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SEQUENTIAL VOTING WITH ENDOGENOUS VOTER FORECASTS

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We investigate a sequential voting model in which voters' forecasts of outcomes on future issues are determined endogenously. Voters are assumed to make decisions in an environment in which future outcomes are uncertain. The uncertainty arises from two possible sources. Voters may be uncertain of other voters' preferences. In addition, events that occur before future issues are decided may affect voters' preferences on future votes. Information available to voters at each point in time is characterized. An equilibrium to the voting problem, if one exists, is one for which the outcome on the issue currently being decided and voters' forecasts of outcomes on future issues are determined simultaneously. We show that an equilibrium exists under a particular voting institution and characterize voters' forecasts for this equilibrium.

Several recent studies have investigated the properties of majority rule voting processes when issues are voted on sequentially. The motivation for this approach is twofold: (1) to investigate the role of institutional structure in influencing outcomes and (2) to determine whether problems of nonexistence of equilibrium in multidimensional voting problems may be overcome by assuming a particular structure for voting decisions (see Denzau and Mackay 1981; Feld and Grofman 1987; Kramer 1977; Plott 1967).

Denzau and Mackay (1981) show that existence of equilibrium and the properties of equilibrium when it exists depend not only on voters' preferences but also on their expectations about outcomes of future votes. In particular, they show that an equilibrium may fail to exist if voters have perfect foresight expectations. Moreover, when preferences are such that an equilibrium exists with perfect foresight expectations, the equilibrium will generally differ from the one that would prevail

under myopic expectations. Their analysis and subsequent analyses (including the present one) assume voters have weighted Euclidean distance (Enelow and Hinich 1984) preference functions.

Enelow and Hinich (1983) study an environment in which voters forecast outcomes on future votes with error. They show that voter preferences on each issue are single-peaked if each voter's forecast of outcomes on future votes does not depend on the outcome currently under consideration. Enelow (1984) shows that this result can be generalized. Preferences are single-peaked if voters' forecasts of outcomes on future votes depend linearly on the issue currently under consideration, and the variance of their forecasts does not depend on the outcome of the issue currently being decided.

Enelow and Hinich (1983) and Enelow (1984) take voters' forecasting rules as given exogenously, that is, outside the model. However, in light of their results, it is natural to ask whether the sequential

voting model can be augmented by informational assumptions that permit voters' forecasting rules to be derived endogenously, that is, within the model. That is the problem studied here. We assume that voters are in an environment in which future outcomes are uncertain. The uncertainty arises from two possible sources. Voters may be uncertain of other voters' preferences. In addition, events that occur before future issues are decided may affect voters' preferences on future votes. We characterize information available to voters at each point in time. If there is an equilibrium to the voting problem, it is one in which, for each issue, the outcome on that issue and voters' forecasts of outcomes on future issues are determined simultaneously. We show that an equilibrium exists under a particular voting institution, and we characterize voters' forecasts for this equilibrium.

We know from Denzau and Mackay (1981) that there are conditions under which equilibrium will fail to exist when voters have full information and voters' forecasts are endogenous. Hence, it is natural to expect that there will be conditions under which existence of equilibrium will fail when voters make endogenous forecasts with imperfect information. In addition to studying a class of problems for which equilibrium exists, we indicate the problems that arise in attempting to extend our analysis beyond that class.

A model with endogenous voter forecasts is important for the following reasons. Over time, voters are continually receiving new information about the preferences of constituents or other voters (opinion polls), the state of the domestic economy, international events, and a variety of other factors that may affect their preferences about issues yet to be decided. A model with endogenous voter forecasts can provide insights into how voters use information and how voters' forecasts and the associated voting out-

comes change as new information becomes available. Hence, a model with endogenous voter forecasts can provide valuable insights into the role of information in political decisions.

