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Relation between damping, current-induced torques, and wall resistance for domain walls in magnetic nanowires

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We show that each mechanism of the Gilbert damping of wall motion corresponds to a current-induced drive torque on the domain wall, based on the same electron processes. For example, the damping theory of Heinrich, Fraitova, and Kambersky is directly related to the theory of (nonadiabatic) drive torques by Zhang and Li. Using momentum conservation, this relation is extended to the wall resistance. This leads to a classification of most existing electron theories of damping and drive torques, and of wall resistance, into only three different kinds. In all cases, the drive torque is derived from the damping torque by replacing the wall speed by an electron drift speed $\simeq R_0 j$, where R_0 is the ordinary Hall constant and j the current density. This drift speed differs from the one which appears in existing theories of adiabatic spin-transfer torques on a wall. Using the present ideas, mechanisms for drive torques and wall resistance have been invented.

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I. INTRODUCTION

In a recent publication,[1] we analyzed the experimental data of domain-wall mobility, of the critical current for current-induced wall motion, and of the wall resistance from 19 different papers and expressed the results for all three quantities in terms of an effective wall mobility μ. The reciprocal of μ represents the intensity of the viscous friction force coupling the wall to the electron gas. The μ values obtained from the three kinds of data were found to be mutually consistent and to depend very strongly on the wall width as predicted in 1984 by Berger.[2]

One purpose of the present paper is to give a better theoretical justification for this kind of analysis. Another purpose is to describe the general relation which exists between electron theories of damping, of current-induced drive torques on a wall, and of the wall resistance. We restrict ourselves to wall speeds smaller than the value corresponding to Walker breakdown, where the wall spins start to precess. Also, we restrict ourselves to the diffusive limit, where the electron mean free path is smaller than the wall width, because it is realized for most walls in real materials. The main effect of electron scattering is[1] to strongly break up the spatial coherence of the wave function inside the wall; thus, instead of a coherent wave-mechanical electron reflection by the wall, we have a problem of classical electron transport.

Let us assume a density $\mathbf{S}(X,t)$ of spin per unit volume, which comes from $3d$ magnetic electrons and is antiparallel to the local magnetization $\mathbf{M}(X,t)$, in a magnetic nanowire containing one tail-to-tail domain wall. Inside the wall, at a point of spatial coordinate X along the length of the nanowire, $\mathbf{S}$ makes an angle $\theta(X,t)$ with the $-X$ direction. Similarly, we have a spin density $\mathbf{s}(X,t)$ per unit volume coming from itinerant conduction electrons. The $\mathbf{s}$ direction is given in a local frame (x,y,z) where the z axis is parallel to $\mathbf{S}$ and the x axis is normal to the X axis. We decompose the density $\mathbf{s}$ into infinitesimal parts $\delta\mathbf{s}$ which include only electrons of velocity along X contained between v_X and $v_X+\delta v_X$. The semiclassical equations of motion for $\delta\mathbf{s}$ are

$$\hbar\frac{d(\delta s_x)}{dt} = 2\mu_B H_{sd}\delta s_y - \hbar\frac{\delta s_x}{\tau_{sr}},$$

$$\hbar\frac{d(\delta s_y)}{dt} + \hbar\frac{d\theta}{dt}\delta s_z = -2\mu_B H_{sd}\delta s_x - \hbar\frac{\delta s_y}{\tau_{sr}}. \quad (1)$$

