Managing Growth to Achieve Efficient Coordination in Large Groups: Theory and Experimental Evidence

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Managing growth to achieve efficient coordination in large groups: Theory and Experimental Evidence

Roberto Weber

Abstract

Several previous experiments using the minimum effort (weak link) coordination game reveal a striking regularity – large groups never coordinate tacitly on the efficient equilibrium. Given the frequency with which large real-world groups, such as firms, face similarly difficult coordination problems, this poses an important question for economics and organization: Why do we observe large successfully coordinated groups in the real world when they are so difficult to create in the laboratory? This paper presents one reason.

A simple model of dynamic adjustment under strategic uncertainty demonstrates that by starting with a small group and then adding only entrants who are aware of the group’s history, one can obtain groups coordinated at higher levels of efficiency than in groups that start off large. Experiments provide support for this result, showing that, even though efficient coordination does not occur in groups that start off large, efficiently coordinated large groups can be “grown.” That is, by starting with small groups that find it easier to coordinate, we can add entrants – who are aware of the group’s history – to create efficiently coordinated large groups. This represents the first experimental demonstration of independent large groups tacitly coordinated at high levels of efficiency. However, the experiments also demonstrate that growth can be “too fast” and that “managers” may not always be aware of the need for slow growth.
Coordination is an important problem for economics and organization (Coase, 1937; March and Simon, 1958; Schelling, 1960; Arrow, 1974; Cooper, 1999). Tacit coordination deals with situations where economic actors attempt to match the actions of others without knowing what these others will do or without having a precise agreement about what to do.¹

This paper addresses the difficulty of tacit coordination in large groups. Considerable experimental evidence – mostly from a game known as the minimum-effort (or weak-link) coordination game – shows that group size has a strong effect on the ability of groups to coordinate (e.g., Van Huyck, Battalio and Beil, 1990; Weber, et al., 2001). Large groups of people – who cannot speak to one another – almost never coordinate successfully, and repetition alone does not solve the problem.²

However, this is inconsistent with the real world, where we observe groups much larger than those in experiments – such as firms and countries – where coordination plays a crucial role, but where these large groups have managed to coordinate successfully. This observation of large, efficiently-coordinated groups outside the laboratory creates a puzzle: if “large” laboratory groups cannot coordinate efficiently in this type of game, how do large countries and firms manage to often do so?

This paper demonstrates that the ability of large groups to coordinate successfully

¹Examples of coordination problems include buyers or sellers searching for a market, workers with complementary production tasks, or consumers purchasing products with network externalities. In game-theoretic terms, coordination problems arise when there are multiple equilibria, and players must tacitly resolve which one to play.

²In fact, even with communication large-group coordination is still very difficult (see, for instance, Weber, et al., (2001) and Chaudhuri, Schotter and Sopher (2002)).
can be critically affected by the group’s growth process. As previous research shows, coordination is easy in small groups. Therefore, members of a small group, such as the founding members of a firm, do not face substantial difficulty in coordinating efficiently. Once they have done so, they can establish a set of self-reinforcing rules or norms governing what actions are appropriate. These norms allow the group or organization to continue to coordinate activity successfully. As the group grows, new entrants’ exposure to these norms allows these entrants to be aware of the appropriate behavior, and creates an expectation for everyone in the group of what everyone else (including the new entrants) will do. Thus, by coordinating efficiently as a small group, growing slowly, and exposing new entrants to the group’s previous norms, a group can become large and efficiently coordinated.

The rest of this paper demonstrates the influence of growth on tacit coordination. A simple model of dynamic adjustment and strategic uncertainty shows why growth should work when entrants are exposed to the group’s history. Two experiments then test the prediction that grown groups – in which entrants are aware of the groups’ history – should be coordinated more efficiently than groups that start off large. In one experiment, the growth path is determined exogenously (by the experimenter) while in the other it is determined by a “manager” (a subject). Both experiments reveal that growth can produce efficiently coordinated large groups. However, both experiments also show that growth can fail, particularly when entrants are not aware of the group’s history or when growth is too fast.
The minimum-effort coordination game

The minimum-effort, or weak-link, coordination game was first studied experimentally by Van Huyck, Battalio and Beil (1990). In the game – which is a seven-effort-level version of the stag hunt game (see Crawford, 1995) – players choose from a set of integers that can be thought of as orderable strategies such as effort or contribution levels. Every player’s payoff is a function of her choice and the minimum choice of all $n$ players (thus the term “weak-link” since every player’s payoff is partially determined by the lowest choice in the group). The game is presented in Table 1, which gives the payoff to each player as a function of her choice and the minimum choice.

<table>
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<tr>
<th>Player’s choice</th>
<th>Minimum choice of all players</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
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<td>7</td>
<td>.90</td>
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</table>

Table 1: Payoffs (in dollars) for minimum-effort game

When everyone makes the same choice and therefore receives the same payoff (represented by cells along the diagonal), the outcome is one of the game’s seven pure-strategy Nash equilibria. The equilibria differ because those corresponding to higher choices also

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3See also Hirshleifer (1983) for an early theoretical discussion of the problem underlying the game.

4The game in Table 1 is the same as Game A in Van Huyck, Battalio and Beil (1990), except every payoff is lower by $0.40. Of course real payoffs also differ due to differences in experimental location and time. However, experiments using the two sets of payoffs produce similar small vs. large group results.
yield higher payoffs. Therefore, more efficient coordination corresponds to all players making higher choices in equilibrium and the Pareto-optimal (or efficient) equilibrium results when all players select the highest choice, 7, and receive $0.90.

Since all symmetric outcomes, including the efficient one, are equilibria this game does not have the incentive problem present in the prisoner’s dilemma. Nonetheless, the efficient equilibrium may not be easy to achieve because players are faced with strategic uncertainty. Everyone may recognize the efficient equilibrium, but may be unsure of what others will do. Therefore, players may choose something other than 7, particularly when they think it is more likely that someone else will choose something other than 7. Simply being unsure about what others will do may lead players to choose something other than 7.

