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Abstract

When a monopoly carrier provides multiple services(voice, data, video) through a single network, its' profit-maximizing pricing policy usually induces resale. In many cases, resale can benefit consumers by providing them with a cheaper alternatives to a high-priced service. Nevertheless, in this paper, we demonstrate that there are situations in which the carrier can use resale to improve its profits at consumers' expense. We also find that even in cases where resale costs the carrier profit, total consumer surplus does not necessarily always increase. In fact, resale always results in higher consumer surplus in for some users and lower consumer surplus for others. Those findings suggest a regulator should exercise caution in defining the policy for resale.

1. Introduction

With telecommunication deregulation in recent years, many new ventures have started to challenge dominant carriers who previously monopolized telecommunications markets. Given disadvantages in customer base and brand recognition, it is usually difficult for start-ups to engage in direct facility-based competition with dominant carriers. Therefore, many new companies choose to compete with incumbent carrier in an indirect way, such as via resale of the incumbent's services. Even for entrants who are building new networks, resale is a good way to start up servicing customers before all facilities are in place([BEAR98]).

To understand how resale can be profitable in telecommunications markets, consider a dominant carrier who, enabled by packet-switching technology, offers two services at prices p_1 and p_2 through a single network. Assume further that demand for service 2 is less elastic than demand for service 1. To maximize profit, the carrier should set a higher price per unit of resource for services with inelastic demand ([WANG97]). Therefore, p_1 should be smaller than _ p_2 even if calls of service 1 consume exactly half as much capacity as service 2. This price difference can be exploited by a reseller who buys two service 1 calls at price $2p_1$, then multiplexes and resells them as one service 2 call at a price between $2p_1$ and p_2. The business is profitable as long as the multiplexing cost is lower than $p_2 - 2p_1$.

Regulatory policy has a strong influence on the availability of resale. The carrier has an incentive to encourage resale when it improves profit, in which case the regulator can choose to allow or forbid resale. When resale reduces profit, the carrier can drive resellers out of the market by denying service to them or discriminating against them

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through high prices. In this case, the regulator can help resellers to survive by requiring the carrier to give equal access to both end users and resellers.

Determining the appropriate resale policy is a problem facing not only regulators in the US and other industrialized countries, but also those in developing countries. For example, in China, the wireline communications network is monopolized by the Ministry of Information Industry, whose price for international telephone calls is three to six times higher than US prices. In January 1999, a charge was brought against a private operator for reselling the ministry's Internet service as international telephone calls. Though the accused were acquitted, the debate goes on as whether his operation should be allowed[4]. Similar situations have arisen in Haiti where, Teleco, the government owned monopoly, accused ACN, the nation's largest ISP, of reselling international telephone service, and took away two thirds of ACN's access to local telephone lines([PEHA99]).

Intuitively, one would expect that resale benefits consumers by offering a cheaper alternative to users of high-priced services([BEAR98], [KASE97]). Given resellers take away customers, one might also think that resale will cause the carrier's profit to drop, so the carrier will block it. Therefore, to protect consumer benefits, the regulator should always favor resale, and enforce it by preventing the carrier from charging different prices to resellers from end users. However, as we will show in this paper, while the intuition is true in many cases, there are situations in which resale can help the carrier and hurt consumers. Consequently, it is the carrier who should encourage and the regulator who should resist resale. Furthermore, even in cases when resale reduces the carrier's profit, overall consumer benefits, as measured by total consumer surplus, do not always increase. Those results suggest that there is no simple answer to whether the regulator should open the resale market and force the carrier to give equal access to resellers. As indicated by our analysis, to make the right decision, the regulator should consider some seemingly unrelated factors such as the extent to which multiplexing costs and consumer willingness to pay are correlated.

The paper is organized as follows: the mathematical formulation of the problem is given in Section 2. Analysis and major results are presented in Section 3, and the paper is summarized in Section 4.

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[4] Source: News report of Guangzhou Daily, 01/24/99. http://www.asia1.com.sg/gzbao

2. Problem Formulation and Assumptions:

We consider an incumbent carrier who offers two constant bit rate services (indexed as services 1 and 2) with the same average call duration. Assume the data rate of service 2 is k times the rate of service 1. We assume in the absence of resale, the carrier chooses p_1, p_2, prices for services 1 and 2, and C_T, network capacity, to maximize the following profit function:

$$\int_0^T [\frac{(1-\beta_1)p_1\lambda_1}{r_1} + \frac{(1-\beta_2)p_2\lambda_2}{r_2}]dt - F(C_T) \qquad (2.1)$$

where λ_1, λ_2 are call arrival rates, which are functions of p_1, p_2, respectively. T is the planning period. $F(C_T)$ is the capacity cost allocated to that period. β_1 and β_2 are blocking rates of services 1 and 2, which are functions of call arrival rates and capacity.

