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# **Hidden Strategic Challenges Posed by Housing Mobility Policy: An Application of Dynamic Policy Modeling**

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## **Abstract:**

Over the past decade, shifts in subsidized and affordable housing policy have led to a greater role for market dynamics and individual choice on the part of program participants and their new neighbors, and a greater awareness of the importance of neighborhood on family outcomes. Given these trends, there is an opportunity for innovative prescriptive planning models to assist in the design of policy related to regional housing mobility. The goal of this paper is to identify, and answer, some housing policy analytic questions with these models.

The fundamental question motivating this paper is the following: over the long run, can middle-class neighborhoods absorb the numbers of low-income families who might take advantage of an expanded housing mobility policy without leading to neighborhood decline through outmigration of affluent families? Fundamental to this inquiry is the notion that a community's "carrying capacity" is central to housing and community development planning and that parsimonious models based on the principles of dynamic control can help policymakers identify policy directions that are sensitive to the current state of the community.

Answers to these strategic questions provide motivation for two complementary prescriptive planning models that can provide specific short-term policy guidance to affordable and assisted housing practitioners and researchers.

## **Keywords:**

Housing mobility, dynamic models, urban policy, residential segregation

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## I. Introduction

Policy research regarding affordable housing has tended to concentrate on evaluations of existing programs. Work in this vein attempts to determine whether a particular initiative is designed correctly, and to determine whether outcomes associated with the initiative are consistent with policymakers' expectations and relevant theory. Doing so requires great attention to questions of program administration, data collection and evaluation frameworks. A larger goal of this research is to inform policymakers' decisions regarding retention and/or expansion of the program under consideration. Achieving this latter goal is more difficult as it requires assumptions regarding changes in likely program outcomes in response to changes in scale and scope, and the choice of program design parameters to optimize various policy objectives. Understandably, scientists are somewhat reluctant to engage in speculative analysis of this sort. Another, more prescriptive approach to policy analysis uses a variety of assumptions about market characteristics and the preferences of various actors in the policy arena in order to identify specific strategies for decisionmakers to pursue. Though this framework has been applied to a number of domains such as crime and public safety, transportation and natural resources (see, e.g. Pollock, Rothkopf and Barnett 1994), housing for the most part has been absent from this list. In this paper we wish to design and apply stylized prescriptive decision models to the problem of affordable/subsidized housing policy design to identify insights of importance to researchers and practitioners.

This policy focus of this paper is motivated by three problems in urban affairs well-known to housing researchers: residential segregation by race and ethnicity, concentration of poverty, especially in neighborhoods with high concentrations of family-based assisted housing, and low quality and economic inefficiency of traditional public housing. In response to these problems the U.S. Department of Housing and Urban Development (HUD) has pursued two types of initiatives: public housing redevelopment, under the HOPE VI program, and housing mobility programs using the Housing Choice Voucher Program as the primary subsidy vehicle, especially the Moving to Opportunity for Fair Housing national experiment. We believe that there is an untapped opportunity for researchers to perform prospective analysis using these two categories of programs as a catalyst. Such an analysis may result in two research innovations: robust estimates of long-term impacts and scale effects associated with these or similar programs, and dynamic and adaptive design of individual programs, as well as specification of a mix of strategies that respond to changes in housing markets and fiscal constraints.

The contribution of this paper is twofold. First, we present a stylized model that is prescriptive in nature yet grounded in descriptive research in affordable housing. This model represents a new means of identifying potential strategies for affordable and assisted housing. Second, we investigate demographic impacts and policy implications of alternative strategies suggested by solutions to this stylized housing planning model; this serves as a means for "simulating" long-term potential impacts and scale effects.

### *Analytical Frameworks*

It is useful here to examine more closely a number of analytical frameworks used in affordable housing research to highlight the differences between the approach of this paper and others. The first approach can be termed retrospective analysis, in that the focus is on collection and analysis of historical data derived from various housing initiatives. This type of analysis is of two types. The first, a positive/descriptive approach, uses tools such as program evaluation and

cost-benefit analysis to quantify and monetize program impacts as precisely as possible. The second, a normative/prescriptive approach, uses well-established economic models in areas such as housing markets (Arnott 1987), housing subsidies (Rosen 1985) and housing mobility (Clarke and Van Lierop 1986, Galster 2002) to determine whether measured outcomes are consistent with theories of individual and market behavior.

The second approach can be termed prospective analysis, in that the focus is on the estimation of future impacts associated with existing or proposed housing programs. Strategic planning models, for example, abstract from specific program and study area characteristics to identify broad trends in policy outcomes over time associated with optimization of a particular policy model. This perspective is the one used here and in companion papers (Caulkins *et al.* 2004a,b). Tactical planning models, in contrast, incorporate more programmatic and spatial specificity with a shorter planning horizon and extensive assumptions regarding decisionmaker preferences and market characteristics. This perspective is used in a paper that identifies alternative strategies for Section 8 counseling across a PHA's study area that jointly optimize multiple efficiency and equity objectives (Johnson and Hurter 2000) and another that identifies strategies for providing housing for military families in specific regions (Forgionne and Frager 1998). Operational planning models focus on short-term decisions made in very specific policy, programmatic and spatial contexts in which the number of alternatives, and attributes characterizing these alternatives may be fairly small. For example, Kaplan (1986) has designed policies for managing waiting lists for rehabilitated public housing, and Johnson (2004) has designed a decision support system for families seeking to relocate using tenant-based subsidies. Another prospective analytic tool that can combine a long time horizon and programmatic and spatial specificity is microsimulation, though this approach is limited to generation and evaluation of specific strategy alternatives and not necessarily choice of a most-preferred alternative.

Given the goals of this paper, the key to choice of an appropriate policy analytic approach is the extent to which it provides support for policy design. We argue that a prospective, normative and prescriptive analysis that abstracts from program specifics yet reflects key institutional realities may be of greatest interest to housing analysts and practitioners. This is the framework that we will use in this paper.

### *Paper Scope*

Our focus on strategic policy models enables us to define the scope of the paper in three ways. First, we address policy design, in an abstract and stylized manner, over a long time horizon. While no analyst would presume to forecast the long-term impacts of a policy initiative with any certainty, our work enables analysts and decisionmakers to identify alternative policy *trajectories* associated with specific behavioral and policy assumptions. Second, our work presumes a "systems" view of housing policy design that incorporates traditional economic notions of stocks and flows of housing and of persons over space and time, and transformation of these entities based on social policy. This view is considerably more flexible than those used in the operational and tactical planning models described above. Finally, our work explicitly addresses the notion of a community's "carrying capacity" of affordable housing program participants in order to identify scale economies and threshold effects associated with the demographic stability and health of communities.