Previous experiments with minimum-effort coordination games established clear regularities. Tacit coordination on the efficient equilibrium is impossible for large groups. Of the seven sessions initially conducted by Van Huyck, et al. (1990) (VHBB) with groups of size 14 to 16, the minimum in all sessions after the third period was the lowest possible choice. For small groups \((n = 2)\) playing in fixed pairs, coordination on the efficient equilibrium was much easier – it was reached in 12 of 14 (86%) of the groups (a result replicated by Camerer and Knez (2000)). Table 2 summarizes the distributions of fifth-period minima in several different experiments, all using the Van Huyck, et al., game in which subjects choose integers from 1 to 7, and choosing 7 is efficient.

The effect of group size could hardly be stronger. Subjects in a group of size 2 are
almost assured to coordinate on the efficient equilibrium. Subjects in larger groups (6 or more) are almost assured to converge to the least efficient outcome in which at least one player chooses 1. Thus, there is a strong negative relationship between a group’s size and the ability of its members to coordinate efficiently.

2 A model of growing efficient coordination

Given the link between coordination in minimum-effort games and coordination problems faced by real-world groups (e.g., Camerer and Knez, 1997; Nanda, 1997), the above work suggests an impossibility to regularly obtaining efficient coordination in large groups outside the laboratory. However, this is inconsistent with the observation that there exist real-world groups that are efficiently coordinated. To see how we might resolve this apparent inconsistency, we need to begin by recognizing that few large groups start off

<table>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>3</td>
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<td>2</td>
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<td>11</td>
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<td>4</td>
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<td>3</td>
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<td>6</td>
<td>10</td>
<td>Knez &amp; Camerer 1994</td>
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<td>8</td>
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<td>0</td>
<td>0</td>
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<td>9</td>
<td>2</td>
<td>Cachon &amp; Camerer 1996</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>14-16</td>
<td>7</td>
<td>VHBB, 1990</td>
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</tbody>
</table>

Table 2: Fifth period group minima (by %) in various 7-action minimum-effort studies (1 = inefficient; 7 = efficient)
at a large size. Most groups in fact begin small, when solving coordination problems is easier according to the above experimental literature. Once successfully coordinated, and once strategic uncertainty has been reduced, these groups might be able to remain coordinated as new entrants are added – particularly if the new entrants are aware of the group’s previous success (and therefore also face reduced strategic uncertainty).

This section presents a simple model demonstrating the above intuition. The model incorporates the possibility of growth by allowing group size to increase, and also allows for two kinds of entrants, those who are aware of the group’s history and those who are not. The model generates two predictions regarding the roles of growth and exposure of new entrants on coordination.

In presenting the model, we first consider the behavior of players when group size is fixed. Let $N$ be a set of $n$ players and, for all $i \in N$, let $x_{it} \in 1, \ldots, m$ be player $i$’s discrete strategy choice in period $t$ (in the game in Table 1, $m = 7$). A player’s payoff in each period $t$ is a function of her choice $x_{it}$ and the minimum choice of all $n$ players, $y_t = \min_i(x_{1t}, \ldots, x_{nt})$, as in Table 1.

In each period, a player’s choice $x_{it}$ is determined by a continuous latent strategy variable, $a_{it} \in \mathbb{R}$. Let $x_{it}$ be weakly increasing in $a_{it}$, for instance by rounding the $a_{it}$ to the nearest $x_{it}$.

Initial choices are determined by a common belief, $\alpha_0$, and an idiosyncratic stochastic term measuring strategic uncertainty:

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7The model is based on Crawford’s (1995) model of adaptive dynamics in order-statistic coordination games (which include minimum-effort games).
where $\epsilon_{it} \sim N(0, \sigma_i^2)$ and $\sigma_i^2 > 0$. Thus, all player’s choices are i.i.d. draws from a common distribution that is independent of group size.\footnote{The above specification implies a common fixed belief, $\alpha_0$. The results in this paper are not changed if this is replaced by a different belief for each player (e.g., $\alpha_i$) as long as beliefs are distributed independently of group size.} To simplify notation, we refer to a random variable drawn from this common distribution as $x_1$.

If group size is fixed, subsequent choices are determined by a simple linear adjustment from the player’s own previous choice towards the previous minimum, as well as by a mean zero error term that captures remaining strategic uncertainty. That is, player $i$’s latent (continuous) strategy choice is given by

$$a_{it} = (1 - \beta)x_{it-1} + \beta y_{t-1} + \epsilon_{it}.$$  \hspace{1cm} (2)

The $\epsilon_{it}$, which represent strategic uncertainty, are distributed normally with mean zero and variance $\sigma_i^2$. Since strategic uncertainty decreases with experience and the observation of others’ behavior, assume that $\sigma_i^2 > 0$ and that the $\sigma_i^2$ evolve according to the following process:

$$\sigma_i^2 = k\sigma_i^2_{t-1},$$  \hspace{1cm} (3)

where $0 \geq k < 1$. That is, when group size is fixed, each period reduces the amount of
strategic uncertainty underlying player’s choices (more precisely, $\sigma_t^2 = k^{t-1}\sigma_{t-1}^2$).

Next, we introduce the possibility of growing groups. To do so, we define a growth path $M$ over $T$ periods, as a sequence of increasing sets of players (i.e., $M = N_1, ..., N_T$, where $N_{t-1} \subseteq N_t$). Let $n_t$ be the number of players in $N_t$.

For any individuals who are not in the set of players in period $t$ (i.e., $i \not\in N_t$ we draw a distinction between sets of “informed observers” ($i \in O^I_t$), those who are aware of the history of minima ($y_1, ..., y_t$), and “uninformed observers” ($i \in O^U_t$), those who are not aware of any of the previous minima.

We now draw a distinction between three possible kinds of players in any period $t$, $i \in N_t$, based on their previous experience with the group and their access to information. Specifically, we define:

- **Incumbents** are players who played the game in period $t - 1$ (i.e., $i \in N_t \cap N_{t-1}$).

- **Informed entrants** are players who did not play the game in period $t - 1$ but observed the full history of minima (i.e., $i \in N_t \cap O^I_{t-1}$).

- **Uninformed entrants** are players who did not play the game in period $t - 1$ and did not observe any of the previous minima (i.e., $i \in N_t \cap O^U_{t-1}$).

