Bond Market Exposures to Macroeconomic and Monetary Policy Risks

Dongho Song

Boston College

July 24, 2015

SEM Conference 2015
Establishing Key Stylized Facts: $\text{corr}(\text{stock}, \text{bond})$

- return on the five-year Treasury bond.
- stock market return (CRSP value-weighted portfolio of stocks).

See Baele, Bekaert, and Inghelbrecht (2010); Campbell, Pflueger, and Viceira (2013); Campbell, Sunderam, and Viceira (2013); David and Veronesi (2013); Burkhardt and Hasseltoft (2012)
Establishing Key Stylized Facts: $\text{corr}(\Delta c, \pi)$

- Use monthly $\Delta c$ and $\pi = \text{cpi}$.
Establishing Key Stylized Facts: 1970s - 1990s

- $\text{corr}(\Delta c, \pi)$
- $\text{corr}(\text{stock}, \text{bond})$
- Yield curve slope

<table>
<thead>
<tr>
<th>Data, Model</th>
<th>$\text{corr}(\Delta c, \pi)$</th>
<th>$\text{corr}(\text{stock}, \text{bond})$</th>
<th>Yield curve slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

- See Wachter (2006), Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013)
Establishing Key Stylized Facts: 1960s and 2000s

<table>
<thead>
<tr>
<th></th>
<th>corr($\Delta c, \pi$)</th>
<th>corr(stock,bond)</th>
<th>yield curve slope</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td><strong>Model 1</strong></td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td><strong>Model 2</strong></td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Why is it important?

• The evidence poses a significant challenge to existing approaches (e.g., Wachter (2006), Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013))

  • $\pi$ is perceived as a carrier of bad good news for future $\Delta c$
  • yet, $\pi$ risks seem to be there: upward sloping yield curve
Why is it important?

- The evidence poses a significant challenge to existing approaches (e.g., Wachter (2006), Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013))
  
  - $\pi$ is perceived as a carrier of *bad* good news for future $\Delta c$

  - yet, $\pi$ risks seem to be there

- Need a new model to study the main drivers of $\pi$, bond, stock risks

  - $E(\pi), \sigma(\pi), \rho(\pi)$ dropped significantly

  - the risks properties of stock and bond changed significantly

  - flattening of the yield curve

  - less violation of the Expectations Hypothesis

  - sign change in the stock-bond correlation

  - the slope of the yield curve remains positive
Objective

• Build a regime-switching general equilibrium model that
  • characterizes the monetary and macroeconomic determinants of inflation, asset prices, and bond-stock return correlation
  • reconciles both old and new stylized facts

• Key model features
  • an endowment economy with long-run risks in $\Delta c$, EZ preferences
  • endogenous $\pi$ dynamics implied by the Taylor rule
  • exogenous regime-switching as potential source of risk variations
  1 in the Taylor rule coefficients
  2 in the covariance of $\Delta c$ with $\pi$
Intuition: Inflation Risks in Simplest Form

• Setting “Taylor Rule” = “Fisher” equation yields

\[ \phi \pi_t = r_t + \mathbb{E}_t[\pi_{t+1}] \]

\[ r_t \text{ is the real rate from Euler equation} \]

\[ \pi_t = \begin{cases} 
  f(\phi, \ldots) r_t, & \text{if } |\phi| > 1 \\
  \text{nonstationary,} & \text{if } |\phi| \leq 1
\end{cases} \]

• Inflation risks depend on the joint determination of

monetary policy aggressiveness: \( \phi \in \{|\phi| > 1, |\phi| \leq 1\} \)

macroeconomic shocks: \( \text{corr}(\pi_t, r_t) \simeq \text{corr}(\pi_t, \Delta c_t) \in \{+, -\} \)
Risk Characterization

| Active    | $|\phi| > 1$ | Countercyclical | $\text{corr}(\pi_t, \Delta c_t) < 0$ |
|-----------|-----------|-----------------|-----------------|
| Passive   | $|\phi| \leq 1$ | Procyclical     | $\text{corr}(\pi_t, \Delta c_t) > 0$ |

