Relation between damping, current-induced torques, and wall resistance for domain walls in magnetic nanowires

Luc Berger

Carnegie Mellon University, lberger@andrew.cmu.edu

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Relation between damping, current-induced torques, and wall resistance for domain walls in magnetic nanowires

L. Berger
Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
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We show that each mechanism of the Gilbert damping of wall motion corresponds to a current-induced drive torque on the domain wall, based on the same electron processes. For example, the damping theory of Heinrich, Fraito, and Kambersky is directly related to the theory of (nonadiabatic) drive torques by Zhang and Li. Using momentum conservation, this relation is extended to the wall resistance. This leads to a classification of most existing electron theories of damping and drive torques, and of wall resistance, into only three different kinds. In all cases, the drive torque is derived from the damping torque by replacing the wall speed by an electron drift speed \(v_{\text{d}}\), where \(v_{\text{d}}\) is the ordinary Hall constant and \(f\) the current density. This drift speed differs from the one which appears in existing theories of adiabatic spin-transfer torques on a wall. Using the present ideas, mechanisms for drive torques and wall resistance have been invented.

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I. INTRODUCTION

In a recent publication,\(^1\) we analyzed the experimental data of domain-wall mobility, of the critical current for current-induced wall motion, and of the wall resistance from 19 different papers and expressed the results for all three quantities in terms of an effective wall mobility \(\mu\). The reciprocal of \(\mu\) represents the intensity of the viscous friction force coupling the wall to the electron gas. The \(\mu\) values obtained from the three kinds of data were found to be mutually consistent and to depend very strongly on the wall width as predicted in 1984 by Berger.\(^2\)

One purpose of the present paper is to give a better theoretical justification for this kind of analysis. Another purpose is to describe the general relation which exists between electron theories of damping, of current-induced drive torques on a wall, and of the wall resistance. We restrict ourselves to the diffusive limit, where the electron mean free path is smaller than the wall width, because it is realized for most walls in real materials. The main effect of electron scattering is\(^1\) to strongly break up the spatial coherence of the wave function inside the wall; thus, instead of a coherent wave-mechanical electron reflection by the wall, we have a problem of classical electron transport.

Let us assume a density \(S(X,t)\) of spin per unit volume, which comes from 3\(d\) magnetic electrons and is antiparallel to the local magnetization \(M(X,t)\), in a magnetic nanowire containing one tail-to-tail domain wall. Inside the wall, at a point of spatial coordinate \(X\) along the length of the nanowire, \(S\) makes an angle \(\theta(X,t)\) with the \(X\) direction. Similarly, we have a spin density \(s(X,t)\) per unit volume coming from itinerant conduction electrons. The \(s\) direction is given in a local frame \((x,y,z)\) where the \(z\) axis is parallel to \(S\) and the \(x\) axis is normal to the \(X\) axis. We decompose the density \(s\) into infinitesimal parts \(\delta s\) which include only electrons of velocity along \(X\) contained between \(v_X\) and \(v_X + \delta v_X\). The semiclassical equations of motion for \(\delta s\) are

\[
\hbar \frac{d(\delta s_x)}{dt} = 2 \mu_B H_{sd} \delta s_x - \hbar \frac{\delta s_x}{\tau_{\text{sr}}},
\]

Here, \(H_{sd}\) is the magnitude of the \(s-d\) exchange field \(H_{sd} = -J_{sd}/\mu_B n_{sd}\) acting on \(\delta s\), \(n_{sd}\) the density of magnetic atoms per unit volume, \(\mu_B\) the Bohr magneton, \(J_{sd}\) the \(s-d\) exchange integral, and \(\tau_{\text{sr}}\) the conduction-electron spin-relaxation time. The derivatives \(d(\delta s_x)/dt, d(\delta s_y)/dt,\) and \(d(\delta s_z)/dt\) are evaluated in a frame moving at velocity \(v_X\). This frame has the advantage that the electrons constituting \(\delta s\) appear to be in a state of equilibrium, so that the spin current nearly vanishes and the equations become simpler. We apply the operation \(d/dt\) to the second equation of Eqs. (1) and combine the resulting equation with Eqs. (1) again. We keep terms containing \(d^2\theta/dt^2\). Finally, we sum over all parts \(\delta s\) to recover equations of motion for the total spin density \(s\) in the limits \(\omega_\tau\tau_{\text{sr}} \gg 1\) and \(\omega_s^2 \tau_{\text{sr}}^{-1} (d(\delta s_x)/dt)/d(\delta s_x)/dt = \omega_s^{-1} d(\delta s_x)/dt\), where \(\omega_s = 2 \mu_B H_{sd}/K\) is the \(s\) precession frequency around the \(s-d\) exchange field \(H_{sd}\) which is antiparallel to \(S\), we obtain

