Incorporating Temporal and Country Heterogeneity in the Measurement of Labour Productivity Growth, TFP and Structural Change

Alicia Rambaldi  
*University of Queensland, a.rambaldi@uq.edu.au*

Antonio Peyrache  
*University of Queensland, r.peyrache@uq.edu.au*

Follow this and additional works at: [http://repository.cmu.edu/sem_conf](http://repository.cmu.edu/sem_conf)

Part of the [Economics Commons](http://repository.cmu.edu/sem_conf/2015/full_schedule/29)

[http://repository.cmu.edu/sem_conf/2015/full_schedule/29](http://repository.cmu.edu/sem_conf/2015/full_schedule/29)

This Event is brought to you for free and open access by the Conferences and Events at Research Showcase @ CMU. It has been accepted for inclusion in Society for Economic Measurement Annual Conference by an authorized administrator of Research Showcase @ CMU. For more information, please contact research-showcase@andrew.cmu.edu.
Incorporating Temporal and Country Heterogeneity in the Measurement of Labour Productivity Growth, TFP and Structural Change

Antonio Peyrache and Alicia N. Rambaldi

SEM annual conference - 24 July 2015
Outline

Introduction

General Representation of the Production Model

Estimation

Growth Accounting

Empirical Application

Data

Empirical Results

Parameter Estimates

Growth Accounting
Schmidt and Sickles (1984) proposed the time invariant model

\[ y_{it} = a + X_{it}\beta - u_i + \varepsilon_{it} \]

- the technology is fixed
- the term \( u_i \) is a fixed effect unobserved heterogeneity; however, it is truncated and interpreted as technical efficiency term
- As \( T \) is larger, the model becomes less appealing.
The production model

- The production technology is represented via a production function, where \( i = 1, \ldots, N \) indexes the number of countries, \( t = 1, \ldots, T \) indexes the number of time periods.

\[
y_{it} = \mu_t + \gamma_{it} + X_{it}\beta_t + \epsilon_{it} \tag{1}
\]

- single output \( y_{it} \) (log of output)
- multiple inputs \( X_{it} \)
- \( \mu_t, \beta_t \) are time varying parameters common to all the countries,
- \( \gamma_{it} \) is a country specific time varying intercept (group specific time trend) and
- \( \epsilon_{it} \) is a normally distributed error term.
- The country specific intercepts are given by \( a_{it} = \mu_t + \gamma_{it} \).
Our Contribution

- Develop a general econometric representation of the production model and then show that it is able to nest a number of commonly used panel data models introduced in the literature which deal with group specific trends.
- Develop a growth accounting decomposition based on the estimated production function in order to separate productivity change from inputs growth effects. This allows us to identify the main drivers behind observed labour productivity growth.
- The framework is illustrated using the EU-KLEMS dataset to identify the main trends (and biases) in productivity growth in the period 1977-2007 for 13 OECD countries and 20 industrial sectors of each economy.
Reparameterization of the Model

- The country specific intercepts $a_{it} = \mu_t + \gamma_{it}$.

$$y_{it} = a_{it} + X_{it} \beta_t + \varepsilon_{it} \tag{2}$$

- In order to identify all the parameters we need to assume $\frac{1}{N} \sum_i a_{it} = \mu_t$, so that $\mu_t$ represents the time variation in the average intercept level (it is an average function or common shock)

- Reparameterize equation (2) using the following transformation:

$$\max \{ a_{it} \} = a_t, \quad u_{it} = a_{it} - a_t$$

(this is a measure of technical inefficiency) which returns the following stochastic production frontier:

$$y_{it} = a_t + X_{it} \beta_t - u_{it} + \varepsilon_{it} \tag{3}$$

- The production frontier embedded in (3) is time varying (due to the time varying coefficients $(a_t, \beta_t)$) and technical inefficiency is time varying
Reparameterization of the Model (cont.)

