Measurements of the branching fractions and helicity amplitudes in $\bar{B}D^*\rho$ decays

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Hadronic decays of heavy mesons are complicated by final-state interactions (FSI) which result from strong rescattering of the products of the weak decay process. FSI effects may be less important if the final state is easy to produce directly via weak decay. It is also argued that if the final-state hadrons separate rapidly, due to a large energy release, there is little time for interaction. The factorization hypothesis, widely used in heavy-quark physics for hadronic two-body decays, assumes that the two hadronic currents may be treated independently of each other, neglecting FSI. In particular, the Bauer-Stech-Wirbel (BSW) model [1] utilizes this approximation in assuming that the short and long distance QCD contributions can be factorized. However, the validity of the factorization hypothesis has not been demonstrated by any rigorous theoretical calculation.

Körner and Goldstein [2] suggest a test of the factorization hypothesis by examining the polarization in B meson decays into two vector mesons. The idea is that, under the factorization hypothesis, certain hadronic decays are analogous to similar semileptonic decays evaluated at a fixed value of the momentum transfer, $q^2 = M^2_{B^*}$. For instance, the polarization of the $D^{*+}$ meson in $B^0 \to D^{*+} \rho^-$ should equal that in $B^0 \to D^{*+} \ell^- \nu$ at $q^2 = M^2_{\rho}$. Specifically,

$$
\frac{\Gamma_L}{\Gamma} (B^0 \to D^{*+} \rho^-) = \frac{\Gamma_L}{\Gamma} (B^0 \to D^{*+} \ell^- \nu)|q^2 = M^2_{\rho}|.
$$

Here, $\Gamma_L/\Gamma$ is the fraction of longitudinal polarization.

The differential decay rate for $B \to D^\pi \rho^-$ can be expressed in terms of three complex helicity amplitudes $H_0$, $H_+$, and $H_-$ as

$$
d^3\Gamma = \frac{9}{32\pi} \times \left\{ 4|H_0|^2 \cos^2 \theta_{D^\pi} \cos^2 \theta_{\rho} + \left(|H_+|^2 + |H_-|^2\right) \sin^2 \theta_{D^\pi} \sin^2 \theta_{\rho} + 2|\text{Re}(H_+ H_0^*)| \cos 2\chi 
- \text{Im}(H_+ H_0^*) \sin 2\chi \sin^2 \theta_{D^\pi} \sin^2 \theta_{\rho} + \left[\text{Re}(H_+ H_0^*) + H_- H_0^*\right] \cos \chi - \text{Im}(H_+ H_0^*) 
- H_- H_0^* \sin \chi \sin 2\theta_{D^\pi} \sin 2\theta_{\rho} \right\},
$$

where $\theta_{D^\pi}$ is the decay angle of the $D^0$ in the $D^*$ rest frame with respect to the $D^*$ line of flight in the $B$ rest frame; $\theta_{\rho}$ is the decay angle of the $\pi^-$ in the $\rho^-$ rest frame with respect to the $\rho^-$ line of flight in the $B$ rest frame; $\chi$ is the angle between the decay planes of the $D^*$ and $\rho$; and Re$(x)$ and Im$(x)$ denote the real and imaginary parts of $x$, respectively. These angles are shown in Fig. 1.

The longitudinal and transverse polarizations are then defined as

Previous measurements have been performed on the \(D^{*}\rho\) system. Using 0.89 fb\(^{-1}\) of \(Y(4S)\) data and performing an unbinned two-dimensional likelihood fit to the joint \((\cos \theta_{D^*}, \cos \theta_\rho)\) distribution, the CLEO Collaboration measured \(\Gamma_{L}/\Gamma = 0.93 \pm 0.05 \pm 0.05\) [3]. Later, using 3.1 fb\(^{-1}\) of \(Y(4S)\) data and performing an unbinned maximum likelihood fit to the joint three-dimensional \((\cos \theta_{D^*}, \cos \theta_\rho, \chi)\) distribution, along with the invariant \(B\) and \(\rho\) mass distributions, CLEO reported a preliminary result of \(\Gamma_{L}/\Gamma = 0.878 \pm 0.034 \pm 0.030\) [4]. Both results are in agreement with the theoretical prediction of 0.895 \pm 0.019 for \(B \rightarrow D^{*}\rho\) [5]. Testing the factorization hypothesis would benefit from further reduction of the experimental uncertainty. We report here an improved measurement using ten times the data of the first measurement. This represents the final update of the second analysis and uses largely the same technique, but with some important improvements in both the event selection and the treatment of acceptance. Our results include the data used in the previous analyses and hence supersede them.