- Key risks in the economy can be characterized by

- Agents in the model know the probability of moving across regimes

Regime uncertainties reflected in today’s bond market prices
Findings

- Estimate the model using asset prices and macroeconomic data
- Quantify the risks: the darker the riskier

Monetary Policy
- Active
  - Countercyclical: CA
  - Procyclical: PA
- Passive
  - Countercyclical: CP
  - Procyclical: PP

- Upward sloping yield curve in all regimes
- Negative stock-bond correlation in procyclical inflation regime
Findings

- Account for several other features of the bond market
  - time varying risk premiums
  - bond return predictability
  - violation of the expectations hypothesis
- Provide the timeline of the regimes (the **darker** the **riskier**)


**Countercyclical Inf.**
**Procyclical Inf.**
**Active MP**
**Passive MP**

Cuban Missile  Oil Shock 1  Oil Shock 2  Chair Volcker  Gulf War 1  Asian Crisis  9/11  Gulf War 2  Credit Crunch
Model Details

- **Real consumption dynamics**
  \[
  \Delta c_t = \mu_c + x_{c,t} + \eta_{c,t} \\
  x_{c,t} = \rho_c x_{c,t-1} + e_{c,t} + \beta(S_t) e_{\pi,t}, \quad e_{j,t} \sim N(0, \sigma_{j,t-1}^2) \\
  \sigma_{j,t}^2 \sim \text{independent AR(1)}
  \]

- **Taylor Rule**
  \[
  i_t = \mu_i(S_t) + \phi_c(S_t)(\Delta c_t - \mu_c) + \phi_\pi(S_t)(\pi_t - \mu_\pi - x_{\pi,t}) + x_{\pi,t} + x_{m,t}
  \]
  \[
  x_{\pi,t}, x_{m,t} \sim \text{AR(1)}
  \]

- **Stochastic discount factor from EZ preferences, \( \Delta c_t \)**
  \[
  m_t = m(\Theta, x_{j,t}, \sigma_{j,t}^2, \beta(S_t), \ldots)
  \]

- **Inflation from Taylor rule**
  \[
  \pi_t = \pi(\Theta, \phi_c(S_t), \phi_\pi(S_t), \beta(S_t), x_{j,t}, \sigma_{j,t}^2, \mathbb{P}, \ldots)
  \]
Intuition: Term Structure

- Nominal bond yields (ignoring monetary policy shock)

\[
y_{n,t} = \frac{1}{n} \left( B_{n,0} + B_{n,1,c} x_{c,t} + B_{n,1,\pi} x_{\pi,t} + B_{n,2,c} \sigma_{c,t}^2 + B_{n,2,\pi} \sigma_{\pi,t}^2 \right)
\]

<table>
<thead>
<tr>
<th>bond price</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>(x_{c,t}) ↓</td>
</tr>
<tr>
<td>↓</td>
<td></td>
</tr>
</tbody>
</table>

- If nominal vol dominates, then risk increases with maturity
  → upward sloping yield curve

- Their relative magnitudes determined by
  1. stochastic volatilities
  2. monetary policy regimes
  3. macroeconomic shock regimes
**Intuition: Term Structure**

- Nominal bond yields (ignoring monetary policy shock)

\[
y_{n,t}^\$ = \frac{1}{n} \left( B_{n,0}^\$ + B_{n,1,c}^\$ x_{c,t} + B_{n,1,\pi}^\$ x_{\pi,t} + B_{n,2,c}^\$ \sigma_{c,t}^2 + B_{n,2,\pi}^\$ \sigma_{\pi,t}^2 \right)
\]

<table>
<thead>
<tr>
<th>bond price</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>( x_{c,t} )</td>
</tr>
<tr>
<td>↓</td>
<td>( \sigma_{c,t}^2 )</td>
</tr>
</tbody>
</table>