Here, H_{sd} is the magnitude of the s-d exchange field $\mathbf{H}_{sd}=-\mathbf{S}J_{sd}/\mu_B n_{at}$ acting on $\delta\mathbf{s}$, n_{at} the density of magnetic atoms per unit volume, μ_B the Bohr magneton, J_{sd} the s-d exchange integral, and τ_{sr} the conduction-electron spin-relaxation time. The derivatives $d(\delta s_x)/dt$, $d(\delta s_y)/dt$, and $d\theta/dt$ are evaluated in a frame moving at velocity v_X. This frame has the advantage that the electrons constituting $\delta\mathbf{s}$ appear to be in a state of equilibrium, so that the spin current nearly vanishes and the equations become simpler. We apply the operation d/dt to the second equation of Eqs. (1) and combine the resulting equation with Eqs. (1) again. We keep terms containing $d^2\theta/dt^2$. Finally, we sum over all parts $\delta\mathbf{s}$ to recover equations of motion for the total spin density $\mathbf{s}$. In the limits $\omega_s\tau_{sr}\gg 1$ and $\omega_s^{-2}d^2(\delta s_y)/dt^2 \ll \omega_s^{-1}d(\delta s_y)/dt$, where $\omega_s = 2\mu_B H_{sd}/\hbar$ is the $\mathbf{s}$ precession frequency around the s-d exchange field $\mathbf{H}_{sd}$ which is antiparallel to $\mathbf{S}$, we obtain

$$\frac{ds_x}{dt} = \omega_s s_y + \frac{1}{\omega_s\tau_{sr}}\frac{ds_y}{dt} + \frac{s_z}{\omega_s\tau_{sr}}\overline{\frac{d\theta}{dt}} - \frac{s_z}{(\omega_s\tau_{sr})^2\omega_s}\overline{\frac{d^2\theta}{dt^2}},$$

$$\frac{ds_y}{dt} = -\omega_s s_x - \overline{\frac{d\theta}{dt}}s_z + \frac{s_z}{(\omega_s\tau_{sr})\omega_s}\overline{\frac{d^2\theta}{dt^2}}. \quad (2)$$

Here, the overbars indicate an average over electrons of various v_X. The first term on the right-hand side of each equation of Eqs. (2) generates the fast precession of $\mathbf{s}$ at frequency ω_s. The other terms cause a small shift of the precession axis away from $-\mathbf{H}_{sd}$, to a direction $\mathbf{s}_G$ given by

$$s_x^G = -\frac{s_z^G}{\omega_s}\overline{\frac{d\theta}{dt}} + \frac{s_z^G}{(\omega_s\tau_{sr})\omega_s^2}\overline{\frac{d^2\theta}{dt^2}},$$

$$s_y^G = -\frac{s_z^G}{(\omega_s\tau_{sr})\omega_s}\overline{\frac{d\theta}{dt}} - \frac{s_z^G}{\omega_s^2}\overline{\frac{d^2\theta}{dt^2}},$$

$$s_z^G \simeq \text{const}. \quad (3)$$

II. TORQUES ON THE MAGNETIZATION

The torque per unit volume applied by $\mathbf{s}$ on $\mathbf{S}$ is the reaction to that applied by $\mathbf{H}_{sd}$ on $\mathbf{s}$ and is $\tau=2\mu_B\mathbf{s}\times\mathbf{H}_{sd}$. We assume $v_F/\omega_s \ll \Delta_0$, where v_F is the Fermi velocity and Δ_0 the wall width, and we average τ over a precession period. Since $\mathbf{s}_G$ [Eqs. (3)] is the $\mathbf{s}$ precession axis, we obtain $\tau=2\mu_B\mathbf{s}_G\times\mathbf{H}_{sd}$ after averaging. The shift of $\mathbf{s}_G$ [Eqs. (3)] away from the direction of $\mathbf{H}_{sd}$ or of $\mathbf{S}$ itself is what allows a nonzero exchange torque to exist between $\mathbf{s}$ and $\mathbf{S}$.

It is well known[3] that only a field component along the easy axis is able to create an actual force on a wall. The torque generated on $\mathbf{M}$ by such a field is called "nonadiabatic torque" and would be along the x axis in the present geometry. Therefore, we have to consider the component τ_x of τ. Both τ_x and τ_y are generated through s-d exchange, but τ_x is smaller and requires an extra ingredient which is listed for each theory in the first column of Table I. Using Eqs. (3), τ_x becomes

$$\tau_x = \frac{\hbar s_z}{\omega_s}\left(\frac{1}{\tau_{sr}}\overline{\frac{d\theta}{dt}} + \overline{\frac{d^2\theta}{dt^2}}\right). \tag{4}$$

