Fine-Grained MSR Specifications for Quantitative Security Analysis

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Abstract. The traditional Dolev-Yao model of security limits attacks to “computationally feasible” operations. We depart from this model by assigning a cost to protocol actions, both of the Dolev-Yao kind as well as non traditional forms such as computationally-hard operations, guessing, principal subversion, and failure. This quantitative approach enables evaluating protocol resilience to various forms of denial of service, guessing attacks, and resource limitation. While the methodology is general, we demonstrate it through a low-level variant of the MSR specification language.

1 Introduction

Security protocols have classically been analyzed with respect to the Dolev-Yao intruder [8, 17], a model which gives the attacker complete access to the network, but limits its decryption capabilities to messages for which he possesses the appropriate keys. There is consensus among practitioners that the basic problems of protocol verification, namely secrecy and authentication, are by now solved for this model, as the most recent tools sweep through the standard Clark-Jacob benchmark [6] in mere milliseconds. Recent research has moved into two directions: apply the current tools to the much larger protocols used in the real world, and investigate intruder models that rely on capabilities beyond Dolev-Yao gentlemen correctness. This work takes the latter path.

The three tenets of the Dolev-Yao model are the symbolic representation of data, so that a key $k$ is seen as an atomic object rather than a bit-string, the unguessability of secret values such as nonces and keys, and black-box cryptography, by which a message $m$ encrypted with $k$ can be recovered only by a principal in possession of $k^{(-1)}$. All three have been weakened in the last few years. Approaches have taken the bit length of messages and keys into account for example to compute the likelihood of key guesses [1], to investigate type confusion attacks [16], and to reason about asymptotic behaviors [12], as often done to assess cryptographic algorithms. Within the symbolic abstraction, effort has been undertaken to include guessing in the intruder’s toolkit [13], and to let recurrent algebraic operations, in particular XOR and Diffie-Hellman exponentiation, out of the black box, allowing the intruder to use them to mount an attack.

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(within the accepted computational bounds, *e.g.*, taking a discrete logarithm is not permitted) [4, 7].

The present work takes a symbolic view of data, but allows the intruder to guess values and perform computationally hard operations. That is, if he is willing and able to pay the price. Indeed, we are not so much interested in a *lucky* intruder breaking the protocol, but in a *tenacious* one, who will spend Herculean effort in order to gain Alice’s confidence or learn Bob’s secret. The proposed methodology assigns a cost to both Dolev-Yao and non-standard intruder operations. Depending on the intended use, this cost can be a physical measurement, such as time, space or energy, a complexity class, or simply one of the two values 0 and ∞ in a purely Dolev-Yao model. This work directly extends Meadow’s quantitative assessment of denial-of-service [15] and Lowe’s analysis of verifiable guesses [13].

Potential applications of this approach include:

- Provide a way for standard analysis methodologies to take intruder effort levels into account. For example, weak secrets are usually modeled as either unguessable or public values. Assigning them an appropriate cost and estimating the resources (or the persistence) of the intruder may help decide whether this secret is too weak for practical purposes. Intruder cost thresholds can be easily integrated into many model checking tools for example.
- Monitoring of network activity, by either an intruder or a law enforcement agency. This entity may then compute the cost of mounting an action against particular communicating agents, using the result to estimate the needed resources for example.
- Gauge the resilience of a system against denial-of-service scenarios. Cost functions have been used for this purpose [15, 18], but mostly limited to legal Dolev-Yao intruder operations.
- Assess the vulnerabilities of agents meant to operate in a potentially hostile environment with very limited resources in terms of computational power, bandwidth, battery life, etc [5]. Typical examples are smart cards, and mobile devices such as PDAs and cellular phones.
- When building a system, compare protocols providing desired functionalities, with respect to their resilience to particular forms of attacks. During the development phase of a protocol, compare alternative designs or parameter choices for optimal resistance to certain attacks. In particular, proposals for denial-of-service protection, *e.g.*, Juels and Brainard’s client puzzles [11] or even the network level proposal of Gunter et al. [10], are good application candidates for this methodology.

As it allows asking “How secure is this protocol?”, rather than “Is it secure or not?”, the proposed methodology can also be seen as complementing performance and quality of service with an additional quantitative dimension on which to evaluate protocols.

We rely on a Fine-Grained variant of the MSR rule-based specification formalism [2, 3] as a vehicle to introduce this work. Fine-Grained MSR isolates individual verification operations and accounts for the possibility of failure. This is achieved by dividing rule application in a pre-screening phase that commits to a rule, and a more thorough check that fully assesses its applicability.
We recall MSR and present a flexible notion of execution trace in Section 2. Fine-Grained MSR is introduced in Section 3. Our cost infrastructure is described in Section 4, and application possibilities are outlined in Section 5.

2 Background

In this section, we recall the syntax and execution semantics of MSR. We will focus on a minimal instance for simplicity, but the reported results do extend to the more general language, and could also be adapted to other formalisms.