Suppose demand for service 1 is more elastic than that for service 2; so to maximize (2.1), the carrier will "over-charge" for service 2 in the sense that $p_2 > kp_1$.

In the presence of resale, resellers will buy service 1 calls at p_1, multiplex/demultiplex and resell them as service 2 calls. Denote the mux/demux cost as h, which is assumed to be a random variable distributed over users. Let $\phi(h|w)$ be the conditional Probability Density Function (PDF) of the mux/demux cost, and $\psi(w)$ be the PDF of willingness to pay, then $\gamma(h,w) = \phi(h|w)\psi(w)$ is the PDF of the joint distribution of consumer willingness to pay and mux/demux cost. Assume the resale market is competitive, so resellers charge users at marginal cost, i.e. $kp_1 + h$.

We divide users of service 2 into four segments: segment 1 are users whose willingness to pay is below both the resellers' and carrier's prices; segment 2 are users whose willingness to pay is below the carrier's price but above the resellers' price; segment 3 are users whose willingness to pay is above both prices, and for whom the mux/demux costs are lower than the price difference; and segment 4 are users whose willingness to pay is also above both prices, but whose mux/demux costs are higher than the price difference. Assume users always make their decisions so as to maximize their utility, which is their willingness to pay minus the price they pay. In the presence of resale, segments 2 and 3 users will buy service 2 from resellers, segment 4 users will buy the service from the carrier, and segment 1 users will not buy service from either of them.

Define $\lambda_{2i}(i=2,3,4)$ as call arrival rates from segment i users, so $\lambda_2 = \lambda_{22} + \lambda_{23} + \lambda_{24}$. Denote maximum mux/demux cost per user as H. Figure 1(a-c) shows the division of those segments in different situations, and the formula following each figure gives call arrival rates from each group in that situation:
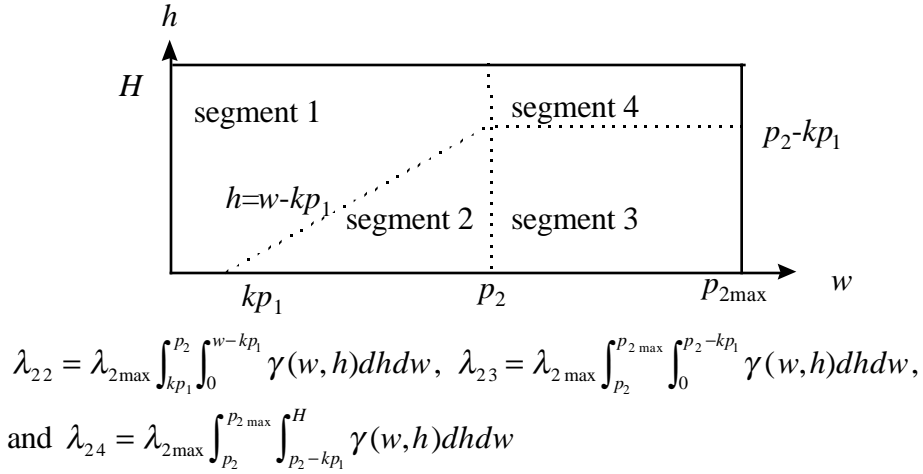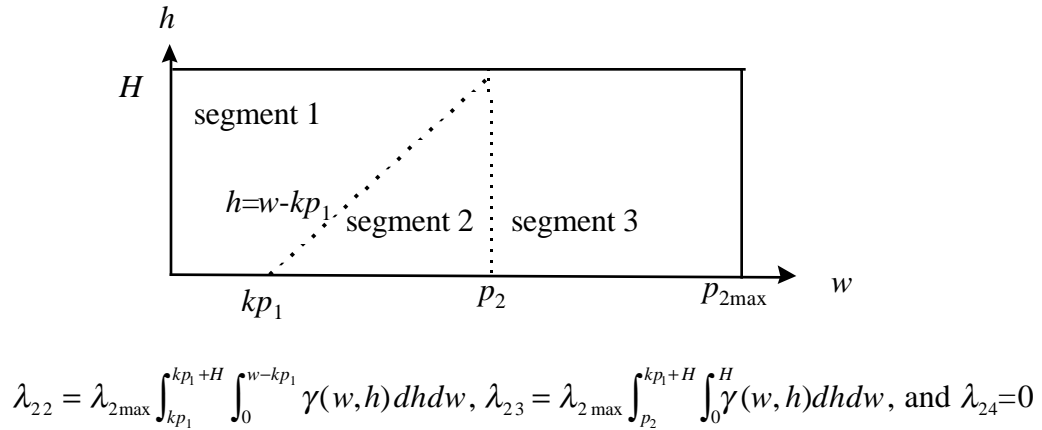
Figure 1 (a)
Segmentation of Users