The second term on the right-hand side of Eq. (4) shows that a nonadiabatic torque can be obtained even in the absence of spin relaxation, provided terms in $d^2\theta/dt^2$ are not neglected. Introducing the wall velocity v_w in the laboratory frame, we have $d\theta/dt=(\partial\theta/\partial X)(v_X-v_w)$ and $d^2\theta/dt^2=(\partial^2\theta/\partial X^2)(v_X-v_w)^2$, where $\partial/\partial X$ and $\partial^2/\partial X^2$ indicate derivation in the laboratory frame. We now write separate spin-up and spin-down densities $\mathbf{s}_\uparrow$ and $\mathbf{s}_\downarrow$, and add the corresponding torques. Also, we replace τ_x and derivatives of θ by those of $\mathbf{M}$ itself. Then, Eq. (4) takes the form of a damping or drive term in the Landau-Lifshitz-Gilbert equation of motion for $\mathbf{M}$:

$$\begin{aligned}\frac{\partial\mathbf{M}}{\partial t} &= -\gamma\tau_x[x] + \cdots = \frac{2\mu_B}{\omega_s\tau_{sr}}[s_z^\uparrow(\bar{v}_X^\uparrow - v_w) \\ &+ s_z^\downarrow(\bar{v}_X^\downarrow - v_w)]\frac{1}{M_s^2}\mathbf{M}\times\frac{\partial\mathbf{M}}{\partial X} + \frac{2\mu_B}{\omega_s}\Big[s_z^\uparrow\overline{(v_X^\uparrow - v_w)^2} \\ &+ s_z^\downarrow\overline{(v_X^\downarrow - v_w)^2}\Big]\frac{1}{\mathbf{M}_s^2}\mathbf{M}\times\frac{\partial^2\mathbf{M}}{\partial X^2} + \cdots .\end{aligned} \tag{5}$$

Here, $[x]$ is the unit vector along x, and the ellipsis represents other terms in the equation of motion. As before, the overbars indicate an average over electrons of various velocities v_X. And γ is the gyromagnetic ratio.

If the spin densities $s_z^\uparrow$ and $s_z^\downarrow$ are uniform, the total effect of the torque in the second line of Eq. (5), on one side of the wall, is exactly compensated by the same torque at points on the other side of the wall, where the sign of $\partial^2\mathbf{M}/\partial X^2$ is reversed. A change of $s_z^\uparrow$ and $s_z^\downarrow$ is needed, caused by local shifts $\Delta\mu_\uparrow(X)$ and $\Delta\mu_\downarrow(X)$ of the Fermi levels, to lift the cancellation. To calculate these shifts, we write the changes $V_\uparrow(X)$ and $V_\downarrow(X)$ of s-d exchange energy associated[1] with the spin-mistracking angle $\gamma(X)$ in the wall:

$$V_\uparrow = -V_\downarrow = \frac{J_{sd}S}{n_{at}}[1-\cos\gamma(X)],$$

$$\gamma(X) = \arctan\left(\frac{\hbar v_F n_{at}}{2SJ_{sd}3^{1/2}}\frac{d\theta}{dX}\right). \tag{6}$$

In the frame moving at velocity $\bar{v}_X$, the occupation number $f_\uparrow(t)$ of a spin-up electron state of wave vector $\mathbf{k}$ and energy $\epsilon_\uparrow(t)$ obeys the relations

$$\frac{df_\uparrow}{dt} = \frac{f_\uparrow(t) - f_0(\epsilon_\uparrow(t))}{\tau_{sr}},$$

$$f_\uparrow(t) = f_0(\epsilon_\uparrow(t) - \epsilon_F - \Delta\mu_\uparrow),$$

$$\epsilon_\uparrow(t) = \epsilon_0^\uparrow(k) + V_\uparrow(\bar{X}(t)), \tag{7}$$

where $f_0(\epsilon)$ is the Fermi function, ϵ_F the equilibrium Fermi level, and $\epsilon_0^\uparrow(k)$ the kinetic energy. The second and third equations of Eqs. (7) show that current and wall motion push the electron gas up the energy barrier $V_\uparrow(X)$, thus causing the change $\Delta\mu_\uparrow$ of the spin-up Fermi level. The first equation of Eqs. (7) shows that this process is limited by spin-flip transitions to the spin-down band. From Eqs. (6) and (7), we obtain for $\gamma(X)\ll 1$ and $\tau_{sr}\ll\Delta_0/v_X$, where Δ_0 is the wall width,