In a model with sequential voting, voter forecasts play a central role in determining the sequence of outcomes. If voter forecasting rules are taken to be exogenous and permitted to change in arbitrary ways between votes, the potential for testing the model is quite limited. Any sequence of observed outcomes could be explained by a suitable choice of parameters in voters' forecasting rules. Hence, formulating a model of how voters' forecasts are determined is an important part of the overall goal of modeling outcomes when issues are decided sequentially. Hence, we believe that making voter forecasts endogenous is the next logical step in building on the insights of Denzau and MacKay (1981), Enelow and Hinich (1983), and Enelow (1984).

Having argued that forecasting rules should be made endogenous, we still have a good deal of latitude in modeling the way in which forecasts are formed. For example, we might simply assume that voters choose forecasting rules before the first issue is decided and adhere to those rules regardless of new information obtained as time passes. Alternatively, we might assume that voters adjust adaptively, changing the coefficients in their forecasting rules in some prescribed way when the outcome they observe differs from their *ex ante* forecast. These and similar approaches have the shortcoming that voters can make systematic errors without revising their approach to the forecasting problem.

Our approach is based on the assumption that voters use their knowledge of the way in which issues are decided to forecast future outcomes. Implementing this approach requires us to be explicit about what information voters have at each point in time about their own preferences,

the preferences of other voters, and other factors relevant to the voting decision. Given this information and their knowledge of the way in which issues are decided, voters can then forecast future outcomes. Their forecasts thus make use of all information available to them.

The Model

A series of $T \geq 1$ issues are to be decided sequentially by N voters. The outcome on each issue is determined by a voting procedure detailed below. Once an issue is decided, it cannot be reconsidered. Decisions are implemented and consumption occurs after all issues have been decided. The order of voting on the issues is fixed.

Individual voters are endowed with private information, ϕ_i , about their own preferences. This information is drawn from a distribution $f(\phi)$. In developing our analysis, we assume that the distribution $f(\cdot)$ is known to all voters. In our remarks, we discuss the implications of relaxing this assumption. After the outcome on each issue has been determined, new information may become available. Let ω_t be the new information received after issue t is decided, and let $\omega = (\omega_1, \omega_2, \dots, \omega_T)$. It is common knowledge (Aumann 1976) that ω is drawn from the distribution $g(\omega)$, and the draw itself is observed by all voters. Preferences over the T -dimensional vector of outcomes θ are given by

$$u(\theta; \phi_i, \omega) = -[\theta - x(\phi_i, \omega)]'A[\theta - x(\phi_i, \omega)], \quad (1)$$

where the vector $x(\phi_i, \omega)$ is the voter i 's ideal point, and A is a positive-definite matrix of salience weights. The functions $x(\cdot)$ are common to all voters.

The following are two examples of special cases conforming to the above structure.

Example 1

A set of N voters is selected at random from a population in which individuals have different ideal points. The distribution of ideal points, ϕ_i , in the population is given by $f(\phi)$. There is no common shock to preferences. In this example, $x(\phi_i, \omega) = \phi_i$.

In this example, the uncertainty about future outcomes arises because voters do not know the preferences of other voters. However, from knowledge of the distribution of preferences in the population, they will be able to form a forecast of outcomes on future votes, as we show below.

In many settings, the outcomes on future votes will be uncertain not only because voters are not entirely sure of other voters' preferences but also because events that may occur before the future votes are decided may affect preferences over future outcomes. A heightened or diminished level of conflict elsewhere in the world may affect willingness to spend for defense. New information about trade deficits or surpluses may affect preferences for protectionist legislation. The emergence of a recession may affect willingness to fund unemployment compensation programs. Disruptions in the supply of oil from abroad may affect willingness to spend resources to develop technologies for alternative energy sources. The next example illustrates how uncertainty about future events might be captured in the structure presented.