The data used in this analysis were collected at the Cornell Electron Storage Ring (CESR) with the CLEO detector in two configurations, known as CLEO II [6] and CLEO II.V [7]. The data consist of an integrated luminosity of 9.1 fb\(^{-1}\) collected at the \(Y(4S)\) resonance, corresponding to 9.7 \times 10\(^{6}\) \(B\bar{B}\) events, as well as 4.6 fb\(^{-1}\) of continuum data at energies just below the \(Y(4S)\) resonance. The latter is used to study the backgrounds due to the nonresonant \(e^+e^-\rightarrow q\bar{q}\) process.

In CLEO II, the momentum measurement of charged particles is carried out with a tracking system consisting of a six-layer straw-tube chamber, a ten-layer precision drift chamber, and a 51-layer main drift chamber. The tracking system operates inside a 1.5 T superconducting solenoid. For charged particles, the main drift chamber also provides a measurement of ionization energy loss \((dE/dx)\), which is used for particle identification. The CLEO II.V detector was upgraded in two main aspects, both affecting charged particles. First, the straw-tube chamber was replaced with a three-layer double-sided silicon vertex detector; and second, the gas in the main drift chamber was changed from an argon-ethane to a helium-propane mixture. Photons are detected with a 7800-crystal CsI(Tl) electromagnetic calorimeter, which is also inside the solenoid. Muons are identified with proportional chambers placed at various depths within the steel return yoke of the magnet.

Charged tracks with momenta greater than 250 MeV/c are required to come from the interaction point and be well-measured (based on the quality of the track fit and the number of hits). Identified electrons and muons are excluded, and pions and kaons are required to have a measured \(dE/dx\) within 2.5 standard deviations \((\sigma)\) of their expected values. To keep the efficiency high, softer tracks are only required to satisfy a looser requirement of consistency originating at the interaction point. The \(\pi^0\) candidates are formed from pairs of photons with an invariant mass within 2.5 standard deviations of the known \(\pi^0\) mass. These pairs are then kinematically fitted with their invariant mass constrained to the known \(\pi^0\) mass. The \(\chi^2\) of this kinematic fit must be less than nine. To suppress background from fake photons, the constituent photons of the \(\pi^0\) must be detected in the central barrel calorimeter (which has the least material shadowing it) and have a minimum energy of 30–65 MeV, depending on the source \((D^{*0}, D^0, \rho^-)\) of the \(\pi^0\).

We reconstruct candidate \(D^{*0}\) and \(D^{*+}\) mesons in the modes \(D^{*0}\rightarrow D^0\pi^0\) and \(D^{*+}\rightarrow D^0\pi^+\), with \(D^0\rightarrow K^-\pi^+\), \(D^0\rightarrow K^-\pi^+\pi^0\), or \(D^0\rightarrow K^-\pi^+\pi^-\pi^+\). Throughout this paper, charge conjugate modes are implied. The reconstructed \(D^*-D\) mass differences and the \(D^0\) invariant mass are required to be within 2.5\(\sigma\) of the nominal values. The resolutions of these quantities are obtained from Monte Carlo simulations, and the \(D^0\rightarrow K^-\pi^+\pi^0\) resolution includes a \(\pi^0\) energy dependence. We also require the \(D^0\rightarrow K^-\pi^-\pi^0\) candidates to come from the more densely populated regions of the Dalitz plot to suppress combinatoric background. Candidate \(\rho^-\) mesons are selected from \(\pi^-\pi^0\) combinations which have an invariant mass within 150 MeV/c\(^2\) of the nominal \(\rho^-\) mass.