### *Key Results*

Having developed a parsimonious but quite stylized one-state dynamic model for housing mobility, we generate values for structural parameters that incorporate as much as possible best knowledge of current housing and demographic research. We solve this calibrated model for a set of “base-case” parameters and find results that are supportive of the notion of housing mobility as a program that may provide opportunity along a number of dimensions to low-income families relocating to more advantaged neighborhoods. Sensitivity analysis results that point to importance of neighborhood health in determining population outcomes associated with housing mobility. Rough calculations using U.S. Census data and dynamic model assumptions that indicate few negative impacts of a housing mobility program operated at scale. Finally, we describe, but do not perform, extensions to the basic dynamic policy model that incorporate more realism regarding housing mobility programs, and which also may enable analysis of complementary initiatives for project-based housing along the lines of the HOPE VI public housing redevelopment program.

### *Paper Outline*

The outline of the remainder of this paper is as follows. Section II introduces the notion of dynamic policy modeling and proposes a specific planning model for affordable housing policy analysis. Section III addresses two specific policy modeling issues: realistic values for model structural parameters, and policy insights associated with base case model results and sensitivity analyses. Section IV presents simple computations that provide rough bounds for the policy insights of our model, discusses the relevance of our model results to policymakers, identifies model limitations and proposes alternative extensions to the policy model used in this paper. Section VI concludes and identifies key policy questions that might be answered using the model presented in this paper and various extensions.

## **II. Model Description**

### *Preliminaries*

To summarize briefly some twenty years of experience with subsidized housing policy in the United States, the broad emphasis has shifted from strategies intended to increase the supply of housing for lowest-income families through place-based investments to those that either shift the mix of place-based investments towards mixed-income stock (HOPE VI) or those that focus on allowing families increased freedom to choose the housing that best suits their needs on the private market with tenant-based subsidies (Housing Choice Voucher Program, formerly Section 8). The stylized policy modeling framework developed in this paper is based on the model of tenant-based subsidies, so our analysis to follow will use HCVP as a baseline; other policy modeling extensions could address place-based investments.

A significant research literature has documented beneficial client outcomes associated with housing mobility programs that have used Section 8 subsidies as a delivery mechanism: the Gautreaux Program (Rubinowitz and Rosenbaum 2000) and Moving to Opportunity (HUD 1999). Though Gautreaux results are tempered by the lack of a true experimental design by which outcomes such as effect of treatment on treated could be measured (Johnson, Ladd and Ludwig 2001), and the most recent MTO results are considerably more mixed than the initial results had led analysts to believe (HUD 2003), we believe that there is a consensus among

housing researchers that housing mobility, as a policy instrument distinct from conventional Section 8, is a worthwhile initiative. Indeed, Johnson (2004) has proposed a spatial decision support system whose purpose is to enable HCVP participants to make improved relocation decisions with housing mobility as a desired outcome.

One important question raised by the viability of housing mobility programs pertains to the scale with which such programs could be pursued. The MTO program was a large-scale experiment, but it was still small scale relative to the number of poor families in the US or even in the 5 cities that participated in MTO. So the favorable benefit-cost results indicated by Johnson, Ladd and Ludwig (2001) based on short-term MTO evaluations are best thought of as indicating a favorable marginal benefit when expanding mobility programs from a baseline scale of zero. It is possible that the marginal benefit of a given expansion in housing mobility is a constant, independent of program scale. If so, then the optimal strategy is a “corner solution”. If the marginal benefit is negative, don’t do the program at all. If it is positive, expand the program until it reaches some physical limit, such as exhausting the pool of poor families eligible to be placed.

Before presuming that the marginal benefits of program expansion are scale-independent, however, one should ask whether there might be diminishing returns, economies of scale, or threshold effects. If so, then the optimal program scale may be positive but short of fully serving all eligible poor families. In that case, it is useful to know at least very roughly how large that optimal scale is. Alternatively, are the diminishing returns so severe that mobility programs will, although beneficial, inevitably serve only a very small fraction of all poor families, dooming them to be a sideshow relative to the “main action” concerning programs for families remaining in poor neighborhoods? Or, at the other extreme, could mobility programs be pursued at such a grand scale that they could essentially drain poor families out of predominantly poor neighborhoods, materially reducing spatial stratification by economic class?

The goal of this paper is to think about these scale issues from the perspective of an important concept that has (reasonably) been neglected in evaluations of mobility programs to date, namely the “carrying capacity” of destination neighborhoods relative to the size of the stock of poor families eligible for mobility programs. The key factor underpinning a notion of carrying capacity is not physical availability of housing. According to the 2000 Census, there are “only” about 4.8 million poor families in the US vs. on the order of 40 million middle class households. Although there are not likely to be 4.8 million vacant houses immediately available in middle class neighborhoods, the housing construction industry would have no trouble increasing the housing stock by 12% over the next decade or so.

Rather, the notion of “carrying capacity” hinges on the maximum number of poor families that could be added to a middle-class neighborhood without inducing middle-class flight. Clearly the cause of economic integration is not well served if many of the pre-existing middle class families depart when poor families enter, or if, in reaction to the increased numbers of poor families in the neighborhood, the number of middle-class in-migrants drops significantly (see for example arguments developed by Gould Ellen 2000). The transplanted poor families might enjoy some a temporary upgrade in the quality of their housing stock, but if being surrounded by poor neighbors is a risk factor for poor children, then merely shifting the location of the concentrated poverty from one neighborhood to another is of little use. Furthermore, there are considerable deadweight costs associated with mass population movements, not to mention political ramifications.

Inasmuch as the premise of housing mobility programs is to enhance upward social mobility, the notion of a neighborhood's carrying capacity is not a single, static number. Suppose that at present, the limit on the number of poor families a neighborhood could absorb were five. One hundred years later, one would hope that all five families would have assimilated into the middle-class to the point of becoming part of the positive stock of middle class families, not part of the "new group" whose presence might trigger flight. One hundred years is so long as to be beyond most officials' planning horizon, but over shorter periods presumably there would be some, although partial, assimilation. The more assimilation, the more "room" there would be to accept a new batch of poor families.

Hence, rather than thinking of a neighborhood's carrying capacity in terms of being able to absorb a given number of families without inducing excessive flight, it is probably more useful to think in terms of absorbing a certain number of poor families per unit time. Even this notion is not entirely appropriate: the current rate at which poor families could be added no doubt depends on the current stock of poor families. Thus, if there are currently no poor families in a given neighborhood, the neighborhood could presumably absorb new poor families at a higher rate without inducing flight than if the neighborhood was already "at capacity".

Thinking through optimal rates, stocks, flows, and state-dependencies is difficult. Fortunately, there is an analytical paradigm that has been developed specifically to deal with these sorts of ideas. That paradigm carries various monikers, including "systems science", "dynamical systems theory", and, our preferred term, "optimal control theory". Indeed, the very idea of thinking about scale issues for mobility programs arose from discussions surrounding construction of an optimal dynamic control model of housing mobility programs.

The remainder of this section will lay out the structure of such a "systems" model of housing mobility and the maximization of social welfare within the framework of that model. Specific functional forms and parameters have been developed in other papers (Caulkins *et al.* 2004a,b); we will examine one instance of this model here.