- Parameterizations (2) and (3) are useful for interpretation.
- Parameterization (3) is used as the post-estimation interpretation of the coefficients of the econometric model.
- Components of productivity are recovered using parameterization (2).
- The general representation is based on specification (1).
Outline

Introduction

General Representation of the Production Model

Estimation

Growth Accounting

Empirical Application
  Data
  Empirical Results
    Parameter Estimates
    Growth Accounting
The General Econometric Model

- The full model representation

\[ y_t = \gamma_t + 1N \mu_t + X_t \beta_t + \varepsilon_t \]  

(4)

\[ \gamma_t = c_\gamma + \phi_{t-1} + \gamma_{t-1} + \eta_{\gamma t} \]
\[ \phi_t = c_\phi + \phi_{t-1} + \eta_{\phi t} \]
\[ \mu_t = c_\mu + \nu_{t-1} + \mu_{t-1} + \eta_{\mu t} \]
\[ \nu_t = c_\nu + \nu_{t-1} + \eta_{\nu t} \]
\[ \beta_t = c_\beta + \tau_{t-1} + \beta_{t-1} + \eta_{\beta t} \]
\[ \tau_t = c_\tau + \tau_{t-1} + \eta_{\tau t} \]

- Noise \( \varepsilon_{it} \sim N \left(0, \sigma^2_\varepsilon\right) \);

- Independent innovations \( \eta_{\mu t} \sim N \left(0, \sigma^2_\mu\right), \eta_{\nu t} \sim N \left(0, \sigma^2_\nu\right), \eta_{\gamma t} \sim N \left(0, \sigma^2_\gamma I_{N \times 1}\right), \eta_{\phi t} \sim N \left(0, \sigma^2_\phi I_{N \times 1}\right), \eta_{\beta t} \sim N \left(0, \sigma^2_\beta I_{K \times 1}\right) \) and \( \eta_{\tau t} \sim N \left(0, \sigma^2_\tau I_{K \times 1}\right) \)

- \( \gamma_t \) is a \( N \times 1 \); \( \phi_t \) is a \( N \times 1 \)

- \( c_\mu \) and \( c_\nu \) allow the nesting of the common practice of using deterministic quadratic trends.

- If both variances, \( \sigma^2_\mu \) and \( \sigma^2_\nu \), are zero the trends are deterministic

- \( \beta_t \) and \( \tau_t \) are \( K \times 1 \)
State-space Representation

- In this model the variance of the country specific intercept comes from two different sources: first, a common shock to all the countries and, second, from a country specific shock.

- One possible state-space representation

\[
\begin{align*}
y_t &= Z_t \alpha_t + \varepsilon_t \\
\alpha_t &= D \alpha_{t-1} + c_t + \eta_t
\end{align*}
\]

where \( \alpha_t = [\gamma_t \phi_t \mu_t \nu_t \beta_t \tau_t]' \),
\[
Z_t = [I_N \ 0_{N \times N} \ 1_N \ 0_N \ X \ 0_{N \times K}],
\]
\[
c_t = [c_\gamma \ c_\phi \ c_\mu \ c_\nu \ c_\beta \ c_\tau]' \]

and \( D \) is a conformable matrix of zeros and ones

- The vector \( \alpha_t \) has size \( \text{size}(\alpha_t) = 2(N + K + 1) = B \). As the vector \( c \) is also size \( B \), there are a total of \( 4(N + K + 1) = 2B \) parameters (plus covariance parameters) to be estimated.
State-space Representation (cont.)

- A more convenient reparameterisation,

$$\alpha_t^* = \begin{bmatrix} \alpha_t - \Gamma_t c \\ c \end{bmatrix}$$  \hspace{1cm} (6)

where the matrix $\Gamma_t$ is defined as:

$$\Gamma_t = \sum_{j=0}^{t-1} D^j = I + D + D^2 + ... + D^{t-1}$$  \hspace{1cm} (7)

- After the transformation the state space becomes:

$$y_t = Z_t^* \alpha_t^* + \epsilon_t$$  \hspace{1cm} (8)

$$\alpha_t^* = D^* \alpha_{t-1}^* + \eta_t^*$$  \hspace{1cm} (9)

where $Z_t^* = \begin{bmatrix} Z_t & (Z_t \Gamma_t) \end{bmatrix}$, $D^* = \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix}$, $\eta_t^* = \begin{bmatrix} \eta_t \\ 0 \end{bmatrix}$, $E(\eta_t^* \eta_t^*) = Q$ and $E(\epsilon_t \epsilon_t') = H$.