$$\Delta\mu_\uparrow = \tau_{sr}\frac{\partial V_\uparrow}{\partial X}(\bar{v}_X^\uparrow - v_w) = \tau_{sr}\frac{\hbar v_F^2}{6\omega_s}\frac{1}{\mathbf{M}_s^2}\left(\frac{\partial\mathbf{M}}{\partial X}\cdot\frac{\partial^2\mathbf{M}}{\partial X^2}\right)(\bar{v}_X^\uparrow - v_w). \tag{8}$$

The uncanceled part of $s_z^\uparrow$ is $\Delta s_z^\uparrow=(3\Delta\mu_\uparrow/4\epsilon_F^\uparrow)n_\uparrow$, where $3n_\uparrow/2\epsilon_\uparrow$ is the spin-up density of states in a free-electron model and $n_\uparrow$ is the spin-up conduction-electron density per unit volume. Also, $\epsilon_F^\uparrow$ is the equilibrium Fermi level counted specifically from the spin-up band bottom. We assume $\epsilon_F^\uparrow \simeq \epsilon_F^\downarrow \simeq \epsilon_F$ for simplicity. Therefore, we replace $s_z^\uparrow$ by this $\Delta s_z^\uparrow$ in the second line of Eq. (5) and $s_z^\downarrow$ by a similar $\Delta s_z^\downarrow$. Also, we assume $v_w \ll v_F$. In the first line of Eq. (5), where there is no such cancellation, we use the relations $s_z^\uparrow=n_\uparrow/2$ and $s_z^\downarrow=-n_\downarrow/2$, and obtain finally from Eqs. (5) and (8)

$$\begin{aligned}\frac{\partial\mathbf{M}}{\partial t} &= \frac{\mu_B(n_\uparrow+n_\downarrow)}{\omega_s\tau_{sr}}[Pv_e - P_n v_w]\frac{1}{M_s^2}\mathbf{M}\times\frac{\partial\mathbf{M}}{\partial X} \\ &+ \frac{\mu_B\tau_{sr}\hbar v_F^4(n_\uparrow+n_\downarrow)}{12\omega_s^2\epsilon_F}[v_e - v_w] \\ &\times\frac{1}{M_s^4}\left(\frac{\partial\mathbf{M}}{\partial X}\cdot\frac{\partial^2\mathbf{M}}{\partial X^2}\right)\left(\mathbf{M}\times\frac{\partial^2\mathbf{M}}{\partial X^2}\right) + \cdots ,\end{aligned} \tag{9}$$

$$v_e = \frac{n_\uparrow\bar{v}_X^\uparrow + n_\downarrow\bar{v}_X^\downarrow}{n_\uparrow+n_\downarrow},$$

$$P = \frac{n_\uparrow\bar{v}_X^\uparrow - n_\downarrow\bar{v}_X^\downarrow}{n_\uparrow\bar{v}_X^\uparrow + n_\downarrow\bar{v}_X^\downarrow},$$

$$P_n = \frac{n_\uparrow - n_\downarrow}{n_\uparrow+n_\downarrow}. \tag{10}$$

Here, P is the current-polarization factor, which is of the order of unity. Also, P_n is the density-polarization factor,

TABLE I. List of electron-based theories of damping and drive torques on a domain wall and of the wall resistance. Theories based on the same physical mechanism are listed on the same line, and the nature of the mechanism is described in the first column. They have the same dependence on wall width and on electron-spin relaxation time, indicated in the last column.

