperature iteration and so it is run at each temperature for some fixed number of times or until a certain number of successful moves is reached.

When simulated annealing is utilized for the continuous problem there is some metric between different states; the same is true for discrete problems with ordered sets. In these cases the mutation space can be reduced as the temperature decreases. There is no metric of nearness between manufacturing processes in the current technique. Thus the mutation in our application is truly random and the simulated annealing algorithm is only a basis for a Monte-Carlo simulation; if convergence is reached then the algorithm may run in fewer iterations than if it were required to complete a fixed number of iterations.

Because there is no metric of nearness and thus the size of the mutation space cannot be reduced, it is possible that the optimal solution will be found and then left at high temperatures (at lower temperatures this becomes more unlikely). During acceptance by the Metropolis algorithm, our algorithm saves the best state that it has found during the run, even if it disposes that state. Thus, although we cannot guarantee to converge on the global minimum in practical time, we can guarantee that if it is generated at any time, it will be retained. Statistically, if the algorithm is run for sufficient time it is likely to have found the optimal state.

```
Begin Discrete-Anneal
    T = l .
    Generate state;
    Best_state = state;
    Evaluate state;
    While T > O do
        Generate temp_state;
        If (verify constraints of temp_state)
        then Begin
            Evaluate temp_state;
            Test temp_state with Metropolis;
            If (accept)
            then
                state = temp_state;
                If state is better than Best_state
                then
                    Best_state = state;
                End
            End
        End
    T = T*reduction_factor,
End
```

Figure 2. Discrete Problem Algorithm

5. Inverted Continuous Problem

Although the algorithm of section 3 is useful for the general, discrete problem, it cannot be guaranteed to converge on the global optimum. Figure 3 shows a sequence of tolerance versus cost curves for different manufacturing processes on a given part. For any given tolerance, there is only one manufacturing process that yields the lowest possible cost; that set of costs is highlighted with the heavy black line of Figure 3 which is also shown in Figure 4. Figure 4 becomes a continuous tolerance versus cost function considering all possible manufacturing processes for a single part. Each part has a similar curve and the optimal tolerance for each part is desired.

Because the cost function of Figure 4 has discontinuous first derivatives, gradient techniques would be inappropriate for solving this problem. Vanderbilt and Louie (1984) showed that simulated annealing could be applied to continuous problems. Also, simulated annealing can keep the solution out of local minima, a useful characteristic when unknown cost functions are utilized. Our second technique applies simulated annealing to converge on the tolerances from the continuous cost curves. As mentioned in section 3, the form of the hyperbola cost curves are appropriate for a wide variety of machining processes on most parts- Thus this method is quite general, although we only demonstrate it for a simple friction wheel problem. If a different manufacturing process is employed or the continuous hyperbola is found not to sufficiently model the cost function of a particular part, then either the new function can be incorporated into our technique or the first technique of section 4 can be employed.