- If real vol dominates, then long-term yields fall in bad times
  \( \rightarrow \) downward sloping yield curve

- Their relative magnitudes determined by
  1. stochastic volatilities
  2. monetary policy regimes
  3. macroeconomic shock regimes
Intuition: Stock-Bond Correlation

- Log price-dividend ratio

\[
pd_t = D_0 + D_{1,c} \chi_{c,t} + D_{1,\pi} \chi_{\pi,t} + D_{2,c} \sigma_{c,t}^2 + D_{2,\pi} \sigma_{\pi,t}^2
\]

- Correlation between stock and bond returns: \( \text{corr}_t[r_{t+1}^m, r_{n,t+1}^s] \)

\[
= \ldots - B_{n,1,c}^s D_{1,c} \sigma_{c,t}^2 - B_{n,1,\pi}^s D_{1,\pi} \sigma_{\pi,t}^2
\]

<table>
<thead>
<tr>
<th>price</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>stock</td>
<td>( \downarrow \chi_{c,t} \downarrow \chi_{\pi,t} \uparrow \sigma_{c,t}^2 \uparrow \sigma_{\pi,t}^2 )</td>
</tr>
<tr>
<td>bond</td>
<td>( \downarrow )</td>
</tr>
</tbody>
</table>

stock-bond correlation is (+) if nominal vol dominates
Intuition: Stock-Bond Correlation

• Log price-dividend ratio

$$pd_t = D_0 + D_{1,c} x_{c,t} + D_{1,\pi} x_{\pi,t} + D_{2,c} \sigma^2_{c,t} + D_{2,\pi} \sigma^2_{\pi,t}$$

mildly–/+ 

• Correlation between stock and bond returns: $$corr_t[r_{m,t+1}, r_{n,t+1}]$$

$$= \ldots - B^S_{n,1,c} D_{1,c} \sigma^2_{c,t} - B^S_{n,1,\pi} D_{1,\pi} \sigma^2_{\pi,t}$$

mildly–/+ 

<table>
<thead>
<tr>
<th>price</th>
<th>state</th>
</tr>
</thead>
<tbody>
<tr>
<td>stock ↓</td>
<td>$x_{c,t}$ ↓</td>
</tr>
<tr>
<td>bond ↑</td>
<td></td>
</tr>
</tbody>
</table>

stock-bond correlation is (–) if real vol dominates
Identification and Estimation

- Apply nonlinear Bayesian method
  - Particle filter + MCMC
- Estimation sample: 1959-2011
  - Quarterly SPF, monthly macroeconomic data
  - Monthly stock price, Treasury bond yields data
- Use mixed-frequency data to pin down the key state variables and identify macroeconomic/monetary policy regime shifts
  - Regime-switching coefficients: $\phi_c(S_t), \phi_\pi(S_t), \beta(S_t)$
  - Stochastic volatilities: $\sigma^2_{c,t}, \sigma^2_{\pi,t}$
  - Latent long-run growth, inflation target, monetary policy shock: $x_{c,t}, x_{\pi,t}, x_{m,t}$
Estimated Regime Transition Probabilities

<table>
<thead>
<tr>
<th>Model</th>
<th>Continuation</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countercyclical Inflation</td>
<td>99.2 %</td>
<td>7.5 years</td>
</tr>
<tr>
<td>Procyclical Inflation</td>
<td>94.1 %</td>
<td>1 years</td>
</tr>
<tr>
<td>Active Monetary Policy</td>
<td>99.0 %</td>
<td>6 years</td>
</tr>
<tr>
<td>Passive Monetary Policy</td>
<td>97.5 %</td>
<td>2.5 years</td>
</tr>
</tbody>
</table>

- Most of the time, $\pi$ shock is large and countercyclical
  - inflation risk premium is positive
  - stock and bond return correlation is positive

- The long-run Taylor principle holds (by construction)
  - the regime in which passive policy is realized is short-lived
  - ensures stationary inflation dynamics
## Model-Implied Correlation between $\Delta c$ and $\pi$

