On Monetary Policy Uncertainty at Zero Interest Rate

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Zero interest rate in U.S.
Introduction
Models
Estimation method
Result
Summary

Zero interest rate
Objective

Economic Policy Uncertainty Index

Baker, Bloom and Davis (2011)
A simple econometric model of monetary policy uncertainty in the presence of zero interest rate.
Objective

- A simple econometric model of monetary policy uncertainty in the presence of zero interest rate.
- Incorporate such measure of policy uncertainty into an empirical asset pricing/term structure model. (in progress)
A State Space Model

- State variables: shadow rate $s_t$ and volatility $x_t$

**Transition equation**

\[
\begin{align*}
    s_t &= \beta_0 + \beta_1 Z_t + e^{\frac{x_t}{2}} \varepsilon_t \\
    x_t &= \phi_0 + \phi_1 x_{t-1} + u_t
\end{align*}
\]
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\end{align*}
\]

- Observed variables: interest rate $r_t$ and economic uncertainty index $v_t$

**Measurement equation**

\[
\begin{align*}
    v_t &= \alpha_0 + \alpha_1 v_{t-1} + \alpha_2 x_t + e_t \\
    r_t &= \max\{s_t, 0\}
\end{align*}
\]
Likelihood Function

\[
p(y_t|\theta, y_{1:t-1}) = \int p(y_t|x_t, \theta)p(x_t|x_{t-1})p(x_{t-1}|y_{1:t-1}, \theta)dx_{t-1}dx_t \quad (1)
\]

\[
p(y_2...y_T|\theta) = \prod_{t=1}^{T-1} p(y_{t+1}|\theta, y_{1:t}) \quad (2)
\]
Particle filter (SIR)

- Draw $N$ particles $x_t \{1 : N\} \sim p(x_t|x_{t-1})$
- Calculate each particle’s weight $w_t^i = p(y_t|x_t^i)$
- Normalize weights: $\tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^{N} w_t^i}$
- Redraw $N$ particles by their weights $\tilde{w}_t^i$
- Each particle has the same weight $1/N$
Particle filter (SIR)

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- Redraw N particles by their weights $\tilde{w}_t^i$
- Each particle has the same weight $1/N$
- Likelihood function:
  - $p(y_t|\theta, y_{1:t-1}) \approx \frac{1}{N} \sum_{i=1}^{N} \tilde{w}_t^i$
A Practical Problem

- Estimate parameters through maximizing log-likelihood
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- Problem: likelihood is not smooth
  - 2-dimensional -log-likelihood by using stratified sampling procedure (Malik and Pitt 2011)
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Data and the Baseline Model

- $s_t = \beta_0 + \beta_1 inf_t + \beta_2 unem_t + \beta_3 D_t \ast inf_t + \beta_4 D_t \ast unem_t + \epsilon_t^{xt} \varepsilon_t$
- $x_t = \phi_0 + \phi_1 x_{t-1} + u_t, \quad u_t \sim N(0, \sigma^2)$
- $r_t = \max\{s_t, 0\}$
- $v_t = \alpha_0 + \alpha_1 v_{t-1} + \alpha_2 x_t + e_t, \quad e_t \sim N(0, \gamma^2)$
## Parameter Estimates

<table>
<thead>
<tr>
<th>parameters</th>
<th>estimator</th>
<th>std error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>13.3237</td>
<td>0.1350</td>
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<tr>
<td>$\beta_1$</td>
<td>0.1580</td>
<td>0.0320</td>
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<tr>
<td>$\beta_2$</td>
<td>-2.0629</td>
<td>0.0233</td>
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<tr>
<td>$\beta_3$</td>
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<td>$\beta_4$</td>
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<td>$\phi_0$</td>
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<td>0.0253</td>
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<tr>
<td>$\phi_1$</td>
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<td>0.0199</td>
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<td>$\sigma$</td>
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<td>0.0748</td>
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<tr>
<td>$\alpha_0$</td>
<td>19.1149</td>
<td>0.6063</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.8645</td>
<td>0.0939</td>
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<tr>
<td>$\alpha_2$</td>
<td>1.9116</td>
<td>0.5432</td>
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<tr>
<td>$\gamma$</td>
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<tr>
<td>log-likelihood</td>
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Shadow Interest Rate

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Policy Uncertainty: filtered $x\{1 : T\}$

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Effect of Zero Lower Bound

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Forecasting Aggregate Stock Market Return