Assume that the composition of the group in any period is common knowledge.

We now define the process generating the $a_{it}$ for each of the three kinds of players in period $t$. First, incumbents update their latent strategy variable in the same manner as players in fixed-size groups (i.e., as in equation 2). Uninformed entrants generate a
random draw from the \textit{i.i.d.} process determining first-period choices (as in equation 1). Finally, informed entrants also generate a draw from the initial distribution, but then perform one iteration of the dynamic adjustment in the direction of the minimum in the last period. Specifically, for informed entrants,

\[ a_{it} = (1 - \beta)x_{1t} + \beta y_{t-1} + \epsilon_{it}. \] (4)

One final, and important, aspect of the model with growth is that the change in the variance of the error term depends on the composition of group in period \( t \). Specifically, if all of the players are aware of the full history of minima (i.e., \( N_t \subseteq N_{t-1} \cup O_{t-1}^I \)), then the \( \sigma_t^2 \) evolve in the same manner as in equation 3. That is, as long as the history of \( y_t \) are commonly known, then the strategic uncertainty is reduced as if everyone in \( N_t \) had been playing the game in periods 1 through \( t - 1 \). However, if there is at least one uninformed entrant, then strategic uncertainty increases for \textit{all players} (i.e., \( \sigma_t^2 > \sigma_{t-1}^2 \)).\footnote{For instance, \( \sigma_t^2 = c \sigma_{t-1}^2 \) where \( c > 1 \) or, alternatively, \( \sigma_t^2 = \sigma_t^2 \).}

That is, if at least one uninformed player enters the group, then everyone faces greater uncertainty about what the minimum will be. (For instance, incumbent players are unsure of what entrants will do, entrants are unsure of what incumbent players will do, incumbent players are unsure of how other incumbent players will react, etc.).

In the following, let \( M \) and \( M' \) denote two (potentially) different growth paths and let the superscript \(^\prime\) differentiate between two groups with these growth paths. Let \( F_r(z) \) denote the cumulative density of random variable \( r \) at \( z \) and note that the cumulative
distribution, at $z$, of the minimum of $d$ draws from that random variable is given by $1 - ((1 - F_r(z))^d$. Moreover, let the terms “stochastically increased” or “stochastically higher” refer to shifts in the distribution of random variables in the sense of first order stochastic dominance (cf. Crawford, 1995).

Given the above model, the following result demonstrates that choices and minima in grown groups (with informed entrants) will be stochastically higher than those in fixed-size groups as long as the size of the grown group never exceeds that of the fixed-size group.

**Proposition 1** For all $M$ and $M'$ such that $N_1 \subset N_t$, $N'_1 = N'_t$, and $n_t \leq n'_t$, for all $t > 1$, and $i \in N_{t-1} \cup O_{t-1}$ for all $i \in N_t$, then $y_i$ will be stochastically higher than $y'_i$ in any period $t$ and $x_{it}$ will be stochastically higher than $x'_{it}$ in any period $t > 1$.

The intuition behind the above result is simple.$^{10}$ A smaller group obtains a higher minimum in period 1, in expectation, than a larger one. If the small group grows by adding only informed entrants and never exceeds the fixed-size group in size, then it maintains this advantage due to the fact that the grown group is never larger and the amount of strategic uncertainty (measured by $\sigma_{it}$) is never greater.

Of course, a key assumption above is that the group grows by only adding informed entrants, meaning that the variance of the error term is equal between the fixed-size group and the growing groups (i.e., the amount of strategic uncertainty is equal in the two groups, or $\sigma^2_i = \sigma'^2_{it}$). The above result does not hold, generally, if groups grow by

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$^{10}$Proofs for both propositions are in the Appendix.
adding uninformed entrants. Specifically, if in any period there are uninformed entrants, then the increase in strategic uncertainty due to the introduction of uninformed entrants in a period increases the variance of choices and stochastically decreases the minimum in that period, meaning that the group that starts small (with a higher minimum) can lose its advantage when new players are added (if the increase in strategic uncertainty due to the introduction of uninformed players is large).\textsuperscript{11}

It is also straightforward to extend the above result to the case in which both groups grow by adding only informed entrants, but one grows more slowly than the other. In this case, the group that grows more slowly (i.e., by starting smaller and never adding more players in any period), will have an advantage. More precisely, the following proposition states that for any two groups that both grow only by adding informed entrants, then if one group grows more slowly than the other (meaning that it starts off smaller and is never larger, and never adds more people in a period), the choices and minima will be stochastically greater in the more slowly grown group.

Before stating the proposition, we define the following relationship between growth paths:

\textbf{Definition 1} Let \( M \) and \( M' \) be any two growth paths. The relationship \( M \) grows more slowly than \( M' \) implies that \( N_1 \subset N_t \), \( N_1' \subset N_t' \), \( n_1 < n_1' \) and \( n_{t-1} - n_t \leq n'_{t-1} - n'_t \), for all \( t > 1 \).

\textsuperscript{11}That is, if \( i \in N_t \cap O_{t-1}^{1'} \) then \( \sigma_i^2 > \sigma_i'^2 \), meaning that it is possible for \( F_{y_i}(z) > F_{y_i}(z)' \) for some \( z \) even if \( F_{x_{it-1}}(z) \leq F_{x_{it-1}}(z)' \) and \( F_{y_{i-1}}(z) \leq F_{y_{i-1}}(z)' \) for all \( z \) and \( n_t < n_t' \).
Thus, if $M$ grows more slowly than $M'$, it starts off smaller and never adds more entrants in any period, which in turn implies that $n_t \leq n'_t$ for all $t$.

We now state our second result. The intuition behind this result, and the proof, are very similar to those for Proposition 1.

**Proposition 2** For all $M$ and $M'$ such that $M$ grows more slowly than $M'$, and that $i \in N_{t-1} \cup O_{t-1}^i$, for all $i \in N_t$ and $i \in N'_{t-1} \cup O'_{t-1}^i$, for all $i \in N_t^i$, then $y_t$ will be stochastically higher than $y_t'$ in any period $t$ and $x_{it}$ will be stochastically higher than $x_{it}'$ in any period $t > 1$.