We acknowledge that there is sufficient uncertainty concerning proper functional forms and parameter values and, more generally, that the models are so stylized that specific solution values may not be of immediate use to policy makers. However, the structuring of the problem in these terms itself is provocative and insightful, as are some observations about the general qualitative nature of some of the solutions to these special cases.

To foreshadow, these models are consistent with those of Schelling (1971) and others and with historical experience inasmuch as they are characterized by thresholds separating conditions under which the neighborhood tips one way or another. Most notably, one can think of threshold strategies separating long-run solutions in which the basic character of the neighborhood is little changed (modest flight which is offset by in-migration and assimilation) from those in which the middle-class population is greatly reduced, altering the basic character of the neighborhood.

If the inflow of poor families is too high for too long, the neighborhood is pushed to a point from which it will not bounce back without exorbitant investment. It is that property that led us to conceptualize issues of scale in terms of a neighborhood's "carrying capacity" and to wonder how large the collective carrying capacity of all middle class neighborhoods is relative to the stock of poor families who are eligible for mobility programs. These sorts of insights can be extracted from extensions to the basic housing mobility framework. For example, one could identify population trajectories associated with differing mixes of housing policies, e.g. place-

based policies concentrated in higher-poverty communities combined with person-based policies concentrated in lower-poverty communities. This modeling extension is however deferred to future work.

### *Model Exposition*

Suppose that the world is defined by two types of neighborhoods: middle-class neighborhoods and poor neighborhoods. We focus exclusively on demographic changes in a single typical middle-class neighborhood. Denote the level, or “stock” of families in the middle-class neighborhood at the current time by  $X$ . We normalize the optimal level of middle-class families in this neighborhood to 1 without loss of generality. Absent all other policy interventions, the number of families in the middle-class neighborhood changes over time according to a logistic growth function

$$\frac{dX}{dt} = a \cdot X(t) \cdot (1 - X(t)), \quad (1)$$

where  $a$  is a constant that represents the speed at which the equilibrium population is approached.

Suppose that a housing mobility program enables  $u$  low-income families to move from poor neighborhoods to middle-class neighborhoods per unit time at a given time point. However, in reaction to the in-migration of low-income families,  $\beta$  middle-income families leave their current neighborhood for other middle-class neighborhoods for each poor in-mover. Thus, the rate of middle-class flight at a given level of in-migration of poor families is  $\beta u$  per unit time.

Suppose as well that the housing mobility program results in beneficial outcomes for low-income family participants that enable them to exhibit, over time, characteristics with respect to educational attainment, labor market participation, criminal offending, etc. similar to middle-income families. Denote this process of changes in life outcomes as “assimilation.” Without much evidence from housing studies or demographics to guide us, we assume that the number of poor families who assimilate to the middle class is proportional to the product of the stock of middle-class families multiplied by the flow of low-income families. Housing mobility program participants do not all enjoy beneficial life outcomes resulting from moving to a middle-class neighborhood, however. Define  $\gamma$  as the fraction of poor in-movers who enter the middle-class. Then the rate of assimilation of low-income in-movers to the middle class is  $\gamma X u$ .

Then the rate of change of the stock of middle-class families at a given time point resulting from a housing mobility program is given by:

$$\dot{X} \equiv \frac{dX}{dt} = a \cdot X(t) \cdot (1 - X(t)) - \beta \cdot u(t) + \gamma \cdot X(t) \cdot u(t). \quad (2)$$

Suppose now that a social planner wishes to design a housing mobility program for a given middle-income neighborhood that will optimize a measure of net social benefit, subject to known dynamics of low- and middle-income families associated with programs of this sort. In particular, the social planner seeks to choose a level of in-migration of low-income families over time,  $u(t)$ , to maximize the net discounted benefit of a housing mobility program, respecting population dynamics.

As a matter of public policy, we assume that the benefits of a housing mobility program accrue both to low-income in-movers and current middle-class residents. Without loss of generality, we assume that the total benefit of this program to low-income participants is \$1 per participant. Also, we assume that the total benefit per unit time of this program to middle-class residents is  $\rho$  per poor family entering the neighborhood.

Suppose that the direct costs of the housing mobility program are measured by counseling costs, administrative costs, property value impacts and deadweight losses associated with the use of in-kind housing subsidies, and these costs are concave. Again, without much evidence from housing mobility programs to guide us, let us assume that these costs are quadratic in the in-migration rate. If the dollar-valued coefficient for this program is given by  $c$ , then program costs equal  $c \cdot u^2(t)$ . Finally, we assume a discount rate of  $r$ .

Thus, then the objective function the planner seeks to maximize is

$$\int_0^\infty e^{-rt} (u(t) - cu(t)^2 + \rho X(t)) dt, \quad (3a)$$

subject to system dynamics

$$\dot{X} = a \cdot X(t) \cdot (1 - X(t)) - \beta \cdot u(t) + \gamma \cdot X(t) \cdot u(t) \quad (3b)$$

and an initial level of middle-class families  $X(0)$ . Additional details regarding mathematical and policy assumptions underlying this model are available in (Caulkins, *et al.* 2004a). We emphasize that model (3a) – (3b) is a highly stylized representation of a complex policy problem. Our goal in presenting it is to identify important policy questions that are difficult to address using conventional analytical methods, and to propose simple ways to address these questions that provide significant insight to policy-makers.

### *Model Solution*

Model (3a) – (3b) is solved using a well-known principle from the theory of optimum control, the maximum principle (Léonard and Van Long 1992). Define an auxiliary variable  $\lambda(t)$ , denoted as a *costate variable*, and formulate a new function called a *Hamiltonian*:

$$H(X(t), u(t), \lambda(t), t) = (u(t) - cu^2(t) + \rho X(t)) + \lambda(t)[aX(t)(1 - X(t)) - \beta u(t) + \gamma X(t)u(t)] \quad (4)$$

Then an optimal solution to (3a) – (3b) must satisfy the following conditions:

$$(i) \quad \text{The control variable } u(t) \text{ maximizes } H(X(t), u(t), \lambda(t), t), \text{ i.e. } \frac{\partial H}{\partial u(t)} = 0 \quad (5a)$$

(ii) the state and costate variables satisfy a pair of differential equations

$$X(t) = \frac{\partial H}{\partial \lambda(t)}, \text{ and} \quad (5b)$$

$$\lambda(t) = -\frac{\partial H}{\partial X(t)}. \quad (5c)$$

Optimal control problems like (3a) – (3b) can in principle be solved by using (5a) and other relations to derive general solutions for equations (5b) and (5c), and using initial conditions to derive particular solutions for (5b) and (5c).

Dynamic systems are *stable* if the state and control variables are finite at all times and if the system converges to some sort of steady state, characterized by equilibrium points. While the various specific definitions of stability are beyond the scope of this paper, there exist numerous tests that can be applied to determine whether a given system is stable or not (see Sengupta and Fanchon 1997 for details).