If ($p_2 \le kp_1$):

$$\lambda_{22}=0, \quad \lambda_{23}=0, \text{ and } \lambda_{24} = \lambda_{2\max} \int_{p_2}^{p_{2\max}} \int_0^H \gamma(w,h)dhdw$$
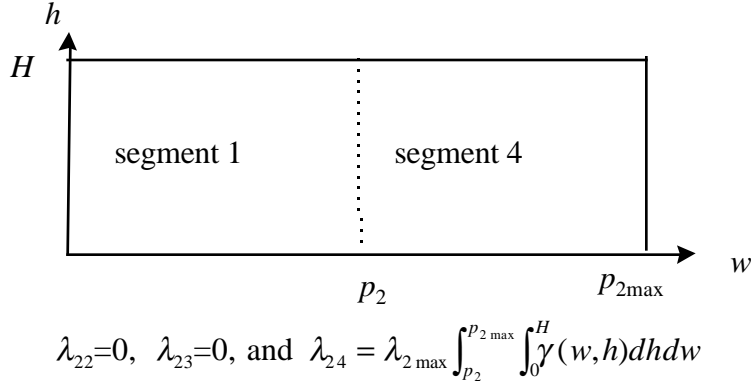
If ($kp_1 \leq p_2 < kp_1 + H$)

Figure 1 (b)
Segmentation of Users

$$\lambda_{22} = \lambda_{2\max} \int_{kp_1}^{p_2} \int_0^{w-kp_1} \gamma(w,h)dhdw, \quad \lambda_{23} = \lambda_{2\max} \int_{p_2}^{p_{2\max}} \int_0^{p_2-kp_1} \gamma(w,h)dhdw,$$

$$\text{and } \lambda_{24} = \lambda_{2\max} \int_{p_2}^{p_{2\max}} \int_{p_2-kp_1}^{H} \gamma(w,h)dhdw$$

If ($kp_1 + H \leq p_2$)

Figure 1 (c)
Segmentation of Users

$$\lambda_{22} = \lambda_{2\max} \int_{kp_1}^{kp_1+H} \int_0^{w-kp_1} \gamma(w,h)\,dhdw, \quad \lambda_{23} = \lambda_{2\max} \int_{p_2}^{kp_1+H} \int_0^H \gamma(w,h)dhdw, \text{ and } \lambda_{24}=0$$

The carrier collects $2p_1$ per minute from segments 2 and 3 users of service 2 and p_2 per minute from segment 4 users. Including both revenue from service 1 and capacity investment cost, the profit function for the carrier can be written as:

$$\int_0^T \{\frac{(1-\beta_1)p_1\lambda_1}{r_1} + \frac{(1-\beta_2)}{r_2}[2p_1(\lambda_{22}+\lambda_{23}) + p_2\lambda_{24}]\}dt - F(C_T) \qquad (2.2)$$

where β_1 and β_2 are blocking rates of services 1 and 2, the values of which we can calculate based on steady-state queuing system analysis, using $\lambda_1, \lambda_2,$ and C_T as inputs ([OZEK90]).

3. Analysis

In the following, we discuss the impact of resale on the dominant carrier and consumers in 3.1 and 3.2 respectively. We demonstrate that resale may or may not reduce the carrier's profit, so the carrier may not always act against it. Furthermore, resale can cause consumer surplus to increase or decrease, depending on the situation. Therefore, whether the regulator should promote resale is a question without unique answer.

Our analysis is based on the model and assumptions developed in section 2. We assume the demand function - *i.e.* call arrival rate as a function of price - takes the following form:

$$\lambda_i = \lambda_{i\,max}[1 - (\frac{p_i}{p_{i\,max}})^{\alpha_i}] \qquad\qquad i=1,2 \quad (3.1)$$

In that formulation, λ_{imax} is the maximum call arrival rate, which equals call arrival rate when price is zero. p_{imax} is the maximum consumer willingness to pay, which equals the lowest price at which there will be no call arrival. α_i is a parameter that characterize how fast demand falls as price increases.

In all cases, we assume the carrier offers two constant bit rate services of 64kbps and 128kbps, respectively. The average call duration is 10 minutes for both services.

3.1 Impact of Resale on the Carrier

Given resellers competing for customers with the monopoly carrier, it is not difficult to imagine that resale can reduce the carrier's profit. However, in this section, we will first demonstrate that there can be situations where resale causes the carrier's profit to increase, and explain why. We will then discuss under what situations, resale always reduces the carrier's profit.

