Indeterminacy and Learning:
An Analysis of Monetary Policy in the Great Inflation

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Reconciling the Narratives of the Great Inflation

- Great Inflation of the 1970s, and the Great Moderation of the 1980s and 1990s, are testing grounds for economic theories

- Various stories:
  - Bad policy: Fed chose policies that led to suboptimal outcomes (Clarida et al., 2000; Lubik and Schorfheide, 2004)
  - Bad luck: 1970s were characterized by adverse shocks (Sims and Zha, 2006)
  - Learning: the Fed did not know the structure of the economy, leading to policy choices that are sub-optimal ex-post (Sargent et al., 2005)
  - Misperception: Fed misinterpreted data and unwittingly chose suboptimal policies (Orphanides, 2001)

- We reconcile these stories: optimal central bank behavior can imply an indeterminate private sector equilibrium because of data misperceptions and uncertainty about the structure of the economy

- We find that the 1970s are characterized by an alternating sequence of indeterminate and determinate equilibria.

- Provide an argument for why central banks paradoxically choose policy rules that imply indeterminate equilibria
Our Conceptual Approach

- Central bank (CB):
  - observes real-time data that are subject to future revisions: Orphanides (2001)
  - does not know the true data-generating process, gathers information by estimating a backward-looking model (least-squares learning): Primiceri (2006)
  - solves for policy from a linear-quadratic dynamic programming problem over an ad-hoc welfare function: Sargent, Williams and Zha (2006)

- Private sector (PS):
  - knows the true monetary policy rule in each period, takes policy as given, forms rational expectations
  - myopic in the sense that policy rule is believed to last forever even if is re-optimized every period

- Each period CB and PS behavior results in a rational expectations model that is solved using standard methods: allow for indeterminate equilibria as in Lubik and Schorfheide (2003)

- Model is estimated using Bayesian methods
Indeterminacy, Learning, and Measurement Error

- Measurement error in real-time data leads to biased coefficient estimates of the CB’s learning model
- As the estimated coefficients of the central bank’s model and the associated policy rule change, so does private sector behavior
- In each period, there is a rational expectations equilibrium that is either unique or indeterminate
- It is the shifts of the policy rule (for fixed private sector parameters) that move the economy across the threshold between the determinate and indeterminate regions of the parameter space
- ‘Bad Policy’ (i.e., indeterminacy) arises not because of intent, but because of data mismeasurement and incomplete knowledge of the economy on behalf of the central bank.
The Central Bank (I)

- Each period the central bank observes its state vector $X_t$, which is a noisy measurement of the true outcomes:
  \[ X_t = X_t^{true} + \eta_t. \]

The measurement error is serially correlated (Orphanides, 2001):
\[ \eta_t = \rho_\eta \eta_{t-1} + \varepsilon_\eta^t. \]

- The CB sets the interest rate target:
  \[ i_{CB}^t = i_t + \varepsilon_i^t. \]

Based on a policy rule of the form:
\[ i_{CB}^t = \sum_{k=1}^{K} \alpha_i^k X_{t-k} + \xi_i^t \delta_{t-1}, \]

where $\varepsilon_i^t$ is a monetary policy implementation error.

- The policy coefficients $\alpha_t$ are chosen each period from an optimal policy problem as in Sargent, Williams, and Zha (2006).
The Central Bank (II)

- The CB minimizes:

$$\mathcal{W}_t = E_t \sum_{j=t}^{\infty} \beta^{(j-t)} \left[ (\pi_j - \pi_{target})^2 + \lambda_y (\Delta y_j - \Delta y_{target})^2 + \lambda_i (i_t - i_{t-1})^2 \right]$$

subject to estimated laws of motion of the following form:

$$\pi_j = c_\pi + a(L)\pi_{j-1} + b(L)y_{j-1} + u_\pi^t$$
$$\Delta y_j = c_y + d(L)\Delta y_{j-1} + \gamma i_{t-1} + u_y^t$$

and the definition of the policy instrument:

$$i_t^{CB} = i_t + \varepsilon_t$$

- $\pi_{target}$ and $\Delta y_{target}$ are fixed inflation and output growth targets that we calibrate
- $a(L), b(L), c_\pi, c_y, d(L), \gamma$ are re-estimated each period and are thus time-varying
The Private Sector

- Similar to Lubik and Schorfheide (2004)

- Hybrid New Keynesian Phillips Curve:

  \[ \pi_t - \bar{\pi}_t = \beta(\alpha_\pi E_t(\pi_{t+1}) + (1 - \alpha_\pi)\pi_{t-1} - \bar{\pi}_t) + \kappa y_t - z_t. \]

- Output dynamics:

  \[ y_t = -\sigma^{-1}(i_t - \bar{i}_t - E_t(\pi_{t+1} - \bar{\pi}_t)) + E_t y_{t+1} + g_t. \]

- Serially correlated demand and supply shocks:

  \[ z_t = \rho_z z_{t-1} + \varepsilon^z_t \]
  \[ g_t = \rho_g g_{t-1} + \varepsilon^g_t \]