Example 2

A set of N voters is drawn at random from a population with direct utility function

$$U(\theta, q_i; z_i, A) = -(\theta - z_i)'A(\theta - z_i) + q_i,$$

where θ is publicly provided goods, q_i is

consumption of private goods, and z_i and A are preference parameters. The publicly provided goods θ will be financed by a proportional income tax. Thus,

$$q_i = y_i(1 - \tau), \tag{2}$$

where y_i is i 's before-tax income and τ is the tax rate. Suppose there are two publicly provided goods and let θ_1 and θ_2 be funds spent on these two activities. The public sector budget constraint is then

$$\tau Y = \theta_1 + \theta_2, \tag{3}$$

where Y is aggregate income. Suppose i 's relative position in the income distribution is given by

$$\frac{y_i}{Y} = k_i + \delta\epsilon_i, \tag{4}$$

where δ is a common shock to income (associated with, say, fluctuations in the aggregate economy), ϵ_i is an individual-specific variable reflecting the sensitivity of individual i 's earnings to business fluctuations, and k_i is an individual-specific variable reflecting the individual's position in the income distribution in the absence of shock δ . Here $\sum k_i = 1$ and $\sum \epsilon_i = 0$.

Solve equation 3 for τ , substitute the result into equation 2, and replace y_i/Y by the right-hand side of equation 4 to yield an indirect utility function of the form in equation 1 (up to an individual-specific additive constant) with ideal points:

$$x_i = z_i - (k_i + \delta\epsilon_i) \frac{A^{-1}}{2} e', \tag{5}$$

where $e = (1, 1)$. In this example, $\phi_i = (k_i, \epsilon_i, z_i)$, $\omega = (\delta, Y)$, and $x(\phi_i, \omega)$ is given by equation 5. The example illustrates how fluctuations in the state of the economy δ and Y affect voters' ideal points over issues.

Analysis of voting with incomplete information poses difficulties not encoun-

tered in models with full information (Ordeshook and Palfrey 1988). In order to characterize equilibrium in the sequential voting problem, we assume the following voting procedure is employed. When an issue is considered, each voter submits a proposal $v_i \in R$. The outcome on the issue is

$$\text{med}_i\{v_i\}.$$

We call the proposal that i submits i 's vote. On each issue each voter observes the outcome. Our results do not depend on whether voters observe the proposals made by other voters. Hence, the results apply to either secret ballot elections or to elections where all votes are revealed.

This voting procedure clearly does not capture the richness of information transmission and agenda formation in committee decision making (Austen-Smith 1988). However, this approach permits us to sidestep some of the complexity of models of agenda formation. The procedure proves to be a useful vehicle for gaining insights into models such as the sequential-voting model we study.

In this model, a player i 's strategy maps the history of the game (the outcomes on past votes) and player i 's type (the vector ϕ_i) into a vote on the current issue. We consider the Nash equilibrium that emerges by solving the model using backward induction. It is a property of our model that a voter's optimal decision at each stage of the game is the same regardless of the voter's beliefs about other voters' types (i.e., the ϕ_i 's of other voters) at that stage of the game. Hence, we do not need to make any particular assumption about how a voter updates beliefs about other voters' types as the game proceeds. It is natural to assume, however, that beliefs are updated using Bayes's rule, in which case, our solution to the game is a sequential equilibrium (Kreps and Wilson 1982).

We next present some definitions and

results that prove useful in the analysis to follow.

DEFINITION. For utility function $U_{1i}(\bullet)$ for voter i , v_i^* is a dominant strategy if

$$U_{1i}(\text{med}\{v_i^*, v_{-i}\}) \geq U_{1i}(\text{med}\{v_i, v_{-i}\})$$

for all v_i, v_{-i} . (6)

Here v_i is i 's vote and v_{-i} is the vector of votes by all other voters.

THEOREM 1 (Black 1958). If $U_{1i}(\bullet)$ is single-peaked, i has a dominant strategy, $v_i^* = \text{Argmax}_x U_{1i}(x)$.

COROLLARY. If $v_j = v_j(\epsilon)$ for $j \neq i$, where ϵ is a random vector and $U_{1i}(\bullet)$ is single-peaked, $v_i^* = \text{Argmax}_x U_{1i}(x)$ is a dominant strategy for i regardless of the process that generates ϵ .

