When Information Improves Information Security

Jens Grossklags   Benjamin Johnson   Nicolas Christin

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CyLab
Carnegie Mellon University
Pittsburgh, PA 15213

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(Extended version)

Jens Grossklags[a] *Benjamin Johnson*[b] *Nicolas Christin*[c]

[a]School of Information
University of California, Berkeley
Berkeley, CA 94720
jensg@ischool.berkeley.edu

[b]CyLab
Carnegie Mellon University
5000 Forbes Avenue
Pittsburgh, PA 15213
johnsonb@andrew.cmu.edu

[c]Information Networking Institute and CyLab
Carnegie Mellon University
5000 Forbes Avenue
Pittsburgh, PA 15213
nicolasc@andrew.cmu.edu

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Abstract

We investigate a mixed economy of an individual rational expert and several naïve near-sighted agents in the context of security decision making. Agents select between three canonical security actions to navigate the complex security risks of weakest-link, best shot and total effort interdependencies. We further study the impact of two information conditions on agents' choices. We provide a detailed overview of a methodology to effectively determine and compare strategies and payoffs between the different regimes. To analyze the impact of the different information conditions we propose a new formalization. We define the *price of uncertainty* as the ratio of the expected payoff in the complete information environment over the payoff in the incomplete information environment.

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1 Introduction

Users frequently fail to deploy, or upgrade security technologies, or to carefully preserve and backup their valuable data [20, 27], which leads to considerable monetary losses to both individuals and corporations every year. This state of affairs can be partly attributed to economic considerations. End users may undertake a cost-benefit analysis and decide for or against certain security actions [13, 31]. However, this risk management explanation overemphasizes the rationality of the involved consumers [19]. In practice, consumers face the task to "prevent security breaches within systems that sometimes exceed their level of understanding" [5]. In other words, the amount of information users may be able to acquire and/or to process, is much more limited than is required for a fully rational choice.

We focus on decision-making in different security scenarios that pose significant challenges for average users to determine optimal security strategies, due to *interdependencies* between users [16, 23]. Interdependencies occur when the actions of a given user have an effect on the rest of the network, in part or as a whole (externalities), or when the status of a given user impacts that of other users. For example, consumers who open and respond to unsolicited advertisements increase the load of spam for all participants in the network. Similarly, choosing a weak password for a corporate VPN system can facilitate compromise of many user accounts.

We anticipate the vast majority of users to be *non-expert*, and to apply approximate decision-rules that fail to accurately appreciate the impact of their decisions on others [2]. In particular, in this paper, we assume non-expert users to conduct a simple self-centered cost-benefit analysis, and to neglect interdependencies. Such users would secure their system only if the vulnerabilities being exploited can cause a direct annoyance to them (e.g., their machine becomes completely unusable), but would not act when they cannot perceive or understand the effects of their insecure behavior (e.g., when their machine is used as a relay to send moderate amounts of spam to third parties).

In contrast, an advanced, or expert user fully comprehends to which extent their and others' security choices affect the network as a whole, and responds rationally. The first contribution of this paper is to study the strategic optimization behavior of such an expert user in an economy of inexperienced end users, using three canonical security games that account for network effects [16].

The second contribution of this paper is to address how the security choices by users are mediated by the information available on the severity of the threats the network faces. We assume that each individual faces a randomly drawn probability of being subject to a direct attack. We study how the decisions of the expert user differ if all draws are common knowledge, compared to a scenario where this information is only privately known. With this approach we provide two important baseline cases for the impact of the expert agent. We further propose a metric to quantify the differences in the total expected payoff between the two information conditions, which we term the "*price of uncertainty*," per analogy with Koutsoupias and Papadimitriou's "price of anarchy" [22].

By evaluating the price of uncertainty for a range of parameters in different security scenarios, we can determine which configurations can accommodate limited information environments (i.e., when being less informed does not significantly jeopardize an expert user's payoff), as opposed to configurations where expert users and non-expert users achieve similar outcomes due to a lack of available information.

We first discuss selected work related to our analytic model (Section 2). In Section 3, we summarize the security games framework we developed in prior work, and detail our assumptions about agent behaviors and information conditions. We present our methodology and formal analysis in Section 4. We discuss the results and their implications in Section 5. Finally, we close with concluding remarks in Section 6.

2 Related work

In our prior work we have reviewed the research area of the economics of security in depth [16]. In this paper we conduct a decision-theoretic analysis for a sophisticated (expert) agent who interacts with a group of users that follow a simple but reasonable rule-of-thumb strategy. Our work significantly differs from prior decision-theoretic approaches. Gordon and Loeb present a model that highlights the trade-off between perfect and cost-effective security [14]. They consider the protection of an information set that has an associated loss if compromised, probability of attack, and probability that attack is successful. They show that an optimizing firm will not always defend highly vulnerable data, and only invest a fraction of the expected loss. Cavusoglu *et al.* [7] consider the decision-making problem of a firm when attack probabilities are externally given compared to a scenario when the attacker is explicitly modeled as a strategic player in a game-theoretic framework. Their model shows that if the firm assumes that the attacker strategically responds then in most considered cases its profit will increase. Schechter and Smith [30] consider the decision-theoretic analysis from the perspective of the potential intruder. They highlight several modeling alternatives for attacker behavior and their payoff consequences. The analytic work on security investments and level of penalties for offenses is complemented by empirical research [29, 33].

We structure the remainder of the review of related literature and background information into three selected areas in which we are making a research contribution.

2.1 Bounded rationality

Acquisti and Grossklags summarize work in the area of behavioral economics and psychology that is of relevance for privacy and security decision-making [2]. Users' decisions are not only limited by cognitive and computational restrictions (i.e., bounded rationality), but are also influenced by systematic psychological deviations from rationality.

Recent research has investigated agents that overemphasize earlier costs and benefits at the expense of their future well-being [1, 3]. Christin *et al.* suggest that agents respond near-rationally to the complexity of networked systems [8]. In their model individuals are satisfied with a payoff within a small margin of the optimal outcome.

Different from the above work that considers all players to act the same, the current paper studies a mixed economy, with expert and non-expert users co-existing. While expert users are as rational as possible, non-expert users deviate from rationality by adopting approximate (rules-of-thumb) decision strategies. In practice, users frequently have to rely on rules-of-thumb when a ”quantitative method to measure security levels” is not available [26]. Economic analysis including rule-of-thumb choices have been discussed outside of the security context, e.g., [10, 11, 24].

2.2 Limited information

In the context of the value of security information, research has been mostly concerned with incentives for sharing and disclosure. Several models investigate under which conditions organizations are willing to contribute to an information pool about security breaches and investments when competitive effects may result from this cooperation [12, 15]. Empirical papers explore the impact of mandated disclosures [6] or publication of software vulnerabilities [34] on the financial market value of corporations. Other contributions to the security field include computation of Bayesian Nash outcomes for an intrusion detection game [25], and security patrol versus robber avoidance scenarios [28].

We conduct a comparative analysis of strategies and payoffs for a sophisticated agent in a security model when the likelihood of a directed attack is either common or private knowledge. In particular, we evaluate the influence of the lack of information given different organizational dependencies [35].

2.3 Heterogeneous agents

In our previous work we have analyzed both the case of homogeneous [16] and heterogeneous agents [17]. When considering heterogeneous agents, however, we have focused on differences in the costs agents may face. We assumed that users differ in the price they have to pay for protection and self-insurance, and that they have different perceived or actual losses associated with successful (uninsured) security compromises. In this paper we analyze the case of agents facing different attack probabilities. Indeed in practice, different targets, even if they are part of a same network, are not all equally attractive to an attacker: a computer containing payroll information is for instance, considerably more valuable than an old “boat anchor” sitting under an intern’s desk.

Given certain differences in the attractiveness of a particular target the question remains how a defender is able to determine a reasonable estimate of the attack probability. Such a problem far exceeds the scope of this paper, whose main goal is to study the impact of information (or lack thereof) on security strategies, and we refer the reader to the threat modeling literature. (See [4] for an introduction and references.)

3 Decision Theoretic Model

We next summarize the security games we analyze, and extend models we previously proposed [16] to the case of an economy consisting of an expert user and several unsophisticated users.

3.1 Basic model

Self-protection and self-insurance. In practice, the arsenal of a defender may include several actions to prevent successful compromises and to limit losses that result from a breach. In Grossklags *et al.* [16] we provide a model that allows a decoupling of investments in the context of computer security. On the one hand, the perimeter can be strengthened with a higher self-protection investment (e.g., implementing or updating a firewall). On the other hand, the amount of losses can be reduced by introducing self-insurance technologies and practices (e.g., backup provisions). Formally, player i chooses an insurance level $0 \leq s_i \leq 1$ and a protection level $0 \leq e_i \leq 1$. $b \geq 0$ and $c \geq 0$ denote the unit cost of protection and insurance, respectively, which are homogeneous for the agent population. So, player i pays be_i for self-protection and cs_i for insurance.

Interdependency. We focus in this work on tightly coupled networks [35].[1] In a tightly coupled network all defenders will face a loss if the condition of a security breach is fulfilled whereas in a loosely coupled network consequences may differ for network participants. We denote H as a "contribution" function that characterizes the effect of e_i on agent's utility U_i, subject to the protection levels chosen (contributed) by *all* other players. We require that H be defined for all values over $(0, 1)^N$. We distinguish three canonical cases that we discussed in-depth in prior work [16]:

- Weakest-link: $H = \min(e_i, e_{-i})$.
- Best shot: $H = \max(e_i, e_{-i})$.
- Total effort: $H = \frac{1}{N}\sum_k e_k$.

where, following common notation, e_{-i} denotes the set of protection levels chosen by players other than i.

Attack probabilities, network size and endowment. Each of $N \in \mathbb{N}$ agents receives an endowment M. If she is attacked and compromised successfully she faces a loss L. We assume that each agent i draws an individual attack probability p_i ($0 \leq p_i \leq 1$) from a uniform random distribution. This models the heterogeneous preferences that attackers have for different targets, due to their economic, political, or reputational agenda. The choice of a uniform distribution ensures the analysis remains tractable, while already providing numerous insights. We conjecture that different distributions (e.g., power law) may also be appropriate in practice.

3.2 Player behavior

At the core of our analysis is the observation that expert and non-expert users differ in their understanding of the complexity of networked systems. Indeed, consumers' knowledge about risks and means of protection

[1] There is an ongoing debate whether researchers should assume full connectivity of a network graph given modern computer security threats such as worms and viruses. (Personal communication with Nicholas Weaver, ICSI.)

with respect to privacy and security can be quite varied [2], and field surveys separate between high and low expertise users [32].