Consider the following example: the dominant carrier serves two groups of users, indexed by *a* and *b*. Both groups have the same demand function for service 1 but different demand functions for service 2. Let $\lambda_j^{(g)}$ represent the demand of group *g* for service *j*. We assume:

$$\lambda_1^{(g)} = 20*[1 - (\frac{p_1}{0.6})^{0.4}] \qquad\qquad g=a,b$$

and

$$\lambda_2^{(a)} = 10*[1 - (\frac{p_2}{1.5})^{0.5}] \text{ and } \quad \lambda_2^{(b)} = 10*[1 - (\frac{p_2}{2})^{0.5}]$$

We also assume that the two groups have different mux/demux costs, *h*. For group *a*, *h*=0, while for group *b*, *h*=2.5. In essence, group *a* consists of users with low willingness to pay and low mux/demux cost, while group *b* consists of users with high willingness to pay and high mux/demux cost.

As formulated in Section 2, we assume the carrier maximizes profit as defined in (2.1) in the absence of resale, and the profit function defined in (2.2) in the presence of resale. Based on solving strategy specified in WANG98, we obtain optimal prices, revenue, profit, and consumer surpluses. Those results are compared in Table 1.

Table 1
Comparisons of Price, Revenues, Profit, and Consumer Surplus

price	service 1	service 2	p_2/p_1	
resale	0.432	1.17	2.71	
no resale	0.399	1.04	2.61	

	revenue		investment	profit
	service 1	service 2		
resale	901	780	832	849
no resale	351	1318	832	838

	consumer surplus			
	service 1	service 2 (group 1)	service 2 (group 2)	total
resale	58	207	263	528
no resale	85	106	360	551

Since resellers buy service 1 from the carrier and resell it as service 2 to end users, it can be expected that in the presence of resale, the carrier's revenue from service 1 increases, and that from service 2 decreases, just as shown in Table 1. What is interesting in this example is that when resale occurs, the carrier's optimal price of service 2 also increases: it is optimal for a carrier to charge an even higher price for service 2 when it faces competitions from resellers than in the monopoly situation. Furthermore, it is also shown in the table that under such a pricing strategy, the carrier's profit increases as a result of resale. Under our assumptions, the carrier is able to increase profits via price discrimination as we explain below.

A company can usually achieve a higher revenue by dividing markets into different segments, and charging a different price to each segment. Sometimes, the price discrimination can be practiced through a third party if the company can't do it itself. For example, fashion designer's may provide goods labeled with a store brand, which helps to divide buyers into a segment that is willing to pay a high price for "brand-names", and a segment that wants cheaper store brands. Those manufacturers can then charge a higher price to the higher willingness to pay segment, while still retain the lower willingness-to-pay segment through the store brand.

Similarly, a carrier can practice price discrimination through resellers. In the above example, group *a* users have lower demand, and group *b* users have higher demand. As a result, the carrier can choose to provide service 2 only to group *b* and leave group *a* to resellers. This segmentation gives the service provider the leverage to increase the price of service 2 since they can still indirectly sell the service to the low willingness to pay users through resellers at a lower price ($2p_1$ instead of p_2).

Like other forms of price discrimination, the strategy works only when the carrier is capable of retaining higher willingness to pay users. Therefore, it is crucial in the above example that users of group b have higher mux/demux costs, and would thus pay a higher total price by switching to resellers. In fact, such a positive correlation between consumer willingness to pay and mux/demux costs is a necessary condition for the carrier to benefit

from resale. In the following proposition, we show that when they are uncorrelated, the carrier can not benefit from resale(see appendix for proof).

Proposition 1 Let $F(C)$ be the incumbent carrier's investment as a function of C, the amount of capacity invested. Let $G(x,y)$ be the minimum capacity required to maintain given blocking rates for two services when call arrival rates are x and y, and let $\phi(h|w)$ be the conditional PDF of mux/demux cost. Let (p_{1a},p_{2a}) and (p_{1p},p_{2p}) be the carrier's profit-maximizing prices in the absence and presence of resale, respectively. $\Pi_a(p_{1a},p_{2a})$ and $\Pi_p(p_{1p},p_{2p})$ are corresponding profits:

    1). $\dfrac{\partial^2 F}{\partial C^2} \leq 0$ for any C,

    2). $G(x, y + \Delta y) \leq G(x, y) + G(x, \Delta y)$ for all x, y, and $\Delta y > 0$,

    3). $\phi(h|w) = \phi(h)$,

    then $\Pi_p(p_{1p}, p_{2p}) \leq \Pi_a(p_{1a}, p_{2a})$.

In summary, we demonstrated in this section that resale may not always cause the carrier's profit to decrease, and derived a necessary condition if the carrier is to improve profit from resale. The discussion indicates two public policy making scenarios: in the case where resale benefits the carrier, the regulator's policy choice is whether or not to ban resale. In the case resale cause the carrier's profit to decrease, the regulator's policy choice then becomes whether or not to enforce resale by requiring the carrier to charge the same price to resellers and end users for the same service. In either scenario, regulator's choice depends upon the impact on consumer benefits, to which now turn our attention.

3.2 Impact of Resale on Consumers

Assume that the regulator's objective is to enhance consumer benefits, and will thus use consumer surplus to evaluate the impact of resale. In this section, we compare total consumer surplus with and without resale, and discuss the policy implications for the regulator.