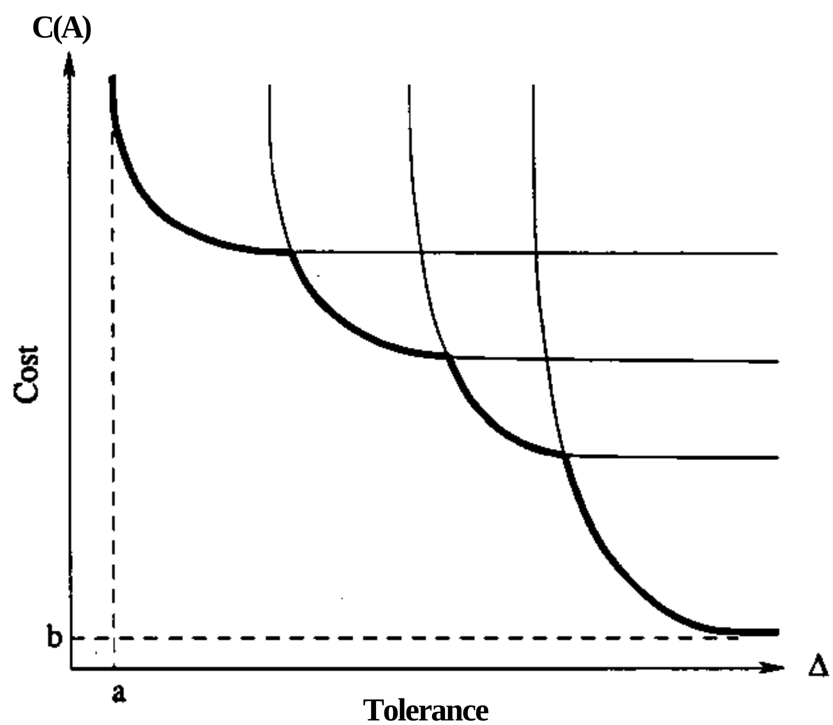

Figure 3. Tolerance versus cost curves for different manufacturing processes

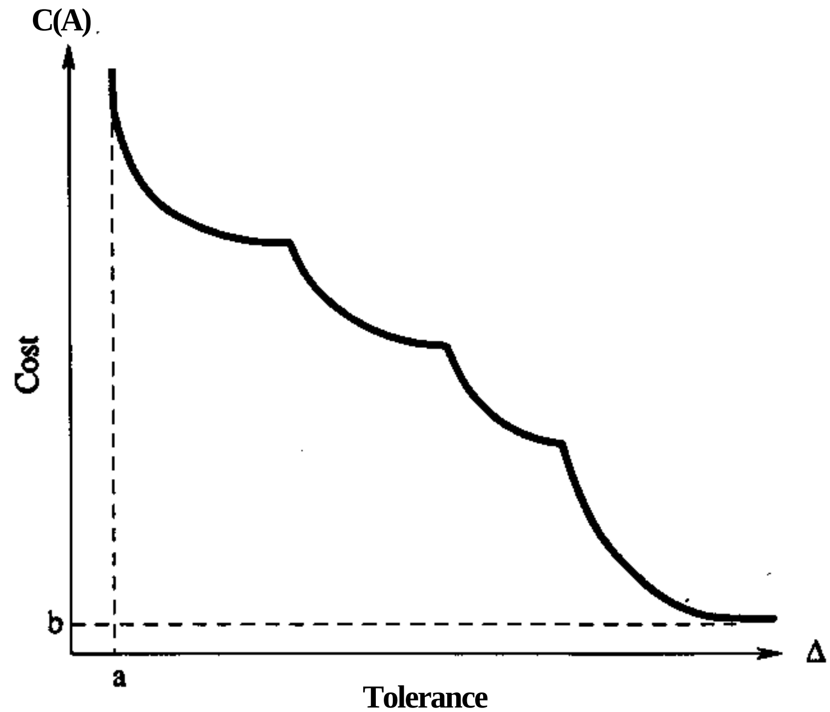

Figure 4, Continuous tolerance versus cost curve derived from Figure 3

The algorithm is found in Figure 5. First the intersection points of the various potential manufacturing processes are obtained and the continuous cost curve is defined by range. Next initial starting tolerances are selected, making sure tolerances sum up to the specified output tolerance. In general this is done by selecting the first of n tolerances, then from the remaining available range selecting the next tolerance, continuing this process for n-1 parts until the final part is assigned the remaining tolerance range. The total cost is evaluated.

The continuous simulated annealing algorithm is then run by randomly generating new tolerances in a neighborhood (e) about the tolerance of each of the first n-1 parts; the final part is again assigned the remaining available tolerance range. This new set of tolerances is then evaluated and the Metropolis algorithm determines whether it is accepted. As the temperature is reduced, so is the range (e) of the mutation space. The algorithm terminates when the cost converges or the temperature reaches 0.