Naïve (non-expert) user. Average users underappreciate the interdependency of network security goals and threats [2, 32]. We model the *perceived* utility of each naïve agent to only depend on the direct security threat and the individual investment in self-protection and self-insurance. The investment levels of other players are *not* considered in the naïve user's decision making, despite the existence of interdependencies. We define the perceived utility for a specific naïve agent j as:

$$PU_j = M - p_j L(1 - s_j)(1 - e_j) - be_j - cs_j \ .$$

Clearly, perceived and realized utility actually differ: by failing to incorporate the interdependencies of all agents' investment levels in their analysis, naïve users may achieve sub-optimal payoffs that actually are far below their own expectations. This paper does not aim to resolve this conflict, and, in fact, there is little evidence that users will learn the complexity of network security over time [32]. We argue that non-expert users would repeatedly act in an inconsistent fashion. This hypothesis is supported by findings in behavioral economics that consumers repeatedly deviate from rationality, however, in the same predictable ways [21].

Sophisticated (expert) user. Advanced users can rely on their superior technical and structural understanding of computer security threats and defense mechanisms, to analyze and respond to changes in the environment [9]. In the present context, expert users, for example, have less difficulty to conclude that the goal to avoid censorship points at a best shot scenario, whereas the protection of a corporate network frequently suggests a weakest-link optimization problem [16]. Accordingly, a sophisticated user correctly understands her utility to be dependent on the interdependencies that exist in the network:

$$U_i = M - p_i L(1 - s_i)(1 - H(e_i, e_{-i})) - be_i - cs_i \ .$$

Binary strategies. We further restrict the actions available to each player (of either type) to make the analysis tractable. Instead of picking a continuous protection level $0 \leq e_i \leq 1$, players only have the choice between $e_i = 0$ ("do not protect") or $e_i = 1$ ("protect") only. Likewise, the parameter space for s_i is restricted to a binary choice $s_i \in \{0, 1\}$. While this may seem a very strong restriction, prior analysis [16, 17] showed that, for all models we look at, Nash equilibria are all of the form $(e_i, s_i) \in \{(0, 0), (0, 1), (1, 0)\}$ (respectively, "passivity," "full insurance," and "full protection") for all players i, even when players can choose e_i and s_i from a continuous spectrum of values. As such, binary choices for e_i and s_i are not necessarily a poor approximation of actual behavior.

3.3 Information conditions

Our analysis is focused on the decision making of the expert user subject to the bounded rational behaviors of the naïve network participants. That is, more precisely, the expert agent maximizes their expected utility subject to the available information about other agents' drawn threat probabilities and their resulting actions. Two different information conditions may be available to the expert agent:

Complete information: Actual draws of attack probabilities p_j for all $j \neq i$, and her own drawn probability of being attacked p_i.

Incomplete information: Known probability distribution of the unsophisticated users' attack threat, and her own drawn probability of being attacked p_i.

Therefore, the expert agent can accurately infer what each agent's investment levels are in the complete information scenario. Under incomplete information the sophisticated user has to develop an expectation about the actions of the naïve users.

4 Analysis methodology

In the remainder of this discussion, we will always use the index i to denote the expert player, and $j \neq i$ to denote the naïve players. For each of the three games, weakest-link, best shot, and total effort, our analysis proceeds via the following five-step procedure.

1. Determine player i's payoff within the game for selected strategies of passivity, full insurance, and full protection. As shown in [16, 17] through a relatively simple analysis, player i can maximize their utility only by relying on (one or more of) these three strategies.

2. Determine the conditions on the game's parameters (b, c, L, N, p_i, and if applicable, p_j for $j \neq i$) under which player i should select each strategy.

3. Determine additional conditions on the game's parameters such that the probability (relative to p_i) of each case, as well as the expected value of p_i within each case can be easily computed.

4. Determine player i's total expected payoff relative to the distribution on p_i and all other known parameters.

5. In the case of complete information, eliminate dependence on p_j for $j \neq i$ by taking, within each parameter case, an appropriate expected value.

Diligent application of this method generates a table recording the total expected payoffs for player i, given any valid assignment to the parameters b, c, L, N. In the process it also generates tables of selection conditions, probabilities, and expected payoffs for each possible strategy; and in the complete information case, gives results for total expected payoffs conditioned on the exact draws of p_j by the other players. The results are presented in Tables 1–15.

In the remainder of this section we illustrate this method by considering, for each step listed above, one game and one parameter case for which we have applied the appropriate step.

Step 1 example: Passivity payoff computation. Let us consider the challenge of determining payoffs for player i's passivity in the best shot game, under the conditions of limited information and parameter constraints $b \leq c$. The general payoff function for the best shot game is obtained by substituting $H(e_i, e_{-i}) = \max(e_i, e_{-i})$ into the general utility function for all games, i.e. $U(i) = M - p_i L(1-s_i)(1 - H(e_i, e_{-i})) - be_i - cs_i$. Doing this, we obtain $U(i) = M - p_i L(1-s_i)(1 - \max(e_i, e_{-i}) - be_i - cs_i$. To get the payoff for player i's passivity we plug in $e_i = s_i = 0$ to obtain

$$U_i = \begin{cases} M - p_i L, \text{ if } \max_{j \neq i} e_j = 0 \\ M, \text{ if } \max_{j \neq i} e_j = 1 \end{cases}.$$

Now in the incomplete information case, we do not know any of the p_j for $j \neq i$, so we do not know all the parameters to compute the required payoff. However, since we assume that the p_j are independently and uniformly distributed in $[0, 1]$, we can compute an expected value for this payoff as follows. The probability (over p_j) that none of the other players protect (i.e. that $\max_{j \neq i} p_j < b/L$) is exactly $(b/L)^{N-1}$, and in this case the payoff would be $M - p_i L$. The probability (over p_i) that at least one of the other players protect (i.e. that $b/L \leq \max_{j \neq i} p_j$) is exactly $1 - (b/L)^{N-1}$, and in this case the payoff would be M. Thus the total expected payoff for selecting the passivity strategy is $(b/L)^{N-1}(M - p_i L) + (1 - (b/L)^{N-1})M$, which simplifies to $M - p_i L(b/L)^{N-1}$. We record this as the payoff result for passivity in the incomplete game, with $b \leq c$, as can be seen in Table 6.

Step 2 example: Strategy selection. Let us next consider the challenge of determining parameter conditions under which we should select player i's strategy in the weakest link game. Since this is a second step, consider the game payoffs in Table 1 as given. We are interested in determining player i's most strategic play for any given parameter case. Select for consideration the case $b \leq c$ with incomplete information. (Note: this is the most difficult case for this game).

To determine the optimal strategy for player i, we must select the maximum of the quantities Passivity: $M - p_i L$, Insurance: $M - c$, and Protection: $M - b - p_i L(1 - (1 - b/L)^{N-1})$. We should choose passivity if it is better than insurance or protection, i.e. $M - p_i L > M - c$ and $M - p_i L > M - b - p_i L(1 - (1 - b/L)^{N-1})$. We should choose insurance if it is better than passivity or protection, i.e. $M - c \geq M - p_i L$ and $M - c > M - b - p_i L(1 - (1 - b/L)^{N-1})$. We should choose protection if it is better than passivity or insurance, i.e. $M - b - p_i L(1 - (1 - b/L)^{N-1}) \geq M - p_i L$ and $M - b - p_i L(1 - (1 - b/L)^{N-1}) \geq M - c$.

Re-writing the above inequalities as linear constraints on p_i , we choose passivity if $p_i \leq c/L$ and $p_i \leq \frac{b}{L(1-(1-b/L)^{N-1})}$; we choose insurance if $p_i > c/L$ and $p_i > \frac{c-b}{L(1-(1-b/L)^{N-1})}$; and we choose protection if $\frac{c-b}{L(1-(1-b/L)^{N-1})} \leq p_i \leq \frac{b}{L(1-(1-b/L)^{N-1})}$.

For simplicity of computation, we would like to have our decision mechanism involve only a single inequality constraint on p_i. To obtain this it is necessary and sufficient to determine the ordering of the three terms: $\frac{c}{L}$, $\frac{b}{L(1-(1-b/L)^{N-1})}$, and $\frac{c-b}{L(1-(1-b/L)^{N-1})}$.

It turns out that there are only two possible orderings for these three terms. The single inequality $c < \frac{b}{(1-b/L)^{N-1}}$ determines the ordering: $\frac{c}{L} < \frac{c-b}{L(1-(1-b/L)^{N-1})} < \frac{b}{L(1-(1-b/L)^{N-1})}$; while the reverse inequality

$\frac{b}{(1-b/L)^{N-1}} \leq c$ determines the reverse ordering on all three terms. This observation suggests we should add sub-cases under $b \leq c$ depending on which of these two inequalities holds. See Table 2.

Within each new sub-case the criterion for selecting the strategy that gives the highest payoff can now be represented by a single linear inequality on p_i. If $c \leq \frac{b}{(1-b/L)^{N-1}}$, then passivity wins so long as $p_i < c/L$; (because the new case conditions also guarantee $p_i < \frac{b}{L(1-b/L)^{N-1}}$). Similarly insurance wins if $p_i \geq c/L$. Protection never wins in this case because we cannot have $\frac{c-b}{L\left(1-(1-b/L)^{N-1}\right)} \leq p_i \leq \frac{b}{L\left(1-(1-b/L)^{N-1}\right)}$ when we also have $\frac{b}{(1-b/L)^{N-1}} < \frac{c-b}{L\left(1-(1-b/L)^{N-1}\right)}$. The computations for the case $\frac{b}{(1-b/L)^{N-1}} < c$ are similar; the results are recorded in Table 2.

Step 3 example: Case determination. Now, consider the challenge of determining additional constraints on parameters in the total effort game, so that in any given case, the total payoffs can be represented by simple closed form functions of the game's parameters. Since this is a third step, we assume the second step has been diligently carried out and consider the strategy conditions given in Table 12 as given. For brevity, we consider only the incomplete information case under the assumption $b \leq c$.

To illustrate the problem we are about to face, consider the condition for selecting passivity in the incomplete game and case: $b + b^2(N-1)/L < c$. The condition here is that $p_i < bN/L$. This condition is possible if and only if $bN < L$. The case conditions determined thus far do not specify which of these is the case; so for subsequent computations, we will need to know which it is, and therefore must consider the two cases separately.