In section 3.1, we demonstrated in our example that when there is a positive correlation between consumer willingness to pay and mux/demux costs, resale can be used by the carrier to improve its profit through price discrimination. While price discrimination is not necessarily always harmful to consumers, in our example, total consumer surplus does decrease as a result of resale. The example shows there can be cases in which resale allows the carrier to profit at consumers expense.

We now consider other scenarios consistent with Proposition 1, in which resale always reduces the carrier's profit because we assume there is no positive correlation between consumer willingness to pay the mux/demux cost. Our analysis below demonstrates that even in these situations, resale may or may not benefit consumers.

Consider the following example: assume the mux/demux cost is independent of consumer willingness to pay, and is uniformly distributed in the interval $[0,H]$, where $H=0.1$. Assume the demand functions for services 1 and 2 are the same as in Equation 3.1. Let $\lambda_{1max} = \lambda_{2max} = 6$, $\alpha_1 = 0.4$, $\alpha_2 = 1.0$, $p_{2max} = 1.5$, and vary p_{1max} from 0.5 to 0.75. The difference in profit with and without resale is compared in Figure 2, the difference in total consumer surplus is compared in Figure 3, and differences in prices of services 1 and 2 are compared in Figure 4.

As indicated by Proposition 1, given there is no positive correlation between mux/demux cost and consumer willingness to pay, the carrier's profit is always lower in

the presence of resale (See Figure 2). Furthermore, unlike the previous case in which the carrier can increase price because of the discrimination effect, the carrier's price of service 2 is always lower with resale. What is interesting is that despite drops in both profit and price, resale does not necessarily always result in higher total consumer surplus. As shown in Figure 3. As shown in Figure 3, when p_{1max} =0.75, total consumer surplus is actually lower in the presence of resale. This phenomena can be explained as follows:
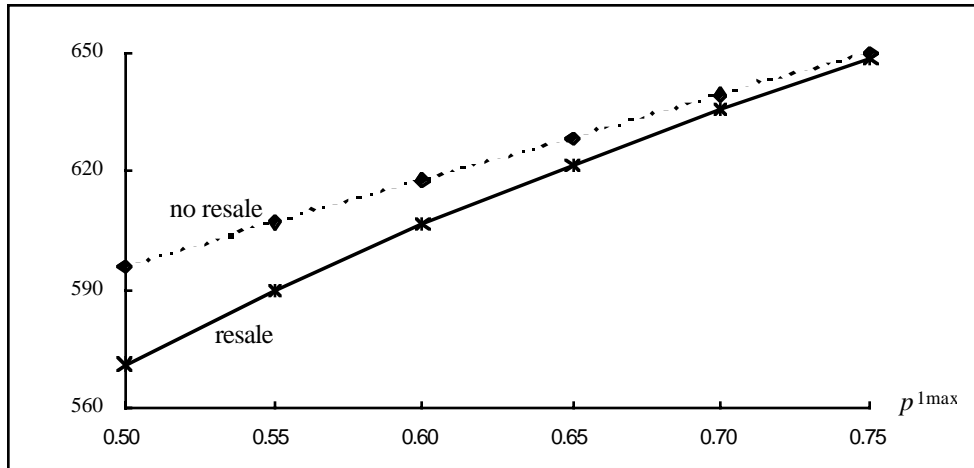
Figure 2
Comparison of Profit