Since equations (5a) – (5c) can be difficult to solve analytically, we often generate two-dimensional graphical qualitative descriptions of trajectories of variables called *phase diagrams*. These diagrams enable one to identify regions in state-control in which variables increase or decrease over time, and to guess at the shapes of trajectories and their relationships to equilibrium points. Such diagrams can also be generated from analytic or simulation-based solutions to the specific problem at hand. In the discussion that follows we will refer to state-control space, though phase diagrams can be generated equally well for state-costate space.

Phase diagrams are typically presented as collections of curves defined with respect to regions in which the signs of the derivatives of the state and costate variables  $X(t)$  and  $\lambda(t)$  change. Thus, these diagrams are composed of isoclines, or loci of points for which  $X(t)=0$  and  $\lambda(t)=0$ . The intersection of the  $X(t)=0$  and  $\lambda(t)=0$  lines is one of a number of possible equilibrium points. Equilibrium points can be stable or unstable, however. An equilibrium point is defined (Sengupta and Fanchon 1997, 25) as:

- a *node* if state trajectories converge towards (stable) or away (unstable) it without crossing the isoclines;
- a *saddle point* if two state trajectories lead towards and away from the point and no other trajectories reach the point;
- a *focus* if all state trajectories cross isoclines to converge towards (stable) or away (unstable) from it
- a *vortex* if all state trajectories are cyclical, neither reaching the point nor diverging from it.

Traditionally, optimal control problems have been assumed to result in convergence to a unique and stable equilibrium (if there is convergence at all). However, Skiba (1978) and subsequent researchers have identified economic optimal control problems in which there are several locally optimal steady states. As a result, in some cases there exists an initial state in which two solutions may originate and converge to different optimal steady states. These initial states are usually referred to as indifference thresholds or “Skiba points”. (See Wagener 2004 for a discussion of the mathematical properties of optimal control problems that generate Skiba points.)

The policy insights in this paper related to housing mobility derive from the existence of Skiba points and associated multiple equilibria associated with the optimal control problem (3a)

– (3b). These multiple equilibria result in two or more strikingly different outcomes based on optimal strategies for the middle-class community under study.

As an example of the many different characteristics of solutions to problem (3a) – (3b), consider the case in which there is no assimilation from low-income in-movers to the middle class, i.e.  $\gamma = 0$ . Suppose that other model parameters are set as follows:  $a$  (housing market adjustment speed coefficient) = 4,  $\beta$  (rate of middle-class flight per poor in-mover) = 0.4,  $r$  (discount rate) = 1,  $\rho$  (value per middle-class family per year relative to value of low-income family placed) = 1, and  $c$  (quadratic cost coefficient of mobility) = 0.5. The phase diagram associated with a numerical solution to our model in state-control space is shown in Figure 1, below.

[Figure 1: Phase Diagram, Housing Mobility Model, No Assimilation]

The isocline  $X(t) = 0$  appears as a parabola expressed functionally as  $u(X) = X(1 - X)\alpha/\beta$ , with  $X$ -intercepts 0 and 1, and the isocline  $u(t) = 0$  is a hyperbola with values according to  $(u - 1/(2c))(a - r - 2aX) = \rho\beta/(2c)$ . Note that  $u > 0$  for feasibility (decreasing income segregation). Also, it can be shown that for the costate variable  $\lambda > 0$ ,  $u < 1/(2c)$  and the Hamiltonian is concave, i.e. that it satisfies the sufficiency condition for optimality. Conversely, for values of  $u > 1/(2c)$ ,  $\lambda < 0$ , i.e. the shadow price of the dynamic constraint is negative. Thus, on economic grounds solutions in which  $u > 1/(2c)$  would be excluded from consideration. For the purposes of this example, however, we will examine solutions for which  $u > 1/(2c)$  as well as  $< 1/(2c)$ .

Based on the parameter values we have defined above, Figure 1 shows that  $u(t) = 0$  crosses  $X(t) = 0$  in three places, that the leftmost equilibrium is a saddle point, the middle equilibrium is an unstable vortex and the rightmost equilibrium is another saddle point. This diagram can be interpreted in the following way: starting from an initial point ( $X = X_{\min}$ ,  $u = 0$ ), the system can evolve towards two different types of equilibria: high- $u$  ( $u > 1/(2c)$ ) and low- $X$  ( $X < 0.2$ ), corresponding to high levels of in-migration of poor families and high rates of middle-class flight, and low- $u$  ( $u < 1/(2c)$ ) and high- $X$  ( $X > 0.8$ ), corresponding to moderate levels of in-migration of poor families and low levels of middle-class flight. The middle point, a vortex corresponding to very-high- $u$  and somewhat lower levels of middle-class flight, is not a stable equilibrium; solutions in this region tend towards the high- $u$  and low- $X$  equilibrium.

It is also possible to estimate the impact on this solution of changes in certain structural parameters. For example, as the cost of program implementation ( $c$ ) increases, the area where the policy is feasible ( $u > 0$ ) and the Hamiltonian is concave ( $u < 1/(2c)$ ) shrinks. This results in higher values for  $X$  as compared to the base case considered above. On the other hand, as the rate at which the housing market readjusts to changes in the middle-class population ( $a$ ) changes, the phase diagram is modified significantly, either towards multiple non-equilibrium solutions ( $a$  large) or multiple optima and no vortexes ( $a$  small). (For a detailed description of these and related phase diagram representations of solutions to (3a) – (3b), please see Caulkins et al. 2004a.)

### III. Policy Analyses

In this section we examine specific policy issues related to the dynamic optimization model (3a) – (3b). First, we develop and justify realistic values for various structural parameters used in the planning model. Next, we interpret various classes of model results (for technical details, see Caulkins *et al.* 2004b). We argue that, given a prescriptive model that is internally consistent and calibrated with plausible values for key parameters, model results can represent important policy implications.

#### *Realistic Values for Model Parameters*

Although we have argued that the single-state dynamic optimization model (3a) – (3b) is quite stylized and inappropriate for specific short-term policy prescriptions, it is useful to examine the range of values for the various structural parameters used by the model, in order to guide future analysis of solutions.

##### 1. Housing Market Adjustment Speed Coefficient, $a$

The uncontrolled population dynamics of the middle-class neighborhood into which low-income families might relocate are assumed to be  $X = aX(1 - X)$ . Parameter  $a$  can be interpreted as the half-life of decay of vacancies when the neighborhood is near its uncontrolled equilibrium. That is, how long, on average, does it take for a house to sell in a healthy middle-class neighborhood ( $X$  close to 1)?