Going beyond this particular example, there are several other values in this table where a similar phenomenon occurs. In particular, we need new cases to determine whether each of the following relations holds: $bN/L \leq 1$, $\frac{c}{b+(L-b)/N} \leq 1$, and $\frac{c-b}{b-b/N} \leq 1$. (See Table 12). To combine these with previous cases in a way that avoids redundancy, we rewrite the conditions involving c as linear inequalities on c; obtaining $c \leq b + (L-b)/N$ and $c \leq 2b - b/N$.

We are thus left to reconcile these additional cases with the current cases $b \leq c \leq b + \frac{b^2}{L}(N-1)$ and $b + \frac{b^2}{L}(N-1) < c$. To do this efficiently we must know the order of the terms $b + \frac{L-b}{N}$, $2b - \frac{b}{N}$, and $b + \frac{b^2}{L}(N-1)$. Fortunately, it turns out that there are only two possible orderings on these terms; and furthermore, which of the two orderings it is depends on the relation $bN < L$ which we already needed to specify as part of our case distinctions. If $bN \leq L$, then $b + \frac{b^2}{L}(N-1) \leq 2b - \frac{b}{N} \leq b + \frac{L-b}{N}$ and if $bL > N$, then the reverse relations hold.

Assuming limited information, $b \leq c$, and dividing all cases according to $bN \leq L$, it requires a total of 5 cases to determine all important relationships among important parameters for this game. We may have $bN \leq L$ and $b \leq c \leq b + \frac{b^2}{L}(N-1)$; $bN \leq L$ and $b + \frac{b^2}{L}(N-1) < c < 2b - \frac{b}{N}$; $bN \leq L$ and$2b - \frac{b}{N} \leq c$; $bN > L$ and $c \leq b + \frac{L-b}{N}$; and $bN > L$ and $b + \frac{L-b}{N} < c$. For reference, see table 15.

Step 4 example: Total payoff computation. Let us determine the total expected payoff for the expert player with incomplete information in the best shot game with $b \leq c$. As intermediate steps we must compute the probability that each strategy is played, along with the expected payoff for each strategy. The total payoff is then given by (Probability of passivity $\cdot$ Expected payoff for passivity) + (Probability of insurance $\cdot$ Expected payoff for insurance) + (Probability of protection $\cdot$ Expected payoff for protection).

The expected probability of passivity in this case is 1, with a payoff of $M - p_i L(b/L)^{N-1}$. To get an expected payoff, we compute the expected value of p_i within this case. Since there is no constraint on p_i and it is drawn from a uniform distribution its expected value is $1/2$. Thus the expected payoff for this case is $M - (L/2)(b/L)^{N-1}$. The total expected payoff is thus $M - (L/2)(b/L)^{N-1}$.

Step 5 example: Eliminating dependencies on other players. Consider the challenge of examining the total expected payoff for player i, who has complete information, and rewriting this payoff in a way that is still meaningful as an expected payoff, but does not depend on any p_j for $j \neq i$. The reason we want to do this last step is so we can compare complete information payoff results with incomplete information payoff results. We can only do this if the direct dependence on privileged information is removed from the complete information case payoff. Our method of information removal involves taking an appropriate expected value.

For this example we consider the best shot game with complete information in the case $b \leq c$. Since this is a fifth step, we should assume that the fourth step – computing the expected payoff for player i as a function of parameters that may include p_j for $j \neq i$ – has been accomplished.

Indeed, by following steps 1–4, the total expected payoffs for player i (conditioned on other players) in the case $b \leq c$ can be derived, subject to two additional sub-cases. If $\max_{j \neq i} p_j \leq b/L$, then the expected payoff is $M - c + c^2/L$; while if $b/L < \max_{j \neq i} p_j$, then the expected payoff is $M - b + b^2/L$.

To generate an appropriate "a posteriori" expected payoff over all choices of p_j, we compute the probability (over choice of p_j) that we are in case $\max_{j \neq i} p_j \leq b/L$ times the payoff for that case, plus the probability (over p_j) that we are in the case $b/L < \max_{j \neq i} p_j$ times the payoff for that case. We obtain $(b/L)^{N-1} \cdot \left[M - c + c^2/L\right] + \left[1 - (b/L)^{N-1}\right] \cdot \left[M - b + b^2/L\right]$. The end result is $M - b\left(1 - b/2L\right)(b/L)^{N-1}$. See Table 10.

Table 1: Weakest link security game: Payoffs for different strategies under different information conditions

Case	Information Type	Payoff Passivity	Payoff Self-Insurance	Payoff Protection
$c < b$	Complete	$M - p_i L$	$M - c$	$M - b - p_i L$
$b \leq c$ and $\min_{j \neq i} p_j < \frac{b}{L}$	Complete	$M - p_i L$	$M - c$	$M - b - p_i L$
$b \leq c$ and $\frac{b}{L} \leq \min_{j \neq i} p_j$	Complete	$M - p_i L$	$M - c$	$M - b$
$c < b$	Incomplete	$M - p_i L$	$M - c$	$M - b - p_i L$
$b \leq c$	Incomplete	$M - p_i L$	$M - c$	$M - b - p_i L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right)$

Table 2: Weakest link security game: Conditions to select protection, self-insurance or passivity strategies

Case	Information Type	Conditions Passivity	Conditions Self-Insurance	Conditions Protection
$c < b$	Complete	$p_i < \frac{c}{L}$	$p_i \geq \frac{c}{L}$	NEVER!
$b \leq c$ and $\min_{j \neq i} p_j < \frac{b}{L}$	Complete	$p_i < \frac{c}{L}$	$p_i \geq \frac{c}{L}$	NEVER!
$b \leq c$ and $\frac{b}{L} \leq \min_{j \neq i} p_j$	Complete	$p_i < \frac{b}{L}$	NEVER!	$p_i \geq \frac{b}{L}$
$c < b$	Incomplete	$p_i < \frac{c}{L}$	$p_i > \frac{c}{L}$	NEVER!
$b \leq c \leq \frac{b}{\left(1-\frac{b}{L}\right)^{N-1}}$	Incomplete	$p_i < \frac{c}{L}$	$p_i \geq \frac{c}{L}$	NEVER!
$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} < c$	Incomplete	$p_i < \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}$	$p_i > \frac{c-b}{L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}$	$\frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}} \leq p_i$, $p_i \leq \frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)}$

Table 3: Weakest link security game: Probabilities to select protection, self-insurance or passivity strategies

	Case	Information Type	Probability Passivity	Probability Self-Insurance	Probability Protection
WC1	$c < b$	Complete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
WC2a	$b \leq c$ and $\min_{j \neq i} p_j < \frac{b}{L}$	Complete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
WC2b	$b \leq c$ and $\frac{b}{L} \leq \min_{j \neq i} p_j$	Complete	$\frac{b}{L}$	0	$1 - \frac{b}{L}$
WI1	$c < b$	Incomplete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
WI2	$b \leq c \leq \frac{b}{\left(1-\frac{b}{L}\right)^{N-1}}$	Incomplete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
WI3	$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} < c$ and $c < b + L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right)$	Incomplete	$\frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}$	$1 - \frac{c-b}{L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}$	$\frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} - \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}$
WI4	$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} < c$ and $b + L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right) \leq c$	Incomplete	$\frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}$	0	$1 - \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}$

Table 4: Weakest link security game: Total expected game payoffs, conditioned on other players

	Case	Information Type	Total Expected Payoff for player i (conditioned on other players)
WC1	$c < b$	Complete	$M - c + \frac{c^2}{2L}$
WC2a	$b \le c$ and $\min_{j \neq i} p_j < \frac{b}{L}$	Complete	$M - c + \frac{c^2}{2L}$
WC2b	$b \le c$ and $\frac{b}{L} \le \min_{j \neq i} p_j$	Complete	$M - b + \frac{b^2}{2L}$
WI1	$c < b$	Incomplete	$M - c + \frac{c^2}{2L}$
WI2	$b \le c \le \frac{b}{\left(1-\frac{b}{L}\right)^{N-1}}$	Incomplete	$M - c + \frac{c^2}{2L}$
WI3	$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} <$ $c < b + L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right)$	Incomplete	$M - c + \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{N-1}} + \frac{(c-b)^2}{2L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}$
WI4	$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} <$ $b + L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right) \le c$	Incomplete	$M - b - \frac{L}{2}\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right) + \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{N-1}}$

Table 5: Weakest link security game: Total expected game payoffs, not conditioned on other players

	Case	Information Type	Total Expected Payoff for player i (not conditioned on other players)
WC1	$c < b$	Complete	$M - c + \frac{c^2}{2L}$
WC2	$b \le c$	Complete	$M - c + \frac{c^2}{2L} + (c-b)\left(1 - \frac{c+b}{2L}\right)\left(1 - \frac{b}{L}\right)^{N-1}$
WI1	$c < b$	Incomplete	$M - c + \frac{c^2}{2L}$
WI2	$b \le c \le \frac{b}{\left(1-\frac{b}{L}\right)^{N-1}}$	Incomplete	$M - c + \frac{c^2}{2L}$
WI3	$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} < c < b + L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right)$	Incomplete	$M - c + \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{N-1}} + \frac{(c-b)^2}{2L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}$
WI4	$\frac{b}{\left(1-\frac{b}{L}\right)^{N-1}} < b + L\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right) \le c$	Incomplete	$M - b - \frac{L}{2}\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right) + \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{N-1}}$
WN1	$c < b$	Naive	$M - c + \frac{c^2}{2}$
WN2	$b \le c$	Naive	$M - b + \frac{b^2}{2L} - \frac{L}{2}\left(1 - \frac{b^2}{L^2}\right)\left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right)$

Table 6: Best shot security game: Payoffs for different strategies under different information conditions

	Case	Information Type	Payoff Passivity	Payoff Self-Insurance	Payoff Protection
	$c < b$	Complete	$M - p_i L$	$M - c$	$M - b$
	$b \leq c$ and $\max_{j \neq i} p_j < \frac{b}{L}$	Complete	$M - p_i L$	$M - c$	$M - b$
	$b \leq c$ and $\frac{b}{L} \leq \max_{j \neq i} p_j$	Complete	M	$M - c$	$M - b$
	$c < b$	Incomplete	$M - p_i L$	$M - c$	$M - b$
	$b \leq c$	Incomplete	$M - p_i L \left(\frac{b}{L}\right)^{N-1}$	$M - c$	$M - b$

Table 7: Best shot security game: Conditions to select protection, self-insurance or passivity strategies

	Case	Information Type	Conditions Passivity	Conditions Self-Insurance	Conditions Protection
	$c < b$	Complete	$p_i < c/L$	$p_i \geq c/L$	NEVER!
	$b \leq c$ and $\max_{j \neq i} p_j < b/L$	Complete	$p_i < b/L$	NEVER!	$p_i \geq b/L$
	$b \leq c$ and $b/L \leq \max_{j \neq i} p_j$	Complete	ALWAYS!	NEVER!	NEVER!
	$c < b$	Incomplete	$p_i < c/L$	$p_i \geq c/L$	NEVER!
	$b \leq c$	Incomplete	ALWAYS!	NEVER!	NEVER!