\[
\frac{ds_x}{dt} = \omega_s s_x + \frac{1}{\omega_s \tau_{\text{sr}}} \frac{ds_y}{dt} + \frac{s_z}{\omega_s \tau_{\text{sr}}} \frac{d\theta}{dt} - \frac{s_z}{(\omega_s \tau_{\text{sr}})^2} \frac{\delta s_x}{dt} - \frac{\omega_s s_x}{(\omega_s \tau_{\text{sr}})^2} \frac{\delta s_x}{dt^2},
\]

Here, the overbars indicate an average over electrons of various \(v_X\). The first term on the right-hand side of each equation of Eqs. (2) generates the fast precession of \(s\) at frequency \(\omega_s\). The other terms cause a small shift of the precession axis away from \(-H_{sd}\) to a direction \(s_G\) given by

\[
\frac{s_x^G}{\omega_s} = \frac{\delta s_x}{\omega_s} + \frac{s_z^G}{\omega_s} \frac{d\theta}{dt},
\]

where

\[
s_x^G = -\frac{s_x^G}{\omega_s} \frac{d\theta}{dt} + \frac{s_z^G}{\omega_s} \frac{d^2\theta}{dt^2},
\]

\[
s_z^G = \frac{s_z^G}{(\omega_s \tau_{\text{sr}})^2} \omega_s \tau_{\text{sr}} \frac{d\theta}{dt} + \frac{s_z^G}{\omega_s} \frac{d\theta}{dt},
\]

\[
s_z^G \approx \text{const.}
\]
II. TORQUES ON THE MAGNETIZATION

The torque per unit volume applied by $s$ on $S$ is the reaction to that applied by $H_{sd}$ on $s$ and is $\tau = 2 \mu_s s \times H_{sd}$. We assume $v_F/\omega \ll \Delta_0$, where $v_F$ is the Fermi velocity and $\Delta_0$ the wall width, and we average $\tau$ over a precession period. Since $S_\tau$ [Eqs. (3)] is the $\tau$ precession axis, we obtain $\tau = 2 \mu_s S_\tau \times H_{sd}$ after averaging. The shift of $S_\tau$ [Eqs. (3)] away from the direction of $H_{sd}$ or of $S$ itself is what allows a nonzero exchange torque to exist between $s$ and $S$.

It is well known that only a single component along the easy axis is able to create an actual force on a wall. The torque generated on $M$ by such a field is called “nonadiabatic torque” and would be along the $x$ in the present geometry. Therefore, we have to consider the component $\tau_x$ of $\tau$. Both $\tau_x$ and $\tau_y$ are generated through $s$-$d$ exchange, but $\tau_x$ is smaller and requires an extra ingredient which is listed for each theory in the first column of Table I. Using Eqs. (3), $\tau_x$ becomes

$$\tau_x = \frac{h\hat{z}}{\omega} \left( \frac{1}{\tau_{sr}} \frac{d\theta}{dt} + \frac{d^2\theta}{dt^2} \right).$$

The second term on the right-hand side of Eq. (4) shows that a nonadiabatic torque can be obtained even in the absence of spin relaxation, provided terms in $d^2\theta/dt^2$ are not neglected. Introducing the wall velocity $v_w$ in the laboratory frame, we have $d\theta/dt = (\partial \theta / \partial X)(v_X - v_w)$ and $d^2\theta/dt^2 = (\partial^2 \theta / \partial X^2)(v_X - v_w)^2$, where $\partial \theta / \partial X$ and $\partial^2 \theta / \partial X^2$ indicate derivative in the laboratory frame. We now separate spin-up and spin-down densities $s^\uparrow$ and $s^\downarrow$, and add the corresponding torques. Also, we replace $\tau_x$ and derivatives of $\theta$ by those of $M$ itself. Then, Eq. (4) takes the form of a damping or drive term in the Landau-Lifshitz-Gilbert equation of motion for $M$:

$$\frac{\partial M}{\partial t} = - \gamma \tau_x[x] + \cdots = \frac{2 \mu_B}{\omega \tau_{sr}} \left[ s^\uparrow \bar{v}^\uparrow - v_w \right]$$

$$+ \frac{s^\downarrow (\bar{v}^\downarrow - v_w)}{M^\perp} \frac{\partial M}{\partial X} + \frac{2 \mu_B}{\omega s} \left[ s^\uparrow \bar{v}^\uparrow - v_w \right]^2$$

$$+ \frac{s^\downarrow (\bar{v}^\downarrow - v_w)}{M^\perp} \frac{\partial M}{\partial X} + \cdots.$$

Here, $[x]$ is the unit vector along $x$, and the ellipsis represents other terms in the equation of motion. As before, the overbars indicate an average over electrons of various velocities $v_X$. And $\gamma$ is the gyromagnetic ratio.