- There are two advantages of this representation,
  - We can now show that it nests a number of other models, and
  - We can now use standard state-space estimation algorithms.
Nested Models

- Generic set of $J$ linear restrictions on the parameters of the model:

$$R_t \alpha_t^* = 0$$

- The following models are nested:
  - Fixed Effects (FE) (and FE with deterministic trends)
  - Cornwell Schmidt and Sickles (1990) model (CSS) - and Battese and Coelli (1992)
  - Simple stochastic trend model
Fixed effects (FE)

- The model

\[ y_t = \gamma_t + X_t \beta_t + \varepsilon_t \]
\[ \gamma_t = \gamma_{t-1} = \gamma \]
\[ \beta_t = \beta_{t-1} = \beta \]

\[ R_t = \begin{bmatrix} 0_{(N+2) \times N} & I_{N+2} & 0_{(N+2) \times (2K+B)} \\ 0_{(K+B) \times (2+2N+K)} & I_{K+B} \end{bmatrix} \]

one obtains the following restrictions:

\[ \begin{bmatrix} \phi_t - t c_{\phi} \\ \tau_t - t c_{\tau} \\ \mu_t - t c_{\mu} - \frac{t(t-1)}{2} c_v \\ v_t - t c_{v} \\ c \end{bmatrix} = 0 \]

The last \( B \) restrictions impose all elements of \( c \) to be zero. The other restrictions impose \( \phi_t = 0, \mu = v = 0, \tau = 0 \).
The Cornwell, Schmidt and Sickles (1990) model (CSS)

Their model,

\[ y_{it} = \gamma_{it} + X_{it}\beta + \varepsilon_{it} \]

with \( \gamma_{it} = \delta_{0i} + \delta_{1i}t + \delta_{2i}t^2 \).

The restrictions on the general framework are \( Q = 0 \) and

\[
R_t = \begin{bmatrix}
0_{2 \times 2N} & I_2 & 0_{2 \times (2K+B)} \\
0_{K \times (2+2N+K)} & I_K & 0_{K \times B} \\
0_{(2+2K) \times (B+2N)} & I_{2+2K}
\end{bmatrix}
\]

imply the following model:

\[
\begin{align*}
\gamma_t &= \gamma_t + X_t\beta_t + \varepsilon_t \\
\gamma_t &= c_\gamma + \phi_{t-1} + \gamma_{t-1} \\
\phi_t &= c_\phi + \phi_{t-1} \\
\beta_t &= \beta_{t-1}
\end{align*}
\]

Battese and Coelli (1992) is a special case of this model.
A simple stochastic trend model (TV)

- The model,

\[
\begin{align*}
\gamma_{it} &= \gamma_{it-1} + \eta_{\gamma_{it}} \\
\beta_t &= \beta_{t-1} + \eta_{\beta_t}
\end{align*}
\]

\[
\begin{align*}
\gamma_{it} &= \gamma_{it} + \chi_{it} \beta_t + \epsilon_t \\
\gamma_{it} &= \gamma_{it-1} + \eta_{\gamma_{it}} \\
\beta_t &= \beta_{t-1} + \eta_{\beta_t}
\end{align*}
\]  

(11)

\[
Q = \begin{bmatrix}
\sigma_\gamma^2 I_N & 0_{(N+2) \times (N+2)} \\
0_{(N+2) \times (N+2)} & \sigma_\beta^2 I_K \\
\end{bmatrix}
\]

\[
R_t = \begin{bmatrix}
0_{(N+2) \times N} & I_{N+2} & 0_{(N+2) \times (2K+B)} \\
0_{(K+B) \times (2N+2+K)} & I_{K+B}
\end{bmatrix}
\]
Outline

Introduction

General Representation of the Production Model

Estimation

Growth Accounting

Empirical Application
  Data
  Empirical Results
    Parameter Estimates
    Growth Accounting
Estimation

- In (1) the local level, $\gamma_t$, (which captures individual country specific trends) and the slope, $\beta_t$ can be correlated with $X_t$ and thus is an extension of the fixed effects model to the case of time-varying parameters.

- To estimate any of the nested members of the family one can incorporate the corresponding set of restrictions and estimate the corresponding state-space form.
  - However, computation can be approached also by estimating each nested model (ie FE, CSS, TV, etc.) using other estimation approaches that are consistent for that model.