Table 8: Best shot security game: Probabilities to select protection, self-insurance or passivity strategies

	Case	Information Type	Probability Passivity	Probability Self-Insurance	Probability Protection
BC1	$c < b$	Complete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
BC2a	$b \leq c$ and $\max_{j \neq i} p_j < \frac{b}{L}$	Complete	$\frac{b}{L}$	0	$1 - \frac{b}{L}$
BC2b	$b \leq c$ and $\frac{b}{L} \leq \max_{j \neq i} p_j$	Complete	1	0	0
BI1	$c < b$	Incomplete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
BI2	$b \leq c$	Incomplete	1	0	0

Table 9: Best shot security game: Total expected game payoffs, conditioned on other players

	Case	Information Type	Total Expected Payoff
BC1	$c < b$	Complete	$M - c + \frac{c^2}{2L}$
BC2a	$b \leq c$ and $\max_{j \neq i} p_j < \frac{b}{L}$	Complete	$M - b + \frac{b^2}{2L}$
BC2b	$b \leq c$ and $\frac{b}{L} \leq \max_{j \neq i} p_j$	Complete	M
BI1	$c < b$	Incomplete	$M - c + \frac{c^2}{2L}$
BI2	$b \leq c$	Incomplete	$M - \frac{L}{2} \left(\frac{b}{L}\right)^{N-1}$

Table 10: Best shot security game: Total expected game payoffs, not conditioned on other players

	Case	Information Type	Total Expected Payoff
BC1	$c < b$	Complete	$M - c + \frac{c^2}{2L}$
BC2	$b \leq c$	Complete	$M - b\left(1 - \frac{b}{2L}\right)\left(\frac{b}{L}\right)^{N-1}$
BI1	$c < b$	Incomplete	$M - c + \frac{c^2}{2L}$
BI2	$b \leq c$	Incomplete	$M - \frac{L}{2} \left(\frac{b}{L}\right)^{N-1}$
BN1	$c < b$	Naive	$M - c + \frac{c^2}{2}$
BN2	$b \leq c$	Naive	$M - b + \frac{b^2}{2L}$

Table 11: Total effort security game: Payoffs for different strategies under different information conditions

	Case	Information Type	Payoff Passivity	Payoff Self-Insurance	Payoff Protection
	$c < b$	Complete	$M - p_i L$	$M - c$	$M - b - p_i L\,(1 - 1/N)$
	$b \leq c$	Complete	$M - p_i L\,(1 - K/N)$	$M - c$	$M - b - p_i L\,(1 - (K+1)/N)$
	$c < b$	Incomplete	$M - p_i L$	$M - c$	$M - b - p_i L\,(1 - 1/N)$
	$b \leq c$	Incomplete	$M - p_i\,(b + (L - b)/N)$	$M - c$	$M - b - p_i\,(b - b/N)$

Table 12: Total effort security game: Conditions to select protection, self-insurance or passivity strategies

	Case	Information Type	Conditions Passivity	Conditions Self-Insurance	Conditions Protection
	$c < b$	Complete	$p_i < \frac{c}{L}$	$p_i \geq \frac{c}{L}$	NEVER!
	$b \leq c \leq b(N-K)$	Complete	$p_i < \frac{c}{L\left(1-\frac{K}{N}\right)}$	$p_i \geq \frac{c}{L\left(1-\frac{K}{N}\right)}$	NEVER!
	$b(N-K) < c$	Complete	$p_i < \frac{bN}{L}$	$p_i > \frac{c-b}{L\left(1-\frac{K+1}{N}\right)}$	$\frac{bN}{L} \leq p_i \leq \frac{c-b}{L\left(1-\frac{K+1}{N}\right)}$
	$c < b$	Incomplete	$p_i < \frac{c}{L}$	$p_i \geq \frac{c}{L}$	NEVER!
	$b \leq c \leq b + \frac{b^2}{L}(N-1)$	Incomplete	$p_i < \frac{c}{b+\frac{L-b}{N}}$	$p_i \geq \frac{c}{b+\frac{L-b}{N}}$	NEVER!
	$b + \frac{b^2}{L}(N-1) < c$	Incomplete	$p_i < \frac{bN}{L}$	$p_i > \frac{c-b}{b-\frac{b}{N}}$	$\frac{bN}{L} \leq p_i \leq \frac{c-b}{b-\frac{b}{N}}$

Table 13: Total effort security game: Probabilities to select protection, self-insurance or passivity strategies

	Case	Information Type	Probability Passivity	Probability Self-Insurance	Probability Protection
TC1	$c < b$	Complete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
TC2	$bN \leq L$ and $b \leq c \leq b(N-K)$	Complete	$\frac{c}{L\left(1-\frac{K}{N}\right)}$	$1 - \frac{c}{L\left(1-\frac{K}{N}\right)}$	0
TC3	$bN \leq L$ and $b(N-K) < c < b + L\left(1-\frac{K+1}{N}\right)$	Complete	$\frac{bN}{L}$	$1 - \frac{c-b}{L\left(1-\frac{K+1}{N}\right)}$	$\frac{c-b}{L\left(1-\frac{K+1}{N}\right)} - \frac{bN}{L}$
TC4	$bN \leq L$ and $b + L\left(1-\frac{K+1}{N}\right) \leq c$	Complete	$\frac{bN}{L}$	0	$1 - \frac{bN}{L}$
TC5	$L < bN$ and $b \leq c < L\left(1-\frac{K}{N}\right)$	Complete	$\frac{c}{L\left(1-\frac{K}{N}\right)}$	$1 - \frac{c}{L\left(1-\frac{K}{N}\right)}$	0
TC6	$L < bN$ and $L\left(1-\frac{K+1}{N}\right) < c$	Complete	1	0	0
TI1	$c < b$	Incomplete	$\frac{c}{L}$	$1 - \frac{c}{L}$	0
TI2	$bN \leq L$ and $b \leq c \leq b + \frac{b^2}{L}(N-1)$	Incomplete	$\frac{c}{b+\frac{L-b}{N}}$	$1 - \frac{c}{b+\frac{L-b}{N}}$	0
TI3	$bN \leq L$ and $b + \frac{b^2}{L}(N-1) < c < 2b - \frac{b}{N}$	Incomplete	$\frac{bN}{L}$	$1 - \frac{c-b}{b-\frac{b}{N}}$	$\frac{c-b}{b-\frac{b}{N}} - \frac{bN}{L}$
TI4	$bN \leq L$ and $2b - \frac{b}{N} \leq c$	Incomplete	$\frac{bN}{L}$	0	$1 - \frac{bN}{L}$
TI5	$L < bN$ and $b \leq c < b + \frac{L-b}{N}$	Incomplete	$\frac{c}{b+\frac{L-b}{N}}$	$1 - \frac{c}{b+\frac{L-b}{N}}$	0
TI6	$L < bN$ and $b + \frac{L-b}{N} \leq c$	Incomplete	1	0	0

Table 14: Total Effort security game: Total expected game payoffs, conditioned on other players

	Case	Information Type	Total Expected Payoff
TC1	$c < b$	Complete	$M - c + \frac{c^2}{2L}$
TC2	$bN \leq L$ and $b \leq c \leq b(N-K)$	Complete	$M - c + \frac{c^2}{2L\left(1-\frac{K}{N}\right)}$
TC3	$bN \leq L$ and $b(N-K) < c < b + L\left(1 - \frac{K+1}{N}\right)$	Complete	$M - c + \frac{b^2 N}{2L} + \frac{(c-b)^2}{2L\left(1-\frac{k+1}{N}\right)}$
TC4	$bN \leq L$ and $b + L\left(1 - \frac{K+1}{N}\right) \leq c$	Complete	$M - b - \frac{L}{2}\left(1 - \frac{K+1}{N}\right) + \frac{b^2 N}{2L}$
TC5	$L < bN$ and $b \leq c \leq L\left(1 - \frac{K}{N}\right)$	Complete	$M - c + \frac{c^2}{2L\left(1-\frac{K}{N}\right)}$
TC6	$L < bN$ and $L\left(1 - \frac{K}{N}\right) < c$	Complete	$M - \frac{L}{2}\left(1 - \frac{K}{N}\right)$
TI1	$c < b$	Incomplete	$M - c + \frac{c^2}{L}$
TI2	$bN \leq L$ and $b \leq c \leq b + \frac{b^2}{L}(N-1)$	Incomplete	$M - c + \frac{c^2}{2\left(b+\frac{L-b}{N}\right)}$
TI3	$bN \leq L$ and $b + \frac{b^2}{L}(N-1) < c < 2b - \frac{b}{N}$	Incomplete	$M - c + \frac{b^2 N}{2L} + \frac{(c-b)^2}{2\left(b-\frac{b}{N}\right)}$
TI4	$bN \leq L$ and $2b - \frac{b}{N} \leq c$	Incomplete	$M - b - \frac{1}{2}\left(b - \frac{b}{N}\right) + \frac{b^2 N}{2L}$
TI5	$L < bN$ and $b \leq c < b + \frac{L-b}{N}$	Incomplete	$M - c + \frac{c^2}{2\left(b+\frac{L-b}{N}\right)}$
TI6	$L < bN$ and $b + \frac{L-b}{N} \leq c$	Incomplete	$M - \frac{1}{2}\left(b + \frac{L-b}{N}\right)$

5 Results

5.1 Strategies and payoffs

Our results provide us with insights into security decision-making in networked systems. We can recognize several situations that immediately relate to practical risk choices. We start with basic observations that are relevant for all three games, before discussing the different games and information conditions in more detail.