Over the last 15 years, the average length of time for new homes to sell across the United States has been between 3.6 and 6.9 months (U.S. Census Bureau 2004a). The average of those averages is 4.89, suggesting  $a = (12/4.89)*\ln(2) = 1.7$ , which we round up to a base case of 2. However, summary data from the 2001 American Housing Survey (U.S. Census Bureau 2004b) indicates that there is considerable variation in time spent on the market. Since lower values of parameter  $a$  can yield qualitatively different results, we also explore the consequences of introducing housing mobility programs to neighborhoods with weaker real estate markets. The AHS data indicate that 13.4% of units were on the market for more than 2 years. The data are reported in categories, so we do not know exactly what vacancy period is the 90th percentile. It is clearly above 24 months and might be on the order of 36 months ( $a = 0.231$ ). For a round number we take  $a = 0.2$  (vacancy period 41.6 months) because that is our base case value ( $a = 2$ ) divided by 10.

##### 2. Assimilation Coefficient $\gamma$

The parameter  $\gamma$  reflects the “success” or “assimilation” rate for persons who participate in housing mobility programs, i.e. the proportion of families initially placed in low-poverty or low-percent minority (what we call “middle-class”) neighborhood who stay for an extended period of time and who enjoy life outcomes consistent with a transition to the middle class.

The MTO Interim Impacts Evaluation (HUD 2003) reports that 35% of all “experimental group” families who successfully found rental housing between 1994 and 1998 were recorded in 2002 as living in Census tracts with Census 2000 poverty rates of 20% or less. Shroder’s (2001) analysis of MTO outcomes reports that 64% of treatment group started on welfare. 13 quarters later it fell to 34%, suggesting an assimilation rate of  $1 - 34/64 = 47\%$ . DeLuca and Rosenbaum

(2003) concluded that for the Gautreaux Program, after an average of 17 years post-move, about 57% of suburban movers remain in the suburbs. Also, about 37% of participants who moved to neighborhoods that were 15% black or less initially (most of whom were probably suburban movers) remain in neighborhoods that are 30% black or less. This would imply a success rate for the Gautreaux Program of between 37% and 57%. Given these three figures, we set  $\gamma = 45\%$  in our base case.

### 3. Flight Coefficient $\beta$

The parameter  $\beta$  represents the number of middle-class families who leave the current neighborhood per poor in-mover, or the level of middle-class “flight.” Flight is not the only possible reaction of middle-class families to the presence of poor families; middle-class families may choose not to move into a community whose levels of poor families are high and/or increasing. We do not model this “avoidance” dynamic, however.

Betts and Fairlie (2003) find that one native-born person moved out of the school district for every four immigrants entering. That suggests that  $\beta = 0.25$  in their context. Flight from lower-class could be stronger than flight from immigrants, suggesting somewhat larger values of  $\beta$  in our context. Gould Ellen (2000, p.124) reports that “The probability of the typical white homeowner moving is between 2.0 and 3.5 percentage points higher when the black population has grown by 10 percentage points over the decade.” For a neighborhood around the uncontrolled steady state, an increase of 10% over the decade means  $\gamma u = 0.01$ . If that inflow leads to a 0.02 to 0.035 increase in the per capita outflow rate, that means  $\beta \cdot u$  is in the neighborhood of 0.02 to 0.035 and hence,  $\beta = 2 \gamma$ . Given  $\gamma = 0.45$ , that implies  $\beta$  in the range of 0.9 - 1.575. In light of this, we take  $\gamma = 0.5$  as our base case, but also consider sensitivity analyses with smaller ( $\gamma = 0.25$ ) and larger ( $\gamma = 1.0$ ) values.

### 4. Objective Function Coefficient for X, $\rho$

Recall that  $\rho$  is the value per middle-class family per year relative to the value per low-income family placed, so first we need the value per low-income family placed. The U.S. Office of National Drug Control Policy is an agency that offers an estimate of the monetary value of saving a high-risk youth (based on Cohen's (1995) work). For a 5% discount rate, the values would be \$1.1 - 1.6 M (ONDCP 2002, Table 13).

Shroder (2001) reports that for kids in MTO treatment vs. control: (1) 21% vs. 35% had trouble with teachers; (2) 21% vs. 32% were disobedient at home; (3) 5% vs. 19% were “mean or cruel to others”, and (4) 16% vs. 28% were “unhappy, sad, or depressed”. These short-term findings might suppose suggest that about 12% of kids will be “saved”, but gains can erode over time. The MTO experiment is too recent for there to be long-run results, but Behrens *et al.* (2002) found that in the context of drug prevention, in percentage terms, long-term gains were about one-fifth of short-term gains. So the social benefit per family placed might be something on the order of  $(\$1.1M - 1.6M) * 12 \% * 2.5 * 20\% = \$66,000 - \$96,000$ , or an average of about \$80,000. The average cost per family placed is roughly the \$70M Congress appropriated for the MTO program divided by 2,414 households = \$29,000 per household placed, or a net benefit of \$80,000 - \$29,000 or about \$50,000.

When  $X$  declines in this model, it is the result of middle class flight, not middle class death, so it is relevant to focus on the local municipality's marginal loss of local tax revenue. Personal income in the US in 1995 was about \$4.3 billion (counting wages, salary, other labor

income, and proprietor's income, but not rental, dividend, or interest income because many states tax only earned income), or about \$43,000 per household. Assuming a 2.5% municipal income tax, that is roughly \$1,000 per year in foregone tax revenues, suggesting values for  $\rho$  of  $\$1,000/\$50,000 = 0.02$ , although clearly this is a parameter whose estimate is subject to considerable uncertainty.

## 5. Quadratic Cost Coefficient, $c$

Recall that the parameter  $c$  represents total short-term and long-term administrative and social costs associated with the housing mobility program. Administrative costs include counseling, management and community outreach; social costs include deadweight losses, property value impacts and other negative externalities.

There is currently no empirical basis for estimating parameter  $c$  using data currently available for MTO and the Gautreaux Program. The inflow  $u$  in the five MTO cities was about 0.24 per 1,000 current residents or  $u = 0.0002$ . This rate is so small that the  $c \cdot u^2$  term is negligible and, indeed, there is no discernable relationship between costs per person placed and placement intensity across the five cities. Hence, the most appropriate value for parameter  $c$  can be identified by reference to the instantaneous part of the objective function pertaining to the control:  $u - c \cdot u^2$ . This says that, leaving aside the  $\rho \cdot X$  term, the instantaneous satisfaction the policy maker derives is maximized when  $u = 1/(2c)$  and is driven to zero if  $u=1/c$ . In other words, when focusing only on short-run considerations, the convex program costs make the preferred rate of poverty deconcentration in the short run  $u = 1/(2c)$  and by the time  $u=1/c$  they would offset all of the poverty deconcentration benefits. A value of  $c = 20$  implies that these rates are placing one poor family per 40 middle class families per year and one poor family per 20 middle class families per year, respectively.

At one level these seem about right. The first might be a good target; the second might be overly aggressive. However, those judgments are probably tempered by long-run considerations including flight and assimilation. Focusing only on the short-run considerations driven by the convexity of the program cost structure, the optimal and maximum desirable placement rates might be higher, so we also consider examples below with lower values of the parameter  $c$ .