The \(B^-\) and \(\bar{B}^0\) mesons are reconstructed by combining the \(D^{*0}\) or \(D^{*+}\) candidates with the \(\rho^-\). We calculate a beam-constrained \(B\) mass by substituting the beam energy
(E_R) for the measured B^- or B^0 candidate energy (Σ,E_i): $M = \sqrt{E_R^2 - p_R^2}$, where $p_R$ is the measured momentum of the B candidate. This improves the M resolution by one order of magnitude, to about 3 MeV/c^2. The difference between the reconstructed energy of the B^- or B^0 candidates and the beam energy, $ΔE = Σ,E_i - E_R$, is required to be 0 to within 2.5σ. The resolution of the energy difference varies from 10 MeV to 35 MeV, depending on the decay mode, and is also obtained from Monte Carlo simulations. We also account for the dependence of our $ΔE$ resolution on the π^0 (from the $ρ^-$ decay) energy, parametrizing it as a function of $cos \theta_ρ$.

To suppress background from the continuum under the Y(4S) resonance, only events with a ratio of Fox-Wolfram moments [8] $R_2<0.5$ are used, taking advantage of the fact that this ratio is larger for the more jet-like events from the continuum than the more spherical events from BB̄ decays of the Y(4S). We also require that the polar angle of the reconstructed B satisfies $|cos θ_0|<0.95$, given the known $sin^2θ_0$ distribution for Y(4S) decay. Finally, a requirement is made on the cosine of the sphericity angle, $θ_s$, defined as the angle between the sphericity axis of the B decay products and the rest of the particles in the event. Because of their two-jet structure, continuum events peak strongly at $|cos θ_s|=1$, while signal events are flat. Only candidates with $|cos θ_s|$ below a maximum allowed value, dependent on the $D^0$ decay mode, are retained.

To measure both the branching fractions and the helicity amplitudes in the decay of $B→D^0 ρ$, we perform two unbinned maximum likelihood fits. The first fit is an extended likelihood fit to extract the number of signal and background events. The subsequent fit fixes the number of signal and background events and extracts the helicity amplitudes (possible variations in the background level are treated as a systematic uncertainty). Detector smearing is implicitly included in our definition of acceptance: candidates reconstructed in a given kinematic bin, regardless of origin, divided by the number generated in that same bin. The Monte Carlo simulation models the detector smearing very well with any possible inadequacies included later as systematic errors.

The dependence of the acceptance on the decay angles, combined with the effects of detector resolution, are determined from Monte Carlo simulations. However, we need to know the true helicity amplitudes to correctly determine the average acceptance necessary to extract the branching ratios. In addition, we need the correct amplitudes to determine the acceptance in the helicity amplitude fit itself since the distribution of the data affects the acceptance due to smearing of the reconstructed quantities. The fits are therefore iterated until convergence is achieved; we have checked that this convergence to the correct result is rapid and independent of the initially assumed amplitudes.

The generation of large numbers of signal Monte Carlo events is time intensive; we therefore reweight our Monte Carlo sample in order to calculate the correct acceptance for any desired set of helicity amplitudes. The weighting factor is the ratio of the decay rates given by Eq. (2) for the desired and generated amplitudes.

Events from the three $D^0$ decay modes are combined in the fit. The likelihood function, $L$, has the form

$$L = \prod_{j=1}^{3} \prod_{i=1}^{n_j} \prod_{j=1}^{n_i} e^{-n_j} \frac{n_j!}{n_i!} \frac{p_{ji}^S(M,m,cos θ_{D^0,ρ},X) + n_j^B \cdot p_{ji}^B(M,m,cos θ_{D^0,ρ},X)}{n_j^S + n_j^B},$$

where $m$ is the invariant mass of the candidate $ρ^-$ meson, $n_j^S$ and $n_j^B$ are the number of signal and background events for the $j$-th $D^0$ decay mode, respectively, and $n_j=n_j^S+n_j^B$ is the total number of data events for the $j$-th $D^0$ decay mode. The fitting method has been tested with numerous Monte Carlo samples to verify correct performance. Distributions of normalized deviations between fitted parameters and input values are consistent with unit-width Gaussians centered at zero and indicate no discernible bias, as well as a correct evaluation of statistical errors.