General observations applicable to all three security games. Every scenario involves simple cost-benefit analyses for both sophisticated and naïve agents [13]. Agents remain passive when the cost of self-protection and self-insurance exceeds the expected loss. Further, they differentiate between the two types of security actions based on their relative cost. This behavior describes what we would usually consider as basic risk-taking that is part of everyday life: It is not always worth protecting against known risks.

One important feature of our model is the availability of self-insurance. If $c < b$ the decision scenario significantly simplifies for all games and both information conditions. This is because once insurance is applied, the risk and interdependency among the players is removed. The interesting cases for all three games arise when $b \leq c$ and protection is a potentially cost-effective option. Within this realm insurance has a more subtle effect on the payoffs.

There are important differences between the two agent types. The expert agent considers the strategic interdependencies of all agents' choices. Therefore, given the same draw of an attack probability she

Table 15: Total effort security game: Total expected game payoffs, not conditioned on other players

	Case	Information Type	Total Expected Payoff
TC1	$c < b$	Complete	$M - c + \frac{c^2}{2L}$
TC2-4	$bN \leq L$ and $b \leq c$	Complete	$*\sum_{k=0}^{\lfloor N-\frac{c}{b}\rfloor} Pr[k] \cdot \left(M - c + \frac{c^2}{2L\left(1-\frac{k}{N}\right)}\right)$ $+ \sum_{k=\lfloor N-\frac{c}{b}+1\rfloor}^{\lfloor N-1-\frac{N}{L}(c-b)\rfloor} Pr[k] \cdot \left(M - c + \frac{b^2 N}{2L} + \frac{(c-b)^2}{2L\left(1-\frac{k+1}{N}\right)}\right)$ $+ \sum_{k=\lfloor N-\frac{N}{L}(c-b)\rfloor}^{N-1} Pr[k] \cdot \left(M - b - \frac{L}{2}\left(1 - \frac{k+1}{N}\right) + \frac{b^2 N}{2L}\right)$
TC5-6	$L < bN$ and $b \leq c$	Complete	$*\sum_{k=0}^{\lfloor N-\frac{cN}{L}\rfloor} Pr[k] \cdot \left(M - c + \frac{c^2}{2L\left(1-\frac{k}{N}\right)}\right)$ $+ \sum_{k=\lfloor N-\frac{cN}{L}+1\rfloor}^{N-1} Pr[k] \cdot \left(M - \frac{L}{2N}(N-k)\right)$
TI1	$c < b$	Incomplete	$M - c + \frac{c^2}{L}$
TI2	$bN \leq L$ and $b \leq c \leq b + \frac{b^2}{L}(N-1)$	Incomplete	$M - c + \frac{c^2}{2\left(b+\frac{L-b}{N}\right)}$
TI3	$bN \leq L$ and $b + \frac{b^2}{L}(N-1) < c < 2b - \frac{b}{N}$	Incomplete	$M - c + \frac{b^2 N}{2L} + \frac{(c-b)^2}{2\left(b-\frac{b}{N}\right)}$
TI4	$bN \leq L$ and $2b - \frac{b}{N} \leq c$	Incomplete	$M - b - \frac{1}{2}\left(b - \frac{b}{N}\right) + \frac{b^2 N}{2L}$
TI5	$L < bN$ and $b \leq c < b + \frac{L-b}{N}$	Incomplete	$M - c + \frac{c^2}{2\left(b+\frac{L-b}{N}\right)}$
TI6	$L < bN$ and $b + \frac{L-b}{N} \leq c$	Incomplete	$M - \frac{1}{2}\left(b + \frac{L-b}{N}\right)$
TN1	$c < b$	Naive	$M - c + \frac{c^2}{2}$
TN2	$b \leq c$	Naive	$M - b - \frac{1}{2}\left(b - \frac{b}{N}\right) + \frac{b^2}{L}\left(1 - \frac{1}{2N}\right)$

$*Pr[k] = \binom{N-1}{k}\left(1 - \frac{b}{L}\right)^k \left(\frac{b}{L}\right)^{N-1-k}$ is the probability that exactly k players other than i choose protection.

sometimes prefers to self-insure, or remain passive when naïve agents would always protect without further consideration. However, the expert agent never protects when the naïve would not (given the same p_i).

The naïve agents face a payoff reduction as a result of their limited understanding of correlated threats, but even the sophisticated agent can experience a similar payoff reduction due to limited information. On the one hand, she might invest in self-protection or self-insurance when it is not necessary because the naïve agents collectively or individually secured the network. On the other end, she may fail to take a security action when a (typically low probability) breach actually occurs. It is important to mention that she acted rationally in both situations, but these additional risks remain.

Basic payoffs for different security actions: We can immediately observe that the additional risk due to limited information results from different mechanisms for each security scenario. In the weakest-link game (Table 1) we find that self-protection carries a risk for the expert agent with limited information that at least one naïve agent chooses not to protect. This would result in a break-down of system security and a waste of self-protection expenditure. In contrast, in the best shot game (Table 6) the investment in preventive action always secures the network but with limited information this may be a duplicative effort. In the total effort game these risks are more balanced (Table 11). The expert can add or withhold her N-th part of the total feasible security contribution. Depending on the cost of security she has to estimate the expected number of naïve contributors K in order to respond adequately.

Conditions for choice between different security actions: In the weakest-link game and complete information, the expert agent can utilize the lowest attack probability that any naïve agent has drawn. If this value is below the required threshold for protection, (i.e. if $\min_{j \neq i} p_j < b/L$), then the sophisticated agent will never protect. Otherwise, depending on her own draw she will make or break a successful defense. Under incomplete information she has to consider the likelihood $(1 - b/L)^{N-1}$ that all naïve agents protect. In all cases there is now a residual likelihood that she might self-insure. See Table 2.

In the best shot game the fully informed expert can simply determine the highest likelihood of being attacked for any naive agent to decide whether she should contribute to system protection. With full or limited information, it is obvious that she will only have to contribute very rarely, and can mostly rely on others' efforts. Nevertheless, it is surprising to find that in the incomplete information scenario the expected payoff from passivity always dominates the expected payoff for protection, even when the expected loss is near total ($p_i \sim 1$). The sophisticated user with limited information will never protect. Under neither information condition is it optimal to self-insure if $b \leq c$. See Table 7 for details.

Next consider the the total effort game (Table 12). Under full information with $b \leq c$, all decision conditions depend non-trivially on K, the number of contributors to protection. Under incomplete information the expert must compute the expected value of K, which is $(1 - b/L)(N - 1)$. The case differences between complete and incomplete conditions reflect the replacement of K with $E[K]$, and subsequent simplification. In all cases, the critical factor for the decision to protect is whether the potential loss is N times greater than the cost of protection (i.e. $p_i L \geq bN$).

Case boundaries for choice between different security actions: In Figure 3, we plot the cases used to record total expected payoffs for the expert agent in Tables 5, 10, and 15. The associated tables for the probabilities of self-protection, self-insurance and passivity (within each case) are available in the companion technical report [18].

In the weakest-link game only cases 3 and 4 allow for investments in self-protection. We find that increasing the number of agents, N, results in a shrinkage of both cases 3 and 4 to the benefit of case 2. In contrast, the determination of case boundaries in the best shot game is independent of the size of the network. Finally, in the total effort game only cases 3 and 4 allow for rational self-protection investments. Again an increase in the network size reduces the prevalence of these cases (since $bN \leq L$ is a necessary condition).

Payoffs: Tables 5, 10, and 15 contain the total expected payoff for decisions made by the sophisticated agent, but also for the naïve agents.

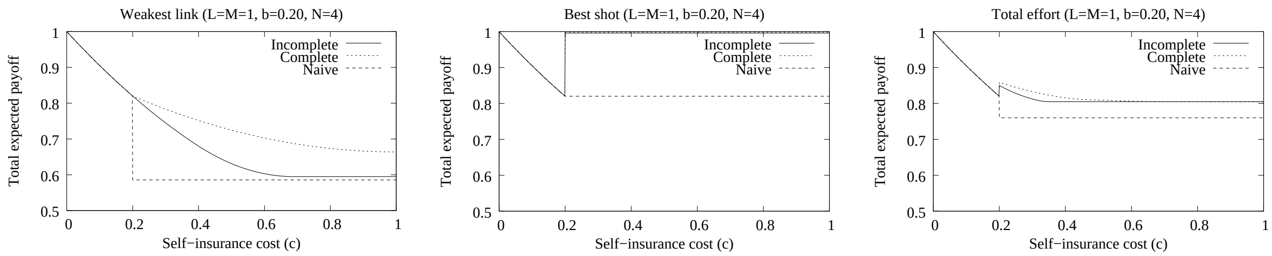
We have already highlighted that for $c < b$ all agents follow the same simple decision rule to decide between passivity and self-insurance. Therefore, payoffs in this region are identical for all agent types in the case of homogeneous security costs. But, there are payoff differences among all three information conditions for some parts of the parameter range when $b \leq c$.

Consider the graphs in Figure 1. We plot the payoff functions for sophisticated agents types under the different information conditions, as well as the payoff output for the non-expert agent. It is intuitive that the naïve agents suffer in the weakest-link game since they do not appreciate the difficulty to achieve system-wide protection. Similarly, in the best shot game too many unsophisticated agents will invest in protection lowering the average payoff. In the total effort game, sophisticated agents realize that their contribution is only valued in relation to the network size. In comparison, naïve agents invest more often. Further, the payoff profile of the unsophisticated agents remains flat for $b < c$. This reflects the fact that the naïve agent ignores the insurance option whenever protection is cheaper.

We can observe that the sophisticated agents will suffer from their misallocation of resources in the weakest-link game when information is incomplete. In the best shot game this impact is limited, but there is a residual risk that no naïve agent willingly protects due to an unlikely set of draws. In such cases the fully informed expert could have chosen to take it upon herself to secure the network. In the total effort game we observe a limited payoff discrepancy for expert users as a result of limited information.

5.2 Price of uncertainty

From a system design perspective it is important to select parameter settings (e.g., making available specific security technologies) that maximize user utility and are robust to changes in the environment. The security games we analyze in this paper are a significant challenge in both aspects. In particular, from Figure 1 we can infer that the penalty for the lack of complete information about attack threats can be highly variable depending on the system parameters. We argue that the reduction of this disparity should be an important



(a) Weakest link (b) Best shot (c) Total effort

Figure 1: Total expected payoffs for the strategic player under different information conditions, compared with that of the naïve agents, expressed as a function of the self-protection cost c. $L = M = 1$, $N = 4$, and b is fixed to $b = 0.20$ in this set of examples.

design goal. To further this goal we propose a mathematical formulation that we call the *price of uncertainty*. We then apply this measure to the analysis of security games.