## 6. Discount rate $r$

The annual social planner discount rate is customarily between 3% and 7%. We take as our base value  $r = 0.05$ .

Table 1 summarizes the parameter values used in numerical experiments. We use the values of  $\rho = 0.02$  and  $\gamma = 0.45$  throughout, but we perform comparative static analysis allowing  $a$ ,  $c$  and  $\beta$  to vary.

[Table 1: Structural Parameter Estimates for Housing Mobility Model]

## *Policy Impacts of Model Solutions*

### 1. Generic and base-case results

If no poor families relocate to the middle-class community ( $u = 0$ ), then the model generates a stable steady state at  $X = 1$ . This is consistent with our modeling assumptions.

However, if there is a positive inflow of families ( $u > 0$ ), the model generates three types of saddle point equilibria that depend on the relative and absolute magnitudes of middle-class growth ( $aX(1 - X)$ ), flight due to externality ( $-\beta u$ ) and assimilation via the bilinear term ( $\gamma Xu$ ).

The first generic result has low values for  $u$  and values for  $X$  close to 1 (“low  $u$ , average  $X$ ”): rates of assimilation and flight are roughly equal, and the poverty deconcentration program does not dramatically alter the character of the neighborhood. The second has somewhat higher values for  $u$  and values for  $X$  much less than 1 (“small  $X$ , modest  $u$ ”): population growth arises mostly from low-income families not part of the mobility program which balances substantial middle-class flight. This outcome, while unhealthy for the community in the long run, could be justified if the decisionmaker has a high discount rate and highly values poverty deconcentration. The last result has values for  $u$  closer to 1 and values for  $X$  much greater than 1 (“high  $X$ , high  $u$ ”): many poor families come to the neighborhood and are assimilated, and the middle-class population is substantial and very transient. This result is reminiscent of dense urban neighborhoods in cities such as New York City known to be “ports of entry” for immigrants.

Base-case model results are derived using the following values for model parameters:  $r = 0.05$ ;  $a = 2$ ;  $\beta = 0.5$ ;  $\gamma = 0.45$ ;  $\rho = 0.02$  and  $c = 2$ . Model (3a) – (3b) generates saddle points for all three equilibria described above; the middle saddle with  $X^* \approx 1$  dominates (see Figure 2, below). The smallest value for  $X^*$  ( $\approx 0.01$ ) can never serve as a rational policy alternative because such policies require using mobility programs with values of  $u > 1/2c$  in which costs that exceed benefits (see Caulkins *et al.* 2004a for details). The largest value for  $X^*$  ( $\approx 1.45$ ) can never serve as a long-run optimum, due to a proposition stated in Caulkins *et al.* (2004b) and proved in Caulkins *et al.* 2004a) that trajectories associated with a critical point  $X^* > 1$  move from right to left towards an unstable node in the middle of the saddle points. These results appear to be robust to parameters governing mobility program costs.

[Figure 2: Phase Diagram, Housing Mobility Model with Assimilation,  
Base-Case Values for Structural Parameters]

As a result, a mobility program administered in an environment consistent with model (3a) – (3b) and using base-case values for structural parameters is likely have minimal impacts on the host neighborhood.

## 2. Sensitivity results for flight parameter $\beta$

A decrease in  $\beta$  from the base-case value of 0.5 to 0.25 reinforces the strength of the middle-type saddle, while an increase in  $\beta$  from 0.5 to 1 with  $c = 20$  reduces the size of the long-run equilibrium by only a little.

## 3. Sensitivity results for housing market parameter $a$

Values of  $a$  ranging from 0.2 to the base-case value of 2, with values of other parameters unchanged, preserve the basic results: a middle-sized city whose optimal size is equivalent to the uncontrolled or natural state.

A value of  $a = 0.04$  and placement cost  $c = 20 >$  base case value of 2 results in four equilibria, three of which are saddle points. An unstable origin node ( $X = 0.16$ ,  $u = 0.012$ )

represents a Skiba point, i.e. a point from which trajectories to two stable optima originate. This point represents indifference between low-equilibrium/complete destruction of the city ( $X^* = 0$ ,  $u^* = 0$ ) and a stable middle-sized city ( $X^* = 0.95$ ,  $u = 0.024$ ); a large city ( $X^* = 1.20$ ,  $u^* = 0.023$ ) is never optimal. An interpretation of this solution is that cities that are initially very small and whose housing markets are weak should not participate in a housing mobility program; induced middle class flight exceeds assimilation of program participants.

A value of  $a = 0.05$  and placement cost  $c = \text{base value of } 2$  generates equilibrium solutions corresponding to a small city ( $X^* = 0.16$ ,  $u^* = 0.016$ ) and an oversized city ( $X^* = 2.26$ ,  $u^* = 0.27$ ) with a Skiba point at  $X(0) \sim 1.2$ . The small city receives relatively few poor in-movers but does not have a strong enough housing market to converge to a normal size; the large city receives many poor in-movers who assimilate to the middle-class, but as well a relatively rapid outflow of existing middle-class residents in response to both poor families entering and general population pressures. From a policy perspective, a decisionmaker should avoid these outcomes unless a solution distinct from the long-term natural equilibrium size for the city is desirable.

## IV. Discussion

### *Real-world implications of model results*

In order to interpret some of the results of the previous section it is necessary to estimate values for the variables used in the housing mobility model. We begin with the demographic constructs “low-income” (or “poor”) and “middle-class”. The definition of “poverty” is well-known: a person is defined to be poor if that person’s family income is less than a dollar standard that is a function of family size and which has its roots in estimates of the yearly income needed to purchase adequate food to provide a minimum level of nutrition (U.S. Census Bureau 2003a). For example, the 1999 poverty threshold for a 3-person family with one member under age 18 was \$13,410. For the purposes of this paper, we define the population of “poor” families—those who might relocate to more advantaged and/or less segregated communities under a housing mobility program—as those who live in metropolitan areas and whose incomes fall below the poverty line. According to the U.S. Census Bureau there were about 4.8 million such families in 2000, 1.1 million of whom lived in the central cities of 10 large metropolitan areas (U.S. Census Bureau 2004c,d). Another definition of the population that might participate in a housing mobility program “at scale” might be the 1.8 million extremely low-income households (incomes less than 40% of the area median income) who have been estimated not have access to affordable housing (Millenial Housing Commission 2002). We estimate then that the population of “poor” families treated as in-movers to middle-class communities in our planning model ranges between 1 million – 5 million people.

The U.S. Census Bureau does not provide a standard definition of “middle class” (U.S. Census Bureau 2004e). Instead, it performs analysis of income distributions on the basis of income quintiles as for example used by the Survey of Income and Program Participation (SIPP). For the purposes of this paper, we define “middle class” families as those in the middle and upper fourth income quintiles as based on the SIPP, whose sample is defined to cover about 100 million families between 1996 – 1999 (U.S. Census Bureau 2004f). Therefore we estimate the size of the “middle class” as 40 million families (no disaggregation of these data are available for families in metropolitan areas alone). This paper has defined the health of middle-class

neighborhoods as the number of families that live there, represented by a variable whose steady state value is normalized to one, synonymous with “carrying capacity”.