The normalized signal probability distribution function, $p_{ji}^S(M,m,cos θ_{D^0,ρ},X)$, is composed of two parts. The mass distribution part is a product of the beam-constrained $B$ invariant mass distribution (assumed to have a Gaussian probability distribution) and a Blatt-Weisskopf form-factor-modeled [9] Breit-Wigner shape for the $ρ^-$ invariant mass distribution. The angular distribution part is given by an appropriately normalized product of Eq. (2) and the detector acceptance, $ε(cos θ_{D^0,ρ},X)$.

The normalized background probability distribution function, $p_{ji}^B(M,m,cos θ_{D^0,ρ},X)$, also has two components: the product of an ARGUS-type background function [10] for the beam-constrained $B$ invariant mass and a flat distribution for the $π^-π^0$ invariant mass distribution; and an angular distribution for the background determined from events in the $B$ mass sideband, defined as $5.200<M<5.265$ GeV/c^2. (A flat $π^-π^0$ invariant mass distribution is an adequate description of the background based on the distribution of this quantity for both data in the $B$ mass sideband and our larger sample of Monte Carlo events.) The background angular shape is parametrized as a product of second-order polynomials in $cos θ_{D^0}$ and $cos θ_ρ$ and the function $1 + P_χ cos(χ + χ_0)$ in χ, where $P_χ$ and $χ_0$ are allowed to vary in the fit to the sideband data.

To extract the number of signal and background events, the reconstructed candidates with $5.200<M<5.30$ GeV/c^2 are fit, with the angular distributions in both the signal and background probability density functions being ignored. Fig-
FIG. 2. $B^-$ (top) and $B^0$ (bottom) candidate mass distributions from the data, along with the results of the fits. Dashed curves indicate the ARGUS-type background.

TABLE I. Number of signal events and the efficiencies for $B^-$ and $B^0$. The uncertainties shown are statistical only.

<table>
<thead>
<tr>
<th>$B$ type</th>
<th>$D^0$ decay mode</th>
<th>$n^3$</th>
<th>$\varepsilon (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^-$</td>
<td>$K^-\pi^+$</td>
<td>148.9±13.8</td>
<td>6.56±0.04</td>
</tr>
<tr>
<td></td>
<td>$K^-\pi^+\pi^0$</td>
<td>177.4±16.6</td>
<td>2.20±0.02</td>
</tr>
<tr>
<td></td>
<td>$K^-\pi^+\pi^-\pi^+$</td>
<td>136.0±15.2</td>
<td>3.04±0.03</td>
</tr>
<tr>
<td>$B^0$</td>
<td>$K^-\pi^+$</td>
<td>196.3±14.6</td>
<td>10.88±0.05</td>
</tr>
<tr>
<td></td>
<td>$K^-\pi^-\pi^+$</td>
<td>196.1±16.4</td>
<td>3.67±0.03</td>
</tr>
<tr>
<td></td>
<td>$K^-\pi^+\pi^-\pi^+$</td>
<td>170.6±13.9</td>
<td>4.46±0.03</td>
</tr>
</tbody>
</table>

TABLE II. The measured helicity amplitudes for $B^-$ and $B^0$ decay modes for the three $D^0$ decay modes. The uncertainties shown are statistical only.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$B^-&gt;D^{(*)0}\rho^-$</th>
<th>$B^0-&gt;D^{(*)+}\rho^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>0.944±0.009±0.009</td>
<td>0.941±0.009±0.006</td>
</tr>
<tr>
<td>$H_+$</td>
<td>1.22±0.040±0.010</td>
<td>1.07±0.031±0.011</td>
</tr>
<tr>
<td>$\alpha_+$</td>
<td>1.02±0.28±0.11</td>
<td>1.42±0.27±0.04</td>
</tr>
<tr>
<td>$H_-$</td>
<td>0.306±0.030±0.025</td>
<td>0.322±0.025±0.016</td>
</tr>
<tr>
<td>$\alpha_-$</td>
<td>0.65±0.16±0.06</td>
<td>0.31±0.12±0.04</td>
</tr>
</tbody>
</table>

other non-$\rho^-$ components are small [3] and neglected. The contribution from the helicity amplitude dependence (for variations within our statistical errors) to the uncertainty on the acceptance is less than 11% of the corresponding contribution from the Monte Carlo statistics, and hence is also ignored.