2.1 MSR Protocol Specifications

In MSR, a protocol is specified as a number of roles. A role corresponds to the abstract sequence of actions executed by each participating principal. Roles are also used to describe the intruder capabilities. A role itself is given as a sequence of multiset rewrite rules, which describe each individual action. Each rule represents a local transformation of the execution state. It has a left-hand side that describes what should be taken out of the state, and a right-hand side denoting what it should be replaced with. State objects are modeled using first-order atomic predicates. They include messages in transit ($N(m)$), public information ($M^\ast(m)$), private data of a principal ($M_A(m)$), and a record of the status of every executing role ($L^v(m)$ — $v$ acts as a program counter, and $m$ is synchronization data). The right-hand side of a rule can additionally mention existentially bound variables to model the creation of nonces and other fresh data. A role can be prefixed by existential binders over the superscripts $v$ of the $L$ predicates to ensure locality during execution. Altogether, an MSR protocol specification is given by the following grammar.

\[
\begin{align*}
\text{(Messages)} & \quad m & := x \mid c \mid \op(m) \\
\text{(Left-hand sides)} & \quad \lhs & := \cdot \mid N(m), \lhs \mid L^v(m), \lhs \\
& & \mid M_A(m), \lhs \mid M_s(m), \lhs \\
\text{(Right-hand sides)} & \quad \rhs & := \lhs \mid \exists x.\rhs \\
\text{(Rules)} & \quad r & := \lhs \rightarrow \rhs \\
\text{(Roles)} & \quad \rho & := \cdot \mid \rho, \rho \mid \exists v.\rho \\
\text{(Protocols)} & \quad \mathcal{P} & := \cdot \mid \mathcal{P}
\end{align*}
\]

Here, $x$ is a variable and $c$ stands for atomic terms, which shall include principal names (usually written $a$ possibly subscripted), nonces ($n$), keys ($k$), etc. Composite terms are formed using constructors $\op$, which shall include concatenation ($\cdot$), encryption ($\{}$, and more. We write $x$ for a sequence $x_1, \ldots, x_n$. We will occasionally write a composite term as $\op_{m'}(m)$ to isolate terms that must be given ($m'$) in order to produce arguments $m$ (e.g., the key in a decryption). We denote the empty sequence as "\cdot", and often keep it implicit.

2.2 MSR Execution

The execution semantics of MSR operates by transforming configurations of the form $$\langle S \rangle^S_\Sigma$$, where the state $S$ is a multiset of ground predicates, the signature $\Sigma$ keeps track
of the symbols in use, and the active role set \( R = (\rho_1^{a_1}, \ldots, \rho_n^{a_n}) \) records the remaining actions of the currently executing roles (\( \rho_i \)), and who is executing them (\( a_i \)). The basic execution step is expressed by judgments of the form \( \mathcal{P} \triangleright C \rightarrow C' \) where \( \mathcal{P} \) is the protocol specification, and \( C \) and \( C' \) are consecutive configurations. This judgment is defined by the following two rules:

\[
\frac{\mathcal{P} \triangleright C \rightarrow C'}{\mathcal{P} \triangleright \langle S_{\mathcal{P}}[\theta], \underline{\text{rhs}_S} \rangle \Sigma^{r \mathcal{P}}_{\mathcal{R}, \rho} \longrightarrow \langle S_{\mathcal{P}}[\theta, c/\underline{x}], \text{rhs}_S \rangle \Sigma^{r \mathcal{P}}_{\mathcal{R}, \rho}},
\]

The first rule prepares a role for execution by inserting it in the active role set: it instantiates the existentials \( \exists v_i \) appearing in it by means of the substitution \( \xi = u/v \) for new constants \( u \), and associates it with the principal \( a \) that will be executing it. The second inference describes the application of a rule \( r = \text{lhs} \rightarrow \text{rhs} \): if an instance \( [\theta] \text{lhs} \) of its left-hand side appears in the state, it is replaced by the corresponding instantiation of the right-hand side after instantiating the existential variables \( x \) with new constants \( c \). The rule \( r \) is removed from the active role set, and \( c \) is added to the signature.

### 2.3 Traces and Scripts

In preparation to adding costs to this framework, it is useful to take a higher-level view of execution. This will also act as an abstract interface where other formalisms can experiment with the techniques in this paper.

An abstract execution step is a quadruple \( C \xrightarrow{\alpha, \iota} C' \), where \( C \) and \( C' \) are consecutive configurations, \( \alpha \) identifies the rule \( r \) from \( \mathcal{P} \) together with the role substitution \( \xi \), and \( \iota \) stands for the substitution we denoted earlier with \( (\theta, c/\underline{x}) \). An abstract execution step is just a compact yet precise way to denote rule application. It is reasonable to think about it as a partial function from \( C, \alpha \) and \( \iota \) to \( C' \). We say that \( \alpha \) is applicable in \( C \) if there is a substitution \( \iota \) and a configuration \( C' \) such that \( C \xrightarrow{\alpha, \iota} C' \) is defined. A trace \( T \) is then a sequence of applications

\[
C_0 \xrightarrow{\alpha_1, \iota_1} C_1 \xrightarrow{\alpha_2, \iota_2} \cdots \xrightarrow{\alpha_n, \iota_n} C_{n+1}
\]

While we rely on the notion of sequence here, this definition could be generalized to a lattice with minimum \( C_0 \) and maximum \( C_n \) to account for action independence. We will however stick to sequences for simplicity.

At this point, a protocol requirement for a safety property such as secrecy or authentication is simply given by a set \( \mathcal{S}_I \) of initial configurations and a set \( \mathcal{S}_A \) of attack configurations, or some finite abstraction of them. A verification procedure decides, for a given protocol, whether there exists a valid trace from an initial to an attack configuration.