We, therefore, obtain two main results. If groups grow only by adding informed entrants, then a group that grows to any size $n$ will have stochastically higher minima and choices than a group that starts off at that size and a group that grows to that size more rapidly. This presents a clear prescription for alleviating large-group coordination failure. By starting a group off at a small size and adding only players who are aware of the group’s history, one should be able to produce a group of any size with higher minima, on average, than groups that start off at the larger size. However, the ability of new entrants to observe the group’s history is critical, since it underlies the decrease in strategic uncertainty. This implies that growth without such “exposure” of new entrants should not be successful in producing large groups with higher minima. The experiments in the next section directly test the effectiveness of growth with and without such exposure.
3 Growing efficient coordination in the laboratory

The experiments are conducted similarly to previous experiments by Van Huyck, et al. (1990) and Weber, et al. (2001). There are two almost identical experiments. In the first experiment, the rate of growth was fixed and pre-determined by the experimenter. This experiment tests the main hypothesis that growth, coupled with exposure of new entrants to the group’s history, can produce efficiently coordinated large groups relative to control groups that start off at a large size and to groups in which entrants are not aware of the group’s history. The results are clear: while none of the control groups or “no history” groups manages to coordinate on a minimum above one, the grown groups frequently manage to coordinate at higher levels of efficiency, including several on the highest possible minimum.

As a result of the successful, though fragile, growth in the first experiment, a second experiment looks at whether subjects themselves are aware of the need for slow growth. In this experiment, groups were again grown as before and entrants knew the group’s history, but the rate of growth was endogenous and not determined until the experiment – in each period, a “manager” determined the size of the group. This experiment provides additional evidence of successful growth. In addition, while limited by a very small number of observations due to the high cost of conducting a session (each data point involves using 13 paid subjects for 2 hours), the experiment reveals that subjects in the role of manager are not initially aware of the need for careful growth and tend to grow the groups too quickly, but that some of them learn to start over and are successful with
slower growth.

3.1 Exogenous pre-determined growth

3.1.1 Experimental Design

Since large groups of ten or more subjects have never consistently coordinated efficiently in previous experiments this first experiment was designed to explore whether a slow, controlled growth rate determined by the experimenter could create large groups that coordinated efficiently. In the experiment, groups of 12 students (at Stanford, UC Santa Cruz, or Carnegie Mellon) were assembled in one room. Subjects were presented the game in Table 1, though it was framed in the context of a report completion as in Weber, et al. (2001). Subjects were informed about how growth would take place (see below). They then played 22 periods in which the group was grown from a size of 2 to 12 players, except in the control sessions, where they played 12 periods at a fixed group size of 12 and there was no mention of growth. Instructions were read aloud and, before playing the game, subjects answered several questions to check their comprehension of the instructions and the game. At the end of the experiment, subjects were paid their earnings in cash.

In sessions where the group was grown, two treatments tested the effect of entrants’ awareness of the group’s history. In the history condition, new entrants observed the

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The context presents each player’s choice as the choice of a time at which to contribute a section of a report. Each player receives a bonus depending on when the report is completed (when the last section is contributed) and pays a cost (which is higher for contributing earlier). The game was presented both in this context and also by giving the subjects Table 1 with an explanation of the payoffs. Weber, et al. (2001) included experiments both with and without this context and found no difference in the results.
group’s history (the minima in all previous periods) and this was common knowledge. In the no history condition, new entrants were not aware of any of the previous minima. This treatment serves as a simple metaphor for the extensive training, socialization, and acculturation often required of new entrants to a firm or country. Without such exposure growth is not expected to work (Weber, 2000).

In each experimental session, subjects were anonymously assigned participant numbers. Each of the growth sessions consisted of 22 periods. In the first several periods, only participants 1 and 2 played the game while the other subjects sat quietly (history condition) or were in another room (no history condition). Participants were told that they would receive a fixed, positive, “fair” amount for periods in which they were not playing the game, but that the exact amount would not be revealed until the end of the experiment. In each period, participating subjects recorded a number from 1 to 7 (indicating the contribution time for their section of the report) on a piece of paper and handed it to the experimenter. The experimenter then calculated the minimum, wrote

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13 This was done because of the concern that announcing the per-period amount might create a focal point that could influence participants’ choices. Subjects were told that the reason they were not informed of the amount was because “we do not want this to influence your choices.” To ensure that they believed the amount was fixed – and not dependent on what happened in the experiment – the experimenter placed a large envelope at the front of the room and told subjects that the amount was written on a sheet inside the envelope. This sheet was shown to subjects at the end of the experiment.

14 To prevent players from knowing which others were participating in the history condition, all players handed in slips of paper; non-participants simply checked a box saying they were not participating. In the no history condition, subjects knew the participant number of subjects 3-12 since they entered the room once they were added (though participants already in the group were seated facing away from the door so that they could not look at or make eye contact with the new entrant.) The main reason for this difference is that the history and control sessions (with anonymity) were designed and conducted before deciding to run the no history condition (where anonymity is difficult) at the suggestion of two anonymous reviewers. It would have been possible to maintain anonymity in the no history sessions by, for instance, using a code to report the group’s history or reporting history via private messages. However, both of these procedures eliminate common knowledge (among those in the group) of outcomes, creating a likely even more important difference between the two conditions.
this number on the board, asked subjects to calculate their payoffs and then checked to make sure that they had done so correctly.\(^{15}\) Before proceeding to the next round, the experimenter erased outcome information from the board.

At various preannounced and commonly known periods, other participants joined the group of those actively playing the game. Participant numbers of those playing the game were written on the board. In the no history condition, the new entrant was brought in from the other room. For each session, there was a schedule of such additions – referred to as a *growth path* – that was handed to all subjects at the beginning of the experiment. For example, in one of the growth paths a third participant (#3) was added in period 7, joining the first two participants (#1 and #2) who continued to participate. Subjects all knew the predetermined growth path, and they knew that earlier participants always continued to participate. In all growth paths, 12 subjects were participating by the last few periods.