Physical mechanism	Damping theory	Drive-torque theory	Wall resistance theory	Dependence
Electron-spin relaxation	Turov (1961) Heinrich-Fraitova-Kambersky (1967) Zhang-Li (2004)	Zhang-Li (2004)	Simanek (2001)	$1/(\tau_{sr}\Delta_0)$
Energy of mistracking	Berger (1984)	Berger (1984)	Berger (1986)	τ_{sr}/Δ_0^5
Anisotropic *s-d* exchange	Kambersky (1970)	Novel	Novel	τ_{sr}/Δ_0

which is probably smaller than that. And v_e represents the average drift velocity of the electron gas, where the average over spin-up and spin-down velocities is weighted according to the densities $n_\uparrow$ and $n_\downarrow$ of spin-up and spin-down electrons. If all carriers are electronlike, Eqs. (10) show that v_e is related to the current density j by $v_e=-j/(n_\uparrow+n_\downarrow)e$. This implies $v_e \simeq R_0 j$, where R_0 is the ordinary Hall constant, thus providing a rough but easy way to determine experimentally this drift velocity.

In Eq. (9), let us consider the first line. In it, the term in v_w is consistent with Gilbert damping, with a torque proportional to $\partial \mathbf{M}/\partial X$—i.e., to Δ_0^{-1}. This term is to be identified with the classical damping theory of Turov,[4] as well as with the equivalent quantum damping theory of Heinrich, Fraitova, and Kambersky[5] and also with the wall damping theory of Zhang and Li.[6] All three theories predict a damping torque proportional to Δ_0^{-1} and τ_{sr}^{-1} (see the first line of Table I). The term in v_e in the first line of Eq. (9) is also proportional to Δ_0^{-1} and τ_{sr}^{-1}, and is to be identified with the Zhang-Li theory[6] of (nonadiabatic) current-induced drive torques on a wall (Table I).

As shown by Eq. (9), the drive torque is obtained from the damping torque simply by replacing the wall velocity v_w by the electron drift velocity v_e, defined in Eqs. (10). In addition, when present, the dimensionless polarization factor P_n is replaced by P. This is one of the main results of the present paper and is more general than the derivation given here.

Equation (9) shows that nonadiabatic torques on the wall depend only on the relative motion of wall with respect to electron gas, as represented by the difference between v_e and v_w. This idea was already proposed in Ref. 1 and shown to be consistent with an extensive array of experimental data for walls in nanowires. The fact that spin-up and spin-down gases move at somewhat different rates is taken into account through the definition of the average drift speed v_e [Eqs. (10)]. The polarization factors P and P_n are needed in the first line of Eq. (9) because that line describes theories where spin-up and spin-down electrons generate torques of opposite signs.

Thiaville *et al.*[7] have proposed a somewhat similar relation between damping and drive torques, represented by adjustable phenomenological coefficients α and β, but without microscopic justification. Note also that the electron drift velocity v_e of Eq. (10), which is associated with nonadiabatic torques, differs significantly from the one which appears in Ref. 7 as well as in existing theories[8] of adiabatic torques. The latter velocity is $u=-jP\mu_B/eM_s$. In order to obtain our v_e from this u, a factor of M_s/μ_B downstairs has to be replaced by the electron density $n_\uparrow+n_\downarrow$.

Now, we turn to the second line of Eq. (9). It is not consistent with the usual Gilbert damping and contains a second derivative of $\mathbf{M}$ and is highly nonlinear. Under the present assumption $\gamma(X) \ll 1$, it gives a torque proportional to Δ_0^{-5} and τ_{sr} (second line of Table I) and is to be identified with the Berger theory[2] of damping and drive forces on a wall. Again, the drive torque is obtained from the damping torque [Eq. (9)] simply by replacing v_w by v_e. But there are no polarization factors here, because the spin-up and spin-down torques have the same sign.

As we showed earlier,[1] only through such high-order, nonlinear terms as found in the second line of Eq. (9) can one explain the fast dependence of damping and drive torques, and of the wall resistance, on Δ_0 found[1] in existing data for walls in magnetic nanowires.