- The importance of the result: It is possible to make a systematic statistical comparison among them using the AIC and BIC measures of fit.

  \[
  AIC = \log \left( \frac{e' e}{N T} \right) + \frac{2K}{N T}, \quad BIC = \log \left( \frac{e' e}{N T} \right) + K \frac{\log NT}{N T}.
  \]
Outline

Introduction

General Representation of the Production Model

Estimation

Growth Accounting

Empirical Application
  Data
  Empirical Results
    Parameter Estimates
    Growth Accounting
Growth Accounting

- Model is a translog production function at any point in time, but it allows the parameters to move in time in a non-translog way.
- **Output growth** between two time periods using the reparameterization of the production model in (2):

  \[ y_{it+1} - y_{it} = a_{it+1} + X_{it+1} \beta_{t+1} - a_{it} - X_{it} \beta_t \]  

(12)

- Decompose into two effects that are exhaustive and mutually exclusive:

  \[ y_{it+1} - y_{it} = TFP + FA \]  

(13)

- Then, decompose further \( TFP = UTC + BTC \).
  - The change in the country specific intercept level \( UTC \)
  - The bias in technical change deriving form the time variation of the input coefficients \( BTC \)
Productivity Growth (TFP)

- Productivity growth can be measured keeping the level of inputs at the base period level, obtaining the equivalent of the base period Malmquist productivity index:

\[
TFP_t = a_{it+1} - a_{it} + X_{it} (\beta_{t+1} - \beta_t)
\]  

(14)

Keeping the level of inputs at the comparison period value, we obtain the equivalent of the comparison period Malmquist productivity index:

\[
TFP_{t+1} = a_{it+1} - a_{it} + X_{it+1} (\beta_{t+1} - \beta_t)
\]  

(15)

and to avoid the arbitrariness of choosing the base, we use the geometric mean of these two indexes in order to get an index of productivity growth:

\[
TFP = \frac{1}{2} \left( TFP^t + TFP^{t+1} \right) =
\]

\[= a_{it+1} - a_{it} + \frac{1}{2} (X_{it} + X_{it+1}) (\beta_{t+1} - \beta_t)
\]

(16)
Factor Accumulation (FA)

- The input growth effect or factor accumulation effect (FA) can be computed using the same logic. The base period index is:

\[ FA^t = (X_{it+1} - X_{it}) \beta_t \]  

(17)

- The comparison period index is:

\[ FA^{t+1} = (X_{it+1} - X_{it}) \beta_{t+1} \]  

(18)

- Finally, we use the geometric mean as a measure of the input growth effect:

\[ FA = \frac{1}{2} (FA^t + FA^{t+1}) = \frac{1}{2} (\beta_t + \beta_{t+1})(X_{it+1} - X_{it}) \]  

(19)
Further we decompose

- The change in the country specific level

\[ UTC = a_{it+1} - a_{it} \]

- The bias in technical change deriving from the time variation of the input coefficients:

\[ BTC = \frac{1}{2} (X_{it+1} + X_{it}) [\beta_{t+1} - \beta_t] \] (20)

Thus the overall growth accounting decomposition is given by:

\[ y_{it+1} - y_{it} = TFP + FA = UTC + BTC + FA \] (21)

where \( UTC + BTC = TFP \).
Growth Accounting (cont.)

- All these components are country and time specific.
- A positive (negative) BTC measures the extent to which technical change has been biased (for example, capital-using or capital saving).
- UTC incorporates the effect of the change in the country specific intercept (which is a measure of the productivity level).
- FA is the contribution of the increase (decrease) in the inputs endowment. This decomposition is exhaustive (does not leave any residual) and the different components are independent from each other.
  - UTC and BTC depend on different coefficients of the model
  - FA depends on how much a country is investing and accumulating factors of production.
Outline

Introduction

General Representation of the Production Model

Estimation

Growth Accounting

Empirical Application
   Data
   Empirical Results
   Parameter Estimates
   Growth Accounting
Data

- The EU-KLEMS dataset input and output data on prices and quantities for 26 industrialized countries in the time span 1970-2007 (see O’Mahony and Timmer (2009)).

- For each industry:
  - value data on gross output, capital compensation, intermediate inputs (materials and energy) along with fixed base price and quantity index numbers (1995=100).

- We use the amount of total hours worked by persons engaged as a proxy for the quantity of labour.

- We use PPPs to adjust for cross-sectional differential in the general level of prices.
  - PPPs indexes use US as benchmark (US=100, 1995=100), are sector specific (i.e., each sector has different PPPs) and different for sectoral output, intermediate inputs and capital services (i.e., there are three sets of PPPs).