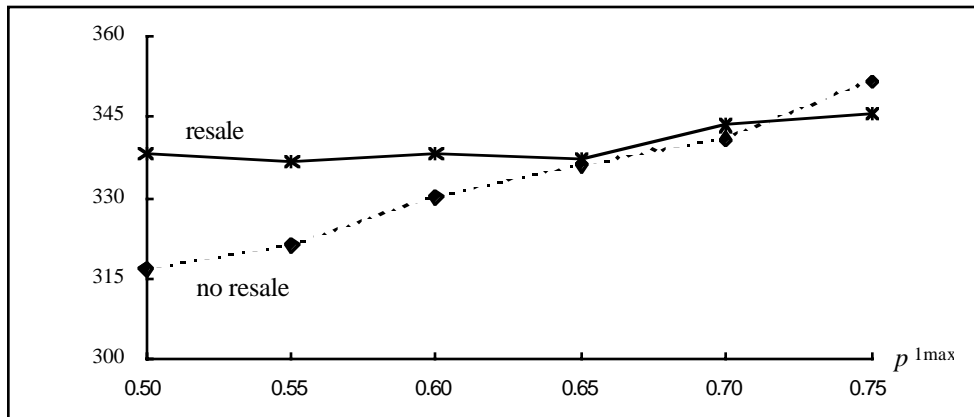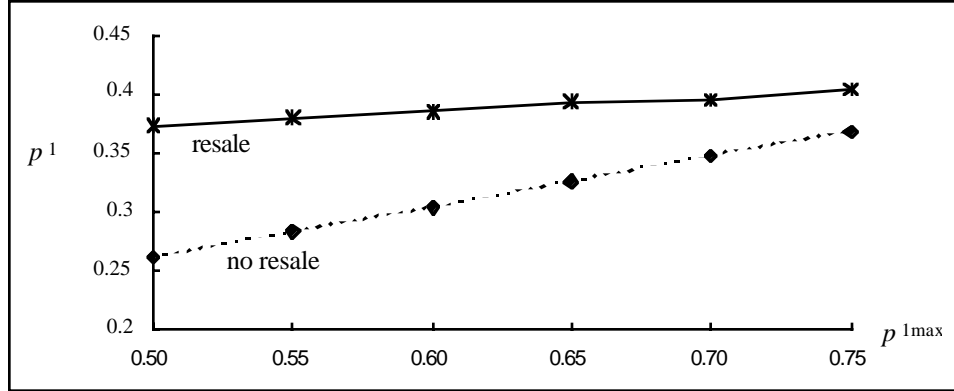**Figure 3**
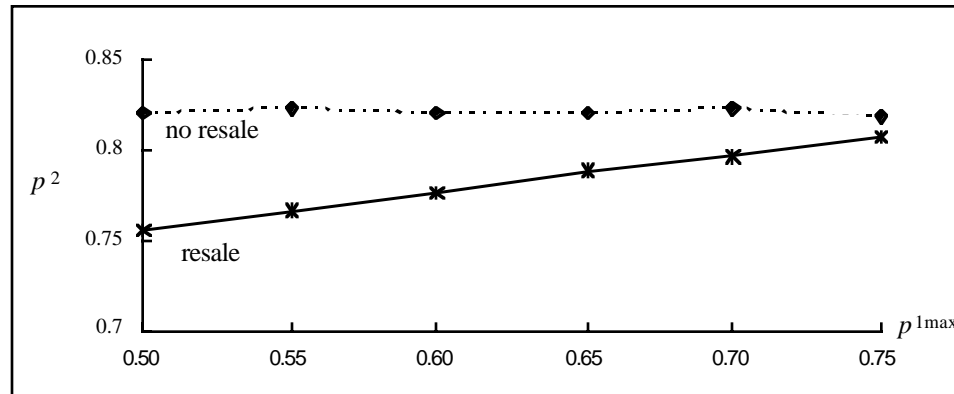**Comparison of Total Consumer Surplus**

Figure 4 (a)
Comparison of Price of Service 1

To compete with resellers, the carrier has to reduce the discrepancy in the price per unit of capacity between the two services, which can be accomplished either by increasing the price of service 1 or decreasing the price of service 2. Given that the original prices are optimized for the situation without resale, increasing price 1 and reducing price 2 will cause the carrier's revenues from services 1 and 2 to fall, respectively. From the carrier's perspective, changes in the two prices should be balanced to minimize the decrease in total profit. From the consumers' perspective, increasing price 1 results in a smaller consumer surplus from service 1 and decreasing price 2 leads to greater consumer surplus from service 2. Therefore, whether or not resale can cause total consumer surplus to increase depends on how much price 1 is increased versus how much price 2 is decreased.

For the demand function assumed by Equation 3.1, a smaller value for p_{1max} implies a greater elasticity of demand of service 1. This means the same percentage increase of price for service 1 leads a larger percentage decrease in demand, and thus revenue. Therefore, the carrier's optimal response to resale is to decrease price 2 more when p_{1max} is smaller, and increase price 1 more when p_{1max} becomes larger. As p_{1max} keeps increasing, the carrier relies more on increasing price 1 to eliminate the difference of price per unit of capacity between the two services. Eventually, the decrease of consumer surplus due to resale of service 1 outweighs the increase of consumer surplus of service 2, and total consumer surplus starts to decrease.

The analysis has the following two implications for the regulator. First, the fact that the price of service 1 is always higher with resale (see Figure 4.4a) implies there is a tradeoff for the regulator, *i.e.* opening up the resale market will inevitably hurt some users

even if it helps to improve overall consumer welfare. As a result, if a regulator considers that the welfare of some users are more important than others, s/he may still choose not to enforce resale. For example, if the regulator's first priority is to keep down household phone bills, then s/he will not authorize reselling of residential POTS lines as business lines, even if doing so would increase the combined welfare of residential and business customers.

More importantly, even if the regulator is indifferent to distributional effects and choose to maximize total consumer surplus, resale may still be unattractive. This happens in two situations. First, when there is a positive correlation between consumer willingness to pay and mux/demux costs, resale can be used by the carrier as a means of price discrimination, which may lower total consumer surplus. When this happens, the carrier will embrace resale, but the regulator should forbid it. Second, even if there is no such positive correlation and resale always hurts the carrier's profit, the carrier may still increase the price of service bought by resellers to such an extent that total consumer surplus decreases. In both cases, the regulator may wish to eschew resale as a means to constrain pricing by a monopolist.