Proof. Since v_i^* satisfies equation 6 for all v_{-i} , it follows that $E U_{1i}(\text{med}\{v_i^*, v_{-i}(\epsilon)\}) \geq E U_{1i}(\text{med}\{v_i, v_{-i}(\epsilon)\})$. QED

LEMMA 1. If y is a random variable that emerges when all players follow their equilibrium strategies in the future, i expects all players to pursue their equilibrium strategies in the future, and i 's utility over present and future outcomes can be written

$$U_i(\text{med}\{v_1, v_2(\epsilon), \dots, v_n(\epsilon)\}, y) = U_{1i}(\text{med}\{v_1, v_2(\epsilon), \dots, v_n(\epsilon)\}) + U_{2i}(y)$$

where $U_{1i}(\bullet)$ is single-peaked, then $v_i^* = \text{Argmax}_x U_{1i}(x)$ is i 's best proposal for the current issue.

Proof. Obvious extension of the corollary. QED

To illustrate the essential features of our analysis, we begin by solving Example 1 for the case in which only two issues are to be decided. For notational convenience, let $x_i = x(\phi_j) = \phi_j$.

At the time that issue 2 is decided,

the outcome on issue 1, θ_1 is known. With θ_1 given, the utility function in equation 1 is concave in θ_2 and, hence, single-peaked. By the corollary to Theorem 1, i 's dominant strategy is to vote the value that maximizes equation 1 with respect to θ_2 . Voters i 's most preferred outcome on issue 2, given outcome θ_1 on issue 1, is found by setting the derivative of equation 1 with respect to θ_2 equal to zero and solving to obtain

$$v_{2i}^* = \alpha\theta_1 + \beta_i \tag{7}$$

where

$$\alpha = -(a_{12}/a_{22}) \tag{8}$$

and

$$\beta_i = -\alpha x_{1i} + x_{2i}. \tag{9}$$

The median outcome of the period 2 vote is then

$$\bar{\theta}_2 = \text{med}_i(\alpha\theta_1 + \beta_i). \tag{10}$$

Since α and θ_1 do not depend on i , this may be rewritten as

$$\bar{\theta}_2 = \alpha\theta_1 + \text{med}_i \beta_i \tag{11}$$

Now $\bar{\theta}_2$ is a random variable, the distribution of which is derived from the distribution of the x_i . The voter's first-period optimization problem is obtained by inserting $\bar{\theta}_2$ in place of θ_2 in equation 1 and taking expectations. Voters have differing information (each voter knows his or her own preferences but not the preferences of others). Hence, the perceived distribution of future outcomes conditional on the information each voter has may differ across voters. In light of the results of Enelow and Hinich (1983) and Enelow (1984), it is interesting to observe that the mean of $\bar{\theta}_2$ is linear in θ_1 .

It is also of interest to note how the structure of the voting problem affects the forecasting equation 11. In particular,

from equation 8, the coefficient of θ_1 in equation 11 is determined by elements of the salience matrix A . Also, the random component in equation 11 arises from voters' uncertainty about other voters' preferences, as is evident from equation 9. Hence, the distribution of voter forecasts in this example is determined by the distribution of voter preferences through equation 11.

Voter i 's most-preferred outcome on issue 1 is obtained by choosing v_i to maximize:

$$-E\{[a_{11} - (a_{12}^2/a_{22})](\theta_1 - x_{1i})^2 - Ea_{22}[\beta - x_{2i} - (a_{12}/a_{22})x_{1i}]^2, \quad (12)$$

where the expectation in equation 12 is taken conditional on information available to voter i and

$$\beta = \text{med}_i \beta_j.$$

In particular, voter i knows $f(\bullet)$ and $g(\bullet)$ as well as his or her own preferences. The second expression in equation 12 does not depend on i 's vote on issue 1. The first expression in equation 12 is single-peaked in θ_1 . Hence, Lemma 1 applies. Voter i 's most-preferred outcome on issue 1 is $v_{1i}^* = x_{1i}$, and the outcome on issue 1 is

$$\tilde{\theta}_1 = \text{med}_i \{x_{1i}\}. \quad (13)$$

In this example, the outcome on the first issue to be voted on (issue 1) is simply the median of the most-preferred values on that issue (equation 13). By contrast, the outcome on the second issue to be voted on (issue 2) depends on the outcome on the first issue (equation 11). Thus, the order of voting will often matter. It follows that the sequential voting outcomes need not be the same as outcomes obtained if the issue space were rotated so that voters' salience weights in the rotated space were diagonal. In the rotated space voting outcomes would be invariant to order while in the original space outcomes will often depend on order.