A script is a parametric sequence of actions \( (r_1, \sigma_1), \ldots, (r_n, \sigma_n) \), where the codomain of the \( \sigma_i \)’s may mention variables. A script is realizable if there are configurations \( C_0, \ldots, C_{n+1} \), role substitutions \( \xi_1, \ldots, \xi_n \), and grounding substitutions \( \gamma_1, \ldots, \gamma_n \) such that \( \alpha_i = [\xi_i][r_i] \) and \( \iota_i = \sigma_i \circ \gamma_i \) (for \( i = 1..n \)) and \( C_0 \xrightarrow{\alpha_1 \iota_1} \cdots \xrightarrow{\alpha_n \iota_n} C_{n+1} \) is a trace. Scripts describe patterns of execution.
In general, there are two types of scripts of interest: the ones corresponding to the expected runs of the protocol (written $T_{ER}$), and the scripts that an intruder devises to mount an attack. For our purposes, the latter are more interesting, and we shall extend their syntax for flexibility. An attack script is then given by the following grammar:

$$A ::= \cdot \quad \text{(Empty script)}$$

$$| A (r, \sigma) \quad \text{(Extension with an action)}$$

$$| ! n A \quad \text{(Script iterated n times)}$$

$$| A + A \quad \text{(Alternative scripts)}$$

We are particularly interested in attack scripts that are realizable in an initial configuration and end in an attack configuration.

3 Fine-Grained MSR

It is possible to use the definitions in Section 4 to endow MSR with a notion of cost. This would however not be a very precise model, in particular as far as rule application failure is concerned. Therefore, we dedicate this section to defining a finer-grained version of MSR. We isolate the verification operations implicit in an MSR left-hand side as separate rules. During execution, we split rule application into two steps: pre-screening commits to a rule, while left-hand side verification decides if it should succeed or fail (typically when messages have been tempered with). For space reasons, we describe the compilation of an MSR specification into fine-grained MSR only intuitively.

3.1 Fine-Grained Specifications

Fine-grained MSR inherits its language of messages from MSR. However, it makes two changes to the set of available predicates. First, it extends the network predicate with a header $h$, giving it the template $N^h(m)$. The header is meant to identify precisely a message within a protocol instance: it will typically contain the postulated sender and intended recipient, the name and version number of the protocol and a step locator. An attacker can alter the header at will. The second change is the introduction of predicates $R_v(m)$, which will act as local registers during a verification step. Similarly to the local state predicates $L_v(\cdot)$, the dynamically created superscript $v$ is intended to prevent confusion.

An MSR rule application consists of two distinct phases: the left-hand side mandates a number of verification operations on incoming and retrieved messages, while the right-hand side prescribes how to construct out-going or archived messages. Both are represented succinctly in MSR, yet they can be very complex. Fine-Grained MSR replaces each MSR rule $r = (lhs \rightarrow rhs)$ with a number of verification rules, each corresponding to an individual verification step in $lhs$, and a single building rule, which produces $rhs$. Reducing $rhs$ to atomic steps is not necessary since construction cannot fail once verification has succeeded. Registers are used to serialize these rules in a collection that we call rule target.

We will now examine more closely the structure of verification and building rules, as specified by the grammar in Figure 1. Verification rule formats ($vr$) are given in
productions (1–8). The first four simply load the messages appearing on a network, local state, public, and private memory predicate into registers, respectively. Within a rule, register occurrences never share a superscript $v$, although this could be slightly optimized. Rules (5) verifies that the message stored in register $R_v'$ has $\text{op}$ as its top-level operation; necessary input values (e.g., keys for a decryption) are given in registers $R_v$. If the check succeeds, the sub-messages of $\text{op}$ are loaded into registers $R_v''$. If it fails, however, the clean-up rule $cr$ is executed (it will remove registers and local state predicates from the current configuration in preparation for the role’s dismissal). Rule (6) verifies the equality of two values. Rule (7) duplicates a value, while rule (8) builds a message out of stored values (this is useful for example to verify a hash). Note that only rules (5) and (6) can fail, and therefore only these have an associated clean-up clause.

The left-hand side of a building rule mentions the register values that are needed in the right-hand side, and the local memory predicates of the original MSR rule (we do not always want them to be removed from the state in case of verification failure, hence the format of rule (3)). $\text{rhs}^{(h)}$ corresponds exactly to an MSR right-hand side, with the exception that headers have been added to outgoing network messages (hence the superscript $h$).

3.2 Fine-Grained Execution

The execution semantics of Fine-Grained MSR keeps rule inst, but substitute $rw$ with the infrastructure outlined in this section.

In order to account for failure, we must split rule application into two stages. During the pre-screening phase, a rule is selected based uniquely on the predicate names (in-
cluding headers and superscripts) appearing in its left-hand side and in the current configuration. In particular, the arguments are not considered. We commit to the selected rule. Then, the verification phase checks whether the arguments have the expected form. In case of success, the next configuration is computed as in MSR. In case of failure, the clean-up clause is invoked and the entire role this rule belonged to is removed.