Once convergence is reached then the particular manufacturing process of each part is known. Depending on the cooling schedule, the optimal solution may need to be fine-tuned. Either the analysis of section 6 for hyperbolic cost functions can be employed or a gradient-based numerical technique with Lagrange Multipliers can be utilized. Note that in

the example of section 6, **the** simulated annealing solution is essentially identical to the **analytical solution.**

```
Begin Continuous-Anneal
    Determine cost function;
    T = 1;
    Generate state
    Evaluate ∴u ∴
    While T > 0 do
        Generate temp_state by mutation where range is function
            of T;
        If (verify constraints of temp_state)
        then Begin
            Evaluate temp_state;
            Test temp_state with Metropolis;
            If (accept)
            then
                state = temp_state;
            End
        End
        T = T*reduction_factor,
    End
    Fine-tune solution with gradient technique if required;
End
```

Figure 5. Continuous Problem Algorithm

6. Friction Wheel Example

6.1 Problem Definition

In this section we apply the simulated annealing algorithms described in sections 4 and 5 to the design and manufacturing of a system of friction wheels. The engineering cost, tolerancing, and manufacturing information described in section 3 is used to **determine actual** design numbers.

Information pertaining to the machining parameters is based on values obtained from manufacturers and Machinability Data Center (1980). For demonstrative purposes, the recommended feeds, speeds, and cut depths assume the machined material is 1117 low carbon resulfurized free machining wrought steel, cold drawn with a hardness of approximately 200 BHN. Machining time for other materials may be computed with the formulas developed by substituting their particular recommended machining parameters. These parameters are available for a wide variety of materials.

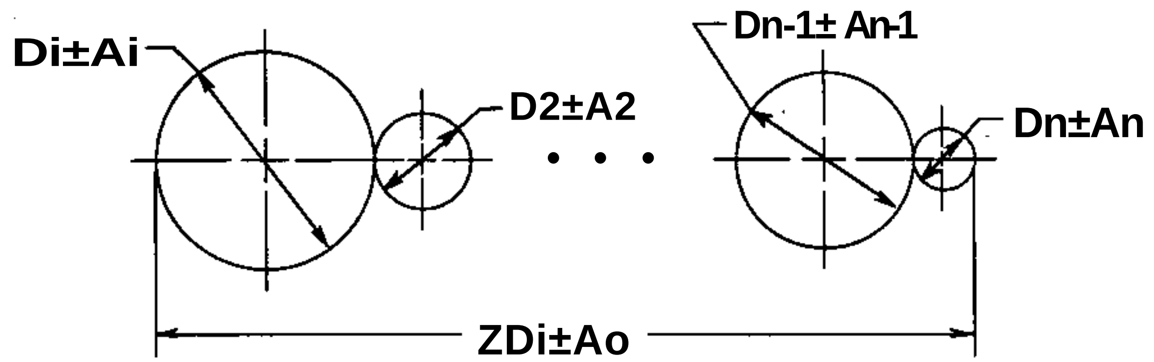

Figure 6. General system of n friction wheels

Consider a system of n friction wheels as shown in Figure 6. The cost function for a single wheel is given in equation (9). The total cost (Or) of all the wheels is then the sum of each individual wheel costs. The performance criteria is specified by a given output tolerance ($A_O$ut)»' the stack-up tolerance on the system must be bounded by A_{ou}t- The optimization problem is formulated as:

$$
\begin{array}{ll}
\text{nun:} & C_T \\
\text{s.t.:} & \\
h_i: & C_T = \sum_{i=1}^{n} Q, \\
h_2: & C_i = \dfrac{K_i}{\Delta_i - a_i} + b_i \,, \\
g_1: & \sum_{i=1}^{n} \Delta_i \le \Delta_{out} \,.
\end{array}
$$

Monotonicity analysis (Papalambros and Wilde, 1988) reveals that both equality constraints hi and h2 and inequality constraint gi must be active and relevant. For given parameters ai, bi, and Kj, and n wheels, this problem has n-1 degrees-of-freedom (DOF). By using all relevant information the minimum can be determined by applying the first-order necessary conditions of optimality (setting the partial derivative of the cost function to zero for each Ai). Algebra then leads to the following solution for each Aj:

$$
\Delta_{i\,,\,i=1\ldots n-1} = \frac{\left(\Delta_{out} - \sum_{j=1}^{i-1} \Delta_j - \sum_{j=i+1}^{n-1} \Delta_j - a_n\right) K_n^{-1/2} + a_i K_i^{-1/2}}{K_i^{-1/2} + K_n^{-1/2}} \,. \tag{12}
$$

Note that there are n-1 simultaneous equations which must be solved to determine the optimal tolerances. The n* tolerance (An) is determined from constraint gj.