We now generalize to the case of an arbitrary number of issues. We will make use of the following linear algebra result.

LEMMA 2. Consider the quadratic form

$$Q(y, z) = [(y - \mu_y)', (z - \mu_z)'] \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \begin{bmatrix} y - \mu_y \\ z - \mu_z \end{bmatrix}$$

where Σ_{11} is nonsingular and $\Sigma'_{12} = \Sigma_{21}$. This quadratic form can be rewritten

$$Q(y, z) = (y - d)' \Sigma_{11} (y - d) + (z - u_z)' \Sigma^{22} (z - \mu_z),$$

where

$$d = \mu_y - \Sigma_{11}^{-1} \Sigma_{12} (z - \mu_z)$$

and

$$\Sigma^{22} = \Sigma_{22} - \Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12}$$

Proof. Substitute d and Σ^{22} in the second expression. Straightforward algebra yields the first expression. QED

Lemma 2 has a statistical interpretation. The first expression for $Q(y, z)$ appears in the exponent of a jointly normal distribution for (y, z) , with mean (μ_y, μ_z) and covariance matrix

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

The second expression for $Q(y, z)$ appears in the exponent of the marginal distribution for z and the conditional distribution of y , given z (see, e.g., DeGroot 1970, 54-56).

For simplicity, we present our proof for the case in which there is no common shock, ω . When the proof is complete, we indicate how the result can be generalized to include the common shock. We next present several definitions used in proving our major result.

Sequential Voting

Define matrices B^s , $s = 1, 2, \dots, S$ recursively as follows:

$$B^s = A$$

$$B^{s-1} = B_{22}^s - B_{21}^s (B_{11}^s)^{-1} B_{12}^s, \\ s = 2, \dots, S$$

where

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s \\ B_{21}^s & B_{22}^s \end{bmatrix}, s = 1, 2, \dots, S.$$

B_{11}^s has dimension 1×1 , B_{21}^s has dimension $(s-1) \times 1$, B_{12}^s has dimension $1 \times (s-1)$, and B_{22}^s has dimension $(s-1) \times (s-1)$. Note that $B^1 = B_{11}^1$. Hence, define $B_{21}^1 = B_{12}^1 = B_{22}^1 = 0$.

Let

$$\Theta_s = [\theta_s, \theta_{s-1}, \dots, \theta_1, \theta_0], \\ s = 0, 1, 2, \dots, S, \text{ where } \theta_0 \equiv 0.$$

Let

$$X_{is} = [x_{is}, x_{is-1}, \dots, x_{i1}, x_{i0}], \\ \forall i, s = 0, 1, 2, \dots, S,$$

where $x_{i0} \equiv 0 \forall i$. Let

$$m_{is} = x_{is} + (B_{11}^s)^{-1} B_{12}^s x_{is-1} \\ \forall i, s = 1, 2, \dots, S \quad (14)$$

$$m_s = \text{med}\{m_{is}\}, s = 1, 2, \dots, S \quad (15)$$

$$M_{is} = \begin{cases} \sum_{t=s}^{s-1} (m_t - m_{it})' B_{11}^t (m_t - m_{it}), \\ 0 \end{cases} \\ \forall i, s = 1, 2, \dots, S-1 \quad (16) \\ s = S$$