### 3.1.2 Results: Control sessions

Five control sessions (n = 60) were conducted using undergraduates at Stanford (sessions C1 & C2), Caltech (sessions C3 & C4), and Carnegie Mellon (session C5). The results are reported in Figure 1, which presents the minimum choice across all 12 periods for each

\(^{15}\)In all conditions, the action chosen by any participant was anonymous – except for choices by participants 1 and 2 in the no history condition, where a subject choosing a higher number could infer the precise choice of the other subject (which would be the minimum). However, since the results for two-person groups are similar in both the history and no history conditions, this appears not to have influenced behavior.
In addition, the two thicker lines present the average of the session minima and the average choice across all sessions.

(Figure 1 about here)

Overall, the results replicate previous experimental results on large groups playing the minimum-effort coordination game. The minimum converges to 1 in all five control groups. Both the average choice and the average of the minima consistently decrease and end up at or near one by the final periods.

3.1.3 Results: Growth sessions

Twelve sessions with pre-determined growth paths (n = 144) were conducted at Stanford (sessions 1 - 4), UC Santa Cruz (sessions 5 - 7), and Carnegie Mellon (sessions 8 - 12). Of these, nine sessions (1 - 9) were in the history condition, while three (10 - 12) were in the no history condition.

The purpose of this experiment is to explore the possibility that growth and exposure of new entrants to a group’s history may produce efficiently coordinated large groups. Therefore, each of the growth paths used was selected with the hope that it would be slow enough to create a large group that coordinated more efficiently than large groups had in earlier experiments and in the control sessions. The principles driving the choice of these growth paths were based on the intuition that successful early coordination followed

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16The entire dataset of choices can be obtained from the author’s website: http://www.andrew.cmu.edu/user/rweber.
by slow growth probably allow the best chance of obtaining efficient coordination (see Weber (2000)). This implies that good candidates for successful growth paths should only add a few players at a time and should allow time between growth periods or "spurts," particularly initially. Therefore, the growth paths were designed to first allow the establishment of successful coordination in a two-person group (by allowing several periods before adding more participants) and then to add players in a slow and regular manner. Thus, with one exception (the last two players added in growth path 3), the growth paths add only one player at a time. Figure 2 shows the three growth paths used in the experiment.

(Figure 2 about here)

Figures 3a through 3c present the minimum choices for sessions 1 through 9 (history), while Figure 3d presents the minimum choices for sessions 10 through 12 (no history). Each figure also presents the corresponding growth path. The marker-less solid line in each figure shows the growth path (measured on the left vertical axis), while the remaining lines all present the minima in a session (measured on the right vertical axis). The data are also presented in Table 3.

(Figure 3 about here)

An examination of behavior in individual sessions helps shed light on several behavioral regularities. First, the small groups were able to coordinate efficiently in both history and no history sessions. In all but one of the sessions, the group of size two
was able to coordinate on a minimum of 7 for at least two consecutive periods. In the remaining one (session 7) the group was able to coordinate on a high level of efficiency (minimum = 6).  

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| Growth path 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
|---------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| Sess. 1 (h)   | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Sess. 2 (h)   | 6 | 5 | 7 | 6 | 7 | 7 | 5 | 6 | 5 | 6 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |

**Table 3:** Minima and group size by period for “growth” sessions

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A second observation is that growth does not always work, even with history. In four of the sessions (1, 3, 6, and 7) the minimum by the first period in which the group plays as a 12-person group is 1. Therefore growth does not always produce efficiently

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17In several of these groups the minima in the final two periods (21 and 22) fell to a lower value than that at which the group was coordinated previously. This “end of experiment effect” frequently occurs in these games from one or two subjects changing their choices in the final period or two (perhaps to punish or do better that others, or perhaps because they believe that others will do so – in which case doing so is a best response). The minima are particularly sensitive to only one subject changing her behavior in this manner. While this phenomenon is interesting and frequently occurs in previous experiments, this paper is not concerned with what occurs in the final periods (after growth is completed), but rather with coordination during and immediately after growth.
coordinated groups, even with growth paths designed to do so.

An equally strong regularity, however, is the fact that growth just as often produces large groups coordinated at higher levels of efficiency. In three sessions (4, 8, and 9), the minimum remains at 7 throughout the growth process and these group play at least two periods at a group size of 12 with the highest possible minimum (recall that control 12-person groups never manage to coordinate on a minimum of 7 in any period). In another session (2), the minimum remains at 5 throughout the growth process (falling only in the last period), while in another (5) it does not fall below 3 as the group grows. Therefore, in five out of nine sessions in the history condition, groups reach a size of 12 while coordinated on an equilibrium with a minimum greater than 1, and in three of those they coordinate on the efficient equilibrium. In previous experiments and in the control sessions, large groups (of 6 or more) never coordinated on a minimum greater than 1.

In addition, in all the sessions that end up at a minimum of 1, the minimum is higher at least through a group size of 9. This higher level of efficiency for groups of size 9 is surprising in light of the fact that the minimum was always 1 for the large groups (nine or larger) in Table 3.\(^{18}\) Thus, there is clear support for the hypothesis that starting with a two-person group, which reliably reach efficiency, and then adding players at a slow rate enables much better coordination than starting with a large group.

\(^{18}\)While Table 2 reports the fifth period minima, the first period minima in previous experiments were not as high as in the sessions reported here and there was never a minimum of 7. Note also that in the control sessions the minimum was never above 4 in any period.
An equally important result is that growth does not work in the three no history sessions (10 - 12). In all of these sessions the minimum falls below 7 by the time the group reaches a size of 4 and the minimum is equal to 1 by the time the group reaches a size of 8 and stays there for the remainder of the experiment. Moreover, this failure is never driven entirely by only one subject – in every session at least 6 of the 10 “entrants” chose a number below 7 in their first play.¹⁹

To directly test whether or not growth with history results in successful coordination, we need to compare choices in the control and no history sessions with those in the history sessions. Table 4 compares the distribution of subject choices in the five control sessions, the three no history session, and the nine history sessions in the fourth period in which participants played at a group size of 12.²⁰

The frequency of subjects choosing 1 is high in all three conditions (13 of 60 in the control; 35 of 36 in no history; 33 of 108 in history), but is clearly highest in the no history

¹⁹There appears to be a relationship between changes in group size and a decrease in the minimum. Of the 34 total decreases in minima in Table 3, 24 (71%) coincided with an increase in group size, 6 (18%) occurred after the group reached a size of 12, 3 (9%) occurred before the growth process began, and one (3%) occurred during a “pause” in growth (in session 11). Roughly half of the decreases in minima during growth were due to a decreased choice by an incumbent group member (14 of 24) rather than by a lower choice by the entrant (10 of 24). Interestingly, most of the decreases in minima during growth in the history sessions were due to an incumbent group member lowering his or her choice (14 of 19 decreases during growth), but all of the decreases during growth in the no history sessions were due to a lower minimum choice by the entrant (5 of 5).