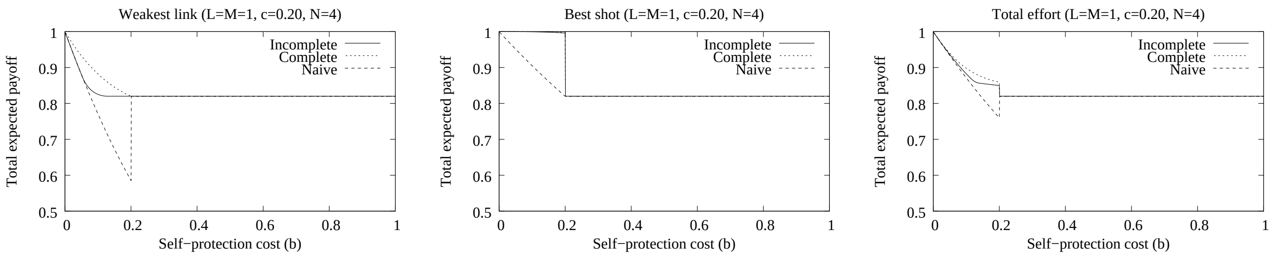
Definition: We are interested in a mathematical measure that allows us to quantify the payoff loss due to incomplete information for sophisticated agents, that can be applied to a variety of decision-theoretic scenarios. It is nontrivial to arrive at a definitive answer for this problem statement; but as a first step towards this goal we are drawing motivation from the work by Koutsoupias and Papadimitriou on worst-case equilibria where they define a quantity called the price of anarchy [22]. We define the price of uncertainty as the ratio:

$$\frac{\text{Expected payoff in the complete information environment}}{\text{Expected payoff in the incomplete information environment}}$$

Observations: Consider Figure 4 which gives, for all three security games, a heat plot for the price of limited information over all choices of b and c with L, M, N fixed at $L = M = 1$ and $N = 4$. The most remarkable feature of these graphs are the different hotspot regions. In the weakest-link game we find that higher price of uncertainty ratios are to be found within the boundaries of cases 3 and 4. Both cases allow for self-protection in the presence of incomplete information and therefore balance the various risks more directly than the remaining cases. (Case 1 and 2 associate zero probability with self-protection.)

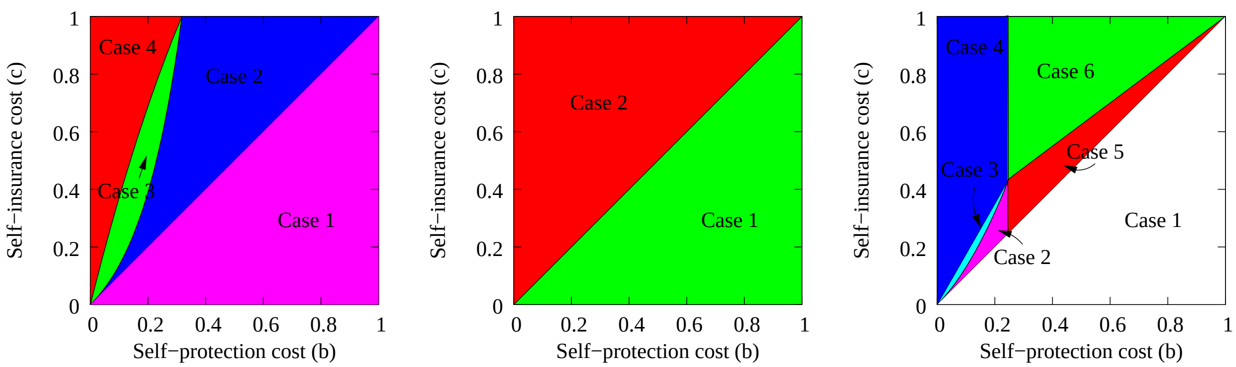
In the best shot scenario the peak region is located trivially within the boundaries of case 2. We know that the expert player will never protect under incomplete information but is subject to the residual risk of a system-wide security failure. For $N = 4$ the likelihood of such a breakdown is already very small, and decreases with N. Still this outcome is feasible and most pronounced for protection costs that are about a half to two-thirds of the loss, L. For higher b the disincentive of buying self-protection and the potential loss are relatively balanced resulting in a lower price of uncertainty.

In the total effort game we observe multiple hotspot regions. Cases 4 and 6 are unaffected by an increased price of uncertainty. They are characterized by the absence of self-insurance as a feasible strategy. This eases the decision-making problem of the expert, and reduces the likelihood of a misspent security investment.



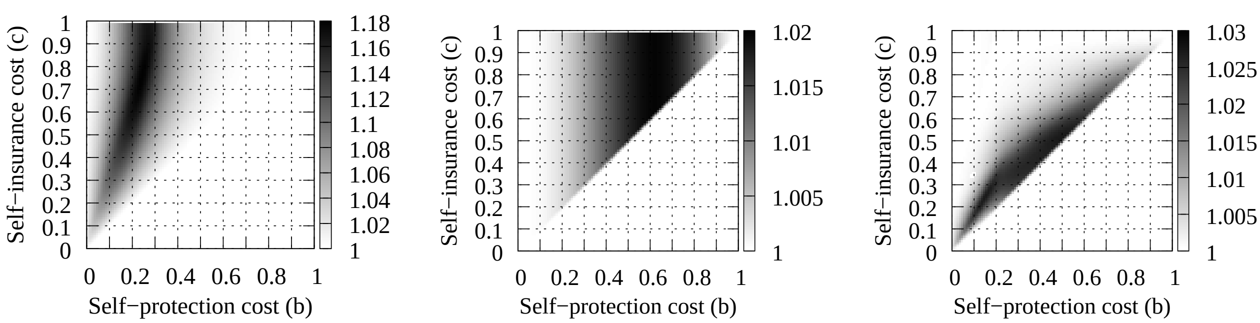
(a) Weakest link (b) Best shot (c) Total effort

Figure 2: Total expected payoffs for the strategic player under different information conditions, compared with that of the naïve agents, expressed as a function of the self-protection cost b. $L = M = 1$, $N = 4$, and c is fixed to $c = 0.20$ in this set of examples.



(a) Weakest link (b) Best shot (c) Total effort

Figure 3: Strategy boundaries in the incomplete information scenario for the sophisticated player. The different cases refer to Tables 5, 10 and 15. $L = M = 1$ and $N = 4$ in this set of examples.



(a) Weakest link (b) Best shot (c) Total effort

Figure 4: Price of uncertainty for the various games. $L = M = 1$, $N = 4$ in this set of examples.

6 Conclusions

In our work we emphasize that security decision-making is shaped by the structure of the task environment as well as the knowledge and computational capabilities of the agents. To that effect, we study security investment choices in three canonical scenarios [16, 35]. Decisions are made from three distinct security actions (self-protection, self-insurance or passivity) to navigate the complex security risks of weakest-link, best shot and total effort interdependencies. In these environments, we investigate the co-habitation of a single fully rational expert and $N-1$ naïve agents. The naïve agents are near-sighted and fail to account for the decisions of other agents, and instead follow a simple but reasonable self-centered rule-of-thumb. We further study the impact of limited information on rational agents' choices. To guide the reader through our analysis, we provide a detailed overview and examples of our methodology to compare strategies and payoffs.

In the kingdom of the blind, is the one-eyed man king? We find that in general, the naïve agents match the payoff of the expert when self-insurance is cheap, but not otherwise. Even with limited information, the sophisticated agent can generally translate her better structural understanding into decisions that minimize wasted protection investments, or an earlier retreat to the self-insurance strategy when system-wide security is (likely) failing. A notable exception is the weakest link game with incomplete information, where the payoff of the sophisticated agent degrades to that of the naïve agent as insurance gets more expensive.

Tragically, her impact on the improvement of system-wide security is never positive (in comparison to her replacement by an unsophisticated agent). While our expert agent is rational, she is not benevolent. Acting selfishly, the set of scenarios for which protection is her best option is always a subset of the set of scenarios for which the naïve agent chooses protection.

Monarchs or netizens? To complement our study we are interest in studying properties of a network with varying fractions of expert to naïve users. Further, we want to address the desire of some computer experts to sacrifice individual resources to improve system resilience to attacks, by introducing *benevolent* agents.

Our work further evidences the limits of independently organized security in fully connected systems. David Clark summarized his vision about Internet governance: "We reject: kings, presidents, and voting. We believe in: rough consensus and running code." While this tenet has been a driver of the sudden growth of large-scale networks, current approaches to overcome the coordination problems need to be organized on the intermediary level. For example, OpenDNS independently released a tool to track the spread of major worms. Other approaches we want to study are vigilante behavior, and peer-to-peer support.

What is the cost of the emperor's lack of information? To analyze the impact of the different information conditions we have proposed a new mathematical formalization guided by prior work on worst-case equilibria [22]. We define the *price of uncertainty* as the ratio of the payoff in the complete information environment to the payoff in the incomplete information environment. Our analysis of Figure 4 is a first step in that direction, however, a more formal analysis is subject to future work.

Finally, a system designer is not only interested in the payoffs of the network participants given different information realities (e.g., due to frequent changes in attack trends). He is also concerned with how well-fortified the organization is against attacks. To that effect we plan to include a more thorough presentation of the parameter conditions that cause attacks to fail due to system-wide protection, and when they succeed (due to coordination failures, passivity, and self-insurance).

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A Technical appendix

A.1 Derivations for weakest link game

Weakest link security game. Derivations for total expected game payoffs, conditioned on other players: The following derivations refer to Table 4.