If the spin densities $s^\uparrow$ and $s^\downarrow$ are uniform, the total effect of the torque in the second line of Eq. (5), on one side of the wall, is exactly compensated by the same torque at points on the other side of the wall, where the sign of $\partial^2 M/\partial X^2$ is reversed. A change of $s^\uparrow$ and $s^\downarrow$ is needed, caused by local shifts $\Delta \mu^\uparrow (X)$ and $\Delta \mu^\downarrow (X)$ of the Fermi levels, to lift the cancellation. To calculate these shifts, we write the changes $V_s(X)$ and $V_x(X)$ of $s$-$d$ exchange energy associated with the spin-misalignment angle $\gamma(X)$ in the wall:

$$V_s = - V_x = \frac{J_{sd} S}{n_{sd}} [1 - \cos \gamma(X)],$$

where $P_s$ is the current-polarization factor, which is of the order of unity. Also, $P_n$ is the density-polarization factor.
which is probably smaller than that. And \( v_e \), represents the average drift velocity of the electron gas, where the average over spin-up and spin-down velocities is weighted according to the densities \( n_1 \) and \( n_1 \) of spin-up and spin-down electrons. If all carriers are electron-like, Eqs. (10) show that \( v_e \) is related to the current density \( j \) by \( v_e = -j/(n_1 + n_1) e \). This implies \( v_e = R_0 j \), where \( R_0 \) is the ordinary Hall constant, thus providing a rough but easy way to determine experimentally this drift velocity.

In Eq. (9), let us consider the first line. In it, the term in \( v_w \) is consistent with Gilbert damping, with a torque proportional to \( \partial \mathbf{M}/\partial \mathbf{X} \)—i.e., to \( \Delta_0^{-1} \). This term is to be identified with the classical damping theory of Turov,\(^4\) as well as with the equivalent quantum damping theory of Heinrich, Fraitova, and Kambersky,\(^6\) and also with the wall damping theory of Zhang and Li.\(^6\) All three theories predict a damping torque proportional to \( \Delta_0^{-1} \) and \( \tau_{sr} \) (see the first line of Table I). The term in \( v_w \) in the first line of Eq. (9) is also proportional to \( \Delta_0^{-1} \) and \( \tau_{sr} \), and is to be identified with the Zhang-Li theory\(^6\) of (nonadiabatic) current-induced drive forces on a wall (Table I).

As shown by Eq. (9), the drive torque is obtained from the damping torque simply by replacing the wall velocity \( v_w \) by the electron drift velocity \( v_e \), defined in Eqs. (10). In addition, when present, the dimensionless polarization factor \( P_n \) is replaced by \( P \). This is one of the main results of the present paper and is more general than the derivation given here.

Equation (9) shows that nonadiabatic torques on the wall depend only on the relative motion of wall with respect to electron gas, as represented by the difference between \( v_e \) and \( v_w \). This idea was already proposed in Ref. 1 and shown to be consistent with an extensive array of experimental data for walls in nanowires. The fact that spin-up and spin-down gases move at somewhat different rates is taken into account through the definition of the average drift speed \( v_e \) [Eqs. (10)]. The polarization factors \( P \) and \( P_n \) are needed in the first line of Eq. (9) because that line describes theories where spin-up and spin-down electrons generate torques of opposite signs.

Thiaville et al.\(^7\) have proposed a somewhat similar relation between damping and drive torques, represented by adjustable phenomenological coefficients \( \alpha \) and \( \beta \), but without microscopic justification. Note also that the electron drift velocity \( v_e \) of Eq. (10), which is associated with nonadiabatic torques, differs significantly from the one which appears in Ref. \(^7\) as well as in existing theories\(^8\) of adiabatic torques. The latter velocity is \( u = -j \mu B / e M_s \). In order to obtain our \( v_e \) from this \( u \), a factor of \( M_s / \mu B \) downstairs has to be replaced by the electron density \( n_1 + n_1 \).