²⁰Selecting a comparison period is somewhat tricky. The control groups played as large groups for 12 periods. The grown groups all started off at small sizes and did not reach a group size of 12 for several periods. The earliest period in which a grown group reached the maximum size was period 18. The key question is when the comparison should be made. A reasonable comparison is to compare the control groups in period t with the t-th period in which the grown groups played as groups of size 12. In this case, subjects in both treatments have t − 1 periods of play in 12-person groups and therefore share a similar history. The analysis here sets t = 4 because this is the greatest value of t for which there is data in all of the growth sessions (growth path 2 has a group size of 12 only in periods 19 through 22), and it allows the groups several periods in which to coordinate.
Table 4: Distribution of subject choice in fourth period as 12-person groups

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>History</th>
<th>No History</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3 (5.0%)</td>
<td>32 (29.6%)</td>
<td>0 (0.0%)</td>
</tr>
<tr>
<td>6</td>
<td>0 (0.0%)</td>
<td>3 (2.8%)</td>
<td>0 (0.0%)</td>
</tr>
<tr>
<td>5</td>
<td>6 (10.0%)</td>
<td>13 (12.0%)</td>
<td>0 (0.0%)</td>
</tr>
<tr>
<td>Choice</td>
<td>4 22 (36.7%)</td>
<td>8 (7.4%)</td>
<td>0 (0.0%)</td>
</tr>
<tr>
<td>3</td>
<td>7 (11.7%)</td>
<td>10 (9.3%)</td>
<td>0 (0.0%)</td>
</tr>
<tr>
<td>2</td>
<td>9 (15.0%)</td>
<td>9 (8.3%)</td>
<td>1 (2.8%)</td>
</tr>
<tr>
<td>1</td>
<td>13 (21.7%)</td>
<td>33 (30.6%)</td>
<td>35 (97.2%)</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td>108</td>
<td>36</td>
</tr>
<tr>
<td>Minima</td>
<td>1,1,1,1,4</td>
<td>1,1,1,1,3,4,5,7,7</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Just as interesting, however, is the difference in the distributions of subjects choosing seven. In the control and no history sessions only 3 of 60 subjects (5%) and 0 of 36 (0%), respectively, did so. However 32 of 108 subjects (30%) in the history sessions chose the action corresponding to the efficient equilibrium. Therefore, the number of subjects playing the efficient strategy is much higher in the grown groups than in the control sessions. The differences in distributions of choices between the history condition and the two other conditions are significantly different in a one-tailed Kolmogorov-Smirnov test.21

21 This aspect of the comparison works against the hypothesis that growth works, since choices in unsuccessfully grown groups are usually almost all 1 even prior to the first period as a 12-person group, while in the control groups many subjects’ choices do not converge to 1 until later periods.
(C-H: \( D = 0.294, p < 0.001; \) NH-H: \( D = 0.667, p < 0.001 \)). A comparison of the mean choice in the history condition (3.89) with the mean choices in the control (3.18) and no history (1.02) conditions using a one-tailed \( t \)-test (with unequal variances) also produces significant differences (C-H: \( t_{161.78} = 1.99, p < 0.02; \) NH-H: \( t_{109.88} = 11.94, p < 0.001 \)).

The above tests rely on the assumption that the observations in each treatment are independent, which is doubtful because the choices of subjects in a session are affected by a shared history. Therefore, the level of significance reported by the statistics is exaggerated. One way to address this problem is to treat each session as the independent unit of analysis and examine only the minima. These minima are reported in the final row of Table 4. Note that the fourth-period minimum in all but one of the control sessions and no history sessions is 1 and that, while the minimum in four of the history sessions is also 1, the minimum is greater than 1 in the remaining five sessions. Moreover, the history sessions produce minima of at least 5 three times, which never occurs in the control sessions. A one-tailed Mann-Whitney \( U \) test of the minima reveals significant differences between control and history sessions (\( z = 1.33, p < 0.1 \)) and no history and history sessions (\( z = 1.55, p < 0.07 \)), while there is no difference between the control and no history sessions (\( z = 0.78 \)). While the \( p \)-values in the above tests are not highly significant, it must be noted that this test is extremely conservative since it treats each group of twelve subjects as just one observation.

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22The difference between the control and no history conditions is also significant in two-tailed versions of the same tests (\( t_{61.09} = 10.28, p < 0.001; \) \( D : 0.756, p < 0.001 \)). This is largely due to the fact that by the fourth period as a 12-person group, groups in the no history condition are almost entirely coordinated on the minimum choice of 1.
The above analysis convincingly shows that growth can solve the problem of large group coordination failure. The history sessions regularly produce independent 12-person groups coordinated at higher minima than 1 (including minima of 7), a result that has never been found in previous experiments on minimum effort coordination games.

The growth sessions in this experiment all use a slow, regular growth path intended to give growth the best chance of producing large, efficiently coordinated groups. It is worth asking how the success of growth might be affected by growth paths that differ. An equally important question is whether subjects themselves are aware of the need for slow, controlled growth. The following experiment provides exploratory evidence addressing both of these issues.

### 3.2 Endogenous dynamic growth

#### 3.2.1 Experimental Design

In a second experiment one participant was randomly selected to act as a “manager” and determine the growth path during the experiment. Otherwise, the experiment was conducted in much the same way as the history sessions in the first experiment. The manager was placed in a separate room from the remaining 12 subjects and an exper-

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23 The results of experiments by Knez and Camerer (1994), which showed that “merging” two three person groups leads to coordination failure (the minimum fell to 1 80 percent of the time), indicate that growth can be too rapid. Note that the minimum for a group of size 6 (which corresponds to the size of Knez and Camerer’s rapidly grown groups) was 1 in only one of the seven sessions in experiment 1.