The pre-screening phase is formalized by the inference rules below. The else clause has been grayed-out in the success situation since not all rules mention it. On the other hand, only rules with an else clause can fail. Note that these rules have identical current configurations, and that \( m \) and \( m' \) can be different.

\[
\begin{align*}
S \triangleright r \triangleright^+ \langle S'' \rangle_{\Sigma'} & \quad \text{srw} \\
\mathcal{P} \triangleright (S', P(m)) & \quad \mathcal{P} \triangleright^R (P(m') \rightarrow \text{rhs else cr}, \rho)^A \quad \rightarrow \quad (S'', R, \rho^A)_{\Sigma', \Sigma'}
\end{align*}
\]

\[
\begin{align*}
S \triangleright r \triangleright^- S'' & \quad \text{frw} \\
\mathcal{P} \triangleright (S', P(m)) & \quad \mathcal{P} \triangleright^R (P(m') \rightarrow \text{rhs else cr}, \rho)^A \quad \rightarrow \quad (S'', R)_{\Sigma}
\end{align*}
\]

The judgments in the premises of these rules represent successful and failed verification. Their semantics is given by the next two rules. Rule \textit{succe} implement the traditional MSR semantics. Rule \textit{fail} is applicable only when there is a mismatch between the actual and the expected state. It then simply invokes a minimally instantiated form of the rewrite judgment on the clean-up clause.

\[
\begin{align*}
S, [\theta] \triangleright \text{lhs} & \quad \triangleright \text{lhs} \rightarrow \exists x.\text{rhs else cr} \quad \triangleright^+ \langle S, [\theta, c/x]\text{rhs} \rangle_c \quad \text{succe} \\
& \quad \exists \theta. [\theta] \triangleright \text{lhs} \subseteq S \quad \triangleright \langle S \rangle, \triangleright \langle S' \rangle_\cdot \quad \text{fail}
\end{align*}
\]

### 3.3 Intruder Model

The intruder capabilities traditionally considered for security protocol verification follow the well-known Dolev-Yao model [8, 17]: the intruder can intercept and generate network traffic, take apart and construct messages as long as it has all the elements to do so in a proper way (e.g., it should know the appropriate key in order to perform a cryptographic operation). This model disallows guessing unknown values and performing operations that are considered “hard” (e.g., recovering a key from a ciphertext). Let \( I \) be a memory predicate belonging to the intruder, so that \( I(m) \) indicates that it knows (or has intercepted) the message \( m \) (this most simplistic setting can be considerably refined). Then, the Dolev-Yao model can be expressed by the following rules:

\[
\begin{align*}
N^h(x) & \rightarrow I(x) & I(x) & \rightarrow N^h(x) \\
M_s(x) & \rightarrow I(x), M_s(x) & I(x) & \rightarrow \exists x. I(x) \\
I(y), I(\text{op}_y(x)) & \rightarrow I(x) & I(x) & \rightarrow I(\text{op}_x) \\
I(x) & \rightarrow I(x), I(x) & I(x) & \rightarrow .
\end{align*}
\]
The first line corresponds to network interception and injection. The second is access to public information and data generation (when allowed). The third abstractly expresses dismantling and constructing messages (of course, some combinations are disallowed). The fourth line contains administrative rules. Note that, unsurprisingly, these capabilities correspond very closely to the rules of the Fine-Grained MSR given in Section 3.1. The correspondence would be even more exact if we had reduced the right-hand side to atomic constructions.

The Dolev-Yao model allows the intruder to perform “easy” operations. Once we explicitly assign cost to actions, we can introduce and reason about intermediate degrees between “easy” and “impossible”, which is really what the Dolev-Yao restrictions boil down to. Indeed, we will allow attacks that involve performing “hard” operations, guessing values, and subverting principals. We will also be able to quantify “easy”, “hard” and levels in between.

The subversion of a principal is easily modeled by another intruder memory predicate, $X(A)$. The first row of rules below represent subversion and rehabilitation of a principal $A$. The others stand for access to $A$’s private data and for the intruder covering its traces.

$$
\begin{array}{c}
\rightarrow X(A) \\
X(A), M_A(x) \rightarrow X(A), I(x) \\
X(A), I(x) \rightarrow X(A), M_A(x)
\end{array}
$$

We model “hard” operations by simply extending the set of patterns allowed in rule template $(I(y), I(op_y(x)) \rightarrow I(x))$ to represent non Dolev-Yao inferences. For example, taking a discrete logarithm is expressed as:

$$I(g), I(g^x) \rightarrow I(x).$$

Clearly there are limitations to this method as it applies only to the inversion of bijections. Other “hard” operations, such as finding hash collisions, can be modeled as guessing problems.