- S&P 500 Index excess return: \( r_t = \log \left( \frac{(D_t + P_t)}{P_{t-1}} \right) \ast 12 - r_{f,t} \)
Forecasting Aggregate Stock Market Return

- S&P 500 Index excess return: \( r_t = \log((D_t + P_t)/P_{t-1}) \times 12 - r_{f,t} \)
- Campbell-Shiller log-linear present value formula

\[
pd_t = E_t \sum \rho^{j-1} \Delta d_{t+j} - E_t \sum \rho^{j-1} \Delta r_{t+j} - E_t \sum \rho^{j-1} \Delta r_{f,t+j}
\]
Forecasting Aggregate Stock Market Return

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\]

- Forecasting regression

\[
\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = a^{(k)} + b^{(k)} pd_t + c^{(k)} xpu_t + \epsilon_{t+k}
\]
## Return-forecasting regressions

### $k = 1$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
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<th>p-value</th>
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<tbody>
<tr>
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<td>2.0413</td>
<td>0.7892</td>
<td>2.5865</td>
<td>0.0105</td>
</tr>
<tr>
<td>$pd_t$</td>
<td>-0.4534</td>
<td>0.1932</td>
<td>-2.3471</td>
<td>0.0200</td>
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<tr>
<td>$xpu_t$</td>
<td>-0.02035</td>
<td>0.0209</td>
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#### $k = 3$

<table>
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<tr>
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<tr>
<td>$xpu_t$</td>
<td>-0.0865</td>
<td>0.0405</td>
<td>-2.1354</td>
<td>0.0341</td>
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Return-forecasting regressions

\( k = 6 \)

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<tbody>
<tr>
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<td>( pd_t )</td>
<td>-2.9882</td>
<td>0.3929</td>
<td>-7.6048</td>
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<tr>
<td>( xpu_t )</td>
<td>-0.1198</td>
<td>0.0548</td>
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## Return-forecasting regressions

- **$k = 6$**

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- **$k = 12$**

<table>
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<tbody>
<tr>
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<td>22.2094</td>
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<td>10.7887</td>
<td>0.0000</td>
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<tr>
<td>$pd_t$</td>
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<td>0.5130</td>
<td>-9.7196</td>
<td>0.0000</td>
</tr>
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<td>$xpu_t$</td>
<td>-0.1091</td>
<td>0.0775</td>
<td>-1.4079</td>
<td>0.1609</td>
</tr>
</tbody>
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Cross-section Pricing

- 30 Industries portfolio from Ken French library.
- \( R_{i,t} - RF_t = \alpha_{xi} + \beta_{xi} x_t + \epsilon_{xi,t} \)
- \( R_{i,t} - RF_t = \alpha_{mi} + \beta_{mi}(R_{mt} - RF_t) + \epsilon_{mi,t} \)
- \( R_i - RF = \alpha + \lambda_m \hat{\beta}_{mi} + \lambda_x \hat{\beta}_{xi} + \eta \)
Cross-section Pricing

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- \[ R_{i,t} - RF_t = \alpha_{xi} + \beta_{xi}x_t + \epsilon_{xi,t} \]
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- \[ R_i - RF = \alpha + \lambda_m \hat{\beta}_{mi} + \lambda_x \hat{\beta}_{xi} + \eta \]

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<tr>
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<tbody>
<tr>
<td>( \bar{\alpha} )</td>
<td>0.7523</td>
<td>0.3078</td>
<td>27.5825</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \bar{\lambda}_m )</td>
<td>0.1697</td>
<td>0.4826</td>
<td>4.0063</td>
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<tr>
<td>( \bar{\lambda}_x )</td>
<td>-0.1120</td>
<td>0.2621</td>
<td>-4.8637</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Macroeconomic Effects

Response to Cholesky One S.D. Innovations ± 2 S.E.

- Response of LOG1P_GAPA to LOG1P_GAPA
- Response of LOG1P_GAPA to CPI_GROWTH
- Response of LOG1P_GAPA to UNCERTAINTY
- Response of CPI_GROWTH to LOG1P_GAPA
- Response of CPI_GROWTH to CPI_GROWTH
- Response of CPI_GROWTH to UNCERTAINTY
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Summary

- Non-linear, non-Gaussian state-space model of the short-term rate via particle filter.
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- Monetary policy uncertainty seems to be a priced risk factor.
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- Negative macroeconomic effects of policy uncertainty shocks.