24 It has been noted that while coordination problems play an important role for firms in several industries, they are often neglected by both managers and organizational researchers (March and Simon, 1958; Lawrence and Lorsch, 1967; Heath and Staudenmayer, 2000).
The minimum effort coordination game that the other 12 participants were to play was described to the manager (again framed in the context of a project completion). In all periods after the first—in which group size was fixed at 2—the manager selected the size of the group that would play the game. The manager’s earnings in each period were determined by the number of active participants and by the group minimum. Table 5 describes the possible earnings for the manager. For any group size, the manager is better off when the group coordinates efficiently. Also, the manager’s payoff is higher when efficiently coordinated groups are larger, but the opposite is true for inefficient groups. Therefore, the manager has an incentive to create a large group, but only if it is efficiently coordinated.

Following the manager’s determination of the group size in each period, a group of up to 12 subjects played the game in the same way as in the first experiment. The instructions for these subjects were the same as in the previous experiment, except they were informed that the number of active participants was not pre-determined and would be determined at the beginning of each period by the manager. To determine the group size in a period, the manager simply wrote down a number between 2 and 12, and then the subjects with participant numbers from 1 to that number played the game in that period.

\[ \pi = \frac{n(\min - 3.5)}{100} + 0.05 \]

except for the payoff when the group size is twelve and the minimum is 7. Since the goal was for managers to attempt to reach this outcome, a large bonus was awarded for achieving it.

The one other difference with experiment 1 was that the manager was given the option, at the
The experiment lasted 35 periods to give the managers plenty of time to experiment with growth. Four sessions (n = 52) were conducted at Caltech. Each session lasted about 2 hours.

3.2.2 Results

The growth paths employed by the four managers (sessions E1 through E4) are presented in Figure 4. The most important thing to note is that all four managers initially grew the groups quickly. In the first period, managers were constrained to a group size of two, but in the next period, all of them added at least three new players. It is also worth noting that two of the four managers (sessions E1 and E4) subsequently implemented beginning of each period, to randomly reassign participant numbers. This was to allow the manager to “restart” the group in case the first few participants became stuck at a bad equilibrium, since previous results indicate that this does occasionally happen (though rarely) in small groups. This option was rarely used.
much slower growth paths.

In order to more closely analyze the behavior of individual managers, we examine the individual session data. Figures 5a through 5d report the results (growth path and minima) for each of the endogenous growth sessions. The left vertical axis presents the group size, while the right vertical axis presents the group minimum.

(Figure 4 & 5 about here)

While there are only four sessions, an examination of the data reveals interesting regularities. First, initially rapid growth by managers resulted in coordination failure. As mentioned above, all four managers added at least 3 more players in the second period. In three of the four sessions (E1, E2, and E4), the minimum was initially high (6 or 7), but this minimum fell when the group reached a size of 9 or larger by the third period. In the other session (E3), the minimum was initially low (3) and did not increase as the manager grew the group. The fact that none of the managers allowed small groups time to coordinate efficiently before growing points to a lack of cognition of the difficulty of coordinating large groups – which is consistent with previous research (see Weber, et al., 2001). Moreover, the fact that the minimum fell when these managers grew quickly indicates the need for slow, controlled growth to solve coordination failure.

Following this initial failure, however, some of the managers learned to grow using a slower and more regular approach. Two of the managers (E2 and E3) continued to grow too quickly – obtaining higher minima at small group sizes, but adding too many players right away, causing the minimum to fall again. The other two managers (E1 and
E4), however, started over with small groups and then grew slowly (never adding more than 2 participants at a time) to create large groups coordinated on minima of 6 and 7. It is interesting to note that the growth path used by the manager in session E1 is very similar to the growth paths used in the first experiment (see Figure 2).

There is also further support in these experiments for the main hypothesis of this paper that slow, regular growth – along with exposure of new entrants to a group’s history – can lead to successful coordination in large groups. In the two sessions in which the managers started over at a small size and grew slowly (E1 and E4), the result was large groups that coordinated on high minima (6 and 7). In addition, the failure to succeed of the other two managers indicates that the rate of growth is important in obtaining efficient coordination in large groups.

4 Conclusion

Previous studies reveal the difficulty involved with obtaining efficient coordination in large groups playing the minimum-effort game. In fact, no previous study has regularly produced independent large groups tacitly coordinated on high levels of efficiency without manipulating payoffs. The results of both experiments in this paper clearly show that efficient coordination in large groups is possible when groups start off small and then grow slowly, as long as new entrants are aware of the group’s history.\(^\text{27}\) Over one-half of

\(^{27}\text{Another way to view this result is as a demonstration of transfer across games (e.g., from n-person games to similar n+1-person games). See Camerer and Knez (2000) and Cooper and Kagel (2004) for experimental evidence of transfer of behavior across related, but different, games.}\)
the 12-person groups in the growth sessions coordinate on a minimum greater than 1, as
do the two groups in the second experiment that start over and grow slowly. However,
growth does not always work, and even the groups that were grown slowly did not always
remain coordinated efficiently.

The results of the second experiment also indicate that subjects may not be aware of
the importance of slow, regular growth. All four of the managers grew quickly initially,
and as a result none were successful at first in maintaining efficient coordination while
growing. However, following this early negative experience, two of the managers were
able to subsequently grow large groups coordinated on high levels of efficiency, using
similar principles to those employed in the growth paths of experiment 1. The results of
these two sessions also suggest that a better approach to growth than the rigid growth
paths of experiment 1 might be one in which a manager waits until some set of conditions
is met before growing. The fact that both of these sessions produced successful growth
points to the potential value of such endogenous rules.