Cases (WC1) and (WC2a) and (WI1):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$
$$= [M - E[p_i] \cdot L] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right] + [M - b - E[p_i] \cdot L] \cdot [0]$$
$$= \left[M - \left(\frac{c}{2L}\right) \cdot L\right] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right]$$
$$= M - \frac{c^2}{2L} - c + \frac{c^2}{L}$$
$$= M - c + \frac{c^2}{2L}$$

Case (WC2b):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$
$$= [M - E[p_i] \cdot L] \cdot \left[\frac{b}{L}\right] + [M - c] \cdot [0] + [M - b] \cdot \left[1 - \frac{b}{L}\right]$$
$$= \left[M - \left(\frac{b}{2L}\right) \cdot L\right] \cdot \left[\frac{b}{L}\right] + [M - b] \cdot \left[1 - \frac{b}{L}\right]$$
$$= M - \frac{b^2}{2L} - b + \frac{b^2}{L}$$
$$= M - b + \frac{b^2}{2L}$$

Case (WI2):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$
$$= [M - E[p_i] \cdot L] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right] + \left[M - b - E[p_i] \cdot L \left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right)\right] \cdot [0]$$
$$= \left[M - \left(\frac{c}{2L}\right) \cdot L\right] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right]$$
$$= M - \frac{c^2}{2L} - c + \frac{c^2}{L}$$
$$= M - c + \frac{c^2}{2L}$$

Case (WI3):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$

$$= [M - E[p_i] \cdot L] \cdot \left[\frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right] + [M-c] \cdot \left[1 - \frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)}\right]$$

$$+ \left[M - b - E[p_i] \cdot L\left(1 - \left(1-\frac{b}{L}\right)^{N-1}\right)\right] \cdot \left[\frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} - \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right]$$

$$= \left[M - \left(\frac{b}{2L\left(1-\frac{b}{L}\right)^{N-1}}\right) \cdot L\right] \cdot \left[\frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right] + [M-c] \cdot \left[1 - \frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)}\right]$$

$$+ \left[M - b - \frac{1}{2}\left(\frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} + \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right) \cdot L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)\right]$$

$$\cdot \left[\frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} - \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right]$$

$$= M - \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{2N-2}} - c + \frac{c^2-bc}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} - b \cdot \left[\frac{c-b}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} - \frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right]$$

$$- \frac{L}{2}\left(1-(1-\frac{b}{L})^{N-1}\right) \cdot \left[\left(\frac{c-b}{L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}\right)^2 - \left(\frac{b}{L\left(1-\frac{b}{L}\right)^{N-1}}\right)^2\right]$$

$$= M - \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{2N-2}} - c + \frac{c^2-bc}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} - \frac{bc-b^2}{L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)} + \frac{b^2}{L\left(1-\frac{b}{L}\right)^{N-1}}$$

$$- \frac{(c-b)^2}{2L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)} + \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{2N-2}}$$

$$= M - c + \frac{c^2-2bc+b^2}{L\left(1-(1-\frac{b}{L})^{N-1}\right)} + \frac{b^2}{L\left(1-\frac{b}{L}\right)^{N-1}} - \frac{(c-b)^2}{2L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}$$

$$= M - c + \frac{b^2}{2L\left(1-\frac{b}{L}\right)^{N-1}} + \frac{(c-b)^2}{2L\left(1-\left(1-\frac{b}{L}\right)^{N-1}\right)}$$

Case (WI4):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$

$$= [M - E[p_i] \cdot L] \cdot \left[\frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right] + [M - c] \cdot [0]$$

$$+ \left[M - b - E[p_i] \cdot L \left(1 - \left(1 - \frac{b}{L}\right)^{N-1} \right) \right] \cdot \left[1 - \frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right]$$

$$= \left[M - \left(\frac{b}{2L\left(1 - \frac{b}{L}\right)^{N-1}} \right) \cdot L \right] \cdot \left[\frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right]$$

$$+ \left[M - b - \frac{1}{2} \left(1 + \frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right) \cdot L \left(1 - \left(1 - \frac{b}{L}\right)^{N-1} \right) \right] \cdot \left[1 - \frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right]$$

$$= M - \frac{b^2}{2L\left(1 - \frac{b}{L}\right)^{2N-2}} - b \cdot \left[1 - \frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right] - \frac{L}{2} \left(1 - (1 - \frac{b}{L})^{N-1} \right) \cdot \left[1 - \left(\frac{b}{L\left(1 - \frac{b}{L}\right)^{N-1}} \right)^2 \right]$$

$$= M - \frac{b^2}{2L\left(1 - \frac{b}{L}\right)^{2N-2}} - b + \frac{b^2}{L\left(1 - \frac{b}{L}\right)^{N-1}} - \frac{L}{2} \left(1 - \left(1 - \frac{b}{L}\right)^{N-1} \right)$$

$$+ \frac{L}{2} \left(1 - (1 - \frac{b}{L})^{N-1} \right) \left(\frac{b^2}{L^2\left(1 - \frac{b}{L}\right)^{2N-2}} \right)$$

$$= M - \frac{b^2}{2L\left(1 - \frac{b}{L}\right)^{2N-2}} - b + \frac{b^2}{L\left(1 - \frac{b}{L}\right)^{N-1}} - \frac{L}{2} \left(1 - \left(1 - \frac{b}{L}\right)^{N-1} \right) + \frac{b^2}{2L\left(1 - \frac{b}{L}\right)^{2N-2}} - \frac{b^2}{2L\left(1 - \frac{b}{L}\right)^{N-1}}$$

$$= M - b - \frac{L}{2} \left(1 - \left(1 - \frac{b}{L}\right)^{N-1} \right) + \frac{b^2}{2L\left(1 - \frac{b}{L}\right)^{N-1}}$$

Weakest link security game. Derivations for total expected game payoffs, not conditioned on other players: The following derivation refers to Table 5. To remove dependence on p_j for $j \neq i$ in case WC2, we simply take a weighted sum of the total payoffs for cases WC2a and WC2b, where the weight is determined by the probability of $\min_{j \neq i} p_j < \frac{b}{L}$ assuming that each p_j is drawn from the uniform distribution over $[0, 1]$ (and assuming $b \leq c$).

We have:

Case (WC2):

Probability[Case (WC2a)] $\cdot$ ExPayoff[Case (WC2a)] $+$ Probability[Case (WC2b)] $\cdot$ ExPayoff[Case (WC2b)]

$$= \left(1 - \left(1 - \frac{b}{L}\right)^{N-1}\right) \cdot \left[M - c + \frac{c^2}{2L}\right] + \left[\left(1 - \frac{b}{L}\right)^{N-1}\right] \cdot \left[M - b + \frac{b^2}{2L}\right]$$
$$= M - c + \frac{c^2}{2L} + \left(c - \frac{c^2}{2L}\right)\left(1 - \frac{b}{L}\right)^{N-1} - \left(b - \frac{b^2}{2L}\right)\left(1 - \frac{b}{L}\right)^{N-1}$$
$$= M - c + \frac{c^2}{2L} + \left(c - b - \frac{c^2 - b^2}{2L}\right)\left(1 - \frac{b}{L}\right)^{N-1}$$
$$= M - c + \frac{c^2}{2L} + (c - b)\left(1 - \frac{c + b}{2L}\right)\left(1 - \frac{b}{L}\right)^{N-1}$$

A.2 Derivations for best shot game

Best shot security game. Derivations for total expected game payoffs, conditioned on other players: The following derivations refer to Table 9.

Cases (BC1) and (BI1):

Payoff$[passivity] \cdot Pr[passivity]$ $+$ Payoff$[insurance] \cdot Pr[insurance]$ $+$ Payoff$[protection] \cdot Pr[protection]$

$$= [M - E[p_i] \cdot L] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right] + [M - b] \cdot [0]$$
$$= \left[M - \left(\frac{c}{2L}\right) \cdot L\right] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right]$$
$$= M - \frac{c^2}{2L} - c + \frac{c^2}{L}$$
$$= M - c + \frac{c^2}{2L}$$

Case (BC2a):

Payoff$[passivity] \cdot Pr[passivity]$ $+$ Payoff$[insurance] \cdot Pr[insurance]$ $+$ Payoff$[protection] \cdot Pr[protection]$

$$= [M - E[p_i] \cdot L] \cdot \left[\frac{b}{L}\right] + [M - c] \cdot [0] + [M - b] \cdot \left[1 - \frac{b}{L}\right]$$
$$= \left[M - \left(\frac{b}{2L}\right) \cdot L\right] \cdot \left[\frac{b}{L}\right] + [M - b] \cdot \left[1 - \frac{b}{L}\right]$$
$$= M - \frac{b^2}{2L} - b + \frac{b^2}{L}$$
$$= M - b + \frac{b^2}{2L}$$

Case (BC2b):

Payoff$[passivity] \cdot Pr[passivity] +$ Payoff$[insurance] \cdot Pr[insurance] +$ Payoff$[protection] \cdot Pr[protection]$

$= [M] \cdot [1] + [M - c] \cdot [0] + [M - b] \cdot [0]$

$= M$

Case (BI2):

Payoff$[passivity] \cdot Pr[passivity] +$ Payoff$[insurance] \cdot Pr[insurance] +$ Payoff$[protection] \cdot Pr[protection]$

$$= \left[M - E[p_i] \cdot L \left(\frac{b}{L}\right)^{N-1}\right] \cdot [1] + [M - c] \cdot [0] + [M - b] \cdot [0]$$

$$= M - \left(\frac{1}{2}\right) \cdot L \left(\frac{b}{L}\right)^{N-1}$$

$$= M - \frac{L}{2} \left(\frac{b}{L}\right)^{N-1}$$

Best shot security game. Derivations for total expected game payoffs, not conditioned on other players: The following derivation refers to Table 10. To remove dependence on p_j for $j \neq i$ in case BC2, we simply take a weighted sum of the total payoffs for cases BC2a and BC2b, where the weight is determined by the probability of $\min_{j \neq i} p_j < \frac{b}{L}$ assuming that each p_j is drawn from the uniform distribution over $[0, 1]$.

We have:

Case (BC2):

Probability[Case (BC2a)] $\cdot$ ExPayoff[Case (BC2a)] $+$ Probability[Case (BC2b)] $\cdot$ ExPayoff[Case (BC2b)]

$$= \left(\frac{b}{L}\right)^{N-1} \cdot \left[M - b + \frac{b^2}{2L}\right] + \left[1 - \left(\frac{b}{L}\right)^{N-1}\right] \cdot [M]$$

$$= M - b\left(\frac{b}{L}\right)^{N-1} + \frac{b^2}{2L}\left(\frac{b}{L}\right)^{N-1}$$

$$= M - b\left(1 - \frac{b}{2L}\right)\left(\frac{b}{L}\right)^{N-1}$$

A.3 Derivations for total effort game

Total Effort security game. Derivations for total expected game payoffs, conditioned on other players: The following derivations refer to Table 14.