These branching fraction measurements and the BSW prediction for $B(B^->D^{(*)0}\rho^-)/B(B^0->D^{(*)+}\rho^-)$ [1,13], can be used to extract the ratio of the effective coupling strengths for color-suppressed modes ($a_2$) and color-enhanced modes ($a_1$) for the $D^0\rho$ final state. The extraction of $a_2/a_1$ is sensitive to the $B^+B^-$ and $B^0\bar{B}^0$ production fractions; we used $f_+ / f_{00}=1.072±0.045±0.027±0.024$ [12]. Our data give $a_2/a_1=0.21±0.03±0.05±0.04±0.04$, where the fourth uncertainty, from $f_+ / f_{00}$, is important here since other experimental systematics partially cancel. This result is in good agreement with the previous CLEO measurement [3] and others [14].

To extract the helicity amplitudes from the data, only the reconstructed $B$ events in the $B$ signal region (defined as $5.27 < M < 5.30$ GeV/$c^2$) are included in the fit. The number of signal and background events for the three $D^0$ decay modes are taken from the previous fit, with the latter appropriately scaled to the smaller $B$ signal mass region.

We must determine an appropriate function to parametrize the acceptance, found from Monte Carlo calculations, and use this function in our unbinned maximum likelihood fit. Our studies show that the acceptance over the angle $\chi$ is quite flat (independent of the other angles). Thus, the acceptance can be factorized as $e(\cos \theta_{D^0}, \cos \theta_{\rho}, \chi) = e_1(\cos \theta_{D^0}, \cos \theta_{\rho})e_2(\chi)$, with $e_1(\chi) = Q_0(1+Q_1\sin \chi + Q_2\cos \chi + Q_3\sin 2\chi + Q_4\cos 2\chi)$. The parameters $Q_1,2,3,4$ are all found to be small; the variation of $e_1(\chi)$ is typically ±3%. We use the following functional form to fit the two-dimensional acceptance from the weighted Monte Carlo events:

\[
e_2(\cos \theta_{D^0}, \cos \theta_{\rho}) = e_0 + \frac{1 + P_1\cos^2 \theta_{D^0} + P_2\cos^2 \theta_{\rho} + P_3\cos^2 \theta_{D^0}\cos^2 \theta_{\rho} + P_4\cos \theta_{D^0}\cos \theta_{\rho}}{1 + P_5\cos^2 \theta_{D^0} + P_6\cos^2 \theta_{\rho} + P_7\cos \theta_{D^0}\cos \theta_{\rho}} \times \exp(-P_8\cos \theta_{D^0} - P_9\cos^2 \theta_{D^0} - P_{10}\cos \theta_{D^0} - P_{11}\cos \theta_{\rho}).
\]

TABLE II. The measured helicity amplitudes for $B^->D^{(*)0}\rho^-$ and $B^0->D^{(*)+}\rho^-$. The phase of $H_0$ is fixed to zero in each mode. $\alpha_+$ and $\alpha_-$ are the phases, in radians, of $H_+$ and $H_-$. $H_\pm = |H_\pm|e^{i\alpha_\pm}$. 

Quantify $B^->D^{(*)0}\rho^-$ | $B^0->D^{(*)+}\rho^-$ |
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<tr>
<td>$H_0$</td>
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</tr>
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</table>
This gives an excellent fit with no discernible pattern of residuals. The first part of the function is a quotient of two simple polynomials. This is motivated by realizing that there will in general be angular dependence in the generated shape and in the detector acceptance. The final exponential terms help describe the acceptance roll-off due to soft particles from the $D^*$ and $\rho$ decays at extreme values of the helicity angles.