Results in the paragraphs above indicate, then, that the ratio of “poor” families to “middle-class” families ranges between 0.025 and 0.125. In other words, excluding consideration of middle-class flight or class transitions, the carrying capacity of middle class neighborhoods is on the order of 2.5% to 12.5% above steady state, values that are much less than those associated with model solutions in which values for middle-class community populations are much larger than uncontrolled steady state, either at a stable or unstable equilibrium.

Suppose now that we wish to incorporate considerations of population dynamics into our calculations. SIPP tabulations of family income mobility (U.S. Census Bureau 2003b) indicate that between 1996 and 1999, 14.8 million poor people exited poverty while 7.6 million non-poor people became poor. Given poverty populations in 1996 and 1999 of 40.9 million and 34.8 million, respectively, we compute a poverty exit rate of  $14.8/40.9 = 36.2\%$  and a poverty entry rate of  $7.6/34.8 = 21.8\%$  respectively, or a net decline of 14.4%. In addition, SIPP tabulations indicate a transition rate from “poor” (lowest quintile) to “middle class” (middle and fourth quintiles) of 11%. Finally, research by Gould Ellen (2000) cited previously indicates an increase in the probability of white flight when the black population has grown. Since our computations indicate a decrease in the poor population (based on 1996 – 1999 data), then assuming that middle-class residents react similarly to the introduction of poor residents, we set the rate of flight to zero.

We conclude then that based on U.S. Census data from 1996 – 2000, a housing mobility program operating according to assumptions underlying the dynamic housing model (3a) – (3b) is not likely to lead to middle-class communities expanding beyond a reasonable estimate of their carrying capacity. That is, MTO-like programs could serve as the primary strategy for meeting housing policy objectives such as poverty deconcentration.

Structurally different results, such as neighborhoods whose equilibrium sizes are much smaller or larger than the uncontrolled equilibrium value depend on assumptions regarding the dynamic resilience of host neighborhoods, program costs and modeling of flight and assimilation.

#### *Model limitations*

The dynamic control model (3a) – (3b) that is the basis for the policy analyses in this paper has a number of limitations that limit the direct applicability of the policy results above. First, there is no consensus in the research literature regarding the most appropriate functional form to represent middle-class population dynamics; we chose the logistic growth curve

$X = aX(1 - X)$  for convenience. Second, the one-state model is insufficient to capture the many subtleties of middle-class flight/avoidance observed by researchers. These include: attitudes of minority and majority groups towards neighborhoods of differing racial/ethnic composition, and changes in these levels over time, affirmative marketing strategies that attempt to generate or preserve levels of majority and minority groups deemed to be in the social interest, differing population dynamics of subgroups, and proximity of the middle-class neighborhood to other majority-minority areas. Third, although our assumption that mobility program costs are concave is reasonable, we have no strong empirical basis for our choice of the quadratic form  $c'u^2$ , made for convenience in model solution.

Fourth, there is no consensus in the research literature regarding functional form for assimilation of poor families into the middle class; we chose the bi-linear relationship  $\gamma X'u$  for convenience. Moreover, our conception of assimilation itself does not capture the many facets of the gradual and challenging adjustment of poor families to middle-class communities as captured for example in research regarding the Gautreaux Program (Rubinowitz and Rosenbaum 2000; DeLuca and Rosenbaum 2003) and MTO (HUD 2002). In particular, our assumption that families either assimilate immediately or depart is particularly strong; we cannot represent the process of gradual assimilation over time using a one-state model.

#### *Alternative modeling approaches*

There are a number of alternative strategies for modeling poverty deconcentration programs, including, but not limited to, those that rely on tenant-based housing subsidies and relocation over space. For example, one might propose a *two-state model* addressing the stock of low-income families in middle-class city ( $Y$ ) along with stock of middle-class families ( $X$ ). The control variable  $u$  representing flows of poor families into middle-class communities is unchanged. In this case, the two controlling differential equations representing population dynamics could be represented as:

- a. Rate of change in  $X$  = change in middle-class dynamics – middle-class flight – low-income families who leave + low-income families who assimilate into middle class
- b. Rate of change in  $Y$  = rate of inflow of poor families – rate of assimilation of poor families to middle-class + natural growth in low-income families in middle-class community – outflow of poor families

Alternatively, a three-state model could represent another policy alternative: construction of high-quality, mixed-income project-based housing in low-income communities, such as the current HOPE VI program. In this case, the three relevant stock variables are: the stock of low-income families in low-income neighborhoods ( $Z$ ); the stock of low-income families in middle-class city ( $Y$ ), and the stock of middle-class families ( $X$ ). Relevant flows would consist of: low-income families into middle-class neighborhoods; low-income families to redeveloped (e.g. HOPE VI) low-to-moderate and/or mixed-income communities, and low-income families from low-income status via death or entering middle class via other means than geographical mobility (education, labor market participation).

Both of these models, while intriguing, are quite difficult to analyze theoretically and numerically using methods employed in Caulkins *et al.* (2004a,b); such extensions are a focus of ongoing research.

## V. Conclusion and Next Steps

This paper represents a summary of current findings and policy implications regarding a planning model for housing mobility programs. This model, which uses the language of optimal control theory, can be classified as a prospective policy-analytic tool that is strategic in nature, combining elements of descriptive and prescriptive models. As such it serves to complement other modeling initiatives related to housing mobility programs: tactical planning models intended to provide detailed guidance over the medium term to agency decisionmakers in well-

defined spatial contexts, and operational planning models such as those that can provide short-term guidance to individual families and housing counselors regarding relocation options.

We have shown that our model, though limited in terms of research justification for specific components, plausibly represents broad policy dynamics related to housing mobility programs and is calibrated using reasonable, conservative values for key structural parameters. Model results derived from base-case parameter values support the core intuition of the designers of housing mobility programs: impacts on host communities are likely to be modest, not changing significantly the demographic character of destination neighborhoods. Changes in the parameter measuring intensity of middle-class flight do not have an appreciable effect on these outcomes. However, neighborhood health, as measured by the length of time to sale for residential housing, appears to a key driver behind adverse outcomes. Values of the associated structural parameter smaller by an order of magnitude than the base case, combined with very high or low placement cost parameter values result in configurations of destination neighborhoods that are unstable and/or much larger or smaller than the uncontrolled equilibrium. Order-of-magnitude computations using U.S. Census data are supportive of the notion that a housing mobility program operated “at scale” in a dynamic environment consistent with our stylized planning model could indeed provide opportunities for many low-income families to live in advantaged communities without compromising the health of their destination communities.