Kambersky[9] introduced a third kind of electronic mechanism for damping, with torque proportional to Δ_0^{-1} and to τ_{sr} (third line of Table I). Again, by replacement of v_w by v_e, we find that we obtain a drive torque on the wall (third line of Table I), also proportional to Δ_0^{-1} and τ_{sr}. This drive-torque mechanism is novel. As in the case of the Berger theory, there is no polarization factors P or P_n in the formulas. While the local energy changes $V_\uparrow$ and $V_\downarrow$ of the Berger theory [Eqs. (6)] arise from spin mistracking and isotropic *s-d* exchange, those appearing in the Kambersky theory require the spin-orbit interaction ("anisotropic *s-d* exchange"[10]), but do not involve mistracking. This explains partly the slower dependence of his torque and wall resistance on Δ_0 (third line of Table I).

The Kambersky damping theory was extended by Korenman and Prange[11] to large values of the dimensionless pa-

rameter Λq, where Λ is the electron mean free path and q the spin-wave wave number. In the case of walls, q is to be identified with Δ_0^{-1}. But since $\Lambda < \Delta_0$ (except for walls in nanocontacts), the ballistic condition $\Lambda q > 1$ is rarely realized. The Tatara-Kohno theory[8] of drive torques on a wall is also based on ballistic assumptions.

III. FORCE ON A WALL

On purpose, we wrote the results of the present theory in the form of terms in the equation of motion for **M** [see Eq. (9)]. We now show how such terms, which represent torques, are related to the force F_X per unit wall area (also called pressure) applied by the electron gas to the wall. This is done by equating the total work of these torques, for a given virtual wall displacement δX, to the corresponding work of F_X. We obtain finally

$$F_X = (\gamma \Delta_0)^{-1} \int_{-\infty}^{+\infty} \frac{\partial M_x}{\partial t} f' dX. \tag{11}$$

Here, $\partial M_x / \partial t$ is given by Eq. (9) and γ is the gyromagnetic ratio. Also, the wall profile is given by $\theta(X) = f((X - X_0)/\Delta_0)$ where X_0 is the wall center and $f'(u)$ is defined as $df(u)/du$.

Equation (11) shows that the dependence of F_X on Δ_0 and τ_{sr} is the same as for the torques of Eq. (9) and is listed for each mechanism in the last column of Table I.

IV. WALL RESISTANCE

We write an equation for the rate of change of the momentum per unit cross-section area $p_\uparrow$ of the spin-up electron gas, in the steady state, in a region of length L along the nanowire which contains one wall:

$$\frac{dp_\uparrow}{dt} = 0 = \sigma_\uparrow^{-1} j_\uparrow n_\uparrow e L + n_\uparrow e V - F_\uparrow. \tag{12}$$

Here, the first term is the force caused by electron scatterers, $\sigma_\uparrow$ is the spin-up conductivity, and $j_\uparrow$ is the spin-up current density. The value of L is assumed to satisfy $L \gg l_{sr}, \Delta_0$, where l_{sr} is the spin-diffusion length. All carriers are assumed to be electronlike. The second term is the force of the electric field, and V is the total voltage difference over the length L. The last term is the reaction on the electron gas of the spin-up force $F_\uparrow$ on the wall. We write a similar equation for the spin-down momentum and use the relation $j = j_\uparrow + j_\downarrow$ to eliminate $j_\uparrow$ and $j_\downarrow$ and to solve for the voltage V. Then, introducing the voltage in the absence of a wall as $V_0 = jL/(\sigma_\uparrow + \sigma_\downarrow)$, we obtain

$$e(V - V_0) = \frac{1+P}{2} \frac{F_\uparrow}{n_\uparrow} + \frac{1-P}{2} \frac{F_\downarrow}{n_\downarrow}. \tag{13}$$

Next, we take $F_\uparrow$ from Eq. (11), where $\partial M_x / \partial t$ is obtained from Eqs. (9) and (10), and $v_w = 0$ is assumed. In the v_e expression of Eq. (10), we must use only the spin-up part in the numerator. Similarly, to derive $F_\downarrow$, we use only the spin-down part in the numerator of v_e. To simplify the results, we assume $n_\uparrow \simeq n_\downarrow$. In the case of the Zhang-Li mechanism[6] [first line of Eq. (9) and of Table I], we obtain finally

$$R_{dw} = \left[\frac{(1+P)^2}{n_\uparrow} - \frac{(1-P)^2}{n_\downarrow} \right] [4PAe^2(n_\uparrow + n_\downarrow)]^{-1} \left(\frac{F_X}{v_e} \right). \tag{14}$$

Here, $F_X = F_\uparrow + F_\downarrow$ and A is the nanowire cross-section area. Note that F_X / v_e [Eqs. (9) and (11)] and R_{dw} [Eq. (14)] can be both negative if $P < 0$.