<table>
<thead>
<tr>
<th>Regime</th>
<th>Data $\text{corr}(\Delta c_t, \pi_t)$</th>
<th>Model $\text{corr}(\Delta c_t, \pi_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA : Countercyclical Inf, Active MP</td>
<td>-0.49  -0.51</td>
<td>[-0.75, -0.28]</td>
</tr>
<tr>
<td>CP : Countercyclical Inf, Passive MP</td>
<td>-0.20  -0.10</td>
<td>[-0.49, 0.35]</td>
</tr>
<tr>
<td>PA : Procyclical Inf, Active MP</td>
<td>0.12  0.02</td>
<td>[-0.09, 0.11]</td>
</tr>
<tr>
<td>PP : Procyclical Inf, Passive MP</td>
<td>0.14  0.02</td>
<td>[-0.12, 0.16]</td>
</tr>
</tbody>
</table>

- When inflation shock is procyclical $\pi$, $\Delta c$ positively comove
- Monetary policy alone cannot change the sign
- Passive monetary policy creates more dispersion
Model-Implied Yield Spread $y_n - y_{3m}, \ n \in \{1y \sim 5y, 10y\}$

$\text{corr}(\pi, \Delta c) < 0$, Active MP

$\text{corr}(\pi, \Delta c) < 0$, Passive MP

$\text{corr}(\pi, \Delta c) > 0$, Active MP

$\text{corr}(\pi, \Delta c) > 0$, Passive MP

- regime-switching NOT allowed
- counter-/procyclicality amplified in passive monetary policy regime
Model-Implied Yield Spread \( y_n - y_{3m}, \; n \in \{1y \sim 5y, 10y\} \)

\[
corr(\pi, \Delta c) < 0, \; \text{Active MP}
\]

\[
corr(\pi, \Delta c) > 0, \; \text{Active MP}
\]

\[
corr(\pi, \Delta c) < 0, \; \text{Passive MP}
\]

\[
corr(\pi, \Delta c) > 0, \; \text{Passive MP}
\]

- upward sloping yield curve due to regime uncertainties
- shifts in monetary policy affect the variance of the yield curve
Model-Implied Stock-Bond Return Correlation

\[ \text{corr}(\pi, \Delta c) < 0, \text{ Active MP} \]

\[ \text{corr}(\pi, \Delta c) > 0, \text{ Active MP} \]

\[ \text{corr}(\pi, \Delta c) < 0, \text{ Passive MP} \]

\[ \text{corr}(\pi, \Delta c) > 0, \text{ Passive MP} \]

- procyclical \( \pi \) shock generates negative stock-bond return correlation
- monetary policy alone cannot change the sign

Bond Market Exposures to Macroeconomic and Monetary Policy Risks

Dongho Song
Conclusion

- I estimate an equilibrium asset pricing model that allows for shifts in
  1. the aggressiveness of the central bank to inflation fluctuations
  2. the covariance between the long-run growth and inflation target

- My Bayesian estimation provides strong evidence of regime changes
  - passive monetary policy: 1970s
  - countercyclical inflation shocks: 1970s - mid1990s

- Main findings
  - upward sloping yield curve due to regime uncertainties
  - positive bond-stock return correlation when $\pi$ shock is procyclical
  - policy shifts affect the 2nd moment of $\pi$ and yield curve

- Future research: extension to the production economy
• Structural shift for all coefficients

\[ z_t = \mu + s_{t-1} + \varepsilon_t, \quad z_t = [\pi_t, \Delta c_t]' \]

\[ s_t = \phi s_{t-1} + \phi K\varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega). \]

1 1970-1990s:

\[
S_t = \begin{bmatrix}
0.96 & 0.14 \\
-0.06 & 0.52 \\
[-0.10, -0.02] & [0.10, 0.20]
\end{bmatrix} S_{t-1} + \varepsilon_t, \quad \text{var}(\phi K\varepsilon_t) = \begin{bmatrix}
1.06 & -0.14 \\
-0.14 & 0.32 \\
[-0.26, 0.52] & [0.01, 0.55]
\end{bmatrix}
\]