- Due to lack of data (missing values) we limit our attention to a subset of data, specifically 13 countries and 20 industrial sectors in the time span 1977-2007.
We build a balanced panel dataset

Let $j = 1, \ldots, 20$ the sector and $i = 1, \ldots, 13$ the country, the index of sectoral output for country $i$ at time $t$ $Y_{it}^j$ is:

$$Y_{it}^j = \frac{GO_{i1995}^j}{PPP_i^j} I_{it}^j$$

(22)

where $GO_{i1995}^j$ is the value of gross output in 1995 for sector $j$ in country $i$;
$I_{it}^j$ is the fixed base index of sectoral output quantity change between time $t$ and the base period 1995;
$PPP_i^j$ is the purchasing power parity of country $i$ in sector $j$. With a similar procedure quantity index numbers are built for intermediate output (materials) and capital services.
Data (cont.)

- With this procedure we obtain a balanced panel data set where
  - cross-sectional (cross-country) comparability is built using PPPs and
  - time comparability is built using fixed base quantity index numbers.

- This procedure guarantees that in each time period countries can be compared.

- The result is a quantity index number that proxies sectoral output production, a quantity index number proxying capital services, a quantity index number that proxies the level of materials used and the number of hours worked for the labour input.

- All the variables obtained with this procedure have been normalized by the sample minimum.
Empirical Model

- The model we estimate

\[ y_{it} = a_{it} + \beta_{kt} k_{it} + \beta_{mt} m_{it} + \beta_{kk} k_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{km} m_{it} k_{it} + \varepsilon_{it} \]

where
- \( y \) is the log of output per worker,
- \( k \) is the log of capital per worker and
- \( m \) is the log of materials per worker.

- We estimate all the models separately for each of the 20 industrial sectors, thus applying them to each sectoral panel dataset individually.

- Nested models estimated:
  - TV
  - CSS
  - FE (with deterministic trends) - also nested see paper

\[ \gamma_{it} = \delta_0 + \delta_1 t + \delta_2 t^2 - u_i \]
\[ \beta_{nt} = \tau_{n0} + \tau_n t, \quad n = 1, \ldots, N \]
## Nested Models (Six Sectors)

<table>
<thead>
<tr>
<th></th>
<th>WOOD</th>
<th>FUEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R^2 )</td>
<td>( R^2 ) Adj.</td>
</tr>
<tr>
<td>TV</td>
<td>0.989</td>
<td>0.989</td>
</tr>
<tr>
<td>CSS</td>
<td>0.997</td>
<td>0.997</td>
</tr>
<tr>
<td>FEDT</td>
<td>0.991</td>
<td>0.991</td>
</tr>
<tr>
<td>PLASTICS</td>
<td>( R^2 )</td>
<td>( R^2 ) Adj.</td>
</tr>
<tr>
<td>TV</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>CSS</td>
<td>0.997</td>
<td>0.997</td>
</tr>
<tr>
<td>FEDT</td>
<td>0.976</td>
<td>0.975</td>
</tr>
<tr>
<td>ELECTRICAL AND OPTICAL</td>
<td>( R^2 )</td>
<td>( R^2 ) Adj.</td>
</tr>
<tr>
<td>TV</td>
<td>0.958</td>
<td>0.956</td>
</tr>
<tr>
<td>CSS</td>
<td>0.999</td>
<td>0.998</td>
</tr>
<tr>
<td>FEDT</td>
<td>0.986</td>
<td>0.985</td>
</tr>
</tbody>
</table>
Electrical and Optical Equipment Sector

TV ($\hat{a}_{it}$)
Electrical and Optical Equipment Sector (TV)

TV \left( \hat{\beta}_{kt}, \hat{\beta}_{mt} \right)
Selected Countries in the Chemical Sector

TV ($\hat{a}_{it}$)
Chemical Sector (TV)
Electrical and Optical (Growth Accounting)
Chemicals (Growth Accounting)
US Growth Accounting

Italy Growth Accounting
Some Patterns

- IT productivity boom in the US (cannot be generalised to all the OECD countries)
- Stable labour productivity growth in Germany (based on a stable flow of investment)
- Industrial crisis of Italy (over the past 15 years)
Current work going forward

- Inputs treated
  - as endogenous
  - measured with error
- Model the joint distribution of outputs and inputs