This general problem to determine a set of tolerances given the manufacturing processes has been programmed in C on a Mac IL The discrete technique is programmed to randomly pick a manufacturing process for each friction wheel and evaluate the geometric tolerance on each wheel and the total cost. The simulated annealing algorithm then repeats this process until the optimal configuration and tolerances are determined.

The continuous problem is solved independent of equation (12) as described in section 5; however, once the optimal manufacturing processes have been determined equation (12) can be used to determine the exact solution.

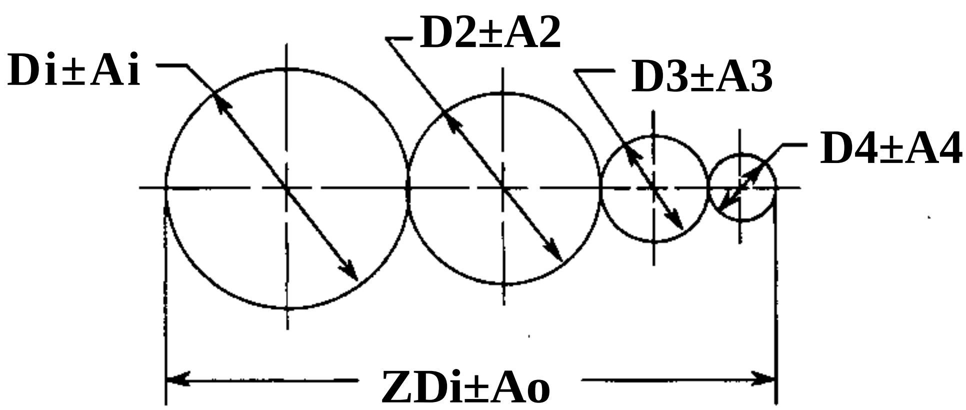

Figure 7. System of four friction wheels

We will demonstrate application of the algorithms to the design of the system of four friction wheels in Figure 7 where each wheel can be manufactured by either turning, grinding, or stock slicing (saw cutting) as discussed in section 3. These results can easily be applied to different manufacturing processes with similar cost functions.

6.1 Tradeoff of Manufacturing Processes

Consider the problem where the friction wheels can be manufactured by three different processes. The wheel diameters and the numerical coefficients for the cost function of equation (9) are given in Table 1. The cost functions of the three processes as a function of tolerance are given in Figure 8 By specifying an output tolerance (A_{ou}t) of 0.08, the optimal configuration has parts 1-4 allocated tolerances of 0.0654, 0.0049, 0.0049, 0.0048, and processes of sawing, sawing, grinding and grinding, respectively;

the system has a total cost of \$6.63. These results are obtained for both the discrete and continuous **methods** introduced in this paper.

Consider the sensitivity of the solution to A_{out}. Figure 9 shows the minimum cost as **a** function of A_{out}; the function is a smooth curve with greater sensitivity for very tight tolerance and relatively small sensitivity for larger tolerances.

In general, although the cost function is smooth with little sensitivity to change in A_{out}, the selection of manufacturing process and allocated component tolerances is quite sensitive. Figure 10 shows the manufacturing process of each component as a function of A_{out}. Note that at tight tolerances all components are ground. As A_{out} increases, the processes progress from grinding to turning to saw cutting, and at loose tolerances all components are saw cut. Note that the larger components (which are more expensive to hold at tight tolerances) rapidly switch to the lathe and then saw cut; the smaller components are ground until much higher tolerances. Note also that all components rapidly switch from the grinder to the saw cut and very few components are ever turned on the lathe; the third wheel actually skips the lathe step altogether.