2 2000s:

\[
S_t = \begin{bmatrix}
0.41 & 0.26 \\
0.07 & 0.83 \\
[-0.03, 0.18] & [0.10, 0.80]
\end{bmatrix} S_{t-1} + \varepsilon_t, \quad \text{var}(\phi K\varepsilon_t) = \begin{bmatrix}
0.78 & 0.29 \\
0.29 & 0.55 \\
[0.01, 2.28] & [0.01, 2.28]
\end{bmatrix}
\]

Drop in \(E(\pi), \sigma(\pi), \rho(\pi)\) and sign switch in covariance
Appendix: Shifts in the Slope of the Phillips Curve

Panel A: Sample Split in mid-1980s, Backward-Looking PC

Panel B: Sample Split in mid-1980s, Forward-Looking PC

Coibon and Gorodnichenko (2013): Is The Phillips Curve Alive and Well After All?
Appendix: Term Premium $t,n = y_{t,n} - \frac{1}{n} \sum_{i=0}^{n-1} E_t(y_{t+i,1})$

$\text{corr}(\pi, \Delta c) < 0$, Active MP

$\text{corr}(\pi, \Delta c) > 0$, Active MP

$\text{corr}(\pi, \Delta c) < 0$, Passive MP

$\text{corr}(\pi, \Delta c) > 0$, Passive MP

• term premium more important in the Active MP regime
Appendix: CP Excess Return Predictive Regression

\[ \text{corr}(\pi, \Delta c) < 0, \text{ Active MP} \]

\[ \text{corr}(\pi, \Delta c) > 0, \text{ Active MP} \]

\[ \text{corr}(\pi, \Delta c) < 0, \text{ Passive MP} \]

\[ \text{corr}(\pi, \Delta c) > 0, \text{ Passive MP} \]

- \textit{higher } R^2 \text{ when Active MP and } \text{corr}(\pi, \Delta c) < 0
Appendix: Expectations Hypothesis (EH) Slope Coefficient

\[ corr(\pi, \Delta c) < 0, \text{ Active MP} \]

\[ corr(\pi, \Delta c) > 0, \text{ Active MP} \]

\[ corr(\pi, \Delta c) < 0, \text{ Passive MP} \]

\[ corr(\pi, \Delta c) > 0, \text{ Passive MP} \]

- Passive MP decreases the degree of violation of EH
Appendix: (EH) Slope Coefficient

\[ y_{t+1,n-1} - y_{t,n} = \alpha_n + \beta_n \left( \frac{y_{t,n} - y_{t,1}}{n-1} \right) + \epsilon_{t+1} \]

- Slope coefficient \( \beta_n = 1 - \frac{cov(\mathbb{E}_t r_{x_{t+1,n}}, y_{t,n} - y_{t,1})}{var(y_{t,n} - y_{t,1})} \rightarrow 1 \)

1. \( cov(\mathbb{E}_t r_{x_{t+1,n}}, y_{t,n} - y_{t,1}) \downarrow \)  
   term spread contains less information about expected excess bond returns

2. \( var(y_{t,n} - y_{t,1}) \uparrow \)  
   variance of the term spread increases

- Passive MP raises \( var(y_{t,n} - y_{t,1}) \uparrow \)
Appendix: Equilibrium Bond Yield Loadings

- Active MP: loadings on level factors are nearly flat across maturities
- Passive MP: loadings on level factors decrease over maturities
Appendix: U.K. Real Bond Yields, $\Delta c, \pi$

Real Consumption Per Capita and CPI Inflation

Real Yields

Bond Market Exposures to Macroeconomic and Monetary Policy Risks

Dongho Song
Appendix: U.K. Real Bond Yields, $\Delta c, \pi$

Rolling Window 2 Years: $\text{corr}(\pi, \Delta c)$

Real Yield Spread: $y_{t,n} - y_{t,5y}, n \in \{6y, ..., 15y\}$
Consumption Reaction to 1 % Point Inflation Surprises