Now, we turn to the second line of Eq. (9). It is not consistent with the usual Gilbert damping and contains a second derivative of \( \mathbf{M} \) and is highly nonlinear. Under the present assumption \( \gamma(X) \ll 1 \), it gives a torque proportional to \( \Delta_0^{-5} \) and \( \tau_{sr} \) (second line of Table I) and is to be identified with the Berger theory\(^2\) of damping and drive forces on a wall. Again, the drive torque is obtained from the damping torque [Eq. (9)] simply by replacing \( v_w \) by \( v_e \). But there are no polarization factors here, because the spin-up and spin-down torques have the same sign.

As we showed earlier,\(^1\) only through such high-order, nonlinear terms as found in the second line of Eq. (9) can one explain the fast dependence of damping and drive torques, and of the wall resistance, on \( \Delta_0 \) found\(^1\) in existing data for walls in magnetic nanowires.

Kambersky\(^9\) introduced a third kind of electronic mechanism for damping, with torque proportional to \( \Delta_0^{-3} \) and to \( \tau_{sr} \) (third line of Table I). Again, by replacement of \( v_w \) by \( v_e \), we find that we obtain a drive torque on the wall (third line of Table I), also proportional to \( \Delta_0^{-3} \) and \( \tau_{sr} \). This drive-torque mechanism is novel. As in the case of the Berger theory, there is no polarization factors \( P \) or \( P_n \) in the formulas. While the local energy changes \( V_1 \) and \( V_1 \) of the Berger theory [Eqs. (6)] arise from spin mistracking and isotropic \( s-d \) exchange, those appearing in the Kambersky theory require the spin-orbit interaction ("anisotropic \( s-d \) exchange")\(^10\), but do not involve mistracking. This explains partly the slower dependence of his torque and wall resistance on \( \Delta_0 \) (third line of Table I).

The Kambersky damping theory was extended by Korenman and Prange\(^11\) to large values of the dimensionless pa-
rameter $\Lambda q$, where $\Lambda$ is the electron mean free path and $q$ the spin-wave wave number. In the case of walls, $q$ is to be identified with $\Delta_0^{-1}$. But since $\Lambda < \Delta_0$ (except for walls in nanocontacts), the ballistic condition $\Lambda q > 1$ is rarely realized. The Tatara-Kohno theory of drive torques on a wall is also based on ballistic assumptions.

III. FORCE ON A WALL

On purpose, we wrote the results of the present theory in the form of terms in the equation of motion for $M$ [see Eq. (9)]. We now show how such terms, which represent torques, are related to the force $F_X$ per unit wall area (also called pressure) applied by the electron gas to the wall. This is done by equating the total work of these torques, for a given virtual wall displacement $\delta X$, to the corresponding work of $F_X$. We obtain finally

$$F_X = (\gamma \Delta_0)^{-1} \int_{-\infty}^{\infty} \frac{\partial M_s}{\partial t} f'(u) dX. \quad (11)$$

Here, $\partial M_s/\partial t$ is given by Eq. (9) and $\gamma$ is the gyromagnetic ratio. Also, the wall profile is given by $t(X) = f(X - X_0)/\Delta_0$, where $X_0$ is the wall center and $f'(u)$ is defined as $df(u)/du$.

Equation (11) shows that the dependence of $F_X$ on $\Delta_0$ and $\tau_{sr}$ is the same as for the torques of Eq. (9) and is listed for each mechanism in the last column of Table I.

IV. WALL RESISTANCE

We write an equation for the rate of change of the momentum per unit cross-section area $\sigma_1$ of the spin-up electron gas, in the steady state, in a region of length $L$ along the nanowire which contains one wall:

$$\frac{d\sigma_1}{dt} = \sigma_1^{-1} j_n eL + n_l eV - F_1. \quad (12)$$

Here, the first term is the force caused by electron scatterers, $\sigma_1$ is the spin-up conductivity, and $j_n$ is the spin-up current density. The value of $L$ is assumed to satisfy $L \gg l_{sr}, \Delta_0$, where $l_{sr}$ is the spin-diffusion length. All carriers are assumed to be electronlike. The second term is the force of the electric field, and $V$ is the total voltage difference over the length $L$. The last term is the reaction on the electron gas of the spin-up force $F_1$ on the wall. We write a similar equation for the spin-down momentum and use the relation $j = j_1 + j_1^{-1}$ to eliminate $j_1$ and $j_1^{-1}$ and to solve for the voltage $V$. Then, introducing the voltage in the absence of a wall as $V_0 = jL/(\sigma_1 + \sigma_1^{-1})$, we obtain

$$e(V - V_0) = \frac{1 + P F_1}{2 n_1} + \frac{1 - P F_1}{2 n_1}. \quad (13)$$

Next, we take $F_1$ from Eq. (11), where $\partial M_s/\partial t$ is obtained from Eqs. (9) and (10), and $u_0 = 0$ is assumed. In the numerator expression of Eq. (10), we must use only the spin-up part in the numerator. Similarly, to derive $F_1$, we use only the spin-down part in the numerator of $u_r$. To simplify the results, we assume $n_i = n_i$. In the case of the Zhang-Li mechanism [first line of Eq. (9) and of Table I], we obtain finally

$$R_{dw} = \left[ \frac{(1 + P)^2}{n_1} - \frac{(1 - P)^2}{n_1} \right] \left[ 4PAe^2(n_1 + n_1) \right]^{-1} F_X. \quad (14)$$