As we follow the transition of the different manufacturing processes for each part, the allocated tolerances also uniquely change as shown in Figure 11. Initially all wheels are kept at a tight tolerance. As A_{out} becomes looser, the larger wheels are allocated looser tolerances while the smaller, less expensive, wheels maintain tight tolerances. The largest wheel tolerance increases until it becomes more expensive for the other wheels to keep tight tolerances than to continue to loosen the tolerance of the largest wheel; the largest wheel tolerance is then slightly tightened as the next largest wheel tolerance becomes significantly looser. The two larger wheels then increase in tolerance until the third largest wheel is allocated a looser tolerance and the two larger wheels obtain a slightly tighter tolerance. This process continues until the final wheel must obtain a looser tolerance. Note that as the wheel diameter is increased, the cost of holding tighter tolerances is also increased. This causes the tolerances on the larger wheels to be relaxed before those of the smaller wheels.

From Figures 10 and 11 we can conclude that manufacturing processes and allocated tolerances do not have smooth transitions. Rather there is great cost sensitivity between the different process and their implied tolerances as A_{out} changes. Accounting for these variations leads to optimal design cost configurations.

Table 1. Parameter values used in equation (9) for three processes for each friction wheel

Wheel Number

Process		1 4.0 inch dia	2 3.0 inch dia	3 2.5 inch dia	4 1.0 inch dia
i	a	0.02	0.02	0.02	0.02
	b	2.0	1.13	0.78	0.13
	K	0.0266134	0.0201421	0.0169064	0.0071994
1	a	0.005	0.005	0.005	0.005
	b	2.67	1.63	1.20	0.3
	K	0.00568620	0.00519498	0.00494938	0.00421255
rind	a	0.0005	0.0005	0.0005	0.0005
	b	2.95	1.89	1.45	0.52
	K	0.0003004	0.0002902	0.0002851	0.0002699