Horizon

Horizon

1959–1997

1998–2011

Bond Market Exposures to Macroeconomic and Monetary Policy Risks

Dongho Song
Inflation Reaction to 1 % Point Inflation Surprises

1959–1997

1998–2011

Horizon

Bond Market Exposures to Macroeconomic and Monetary Policy Risks

Dongho Song
### Bond Market

<table>
<thead>
<tr>
<th></th>
<th>70s, 80s, 90s</th>
<th>00s</th>
<th>Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Correlation between Stock and Bond Return</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(r_m, r_{2y})$</td>
<td>0.16</td>
<td>-0.13</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>Correlation between Spread and Growth</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(y_{5y} - y_{3m}, \Delta c)$</td>
<td>0.33</td>
<td>-0.19</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Excess Bond Return Predictability, $R^2$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{x2y,t+1y}$ onto forward$_t$</td>
<td>34.34</td>
<td>13.60</td>
<td>20.68</td>
</tr>
<tr>
<td><strong>Term Spread Regression, Slope Coefficient</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{2y,t+1y}$ onto $y_{2y,t} - y_{1y,t}$</td>
<td>-0.95</td>
<td>0.89</td>
<td>-0.62</td>
</tr>
</tbody>
</table>
Calibration I: Neutral Inflation

\[
\begin{bmatrix}
\Delta c_{t+1} \\
\pi_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\mu_c \\
\mu_\pi
\end{bmatrix} +
\begin{bmatrix}
x_{c,t} \\
x_{\pi,t}
\end{bmatrix} +
\begin{bmatrix}
\bar{\sigma}_c \eta_{c,t+1} \\
\bar{\sigma}_\pi \eta_{\pi,t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
x_{c,t+1} \\
x_{\pi,t+1}
\end{bmatrix} =
\begin{bmatrix}
\rho_c & 0 \\
0 & \rho_\pi
\end{bmatrix}
\begin{bmatrix}
x_{c,t} \\
x_{\pi,t}
\end{bmatrix} +
\begin{bmatrix}
1 & \chi_{c,\pi}
\end{bmatrix}
\begin{bmatrix}
\sigma_{c,t} e_{c,t+1} \\
\sigma_{\pi,t} e_{\pi,t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sigma_{c,t+1}^2 \\
\sigma_{\pi,t+1}^2
\end{bmatrix} =
\begin{bmatrix}
(0.01) \sigma_{c,t}^2 \\
(0.01) \sigma_{\pi,t}^2
\end{bmatrix} +
\begin{bmatrix}
0.99 \sigma_{c,t}^2 \\
0.99 \sigma_{\pi,t}^2
\end{bmatrix} +
\begin{bmatrix}
\sigma_{w,c} w_{c,t+1} \\
\sigma_{w,\pi} w_{\pi,t+1}
\end{bmatrix}
\]

- regime 1: \( \rho_c = 0.99, \rho_\pi = 0.998 \) and \( \chi_{c,\pi} = 0 \)
- regime 2: \( \rho_c = 0.99, \rho_\pi = 0.6 \) and \( \chi_{c,\pi} = 0 \)

\[
P =
\begin{bmatrix}
0.97 & 0.03 \\
0.04 & 0.96
\end{bmatrix}
\]
Yield Curve, Stock-Bond Correlation: Calibration I

yield curve

regime 1
regime 2

stock-bond correlation

Bond Market Exposures to Macroeconomic and Monetary Policy Risks
Dongho Song
Calibration II: Non-Neutral Inflation