Performing the unbinned maximum likelihood fit, we obtain the helicity amplitudes for the decay $B \to D^*\rho$ listed in Table II. Figure 3 shows the one-dimensional angular distributions and the projections from the fit. The errors quoted in the table are the statistical and systematic uncertainties, respectively.

The sources of systematic uncertainty are the acceptance parametrization, detector smearing, background level and shape, nonresonant $\pi^-\pi^0$ contribution, and the polarization dependence on the mass of the $\rho^-$ meson. Their individual contributions are shown in Table III. For the acceptance parametrization, we use different functional forms for both $e_2(\cos \theta_{\rho \nu}, \cos \theta_{\rho})$ and $e_1(\chi)$ in the maximum likelihood fit. These alternative forms give adequate but somewhat lower quality descriptions of the acceptance; the changes of the helicity amplitudes are taken as the systematic uncertainties.

To gauge the effect of detector smearing, we increase the smearing in the nominal Monte Carlo calculation by a conservative 15% of itself. We increase the number of background events for each $D^0$ decay mode independently by $1\sigma$ and use the observed shifts as the corresponding systematic uncertainties. Our largest systematic uncertainty comes from the shape of the background angular distribution. We compare the nominal fit results to three other fits: one with the background flat in the decay angles, one with the shape fit to the Monte Carlo events in the $M$ sideband, and one with the shape fit to the nonsignal Monte Carlo events in the $M$ peak region. The largest variation among these three is taken as the systematic uncertainty. To account for a possible background contribution from nonresonant $\pi^-\pi^0$ combinations (or a new broad resonance) in the data sample, a new contribution with a flat angular distribution is added to the decay amplitude (thus including interference effects). The $\pi\pi$ mass is removed from this fit to avoid any assumptions on the shape of this distribution. Finally, we include a systematic uncertainty due to our sensitivity to the $q^2$ dependence of the

| TABLE III. Summary of the systematic uncertainties for the helicity amplitudes of $B \to D^{*0}\rho^-$ and $B^0 \to D^{*+}\rho^-$. |
|---|---|---|---|---|---|---|
| $D^{*0}\rho^-$ | Quantity | Accep. | Smearing | Bkg. level | Bkg. shape | Nonres. | $q^2$ dep. | Total |
| $|H_0|$ | 0.08 | 0.06 | 0.13 | 0.83 | 0.02 | 0.07 | 0.85 |
| $|H_+|$ | 0.32 | 0.25 | 0.37 | 0.60 | 0.43 | 0.43 | 1.02 |
| $\alpha_+$ | 2.23 | 1.90 | 2.96 | 9.82 | 0.76 | 1.59 | 10.81 |
| $|H_-|$ | 0.24 | 0.28 | 0.30 | 2.45 | 0.25 | 0.03 | 2.51 |
| $\alpha_-$ | 1.29 | 1.12 | 2.22 | 1.07 | 4.22 | 2.84 | 5.90 |
| $\Gamma_L/\Gamma$ | 0.14 | 0.11 | 0.25 | 1.58 | 0.05 | 0.12 | 1.62 |
| $D^{*+}\rho^-$ | Quantity | Accep. | Smearing | Bkg. level | Bkg. shape | Nonres. | $q^2$ dep. | Total |
| $|H_0|$ | 0.07 | 0.01 | 0.04 | 0.61 | 0.02 | 0.10 | 0.62 |
| $|H_+|$ | 0.25 | 0.01 | 0.09 | 0.92 | 0.57 | 0.09 | 1.12 |
| $\alpha_+$ | 2.17 | 0.18 | 1.64 | 2.12 | 1.14 | 1.35 | 3.88 |
| $|H_-|$ | 0.20 | 0.03 | 0.13 | 1.52 | 0.13 | 0.26 | 1.57 |
| $\alpha_-$ | 0.92 | 0.01 | 0.39 | 3.95 | 1.02 | 0.16 | 4.21 |
| $\Gamma_L/\Gamma$ | 0.13 | 0.02 | 0.07 | 1.14 | 0.04 | 0.19 | 1.17 |
FIG. 4. The fraction of longitudinal polarization in $\bar{B}^0 \rightarrow D^{*+}X$ decays as a function of $q^2 = M_X^2$, where $X$ is a vector meson. Shown are the current $\bar{B}^0 \rightarrow D^{*+} \rho^-$ polarization measurement, and earlier measurements of $\bar{B}^0 \rightarrow D^{*+} \rho^-$ [17], and $\bar{B}^0 \rightarrow D^{*+} \rho^+$ [18]. The shaded region represents the prediction using factorization and HQET, and extrapolating from the semileptonic $\bar{B}^0 \rightarrow D^{*+} \ell^- \nu$ form factor results [19]. The shaded contour shows a one standard deviation variation in the theoretical prediction.