The trivial guessing rule $(\cdot \rightarrow I(x))$ is unrealistic and hard to work with from a cost accounting point of view. Therefore, following the pioneering work of Lowe [13], we require that every guess be backed up by a verification procedure. We express both the guess and its verification as an MSR role of the following form:

$$\exists u, v_1, v_2. \left[ \begin{array}{c}
\ldots \rightarrow G^u(x), \ldots \\
\ldots \rightarrow \ldots, V_1^{v_1}(m_1) \\
\ldots \rightarrow \ldots, V_2^{v_2}(m_2) \\
V_1^{v_1}(y), V_2^{v_2}(y), G^u(x) \rightarrow I(x)
\end{array} \right] \right\} \text{Guess}
\left\{ \begin{array}{c}
\text{Verification}
\end{array} \right\}
$$

On the right, $G$, $V_1$ and $V_2$ are local state predicates (generically called $L$ in Section 2) that hold the guess and two constructions (the verifiers) that should produce the same value if the guess is correct. See [13] for conditions required of acceptable verifiers. The exact format can vary, as illustrated below. Guessing roles are protocol specific, in general. They can be translated into Fine-Grained MSR using the technique outlined above.
As a concrete example, the following role expresses the guess of a Diffie-Hellman exponent:
\[
\exists u, v. \left[ I(g^x) \rightarrow G^u(x'), V^v(g^x, g^{x'}) \right]
\]
Note that, although this role is functionally equivalent to the discrete logarithm specification above, the exponent is explicitly guessed here rather than reverse-engineered as above.

Our final example describes the guess of the shared key \( k \) in the toy protocol informally described to the right of this text.
\[
\exists u, v. \left[ \cdot \rightarrow \exists n. G^u(k), N^h(\{n\}_k), V^v(n) \right]
\]
Here, the intruder generates a nonce \( n \), makes a guess for \( k \) and sends the expected message to \( B \). This copy of \( n \) is the first verifier and is memorized in the predicate \( V \). The second verifier is simply the response from \( B \); if the guess was correct, they will be equal, otherwise it will either come back as a different bit-string, or be dropped by \( B \) altogether if the forgery attempt is uncovered.

4 Cost Model

Traditional approaches to protocol analysis are only interested on whether an action is applicable in a given state. Actions that are not applicable, either because they cannot succeed or because “computationally infeasible”, are unobservable. In this paper, we are concerned with the cost of successful and failed applications. Cost will be measured in terms of whatever resource of interest changes as a result of attempting the action. Primary focuses are time and storage, but other parameters, such as energy, or the lowered randomness of some quantity (that may be used for side-channel attacks, for example) can also be used.

4.1 An Algebra of Cost

We will now define a generic infrastructure for expressing cost, which we will apply to individual actions as well as entire traces in the next sections. The details of the resulting algebra shall be application specific.

We want to associate a value to each type of cost incurred by a principal. A type, denoted with \( \tau \), describes a resource of interest, time, space and energy are typical, but more refined types, e.g., verification vs. construction time, can also be expressed. A cost base relates a cost type \( \tau \) to a principal \( a \). We write it as \( \tau^a \).

How much of a given cost type is incurred by a principal takes the form of a scalar value, which we will denote with \( s \). Scalars can be abstract quantities (e.g., Meadow’s “cheap”, “medium”, “expensive”, etc. [15], or just 0 and \( \infty \) in a Dolev-Yao setting), numbers (in \( \mathbb{N} \) or \( \mathbb{R} \) for example), or even complexity bounds in \( O \)-notation. It is useful that some form of addition (written \( + \)) and a unit (0) be defined on scalars. These could
be just free symbols, but + can also be an actual operation. It is also very useful to have a comparison relation (written <, with the usual variants) among scalars within a cost base. Note that some forms of cost never decrease and + should be monotonic with respect to ≤ for them. Time or energy are examples. This is the only case considered in [15]. Other costs, in particular space, do not need to be monotonic, and this restriction does not apply.

A cost item is a cost base τ^a together with a scalar value s. We denote it as sτ^a. We extend the scalar comparison operators to cost items only when the base is the same. Such an extension rarely makes sense if the cost type is different, and should be evaluated on a case by case basis when the principals are not the same: one byte is one byte for everybody, but performing a decryption will generally take different amounts of time when hardware or implementation varies.

At this point, a cost vector C is simply a collection of cost items s_1τ^a_1, . . . , s_nτ^a_n, which we write \sum_i s_iτ^a_i. Given a cost vector C, we write C^n, C^τ, and C^a for its projections relative to principal a, cost type τ, and their combination, respectively. For example, C^n = \sum_{sτ^a ∈ C} sτ^a. It should be noted that a cost vector can be seen as a generalization of the notion of multiset.

4.2 Cost Assignment for Protocol Operations

In spite of their apparent simplicity, cryptographic protocols comprise a large number of operations and action classes. We will now examine them and comment on their characteristics in term of cost. Most of the issues are discussed relative to Fine-Grained MSR, and transpire also at the level of MSR. Needless to say, similar considerations apply to other specification languages.

Network: The network operations observable in MSR are receiving and sending a message. We denote their associated cost as κ_{N⇒} and κ_{⇒N}, respectively. This generally includes time and storage components. Accounting for other transmission costs such as network latency could be easily accommodated through a simple refinement of (Fine-Grained) MSR.

Storage: Each of public (M_p), private (M_a) and local (L) storage has a temporal and a spatial component. Storage operations include allocating and recording data (e.g., κ_{⇒M_p}), disposal (κ_{M_a⊥}) and look-up (κ_{M_a}). Notice in particular that the spatial component of storage disposal is negative. Note also that some values may be easier to look-up than others, and so κ_{M_a} depends on the actual predicate M.

Registers: We do not associate any cost with register management, preferring to fold it into the operations they participate in.