Another interesting result of these experiments was unintended but merits mention.
In two sessions (H1 and E4) there is clear evidence that subjects form norms about how
to react, not only to the previous minimum, but also to what happened in previous
experiences with growth. For instance, in session H1 (see Figure 3a), the group appears
to develop a precedent that “when the group grows, the minimum falls by 1.” Even more
strikingly, in session E4 (see Figure 5d) the group develops a norm that can be described
as “the minimum falls every time the group grows, but in every period in which the group

29
does not grow the minimum rises by *exactly* 1." The strength of this norm is particularly evident in that all of the subjects changed their choices in exact concordance with this rule in several of the latter periods. These results suggest that players may form “higher-order” precedents based on not just levels of previous play (e.g., “expect the previous minimum to be the minimum again”), but also on the relation between levels of previous play and group sizes or transitions. That is, previous experience with growth may affect how group members subsequently jointly react to a similar event.

Taken together, the results indicate that slow growth, coupled with the exposure of new entrants to a group’s history, is one reason why we observe large, successfully coordinated groups in the real world, and that the growth process itself – and previous experience with growth – may be incredibly important in determining the success or failure of a growing group. Firms, which are often efficiently coordinated large groups, may only get to be so through careful growth.

Of course, the results also reveal that even slow growth and acculturation of new entrants can fail, evidenced by the significant number of groups in the growth with history sessions that fall to a minimum of 1. Moreover, the results in this paper do not provide a prescription for how a firm that is already large and coordinated on an inefficient equilibrium might turn things around. No doubt in practice there are other ways to improve coordination in large groups. For instance, management might publicly emphasize a change in behavior – approximating the common knowledge, public advice treatments in Chaudhuri, Schotter and Sopher (2002). Or they might (temporarily)
modify payoffs to change the game to one in which efficient coordination is easier to obtain.

There are also specific ways in which the results can be extended to provide real-world implications. For instance, the success of slow growth is linked to the exposure of new entrants to the group’s history, which can be thought of as a form of the training or acculturation that new members of an organization frequently undergo (Tichy, 2001).Moreover, the early failure of groups in the endogenous growth sessions appears to produce an instance of the common view in the business world that firms can “grow too fast.” While – as with all experimental results – the value of these kinds of connections lies in the eye of the beholder, the main contribution of this paper is clear: by managing growth, it is possible to create efficient coordination in large groups.

References


28 An even stronger version of such acculturation might involving allowing the incumbents to explicitly tell the entrants what to do. This would capture real world practices such as supervised training or mentoring.


5 Appendix: Proofs

Proof of Proposition 1. We first show that the result holds in the first period and that it then holds, by induction, for all other periods.

For $t = 1$, note that the distribution of $x_1$ is independent of group size and therefore $F_{x_1}(z) = F'_{x_1}(z)$. The cumulative distributions, of first period minima, $F_{y_1}$ and $F'_{y_1}$, are then given by $1 - (1 - F_{x_1}(z))^{n_1}$ and $1 - (1 - F_{x_1}(z))^{n_1'}$, respectively. Thus, $F_{y_1} < F'_{y_1}$ whenever $n_1 < n_1'$, which is true by assumption.

For $t > 1$, note that, for all $t$, in the fixed-size group $a_{it}' = \beta x_{it-1}' + (1 - \beta)y_{t-1}' + \epsilon_{it}$, while for the grown group $a_{it} = \beta x_{it-1} + (1 - \beta)y_{t-1} + \epsilon_{it}$ for incumbents and $a_{it} = \beta x_1 + (1 - \beta)y_{t-1} + \epsilon_{it}$ for entrants. In what follows, we use $x_{it-1}$ to refer only to the choices of incumbents in the grown group in period $t - 1$.

As long as all entrants are informed, the variance of $\epsilon_{it}$ and $\epsilon_{it}'$ are both equal to $k^{t-1}\sigma^2_1$. Since $\beta$ is constant, then $F_{a_{it}'}(z) > F_{a_{it}}(z)$ whenever (i) $F_{y_{t-1}'}(z) > F_{y_{t-1}}(z)$, (ii) $F_{x_{it-1}'}(z) \geq F_{x_{it-1}}(z)$, and (iii) $F_{x_{it-1}'}(z) \geq F_{x_{it}}(z)$. It also follows from the above conditions that $F_{x_{it}'}(z) > F_{x_{it}}(z)$ and, as long as $n_t \leq n_t'$, that $F_{y_{t}'}(z) > F_{y_{t}}(z)$.

We know that (iii) holds for all $t > 1$ because the $x_{it-1}'$ and $x_{it-1}$ are stochastically decreasing in $t$ (i.e., they adjust towards $y_{t-1}$ and $y_{t-1}'$ and $y_{t-1} = min_i(x_{it-1}, ..., x_{n_t-1}t-1)$ and $y_{t-1}' = min_i(x_{it-1}', ..., x_{n_t-1}t-1)$). We have shown that conditions (i) and (ii) hold for $t = 2$ and, therefore, also hold by induction for all $t > 1$.

Q.E.D.
Proof of Proposition 2. The proof is similar to that for Proposition 1. For $t = 1$, the result holds as in Proposition 1.

For $t > 1$, the $x_{it}$ will be stochastically higher than the $x'_{it}$ whenever all entrants are informed (meaning that $\epsilon_{it}$ and $\epsilon'_{it}$ are both equal to $k^{t-1}\sigma_i^2$), $n_{t-1} \leq n'_{t-1}$ (meaning that the set of incumbents is smaller in the “slower” group), and $n_{t-1} - n_t \leq n'_{t-1} - n'_t$ (meaning that $M$ never adds more people in any period than $M'$). As long as these conditions hold, then it is straightforward to show that $F_{a'_{it}}(z) > F_{a_{it}}(z)$, separately, for both incumbents and entrants in any period $t > 1$. Since there are always fewer incumbents and entrants in the “slower” group, it then follows that $F_{x'_{it}}(z) > F_{x_{it}}(z)$ and $F_{y'_{it}}(z) > F_{y_{it}}(z)$.

Q.E.D.
Figure 1. Period minima in control sessions
Figure 2. Pre-determined growth paths
Fig 3a. Growth path 1 and period minima for sessions 1 and 2 (history)

Fig 3b. Growth path 2 and period minima for sessions 3 and 4 (history)
Fig 3c. Growth path 3 and period minima for sessions 5 through 9 (history)

Fig 3d. Growth path and period minima for sessions 10 through 12 (no history)