Define the functions

$$v_{is}(\Theta_{s-1}) = x_{is} \\ - (B_{11}^s)^{-1} B_{12}^s (\Theta_{s-1} - X_{is-1}), \\ \forall i, s = 1, 2, \dots, S$$

$$v_s(\Theta_{s-1}) = \text{med}_i v_{is}(\Theta_{s-1}), \\ s = 1, 2, \dots, S$$

Note that $v_s(\Theta_{s-1})$ can be written

$$v_s(\Theta_{s-1}) = -(B_{11}^s)^{-1} B_{12}^s \Theta_{s-1} \\ + \text{med}_i \{x_{is} + (B_{11}^s)^{-1} B_{12}^s X_{is-1}\}.$$

Thus, using equations 14 and 15, $v_{is}(\Theta_{s-1})$ and $v_s(\Theta_{s-1})$ can be written

$$v_{is}(\Theta_{s-1}) = -(B_{11}^s)^{-1} B_{12}^s \Theta_{s-1} \\ + m_{is}, \forall i, s = 1, 2, \dots, S \quad (17)$$

$$v_s(\Theta_{s-1}) = -(B_{11}^s)^{-1} B_{12}^s \Theta_{s-1} \\ + m_s, s = 1, 2, \dots, S. \quad (18)$$

Let $\tilde{\theta}_0 \equiv 0$. Solve equations 17-18 forward sequentially where, at each $s > 0$,

$$\tilde{\theta}_{is} = v_{is}(\tilde{\Theta}_{s-1}), \tilde{\theta}_s = v_s(\tilde{\Theta}_{s-1}),$$

and, for $s \geq 0$,

$$\tilde{\Theta}_s = [\tilde{\theta}_s, \tilde{\theta}_{s-1}, \dots, \tilde{\theta}_1, \tilde{\theta}_0],$$

where $\tilde{\theta}_0 \equiv 0$. Then,

$$\tilde{\theta}_{is} = -(B_{11}^s)^{-1} B_{12}^s \tilde{\Theta}_{s-1} + m_{is} \quad (19) \\ s = 1, 2, \dots, S$$

$$\tilde{\theta}_s = -(B_{11}^s)^{-1} B_{12}^s \tilde{\Theta}_{s-1} + m_s. \quad (20) \\ s = 1, 2, \dots, S.$$

Define

$$V_{is}(\Theta_s) = -(\Theta_s - X_{is})' B^s (\Theta_s - X_{is}) \\ - M_{is}, s = 1, 2, \dots, S. \quad (21)$$

THEOREM 2. *Let voters have preferences given by equation 1 with $x(\phi_i, w) = x(\phi_i) = x_i$, where x_i is drawn randomly from a distribution $f(x)$. The form of the preference function and the distribution $f(x)$ are known to all voters. For each voter i , the vector of preference parameters x_i is private information. If issues are decided sequentially by the voting procedure we have adopted, an equilibrium exists. The equilibrium sequence of individual votes and the sequence of vote outcomes are given by equations 19 and 20. When voting on issue s with arbi-*

rary history Θ_{s-1} , if voter i expects all voters to pursue their equilibrium strategies in the future, i 's utility over outcomes on the current issue, θ_s , is given by equation 21 and i 's voting strategy is given by equations 17-18.

Proof. The proof is by backward induction. Since $M_{is} = 0$ and $B^s = A$, i 's utility over outcomes on issue S , given history Θ_{S-1} , is given by $V_{is}(\Theta_S)$. From Lemma 2 and equations 17-18 and 21, this can be rewritten

$$\begin{aligned}
 & -[\theta_s - v_{is}(\Theta_{S-1})]B_{11}^s[\theta_s - v_{is}(\Theta_{S-1})] \\
 & - (\Theta_{S-1} - X_{is-1})(B_{22}^s)^{-1} \\
 & (\Theta_{S-1} - X_{is-1}) - M_{is}. \tag{22}
 \end{aligned}$$

The first expression in equation 22 is single-peaked in θ_s , and the remaining terms do not depend on θ_s . By the corollary to Theorem 1, voter i has a dominant strategy when voting on issue S , namely, the strategy that maximizes equation 22 with respect to θ_s . The maximum is given by $v_{is}(\Theta_{S-1})$, defined in equation 17.