\[
\begin{bmatrix}
\Delta c_{t+1} \\
\pi_{t+1}
\end{bmatrix}
= 
\begin{bmatrix}
\mu_c \\
\mu_\pi
\end{bmatrix}
+ 
\begin{bmatrix}
\chi_{c,t} \\
\chi_{\pi,t}
\end{bmatrix}
+ 
\begin{bmatrix}
\bar{\sigma}_c \eta_{c,t+1} \\
\bar{\sigma}_\pi \eta_{\pi,t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\chi_{c,t+1} \\
\chi_{\pi,t+1}
\end{bmatrix}
= 
\begin{bmatrix}
\rho_c & 0 \\
0 & \rho_\pi
\end{bmatrix}
\begin{bmatrix}
\chi_{c,t} \\
\chi_{\pi,t}
\end{bmatrix}
+ 
\begin{bmatrix}
1 & \chi_{c,\pi} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_{c,t} e_{c,t+1} \\
\sigma_{\pi,t} e_{\pi,t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sigma^2_{c,t+1} \\
\sigma^2_{\pi,t+1}
\end{bmatrix}
= 
\begin{bmatrix}
(0.01)\sigma^2_c \\
(0.01)\sigma^2_\pi
\end{bmatrix}
+ 
\begin{bmatrix}
0.99\sigma^2_{c,t} \\
0.99\sigma^2_{\pi,t}
\end{bmatrix}
+ 
\begin{bmatrix}
\sigma_{w,c} w_{c,t+1} \\
\sigma_{w,\pi} w_{\pi,t+1}
\end{bmatrix}
\]

- regime 1: \( \rho_c = 0.99, \rho_\pi = 0.998 \) and \( \chi_{c,\pi} = -0.3 \)

- regime 2: \( \rho_c = 0.99, \rho_\pi = 0.6 \) and \( \chi_{c,\pi} = -0.05 \)

\[
P = 
\begin{bmatrix}
0.97 & 0.03 \\
0.04 & 0.96
\end{bmatrix}
\]
Yield Curve, Stock-Bond Correlation: Calibration II

Bond Market Exposures to Macroeconomic and Monetary Policy Risks
Dongho Song
Calibration III: Non-Neutral Inflation

\[
\begin{bmatrix}
\Delta c_{t+1} \\
\pi_{t+1}
\end{bmatrix} = \begin{bmatrix}
\mu_c \\
\mu_\pi
\end{bmatrix} + \begin{bmatrix}
\chi_{c,t} \\
\chi_{\pi,t}
\end{bmatrix} + \begin{bmatrix}
\bar{\sigma}_c \eta_{c,t+1} \\
\bar{\sigma}_\pi \eta_{\pi,t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\chi_{c,t+1} \\
\chi_{\pi,t+1}
\end{bmatrix} = \begin{bmatrix}
\rho_c & 0 \\
0 & \rho_\pi
\end{bmatrix} \begin{bmatrix}
\chi_{c,t} \\
\chi_{\pi,t}
\end{bmatrix} + \begin{bmatrix}
1 & \chi_{c,\pi} \\
0 & 1
\end{bmatrix} \begin{bmatrix}
\sigma_{c,t} e_{c,t+1} \\
\sigma_{\pi,t} e_{\pi,t+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sigma_{c,t+1}^2 \\
\sigma_{\pi,t+1}^2
\end{bmatrix} = \begin{bmatrix}
(0.01)\sigma_{c,t}^2 \\
(0.01)\sigma_{\pi,t}^2
\end{bmatrix} + \begin{bmatrix}
0.99\sigma_{c,t}^2 \\
0.99\sigma_{\pi,t}^2
\end{bmatrix} + \begin{bmatrix}
\sigma_{w,c} w_{c,t+1} \\
\sigma_{w,\pi} w_{\pi,t+1}
\end{bmatrix}
\]

- regime 1: $\rho_c = 0.99$, $\rho_\pi = 0.998$ and $\chi_{c,\pi} = -0.3$

- regime 2: $\rho_c = 0.99$, $\rho_\pi = 0.6$ and $\chi_{c,\pi} = 0.1$

\[
P = \begin{bmatrix}
0.97 & 0.03 \\
0.04 & 0.96
\end{bmatrix}
\]