4. Summary

To maximize profit, a monopoly carrier always charges a higher price for a service than the cost of providing it. In comparison with a competitive market where price equals marginal cost, the monopoly carrier earns monopoly rents while consumers enjoy a smaller surplus. The difference between price and cost is most significant in service with inelastic demand, and the carrier extracts more rents from users of those services.

To protect consumer interests, regulatory rules have been developed to constrain monopoly pricing. Requiring the carrier to allow resale is one of those approaches. If the carrier is prohibited by regulation from denying service to resellers or charging resellers a different price from ordinary users, it can be profitable to buy one service with elastic demand and resell it as a service with inelastic demand at a lower price. Therefore, one would expect that resale can give cnsumers a cheaper alternative to a previously highly-price service, thereby increasing consumer surplus. Our analysis in this paper demonstrates while the expectation is true in many cases, there are also situations under which consumer benefits can't be improved by promoting resale. Furthermore, there even can be cases where for the benefit of consumers, a regulator should ban rather than require resale.

We present a numerical example that demonstrates that when there is a positive correlation between consumer willingness to pay and mux/demux cost, the carrier may profit from resale. In this case, instead of lowering price to compete with resellers, the carrier can charge a higher price to high willingness-to-pay users who can't switch to resellers because of the high mux/demux cost. Because of resale, the carrier is not punished for losing customers by raising prices, since it can still sell to low willingness-to-pay users through resellers. As a result, in our example, resale causes the carrier's price and profit to increase and total consumer surplus to decrease. Therefore, the carrier will encourage resale even without being required by regulation, and the regulator should ban resale to maximize consumer surplus.

We prove that resale always causes the carrier's profit to decrease if there is no correlation between a reseller's mux/demux costs and consumers' willingness to pay for the service being resold. In those cases, the carrier has no incentive to allow resale, so the regulator has to mandate it. Whether the regulator should do so depends on circumstances. Resale can benefit users of the service with inelastic demand because: 1) users who buy the

service from resellers get a lower price; 2) some users whose willingness to pay is below the carrier's price can get the se4rvice form resellers; and 3) the carrier may reduce the price to retain customers, thus benefiting those who continue use its service. However, resale also causes the price of the service with the more elastic demand to increase and hurts the original users. In some cases, the decrease in consumer surplus from that service exceeds the increase in consumer surplus from other services, so total consumer surplus decreases as a result of resale. Even in cases when resale leads to an increase in total consumer surplus, the regulator may hesitate to mandate it in consideration of the differential impact on consumers of the different services. Resale may improve total consumer welfare only by increasing the welfare of one group more than it decreases the welfare of another.

In summary, while requiring the carrier to allow resale has been viewed as a way to enhance competition and improve consumer welfare, our research identifies some possible pitfalls in this approach. In deciding whether or not to require resale, the regulator should consider the conflict of interests among different groups of users, and be aware that under certain situations, resale can reduce instead of increase consumer welfare.

This work could be extended in several different directions. It would be interesting to consider the impact of resale on other potential forms of competition. For example, one might want to examine whether mandating resale would enhance or retard facility based competition. It would also be interesting to examine the dynamics of resale growth and pricing response over time. The result should be interesting for policy making with respect to the growing market for Internet Telephony, where packet data service is resold as voice service.

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Appendix - Proof of Proposition 1:

Denoting the demand function of service i as $\lambda_i(p)$, $i=1,2$, we start by considering the carrier's profit in four scenarios:

1) Suppose in the absence of resale, p_{ia}, β_{ia} are optimal prices and blocking rates for service i ($i=1,2$), C_a is the optimal capacity, then:

$$\Pi_a = \frac{(1-\beta_{1a})p_{1a}\lambda_1(p_{1a})}{r_1} + \frac{(1-\beta_{2a})p_{2a}\lambda_2(p_{2a})}{r_2} - F(C_a) \tag{A-1}$$

2. Suppose in the presence of resale, p_{ip} and β_{ip} are the optimal prices and blocking rates for services i ($i=1,2$).