Continuing the induction argument, let voter i 's utility function at issue $s < S$ for arbitrary history Θ_{s-1} be $V_{is}(\Theta_{s-1})$. Using Lemma 2, rewrite this function as

$$\begin{aligned}
 & -[\theta_s - v_{is}(\Theta_{s-1})]B_{11}^s[\theta_s - v_{is}(\Theta_{s-1})] \\
 & - (\Theta_{s-1} - X_{is-1})(B_{22}^s)^{-1} \\
 & (\Theta_{s-1} - X_{is-1}) - M_{is}. \tag{23}
 \end{aligned}$$

The first term in equation 23 is single-peaked in θ_s . The second term is a constant known to i at the time that the vote on issue s occurs. The third term, M_{is} , is defined in equations 14-18. When choosing a vote on issue s , i wishes to choose a vote to maximize the expected value of equation 23 conditional on i 's information set, which includes the past history of votes, Θ_{s-1} , i 's own ideal point X_{is} , and any other information that may have been revealed to i prior to submission of the vote on issue s . Let I_{is} denote this information set. Now $E(M_{is} | I_{is})$ does not

depend on i 's vote on issue s . That is, knowledge by i of his or her own vote on issue s reveals no information about M_{is} not already provided by knowledge of I_{is} . Hence, only the first term in equation 23 is a function of i 's vote on issue s . Since the first term is single-peaked in θ_s , Lemma 1 applies, and i chooses the vote that maximizes the first term in equation 23 with respect to θ_s . Hence, i 's vote is given by equation 17, and the vote outcome on issue s by equation 18.

Substituting the vote outcome $v_s(\Theta_{s-1})$ for θ_s in equation 23 and noting that

$$v_s(\Theta_{s-1}) - v_{is}(\Theta_{s-1}) = m_s - m_{is},$$

we obtain that i 's induced utility function over Θ_{s-1} is $V_{is-1}(\Theta_{s-1})$ as given in equation 21. This completes the induction argument.

The claim that equations 19-20 give the equilibrium sequence of votes and vote outcomes then follows by solving equations 17-18 forward sequentially as was done in defining equations 19-20. QED

Remarks

The equilibrium characterized in the theorem is the unique backward induction equilibrium.

The proof can be extended to the case where there is a common shock, ω , in the following way. Replace x_i in the theorem with $x(\phi_i, \omega)$. Interpret the expectation operator at each step of the proof as the expectation conditional on elements of the vector $\omega = (\omega_1, \omega_2, \dots, \omega_T)$ that have been revealed to date. The steps of the theorem can simply be repeated with these changes.

The discussion indicates why the proof remains valid when a common shock is introduced. However, the introduction of a common shock raises an interesting issue. All voters know that common shocks will occur before the sequence of votes. They know that the realizations of those shocks will affect their preferences

over voting choices. One strategy that voters might adopt is to propose policies that are contingent on the realizations of the common shocks. There are many programs that have such contingencies. Social security benefits are indexed for inflation, expenditures for unemployment compensation are contingent on the number of people who are unemployed, disaster relief expenditures are contingent on the occurrence of disasters, and so on. An interesting problem for future research is to investigate the possibility of introducing contingent policies as objects of voting in a sequential voting framework.

In the model we have considered thus far (with or without the common shock to preferences), voters have differing forecasts of future outcomes because they have private information about their own preferences that may not be available to others. If voters have private information about the distribution of other voters' preferences based on private opinion polls, eavesdropping in legislative cloak-rooms, and so on, this would provide a further reason for heterogeneity of voter forecasts. However, the results of the theorem would still hold. In the proof, expectations for each voter are taken to be conditional on the information held by that voter. If voters have private information about other voters' forecasts, the coefficients on past outcomes in voters' forecasting rules would still be common across voters, while the distribution of voter forecasts would continue to differ across voters.

In deriving our results, we have assumed that voters have imperfect knowledge of voters' preferences but that they know the distribution $f(\bullet)$, from which ideal points are drawn. Models in which agents' subjective beliefs about relevant probability distributions are assumed to conform to the objective distributions were first introduced by Muth (1961). He proposed calling expectations formed using this assumption "rational

expectations." McKelvey and Ordeshook's (1985a, 1985b) investigations of formation of voter expectations also uses the rational expectations approach.