Constructor operations: Each constructor op has a number of operations associated with it. We consider its use as a building block of a message (κ_{⇒op}) and during verification. In the latter case, we distinguish between the cost of success (κ_{op√}) and failure (κ_{op⊥}). The cost of performing Dolev-Yao and non Dolev-Yao operations is computed in the same way in our model. What will change is likely to be the magnitude of the scalar values.

Data operations: Atomic values are subject to generation (using Ξ in MSR), and generic values can be tested for equality. We write κ_{Ξ} and κ_{Ξ=} respectively, where ξ represents some notion of type of a value.
Subversion: We write \( \kappa_a \) for the cost of subverting principal \( a \) and \( \kappa_a \) for the cost of its rehabilitation.

Guessing: The cost of a guessing attack can be modeled in two ways. At a high level of abstraction, we can associate a cost to a verification procedure \( \rho_G \) as a whole, which accounts for the cost of the expected number of guesses and verifications until one is successful. We write \( \kappa_{\rho_G} \) for this omnibus, MSR-oriented, cost. Alternatively and at a much lower-level level of detail, we can compile the verification procedure to Fine-Grained MSR, obtaining a role \( \bar{\rho} \), assign a cost to the individual guess itself \( (\kappa_G) \), compute the cost of each guess and verification, \( C(\bar{\rho}) \), as outlined below, and estimate the number of attempts it may take until a successful guess is produced. In general, this type of accounting will have the form \( f(n)C(\bar{\rho}) \), where \( f \) is a function and \( n \) is a parameter such as the length of the data to be guessed.

Each of these operations, with sometimes the exception of guessing, are executed by a single principal, say \( a \) (which may also be the intruder). Each will in general involve several cost components. Therefore, \( \kappa_a \) corresponds to a cost vector relative to \( a \). Guess verification can be performed locally by the intruder, or require exchange of messages with one or more principals. In the latter case, the cost vector will have appropriate components for each of the involved parties.

In general, the accuracy of a cost-based analysis directly depends on the precision of the cost associated with each basic action. For example, a classification into “cheap” and “expensive” forms the basis for a Dolev-Yao investigation, while adding an intermediate “medium” value already provides a setting in which one can start analyzing denial-of-service situations [15]. Moving to numerical classes adds flexibility, but non-trivial problems quickly emerge as accurate physical measurements can be difficult to gather and work with when dependent on hardware, implementation and system load. In this paper, we provide a flexible framework for taking cost into consideration, but have little to say at this stage about how to best determine the granularity and magnitude of basic costs.

4.3 Cost Calculation in MSR

The notion of cost naturally extends from individual operations to traces. First, we define the cost of a Fine-Grained MSR rule by simply adding up the cost of each operation occurring in it. There is little to do in the case of the verification rules (except for (8)), while building rules involve some work. Rule templates (5) and (6) have the option of failing, and therefore both a success and a failure cost is associated with them: we shall consider them as if they were different operations.

Consider now a trace \( T = C_0^{\alpha_1 t_1} \cdots \alpha_n t_n C_{n+1} \). Let \( a_j \) be the principal executing \( (\alpha_j, t_j) \), and \( C^{a_j}(a_j) = \sum_i s_{ij}^a \tau_{ij}^a \) its cost. The cost of the trace is then given by

\[
C(T) = \sum_j C^{a_j}(a_j) = \sum_j \sum_i s_{ij}^a \tau_{ij}^a
\]

The cost calculation for a trace extends naturally to the cost of a script since substitutions do not play any role when computing a cost. The presence of alternatives in an
attack script forces us to define cost for them over (multi-dimensional) intervals rather than points. We have the following definition:

\[
C(\cdot) = I_0 \\
C(A(r, \sigma)) = C(A) + C^{\sigma}(r) \\
C(\lambda A) = \lambda C(A) \\
C(A_1 + A_2) = [\min\{A_1, A_2\}, \max\{A_1, A_2\}]
\]

Here, \(I_0\) is some fixed interval, typically \([0, 0]\). We extend scalar product and addition to intervals by applying these operations to its endpoint, \(i.e., \nu[a, b] = [\nu a, \nu b]\) and \([a, b] + [c, d] = [a + c, b + d]\).

Since most tools for security protocol analysis rely, often symbolically, on traces, the infrastructure we just outlined is compatible with their underlying methodology. Indeed, systems based on explicit model checking can immediately take costs into account, while symbolic approaches need to have the cost model indirectly encoded as part of the problem description. Similar considerations apply to analysis based on theorem proving. In general, how easy it is to extend a tool with cost computation capabilities depends on how deeply the intruder model is ingrained in their implementation. The required modifications include tracking cost and allowing for non Dolev-Yao intruder actions.

Note that any tool natively supporting cost calculation (or even retrofitted to do so) can still perform traditional verification by assigning cost \(\infty\) to non Dolev-Yao intruder actions and abandoning any attack trace as soon as its cost reaches \(\infty\).

5 Quantitative Security Analysis

A first-class notion of cost leads to protocol analysis opportunities that lay far beyond the traditional Dolev-Yao feasibility studies. In this section, we will examine some of the possibilities related to time and space, well aware that many more lay out there, waiting for the imaginative mind to grab them. We elaborate on two non Dolev-Yao forms of verification: threshold analysis tries to determine what attacks are possible given a bound on the resources available to the intruder alone; comparative analysis studies attack opportunities when the resource bounds of all involved parties are taken into consideration. Denial-of-service attacks are a prime example.