In the nominal fit, we ignore this dependence. Instead, we can allow the helicity amplitudes to vary with $q^2$, relating the fit parameters at a momentum transfer $q^2 = M^2_\rho$ to the actual $q^2$ of the events, using the factorization hypothesis (as shown in Fig. 4); we use the shifts as the uncertainty. The total systematic uncertainty is then the sum of the above contributions in quadrature, and is also shown in Table III.

As can be seen from Table II, our results indicate possible nontrivial helicity amplitude phases, $\alpha_+$ and $\alpha_-$. To better gauge the significance of such a conclusion, we use the quantity $\sqrt{\Delta(-2 \ln L_{\text{max}})}$, where $\Delta$ refers to the increase in $-2 \ln L_{\text{max}}$ when both phases are forced to be zero, as compared to the nominal fit with floating phases. Interpreting this quantity as the net statistical significance of nonzero phases, we find $3.19\sigma$ and $2.75\sigma$ for $B^- \rightarrow D^{*0} \rho^-$ and $\bar{B}^0 \rightarrow D^{*+} \rho^-$, respectively. The correlations between the two phases are modest: 0.21 (0.14) for the $B^-$ ($\bar{B}^0$) modes. The likelihood contours in the $\alpha^+, \alpha^-$ plane are only mildly distorted relative to ellipses. The stability of the significance is evaluated by examining the changes in $\sqrt{\Delta(-2 \ln L_{\text{max}})}$ for all of the systematic variations discussed above. The values are quite stable, ranging from $3.09 - 3.54\sigma$ and $2.65 - 2.85\sigma$ for $B^-$ and $\bar{B}^0$, respectively. Previously, indications of FSI phases have also been reported in the $D\pi$ systems [15] and the $J/\psi K^*$ systems [16].

The results for the helicity amplitudes correspond to a longitudinal polarization fraction of

$$\Gamma_L(B^- \rightarrow D^{*0} \rho^-) = 0.892 \pm 0.018 \pm 0.016,$$

$$\Gamma_L(B^0 \rightarrow D^{*+} \rho^-) = 0.885 \pm 0.016 \pm 0.012, \quad (7)$$

where the two uncertainties are statistical and systematic, respectively. Within the uncertainties, the fraction of longitudinal polarization for $\bar{B}^0 \rightarrow D^{*+} \rho^-$ in good agreement with the previous CLEO measurement [3] and with the heavy quark effective theory (HQET) prediction of $0.895 \pm 0.019$ [5] using factorization and the measurements of the semileptonic form factors. Longitudinal polarization as a function of $q^2$ is plotted in Fig. 4 for such a prediction and compared with our new $D^{*+} \rho^-$ result, as well as previous measurements for $D^{*+} \rho^-$ [17] and $D^{*+} D^+ \rho^-$ [18]. The agreement is excellent, indicating that the factorization hypothesis works well at the level of the current uncertainties.

In summary, we have measured both the branching fractions and the helicity amplitudes for $B \rightarrow D^{*0} \rho$. The values of the branching fractions, the ratio $a_1/a_4$, and the degree of longitudinal polarization are in good agreement with previous measurements and with theoretical predictions. The measurement of the fraction of longitudinal polarization confirms the validity of the factorization assumption at relatively low $q^2$. Finally, the measurement of the helicity amplitudes indicates a strong possibility of nontrivial helicity amplitude phases which would arise from final-state interactions. Such phases are of interest since they are required for the observation of direct $CP$ violation in $B$ decay rates [20].

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