Here, $F_X = F_1 + F_1^{-1}$ and $A$ is the nanowire cross-section area. Note that $F_X/v_0$ [Eqs. (9) and (11)] and $R_{dw}$ [Eq. (14)] can be both negative if $P < 0$.

In the case of the Berger mechanism [second line of Eq. (9) and of Table I] and of the Kambersky mechanism (third line of Table I), we obtain instead

$$R_{dw} = \left[ \frac{(1 + P)^2}{n_1} + \frac{(1 - P)^2}{n_1} \right] \left[ 4AEe^2(n_1 + n_1) \right]^{-1} F_X. \quad (15)$$

Here, both $F_X/v_0$ and $R_{dw}$ are always positive. Equations (14) and (15) are similar to Eq. (6) of Ref. 1, but somewhat more complicated because of the polarization factors coming from the two-current model used here [Eq. (12)]. These equations show that the dependence of $R_{dw}$ on $\Delta_0$ and $\tau_{sr}$ is the same as that of $F_X$ and of the torques of Eq. (9), as given in the last column of Table I. In the case of the mechanisms listed in the first line of Table I, the wall resistance theory appears to be that of Simanek,12 with $R_{dw} \propto F_X/v_0 \propto (\tau_{sr} \Delta_0)^{-1}$. In the case of the mechanism in the second line, the resistance theory is that of Berger,13 with $R_{dw} \propto F_X/v_0 \propto \tau_{sr}/\Delta_0$. For the third line, the resistance theory is novel, with $R_{dw} \propto F_X/v_0 \propto \tau_{sr}/\Delta_0$. Because of its faster dependence on $\Delta_0$, the Berger mechanism of resistance probably dominates in the case of very narrow walls in nanowires.

V. OTHER THEORIES

The wall-resistance theory of Simanek and Rebei14 has some similarity to the Berger theories of damping, drive torques, and wall resistance [see Eq. (15) and second line of Eq. (5) of present paper], provided the cancellation argument below the present equation (5) is omitted. For example, as for the torque in the second line of the present equation (5), the wall resistance in their equation (19) is proportional to $\Delta^{-2}$ and $\nu_r^2$, and independent of $\tau_{sr}$.

The wall-resistance theory of Gregg et al.15 and Levy and Zhang16 does not fit into our classification scheme, because their flow of momentum is between wall and lattice, not between wall and electron gas as in our equations (12). In other words, the wall modulates the existing bulk electrical resistivity. These statements also apply to the wall resistance caused by anisotropic magnetoresistance.

VI. CONCLUSIONS

We have shown [Eq. (9)] that most existing theories of current-induced drive torques on a wall are obtained from the corresponding electron theory of damping by a simple replacement of the wall velocity $v_0$ by a certain electron drift velocity $v_\sigma$, defined in Eq. (10). Indeed, the relation between these theories is so close to amount to complete identity. For
example, corresponding damping and drive torques are caused by the same physical mechanism, listed in the first column of Table I. Also, they have the same dependence on wall width and on electron-spin relaxation time, indicated in the last column of Table I. Using momentum conservation, this relation is extended to the wall resistance [Eqs. (14) and (15) and Table I]. The present ideas lead to the invention of mechanisms of drive torques and of wall resistance (third line of Table I).

The condition $\omega_r \tau_p \gg 1$, stated in the Introduction, ensures that spin memory is long enough for our classical $\mathbf{s}$ precession approach [Eqs. (1)–(4)] to be valid. The torques predicted here [Eqs. (4), (9), and (10)] are a simple consequence of the shift of the precession axis $s_G$ away from $H_{sd}$—i.e., from $S$ [Eqs. (3)]. Note that this shift depends primarily on $v_w$ and $v_e$, and is not affected by the existence of scattering: the Ohmic electric field automatically adjusts itself to prevent $v_e$ from relaxing to zero under the influence of scattering.

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6 S. Zhang and Z. Li, Phys. Rev. Lett. 93, 127204 (2004). The relevant torques are the second and last terms on the right-hand side of Eq. (11).