The rational expectations approach implies that an outside observer who wants to study the behavior of voters can use the actual distributions of relevant random variables to predict voters' forecasts. In this way, a link is provided from the theory to actual voter forecasts. This is not the only way that such a link can be made. However, as we argued in the introduction, it is essential that some link be made if the model is to generate implications regarding voter forecasts that may potentially be tested. The rational expectations approach has proven to be a fruitful approach to generating testable implications in other contexts (Lucas and Sargent 1981).

For an equilibrium to exist, however, it is not necessary to invoke the assumption of rational expectations. The proof is valid as long as voters know that the salience matrix (A in our model) in other voters' preference functions is the same as their own. It is not necessary that voters' subjective assessments of the distribution of ideal points be the objective distribution, voters may make systematic errors in predicting outcomes on future votes. However, it is still the case in our model that it is a dominant strategy for each voter to vote his or her ideal point on each issue. Thus, in the case where there is no common shock to preferences, systematic errors in voter forecasts will not affect the sequence of outcomes. If there are common shocks to preferences and voters' subjective assessments of the distribution of the common shock differ from the actual distribution, an equilibrium will exist; but the sequence of outcomes will, in general, be different from the sequence that emerges when the distribution of shocks is common knowledge.

An important assumption that we have made is that the salience matrix, A , in

voter preferences (equation 1) does not differ across voters. This assumption is not easily relaxed. To see why, suppose that matrix A is indexed by i . When the A matrix is indexed by i , the simplification obtained by writing equations 17–18 as a linear function of past outcomes with weights common to all voters is no longer available. This is a critical step in the proof. Problems of nonexistence of equilibrium in Denzau and Mackay's (1981) analysis of the perfect forecast case arise only when the salience matrix A differs across voters. Thus, variation of the A matrix across voters continues to pose a problem for sequential voting models with endogenous voter forecasts. Finding a way to relax the assumption that all voters have the same salience weights is an important issue for future research.

Our model has the property that voters have a dominant strategy on each vote, and our analysis relies heavily on this property. It would be interesting to investigate different voting games, in particular when dominant strategies are not present.

Conclusion

Building on the work of Denzau and MacKay (1981), Enelow and Hinich (1983), and Enelow (1984), we have presented a strategy for deriving endogenous voter forecasts when sequential voting occurs under conditions of uncertainty. Our approach assumes that voters use all available information in making their forecasts. In addition, we assume that voters know the distribution from which preference parameters are drawn. Voters then deduce the distribution of outcomes from their knowledge of this distribution and their knowledge of how decisions are made. Hence, if the voting sequence were repeated with a new draw from the distribution of voters for each sequence, the ex post distribution of outcomes would be the distribution voters use in making their

forecasts. It is in this sense that voters in our models do not make systematic errors.

It is natural to be somewhat skeptical about voters' ability to gather and process the information required to behave in the fashion assumed in our derivation of endogenous forecasts. Indeed, this issue arises in most rational expectations models. One response to this criticism has been the argument that voters behave "as if" they are following the strategy that we characterize in our analysis, whether they perform detailed computations or not. Attention has recently been devoted to the question of assessing whether individuals using simple forecasting strategies may "learn" the forecasting rules that emerge in the rational expectations formulation (Marcet and Sargent 1989). This is one avenue for addressing the concern that an analysis such as ours presumes too much of voters. An alternative strategy is to attempt to characterize how the information that voters have or use falls short of that assumed in our analysis. As we indicated, an equilibrium will exist in our model in cases where voters' beliefs about the relevant probability distributions do not conform to the actual distributions; but their forecasts will be systematically in error and the sequence of outcomes may differ from that in the case where the actual distribution are known. Thus, departing from the rational expectations assumption does not pose a challenge for proving existence of equilibrium but rather for deriving potentially testable propositions from the theory.

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