In the case of the Berger mechanism[2] [second line of Eq. (9) and of Table I] and of the Kambersky mechanism (third line of Table I), we obtain instead

$$R_{dw} = \left[\frac{(1+P)^2}{n_\uparrow} + \frac{(1-P)^2}{n_\downarrow} \right] [4Ae^2(n_\uparrow + n_\downarrow)]^{-1} \left(\frac{F_X}{v_e} \right). \tag{15}$$

Here, both F_X / v_e and R_{dw} are always positive. Equations (14) and (15) are similar to Eq. (6) of Ref. 1, but somewhat more complicated because of the polarization factors coming from the two-current model used here [Eq. (12)]. These equations show that the dependence of R_{dw} on Δ_0 and τ_{sr} is the same as that of F_X and of the torques of Eq. (9), as given in the last column of Table I. In the case of the mechanisms listed in the first line of Table I, the wall-resistance theory appears to be that of Simanek,[12] with $R_{dw} \propto F_X / v_e \propto (\tau_{sr} \Delta_0)^{-1}$. In the case of the mechanism in the second line, the resistance theory is that of Berger,[13] with $R_{dw} \propto F_X / v_e \propto \tau_{sr} / \Delta_0^5$. For the third line, the resistance theory is novel, with $R_{dw} \propto F_X / v_e \propto \tau_{sr} / \Delta_0$. Because of its faster dependence on Δ_0, the Berger mechanism of resistance probably dominates[1] in the case of very narrow walls in nanowires.

V. OTHER THEORIES

The wall-resistance theory of Simanek and Rebei[14] has some similarity to the Berger theories[2,13] of damping, drive torques, and wall resistance [see Eq. (15) and second line of Eq. (5) of present paper], provided the cancellation argument below the present equation (5) is omitted. For example, as for the torque in the second line of the present equation (5), the wall resistance in their equation (19) is proportional to Δ^{-2} and v_F^2, and independent of τ_{sr}.

The wall-resistance theory of Gregg *et al.*[15] and Levy and Zhang[16] does not fit into our classification scheme, because their flow of momentum is between wall and lattice, not between wall and electron gas as in our equations (12). In other words, the wall modulates the existing bulk electrical resistivity. These statements also apply to the wall resistance caused[17] by anisotropic magnetoresistance.

VI. CONCLUSIONS

We have shown [Eq. (9)] that most existing theories of current-induced drive torques on a wall are obtained from the corresponding electron theory of damping by a simple replacement of the wall velocity v_w by a certain electron drift velocity v_e, defined in Eq. (10). Indeed, the relation between these theories is so close to amount to complete identity. For

example, corresponding damping and drive torques are caused by the same physical mechanism, listed in the first column of Table I. Also, they have the same dependence on wall width and on electron-spin relaxation time, indicated in the last column of Table I. Using momentum conservation, this relation is extended to the wall resistance [Eqs. (14) and (15) and Table I]. The present ideas lead to the invention of mechanisms of drive torques and of wall resistance (third line of Table I).

The condition $\omega_s \tau_{sr} \gg 1$, stated in the Introduction, ensures that spin memory is long enough for our classical **s** precession approach [Eqs. (1)–(4)] to be valid. The torques predicted here [Eqs. (4), (9), and (10)] are a simple consequence of the shift of the precession axis $\mathbf{s}_G$ away from $\mathbf{H}_{sd}$—i.e., from **S** [Eqs. (3)]. Note that this shift depends primarily on v_w and v_e, and is not affected by the existence of scattering: the Ohmic electric field automatically adjusts itself to prevent v_e from relaxing to zero under the influence of scattering.

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