There are a significant number of research extensions to this work. One avenue of inquiry involves improved justification for the various functional forms used in the policy model. Another involves model variants that use two or three state variables. We believe that a closer examination of assumptions underlying the current policy model is more likely to generate research results that can provide reliable guidance to decisionmakers, while model variants may be of primary use for theoretical analysis.

Finally, a number of policy questions remain to be addressed, using models similar to or extensions of the one developed in this paper. First, what values of structural parameters for the current model could replicate outcomes associated with conventional tenant-based housing subsidy programs, i.e. the Housing Choice Voucher Program as currently administered by public housing authorities nationally, as compared with “MTO-like” programs incorporating a significant level of counseling to further poverty deconcentration and residential integration by economic class and race/ethnicity? Second, what levels of control variables associated with a dynamic model addressing both housing mobility and project-based assisted housing could result in long-term diminution of low-income communities without inducing excessive middle-class flight at minimum net social cost? Answering these questions is likely to result in additional findings of interest to assisted housing researchers and practitioners.

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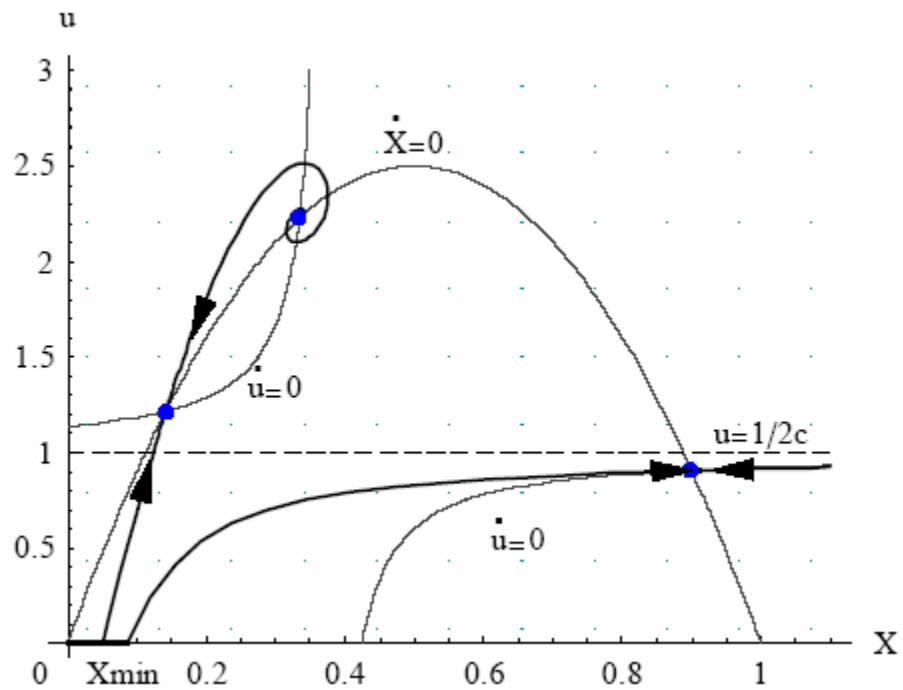
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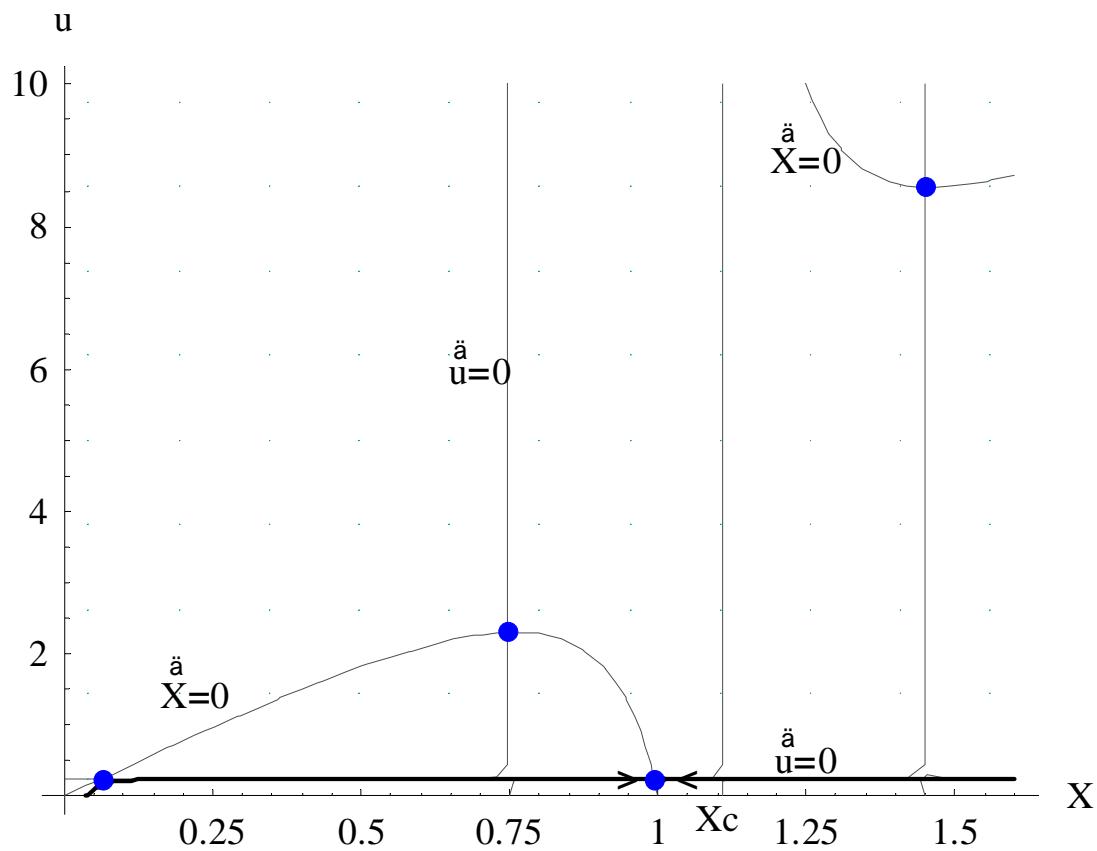
## Tables and Figures



[Figure 1: Phase Diagram, Housing Mobility Model, No Assimilation]

<b>Parameter</b>	<b>Base Case</b>	<b>Range</b>	<b>Interpretation</b>
$a$	2	1 - 4	Housing market adjustment speed coefficient
$\beta$	0.25	0.25 – 0.5	Rate of middle-class flight per poor in-mover
$\gamma$	0.45	0.35 – 0.57	Assimilation (success) rate for poor in-movers
$\rho$	0.02	0.006 - 1	Value per middle-class family per year relative to value of low-income family placed
$c$	20	10 - 40	Quadratic cost coefficient of mobility
$r$	0.05	0.03 – 0.07	Discount rate

[Table 1: Structural Parameter Estimates for Housing Mobility Model]



[Figure 2: Phase Diagram, Housing Mobility Model with Assimilation,  
Base-Case Values for Structural Parameters]