Define $\theta = \int_0^{p_{2a}-kp_{1a}} \phi(h)\,dh$ (A-2)

Define λ_{2r} as the call arrival rate for resellers' services. The call arrival rate of service 2 for the carrier is $(1-\theta)\lambda_2(p_{2p})$. Define C_p as the minimum capacity needed to keep call blocking rates below β_{1p} and β_{2p} when the call arrival rates of services 1 and 2 are $\lambda_1(p_{1p})$ and $\lambda_{2r}+(1-\theta)\lambda_2(p_{2p})$. The carrier's maximum profit in the presence of resale is:

$$\Pi_p = \frac{(1-\beta_{1p})p_{1p}\lambda_1(p_{1p})}{r_1} + \frac{(1-\beta_{2p})[kp_{1p}\lambda_{2r}+(1-\theta)p_{2p}\lambda_2(p_{2p})]}{r_2} - F(C_p) \tag{A-3}$$

3). Suppose in the absence of resale, the carrier charges p_{1p} and p_{2p}, and keeps blocking rates below β_{1p}, β_{2p}. The minimum capacity needed is C_g. The carrier's profit is:

$$\Pi_g = \frac{(1-\beta_1)p_{1p}\lambda_1(p_{1p})}{r_1} + \frac{(1-\beta_{2p})p_{2p}\lambda_2(p_{2p})}{r_2} - F(C_g) \tag{A-4}$$

4). Suppose in the presence of resale, the carrier charges p_{1p} and kp_{1p}, and keeps blocking rates below β_{1p},β_{2p}. The minimum capacity needed is C_y. The carrier's profit is:

$$\Pi_Y = \frac{(1-\beta_1)p_{1p}\lambda_1(p_{1p})}{r_1} + \frac{(1-\beta_{2p})kp_{1p}\lambda_2(kp_{1p})}{r_2} - F(C_y). \tag{A-5}$$

We now prove the theorem ($\Pi_a \geq \Pi_p$) by showing if $\Pi_a < \Pi_p$, then $\Pi_p < \Pi_g$, which contradicts the assumption that Π_p is the maximum profit in the presence of resale.

If $\Pi_a < \Pi_p$, then $\Pi_g \leq \Pi_p$ because in the absence of resale, Π_a is the optimal solution and Π_g is just one feasible solution. By equations (A-3) and (A-4):

$$\Pi_p - \Pi_a = \frac{(1-\beta_{2p})}{r_2}[kp_{1p}\lambda_{2r} - \theta p_{2p}\lambda_2(p_{2p})] - F(C_p) + F(C_g) \geq 0 \tag{A-6}$$

Define $\lambda_{2g}=\lambda_{2r}-\theta\lambda_2(p_{2p})$. From Figures 1(b), $\lambda_{2g}>0$. From (A-6)

$$\frac{(1-\beta_{2p})}{r_2}[kp_{1p}l_{2g} - \theta(p_{2p}-kp_{1p})\lambda_2(p_{2p})] \geq F(C_p) - F(C_g) \tag{A-7}$$

Define $\lambda_{2b}=\lambda_2(kp_1)-\lambda_2(p_2)-\lambda_{2g}$. From Figure 1(b)

$$\frac{\lambda_{2b}}{\lambda_{2g}} \geq \frac{1-\theta}{\theta} \tag{A-8}$$

Multiply $\frac{\lambda_{1b}}{\lambda_{1a}}$ at both sides of (A-7) and apply equation (A-8),

$$\frac{(1-\beta_{2p})}{r_2}[kp_{1p}\lambda_{2g} - (1-\theta)(p_{2p}-kp_{1p})\lambda_2(p_{2p})] \geq \frac{\lambda_{2b}}{\lambda_{2g}}[F(C_p)-F(C_g)] \tag{A-9}$$

C_p, C_g, and C_y are capacities needed to accommodate $\lambda_1(p_1)$ and $\lambda_2(p_2)$, $\lambda_1(p_1)$ and $\lambda_{2r}=\lambda_2(p_2)+\lambda_g$, $\lambda_1(p_1)$ and $\lambda_2(kp_1)=\lambda_2(p_2)+\lambda_{2g}+\lambda_{2b}$, and keep blocking rates at β_{1p},β_{2p}.

Since $G(x,y+\Delta y) \leq G(x,y)+G(x,\Delta y)$ for all x, y, Δy, $\dfrac{\partial^2 F}{\partial C^2} \leq 0$

$$\frac{\lambda_{2b}}{\lambda_{2g}}[F(C_p)-F(C_g)] \geq F(C_y)-F(C_g) \tag{A-10}$$

Combine equations (A-9) and (A-10):

$$\frac{(1-\beta_{2p})}{r_2}[kp_{1p}\lambda_{2b}-(1-\theta)(p_{2p}-kp_{1p})\lambda_2(p_{2p})] \geq F(C_y)-F(C_g) \tag{A-11}$$

From Equations (A-3), (A-5), and (A-11)

$$\Pi_Y - \Pi_p = \frac{(1-\beta_{2p})}{r_2}[kp_{1p}\lambda_{2b}-(1-\theta)(p_{2p}-kp_{1p})\lambda_2(p_{2p})]-F(C_y)+F(C_g) \geq 0,$$

which contradicts the assumption Π_p is the maximum profit in the presence of resale. Therefore, $\Pi_a \geq \Pi_p$