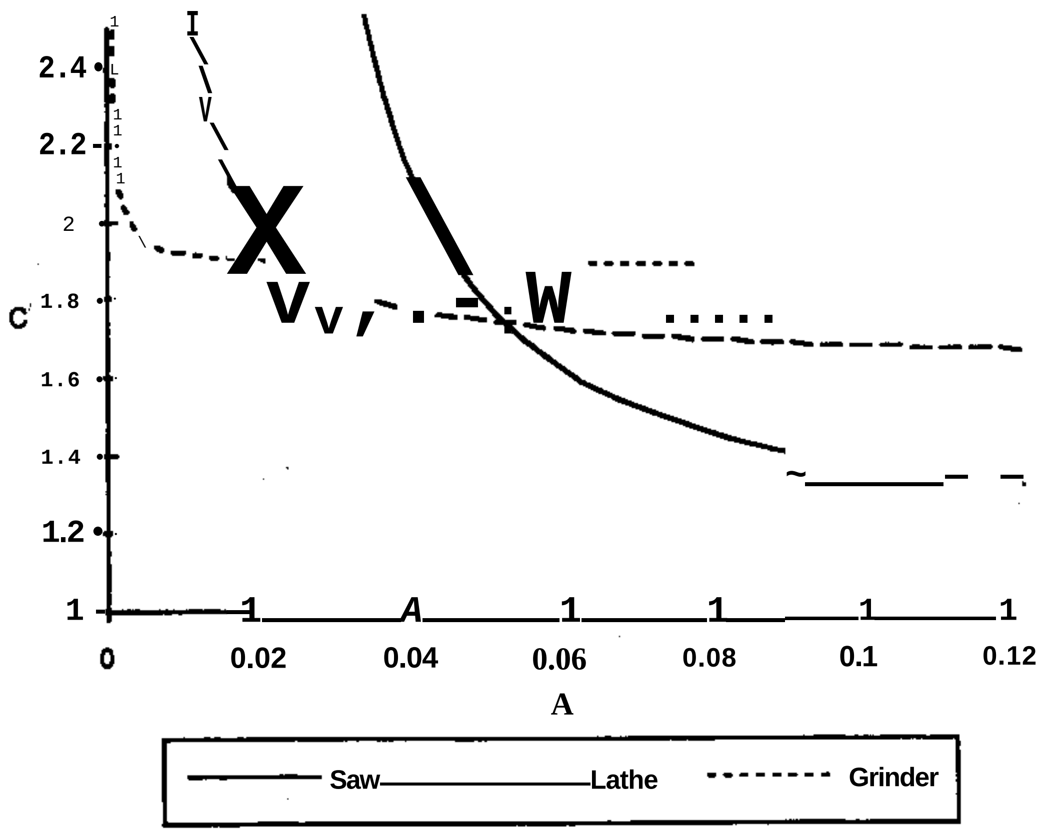

Figure 8. Tolerance versus cost functions for three processes

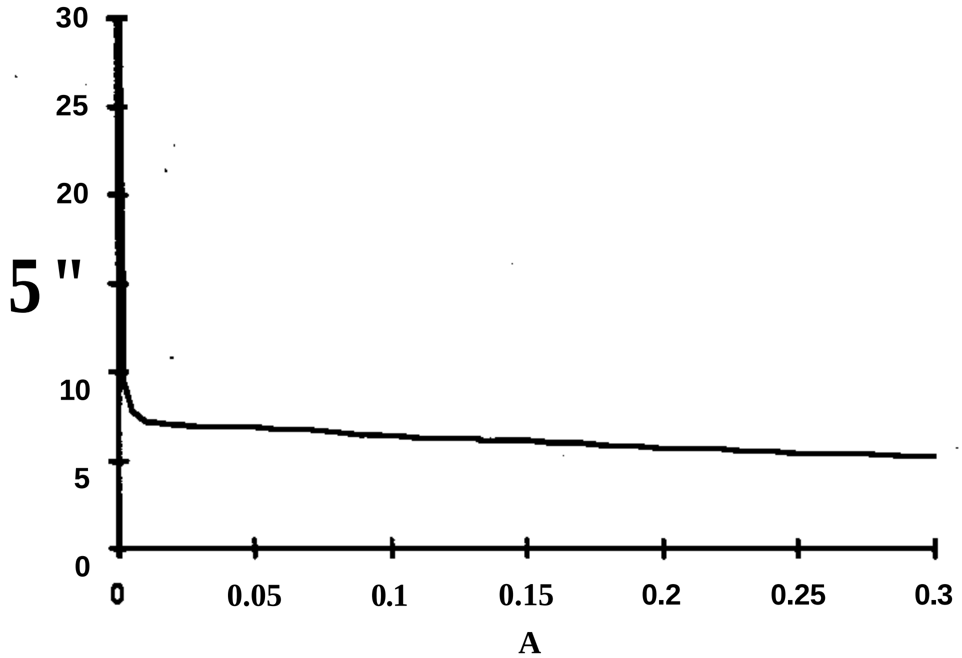

Figure 9. Optimal cost versus Δ_{out}

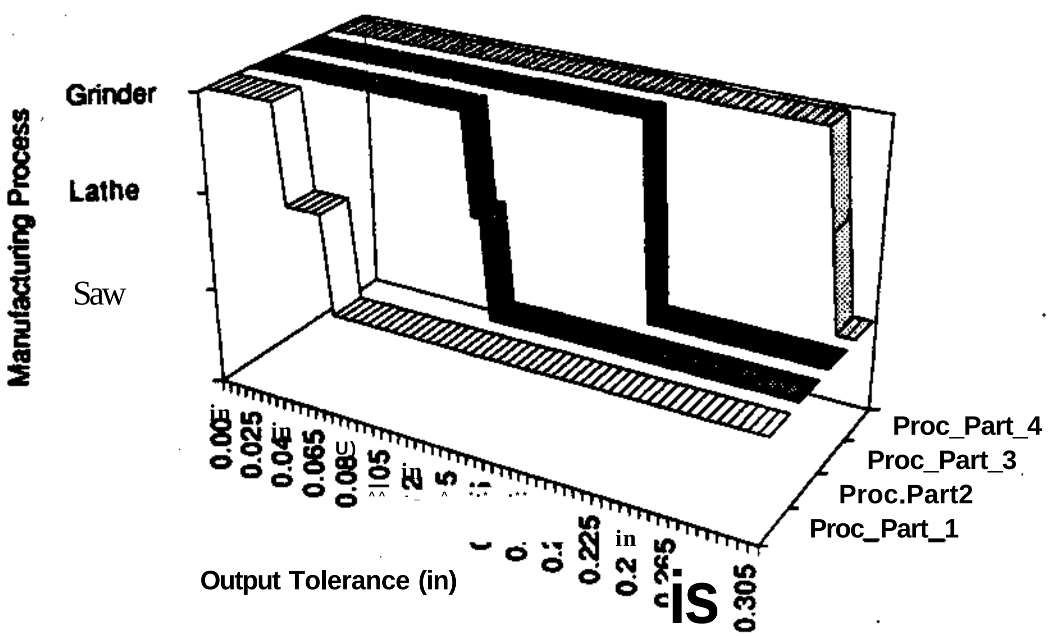

Figure 10. Optimal manufacturing process versus Aout for each component

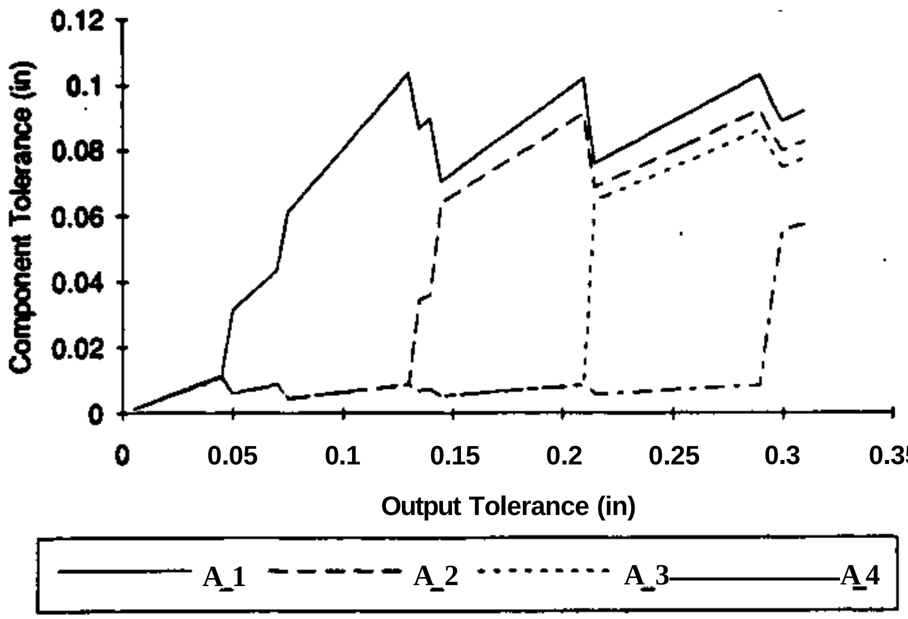

Figure 11. Optimal allocated tolerance versus Aout for each component

7 Conclusions

We have presented two methodologies to determine optimal allocation of tolerances and manufacturing processes to a system of components for minimum cost. Both techniques utilize the stochastic optimization technique of simulated annealing. The first implements a Monte-Carlo simulation for discrete manufacturing processes; the second models the tolerance versus cost function as a continuous function with discontinuous derivatives. The first approach is useful when there is no obvious ordering to the discrete set of processes; the second is useful when there is an ordering and the functions are well known.

We applied the techniques to the design of a system of friction wheels considering the manufacturing processes of grinding, turning, and stock slicing. This example was used to demonstrate the theory, but the approach is more general than for friction wheels alone. The hyperbolic cost functions are useful for numerous other material removal manufacturing processes for a variety of components. However, if the manufacturing process is modeled with a different cost function, the theory still remains valid although the algebraic analysis needs to be updated

8 Acknowledgements

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