5.1 Threshold Analysis

A rather trivial use of cost is to first ascertain that a protocol is secure relative to the Dolev-Yao model, and then compute the amount of resources it requires. This may be useful already in situations characterized by limited capacities, such as protocols implemented on smart-cards. If \(\kappa_{HW}\) is an inventory of the available resources, this problem is abstractly stated as “\(C(T_{ER}) \leq \kappa_{HW}\)”. 

Dually, an intruder can pre-compute the cost of mounting an attack on a discovered vulnerability. This is generally not very interesting in a Dolev-Yao setting where an attack uses the same kind of operations as the protocol itself, and the intruder is implicitly
assumed to have access to resources similar to honest principals. This becomes crucial when the intruder experiments with “computationally infeasible” operations, principal subversion, guessing, or a combination of these non Dolev-Yao operations. Indeed, some protocol analysis tools already allow principals to “lose keys” [14], but do not assign any special status to this operation. The intruder can then calculate the cost of a candidate attack and compare it with its available resources (dictionary attacks on passwords are the simplest instance), in symbols “\( \mathcal{C}(\mathcal{A}) \leq \kappa_1 \)”. A protocol verification tool can similarly discard attack traces as soon as their cost exceeds a predetermined amount of intruder resources.

A protocol designer can go one step further by keeping aspects of the cost calculation as parameters. He can then determine value ranges that would require extravagant amounts of resources from an intruder in order to implement the attack (given foreseeable technology): “\( \min x. \mathcal{C}(\mathcal{A}(x)) \gg \kappa_I \)”. This is how key lengths and other parameters of cryptographic algorithms have traditionally been set. The approach we are promoting extends this form of safe parameter determination in that it takes into account the whole protocol rather than an isolated cryptographic primitives. This is particularly valuable as modern ciphers offer the option of variable key lengths.

5.2 Comparative Analysis

A cost infrastructure can be useful to a designer to choose a protocol among two candidates based on resource usage “\( \mathcal{C}(T^1) > \mathcal{C}(T^2) \)”, or on their resilience to a certain type of attacks: “\( \mathcal{C}(\mathcal{A}_1) > \mathcal{C}(\mathcal{A}_2) \)”. By the same token, an attacker or law enforcement agency can evaluate attack strategies based on their cost.

Denial-of-service (DoS) attacks operate by having a possibly distributed intruder waste a server’s resources with fake requests to the point where legitimate uses cannot be serviced in any useful time frame (or the server crashes). It stresses the bounds on the server’s resources, typically time (or service rate) and storage capacity. A precise cost analysis, like the one proposed here, helps compute actual values for the resources used by both the intruder and the server at different stages of the protocol execution. The statement here is “\( \mathcal{C}^B(\mathcal{A}) > \mathcal{C}^I(\mathcal{A}) \)”. Given assumptions about performance and buffer sizes, it can help determine how many requests can be handled concurrently and in particular by how many compromised hosts. The same calculation can be used to determine the amount of resources needed to withstand a given target level of attack.

Consider the abstract protocol below (left), where a client \( C \) initially contacts the server \( S \) with some message \( m_1 \), is given a challenge \( m_2 \), and receives the requested service \( m_4 \) only after it has provided an adequate response \( m_3 \) to \( m_2 \):

\[
\begin{align*}
C &\rightarrow S : m_1 \\
S &\rightarrow C : m_2 \\
C &\rightarrow S : m_3 \\
S &\rightarrow C : m_4
\end{align*}
\]

The exchange on the right shows the time (\( t_a \)) and space (\( s_a \)) cost incurred by each principal. Let us measure time in seconds and space in bytes. We wrote \( t_{bi}^a \) for the time
An attacker can induce the server to waste time unsuccessfully verifying a fake message \( m_1 \). This time is at most \( t_{v1} \). The server’s verification rate is therefore at least \( 1/t_{v1} \), which must be matched by the intruder in order to successfully attack the server. While this is easily achieved as a fake \( m_1 \) can be an arbitrary string, \( t_{v1} \) is likely to be dominated by networking overhead in a protocol designed with DoS attacks in mind. In any case, our approach allows evaluating this threat.

- A time-out waiting for the reception of \( m_3 \) leads to another potential point of DoS. In this case, the server has spent \( t_{v1} + t_{v2} \) while the attacker has incurred a cost \( t_{b1} \). Again, this gives us a way to compare the attacker’s and the server’s rate.
- Another option for time DoS is the reception of a fake message \( m_3 \) by \( S \). Here \( S \) needs to spend \( t_{v1} + t_{v2} + t_{v3} \) seconds, while the attacker’s cost amounts to \( t_{b1} \) plus the minimal time it takes to produce the counterfeit \( m_3 \) (the intruder is likely to ignore \( m_2 \)). This strategy wastes more server time, but it will release storage earlier unless carefully timed. Moreover, the reception of a large number of garbled messages may trigger countermeasures on the server.

In all these situations, the resilience of the server is given by comparing the service rate as measured above, with the individual attack rate multiplied by the number of attackers. Our methodology can give useful ranges as it takes into account the exact structure of the messages involved, including that of the messages faked by the intruder.