- How do private agents form expectations and views about the steady states?
  - assume agents know policy coefficients: Sargent, Williams and Zha (2006)
  - act as if that policy will not change in the future (just like the central bank when it sets policy)
Model Estimation

- Bayesian inference
- Solving the private sector’s decision problem every period conditional on the policy coefficients $\alpha_t$ gives:
  \[ Z_t = S(\alpha_t)Z_{t-1} + G(\alpha_t)\varepsilon_t^Z \]
  This can be used as the state equation for the Kalman Filter to evaluate the likelihood.
- What happens when the policy coefficients imply indeterminacy?
  - we choose the baseline solution in Sims (2002): sunspot equilibrium without sunspots
Data and Calibration

- Observables:
  - real time estimates of inflation and GDP growth (source: Philadelphia Fed),
  - final data on inflation and GDP growth and the Federal Funds rate
- Quarterly data from 1965:Q2, estimated in growth rates
Figure 1: Real Time and Final Data: GDP Growth Rate
Figure 2: Real-Time and Final Data: Inflation Rate
## Data and Calibration

### Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Target</td>
<td>$\pi^{target}$</td>
</tr>
<tr>
<td>Output Target</td>
<td>$\Delta y^{target}$</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Indexation NKPC</td>
<td>$\alpha_\pi$</td>
</tr>
<tr>
<td>Lag Length in CB regression</td>
<td></td>
</tr>
<tr>
<td>Gain Parameter</td>
<td></td>
</tr>
</tbody>
</table>
Parameter Estimates

Figure 3: Prior and Posterior Parameter Density (Benchmark Specification)
## Parameter Estimates

### Table 2: Posterior

<table>
<thead>
<tr>
<th></th>
<th>5th Percentile</th>
<th>Median</th>
<th>95th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shocks:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>0.91</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.70</td>
<td>0.73</td>
<td>0.76</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>( \sigma_g )</td>
<td>0.011</td>
<td>0.012</td>
<td>0.014</td>
</tr>
<tr>
<td>( \sigma_i )</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td><strong>Measurement:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_\pi )</td>
<td>0.03</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td>( \rho_{g^{\text{growth}}} )</td>
<td>0.41</td>
<td>0.48</td>
<td>0.56</td>
</tr>
<tr>
<td>( \sigma_\pi )</td>
<td>0.0020</td>
<td>0.0022</td>
<td>0.0024</td>
</tr>
<tr>
<td>( \sigma_{g^{\text{growth}}} )</td>
<td>0.0054</td>
<td>0.0059</td>
<td>0.0064</td>
</tr>
<tr>
<td><strong>Structural:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>( \lambda_y )</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>0.54</td>
<td>0.65</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Key Result: Determinacy Indicator

- Draw from the posterior distribution of the estimated model
- For each draw compute equilibrium and record whether (i) determinate, $I = 1$ or (ii) indeterminate, $I = 0$
- Average over the draws, can interpret as posterior probabilities

Figure 4: Determinacy Indicator: Benchmark Specification
Digging Deeper into the Mechanism

- Actual and prescribed Federal Funds Rate Paths
- Estimated policy coefficients:
  - coefficients in the rule: \( \dot{i}_t = \sum_{k=1}^{K} \alpha^k_t X_{t-k} + \xi_t \dot{i}_{t-1} \)
  - long-run coefficients: \( \frac{1}{1-\xi_t} \sum_{k=1}^{K} \alpha^k_t \)
- Determinacy indicator and measurement error for inflation and output growth
- The role of the measurement error
Figure 6: Evolution of the Federal Funds Rate: Actual vs. Prescribed under Optimal Policy
Long-run Policy Coefficients

![Graphs showing the long-run response to inflation, output growth, and the lagged nominal interest rate.](image)

Figure 8: Long-Run Policy Coefficients
The Role of Measurement Errors

- Large ex-post data revisions are associated with equilibrium shifts

Figure 11: Imputed Measurement Error: Inflation Rate
The Role of Measurement Errors

- Large ex-post data revisions are associated with equilibrium shifts

Figure 12: Imputed Measurement Error: Output Growth
Indeterminacy and Learning without Measurement Error

- We re-estimate the model without measurement error: the central bank has access to final data without lag

Figure 13: Determinacy Indicator without Measurement Error
Measurement Errors and CB Decision Making

- We back out the estimated measurement series from the benchmark specification and simulate the model for benchmark parameter estimates and the observed histories of the endogenous variables.

- We scale the measurement series between 1 (benchmark) and 0 (no measurement error) to trace its effect on policy:
  - for observed histories and given measurement error, policy coefficients and policy shocks have to change.
Measurement Errors and CB Decision Making

Figure 14: Determinacy and Measurement Errors
Figure 15: Determinacy and Measurement Errors
Measurement Errors and CB Decision Making

Figure 16: Optimal Policy Paths and Measurement Error
Conclusion

- Learning can help explain the switch from policy rules that imply indeterminacy to policy rules that imply determinacy
- The government chooses to implement a different policy than would be warranted under full information
- Measurement error and data misperception is the key ingredient in this mechanism
- This leads to indeterminate outcomes in the private sector equilibrium: central bank learning as ‘microfoundation’ for indeterminacy
- Robust mechanism