- A time-out on \( m_3 \) is also the target of a space DoS. Let \( B \) be the size in bytes of the buffer where \( S \) stores received bits of \( m_1 \) and generated fragments of \( m_2 \). Then, \( S \) can serve at most \( n(B) = B/(s_1^S + s_2^S) \) concurrent requests: the larger \( B \), the larger the number of parallel attacks the system can withstand. The space allocation rate is given by \( (s_1^S + s_2^S)/(t_{v1} + t_{v2}) \) bytes per second relative to an individual attacker, while the space reclamation rate is at least \( (s_1^S + s_2^S)/(T + t_{v3}) \).

Now, given \( B \), we can calculate optimal values for the time-out \( T \). First, \( T \) should be large enough for all legitimate usage pattern to complete: \( T > t_{\text{min}} \). On the other hand, it should not be so large that an attacker coalition may file more than \( n(B) - 1 \) fake service requests while waiting for time-out on any initial exchange: \( T \leq (t_{v1}^S + t_{v2}^S) \times (n(B) - 1) \). We are looking for the maximum value of \( T \) satisfying these bounds. Concretely, if \( s_1^S + s_2^S = 128 \) bytes, \( t_{v1}^S + t_{v2}^S = 100 \) milliseconds, \( t_{\text{min}} = 90 \) seconds, and the maximum number of expected parallel attacks is 10, 000, we deduce that \( B \) should be at least 1.28 Mb, and that \( T \) can be at most 16 minutes and a few seconds. If this value is too low, then \( B \) should be increased (which would make the system resilient to more concurrent attacks).
5.3 Further Analysis Opportunities

The framework outlined above may allow forms of analysis that have received little or no attention in the cryptographic protocol literature. The reader shall be warned that the following remarks are mere conjectures at this point in time, with no rigorous investigation to support them.

Within our quantitative framework, several classes of interesting queries can be stated as optimization problems. We have already done so when asking for the smallest value of a security parameter $x$ that would require an intruder to possess unrealistic resources to mount a certain attack: $\min x. C(A(x)) \gg \kappa_I$. The space DoS analysis scenario above can also be described as an optimization problem whose objective function may involve finding optimal values for the buffer size $B$ and the time-out $T$ that allow withstanding $n$ parallel attacks for a “safe” $n$. It is our conjecture that both these problems can be solved using conventional operations research techniques, at least in instances involving numerical costs.

Going one step further, an attacker’s objective may be seen as finding a minimally expensive attack script that achieves a given disruption objective. If this objective has a quantitative component, it may additionally ask for maximal disruption. Differently from the scenarios examined in the last paragraph, the attack is now part of the solution rather than the problem statement. It is unclear to us whether existing techniques in operations research can efficiently tackle problems of this sort.

The quantitative analysis of systems that can detect that they are under attack and take appropriate countermeasures immediately assumes a game-theoretic flavor, and may be analyzed with the tools of that trade. In its simplest instance, the attacker’s quest for greatest disruption at the lowest cost can be viewed as a classical mini-max problem, and, if the search-space is manageable, investigated accordingly.

It was recently pointed to us that the framework proposed in this paper may be a starting point in the study of the economics of network security in the presence of a rational attacker. While this would nicely complement the computational complexity considerations examined so far, we are uncertain as to how to go about this investigation.

Moving to a different flavor of security, another reader observed that our framework may also be used to model information flow problems and solutions such as non-interference: safe operations would have a null cost while information leakage from “hi” to “lo” would have cost $\infty$. It is conceivable that finer notions that consider the bandwidth of a covert channel may take advantage of intermediate values. Such an investigation is interesting also in light of recently unveiled relations between security protocol verification and non-interference [9].

6 Conclusions

We have outlined a methodology for assigning precise measures of cost to protocol actions and computing it over traces, in particular attack traces. This allows cost-conscious tools to extend the operations available to an intruder beyond the Dolev-Yao model. We considered here guessing [13], principal subversion, and computationally hard operations such as discrete logarithm. In this, our work extends Meadow’s approach to denial
of service analysis [15]. A security analyst can then determine the resilience of a protocol to such non-conventional attacks by comparing the cost involved in mounting the attack and an estimate of the resources available to the attacker. This may also enable her to fine-tune protocol parameters to maximize the intruder’s effort. We used a low-level variant of the security protocol specification language MSR as a vehicle to illustrate this methodology, but we believe it can be adapted to a large class of formalisms. Efforts in this direction are under way relative to the spi-calculus [18].

We are applying our approach on preliminary experiments with WEP, partially reproducing the analysis of [1], and with the station-to-station protocol, piggy-backing on Meadows work [15]. We will report on them in future publications. We intend to pursue experimentation on resource-conscious protocols, as found in smart-cards or ad-hoc wireless networks, and in protocols designed with denial-of-service in mind, such as JFK. On the theoretical side, a number of interesting directions of future research have been listed in Section 5.3. Additionally, costs expressed as complexity bounds deserve further investigation: while our methodology can already derive basic results, the study of complexity classes closed under common operations, such as the polynomials, may bear particularly interesting fruits. Recent exciting work on measurement functions in domain theory may be relevant to our investigation. Another intriguing direction concerns mixed probabilistic approaches, which would allow, for example, to reason about the level of effort required by an attacker to have a 90% chance of correctly guessing a key.

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