Case (TC1): $c < b$

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$

$$= [M - E[p_i] \cdot L] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right] + \left[M - b - E[p_i] \cdot L\left(1 - \frac{1}{N}\right)\right] \cdot [0]$$

$$= \left[M - \left(\frac{c}{2L}\right) \cdot L\right] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right]$$

$$= M - \frac{c^2}{2L} - c + \frac{c^2}{L}$$

$$= M - c + \frac{c^2}{L}$$

Cases (TC2) and (TC5):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$

$$= \left[M - E[p_i] \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot \left[\frac{c}{L\left(1 - \frac{K}{N}\right)}\right] + [M - c] \cdot \left[1 - \frac{c}{L\left(1 - \frac{K}{N}\right)}\right]$$

$$+ \left[M - b - E[p_i] \cdot L\left(1 - \frac{K+1}{N}\right)\right] \cdot [0]$$

$$= \left[M - \left(\frac{c}{2L\left(1 - \frac{K}{N}\right)}\right) \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot \left[\frac{c}{L\left(1 - \frac{K}{N}\right)}\right] + [M - c] \cdot \left[1 - \frac{c}{L\left(1 - \frac{K}{N}\right)}\right]$$

$$= M - \frac{c^2}{2L\left(1 - \frac{K}{N}\right)} - c + \frac{c^2}{L\left(1 - \frac{K}{N}\right)}$$

$$= M - c + \frac{c^2}{2L\left(1 - \frac{K}{N}\right)}$$

Case (TC3):

Payoff$[passivity] \cdot Pr[passivity] +$ Payoff$[insurance] \cdot Pr[insurance] +$ Payoff$[protection] \cdot Pr[protection]$

$$= \left[M - E[p_i] \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M - c] \cdot \left[1 - \frac{c-b}{L\left(1 - \frac{K+1}{N}\right)}\right]$$

$$+ \left[M - b - E[p_i] \cdot L\left(1 - \frac{K+1}{N}\right)\right] \cdot \left[\frac{c-b}{L\left(1 - \frac{K+1}{N}\right)} - \frac{bN}{L}\right]$$

$$= \left[M - \left[\frac{bN}{2L}\right] \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M - c] \cdot \left[1 - \frac{c-b}{L\left(1 - \frac{K+1}{N}\right)}\right]$$

$$+ \left[M - b - \frac{1}{2}\left[\frac{c-b}{L\left(1 - \frac{K+1}{N}\right)} + \frac{bN}{L}\right] \cdot L\left(1 - \frac{K+1}{N}\right)\right] \cdot \left[\frac{c-b}{L\left(1 - \frac{K+1}{N}\right)} - \frac{bN}{L}\right]$$

$$= M - \frac{b^2N^2}{2L}\left(1 - \frac{K}{N}\right) - c + \frac{c^2 - bc}{L\left(1 - \frac{K+1}{N}\right)}$$

$$- b\left(\frac{c-b}{L\left(1 - \frac{K+1}{N}\right)} - \frac{bN}{L}\right) - \frac{L}{2}\left(1 - \frac{K+1}{N}\right)\left(\left(\frac{c-b}{L\left(1 - \frac{K+1}{N}\right)}\right)^2 - \left(\frac{bN}{L}\right)^2\right)$$

$$= M - \frac{b^2N^2}{2L}\left(1 - \frac{K}{N}\right) - c + \frac{c^2 - bc}{L\left(1 - \frac{K+1}{N}\right)} - b\left(\frac{c-b}{L\left(1 - \frac{K+1}{N}\right)}\right) + \frac{b^2N}{L} - \frac{(c-b)^2}{2L\left(1 - \frac{K+1}{N}\right)} + \frac{b^2N^2}{2L}\left(1 - \frac{K+1}{N}\right)$$

$$= M - c + \frac{(c-b)^2}{L\left(1 - \frac{K+1}{N}\right)} + \frac{b^2N}{L} - \frac{(c-b)^2}{2L\left(1 - \frac{K+1}{N}\right)} - \frac{b^2N}{2L}$$

$$= M - c + \frac{b^2N}{2L} + \frac{(c-b)^2}{2L\left(1 - \frac{K+1}{N}\right)}$$

Case (TC4):

Payoff$[passivity] \cdot Pr[passivity] +$ Payoff$[insurance] \cdot Pr[insurance] +$ Payoff$[protection] \cdot Pr[protection]$

$$= \left[M - E[p_i] \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M - c] \cdot [0] + \left[M - b - E[p_i] \cdot L\left(1 - \frac{K+1}{N}\right)\right] \cdot \left[1 - \frac{bN}{L}\right]$$
$$= \left[M - \left[\frac{bN}{2L}\right] \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M - c] \cdot [0] + \left[M - b - \left[\frac{1}{2}\left(1 + \frac{bN}{L}\right)\right] \cdot L\left(1 - \frac{K+1}{N}\right)\right] \cdot \left[1 - \frac{bN}{L}\right]$$
$$= M - \frac{b^2N^2}{2L}\left(1 - \frac{K}{N}\right) - b\left(1 - \frac{bN}{L}\right) - \frac{L}{2}\left(1 - \frac{K+1}{N}\right)\left(1 - \frac{b^2N^2}{L^2}\right)$$
$$= M - \frac{b^2N^2}{2L}\left(1 - \frac{K}{N}\right) - b + \frac{b^2N}{L} - \frac{L}{2}\left(1 - \frac{K+1}{N}\right) + \frac{b^2N^2}{2L}\left(1 - \frac{K+1}{N}\right)$$
$$= M - b + \frac{b^2N}{L} - \frac{L}{2}\left(1 - \frac{K+1}{N}\right) - \frac{b^2N}{2L}$$
$$= M - b - \frac{L}{2}\left(1 - \frac{K+1}{N}\right) + \frac{b^2N}{2L}$$

Case (TC6):

Payoff$[passivity] \cdot Pr[passivity] +$ Payoff$[insurance] \cdot Pr[insurance] +$ Payoff$[protection] \cdot Pr[protection]$

$$= \left[M - E[p_i] \cdot L\left(1 - \frac{K}{N}\right)\right] \cdot [1] + [M - c] \cdot [0] + \left[M - b - E[p_i] \cdot L\left(1 - \frac{K+1}{N}\right)\right] \cdot [0]$$
$$= M - \frac{L}{2}\left(1 - \frac{K}{N}\right)$$

Case (TI1): $c < b$

Payoff$[passivity] \cdot Pr[passivity] +$ Payoff$[insurance] \cdot Pr[insurance] +$ Payoff$[protection] \cdot Pr[protection]$

$$= [M - E[p_i] \cdot L] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right] + \left[M - b - E[p_i] \cdot L\left(1 - \frac{1}{N}\right)\right] \cdot [0]$$
$$= \left[M - \left(\frac{c}{2L}\right) \cdot L\right] \cdot \left[\frac{c}{L}\right] + [M - c] \cdot \left[1 - \frac{c}{L}\right]$$
$$= M - \frac{c^2}{2L} - c + \frac{c^2}{L}$$
$$= M - c + \frac{c^2}{L}$$

Cases (TI2) and (TI5):

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$

$$= \left[M - E[p_i]\left(b + \frac{L-b}{N}\right)\right] \cdot \left[\frac{c}{b + \frac{L-b}{N}}\right] + [M-c] \cdot \left[1 - \frac{c}{b + \frac{L-b}{N}}\right] + \left[M - b - E[p_i]\left(b - \frac{b}{N}\right)\right] \cdot [0]$$

$$= \left[M - \left(\frac{c}{2\left(b + \frac{L-b}{N}\right)}\right) \cdot \left(b + \frac{L-b}{N}\right)\right] \cdot \left[\frac{c}{b + \frac{L-b}{N}}\right] + [M-c] \cdot \left[1 - \frac{c}{b + \frac{L-b}{N}}\right]$$

$$= M - \frac{c^2}{2\left(b + \frac{L-b}{N}\right)} - c + \frac{c^2}{b + \frac{L-b}{N}}$$

$$= M - c + \frac{c^2}{2\left(b + \frac{L-b}{N}\right)}$$

Case (TI3)

$$\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection]$$

$$= \left[M - E[p_i]\left(b + \frac{L-b}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M-c] \cdot \left[1 - \frac{c-b}{b - \frac{b}{N}}\right] + \left[M - b - E[p_i]\left(b - \frac{b}{N}\right)\right] \cdot \left[\frac{c-b}{b - \frac{b}{N}} - \frac{bN}{L}\right]$$

$$= \left[M - \left(\frac{bN}{2L}\right) \cdot \left(b + \frac{L-b}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M-c] \cdot \left[1 - \frac{c-b}{b - \frac{b}{N}}\right]$$

$$+ \left[M - b - \frac{1}{2}\left(\frac{bN}{L} + \frac{c-b}{b - \frac{b}{N}}\right) \cdot \left(b - \frac{b}{N}\right)\right] \cdot \left[\frac{c-b}{b - \frac{b}{N}} - \frac{bN}{L}\right]$$

$$= M - \frac{b^2N^2}{2L^2}\left(b + \frac{L-b}{N}\right) - c + c\left(\frac{c-b}{b - \frac{b}{N}}\right)$$

$$- b\left(\frac{c-b}{b - \frac{b}{N}} - \frac{bN}{L}\right) - \frac{1}{2}\left(b - \frac{b}{N}\right)\left(\frac{(c-b)^2}{\left(b - \frac{b}{N}\right)^2} - \frac{b^2N^2}{L^2}\right)$$

$$= M - \frac{b^2N^2}{2L^2}\left(b - \frac{b}{N}\right) - \frac{b^2N}{2L} - c + c\left(\frac{c-b}{b - \frac{b}{N}}\right)$$

$$- b\left(\frac{c-b}{b - \frac{b}{N}}\right) + \frac{b^2N}{L} - \frac{(c-b)^2}{2\left(b - \frac{b}{N}\right)} + \frac{b^2N^2}{2L^2}\left(b - \frac{b}{N}\right)$$

$$= M - c + c\left(\frac{c-b}{b - \frac{b}{N}}\right) - b\left(\frac{c-b}{b - \frac{b}{N}}\right) + \frac{b^2N}{2L} - \frac{(c-b)^2}{2\left(b - \frac{b}{N}\right)}$$

$$= M - c + \frac{(c-b)^2}{b - \frac{b}{N}} + \frac{b^2N}{2L} - \frac{(c-b)^2}{2\left(b - \frac{b}{N}\right)}$$

$$= M - c + \frac{b^2N}{2L} + \frac{(c-b)^2}{2\left(b - \frac{b}{N}\right)}$$

Case (TI4)

$$
\begin{aligned}
&\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection] \\
&= \left[M - E[p_i]\left(b + \frac{L-b}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + [M-c] \cdot [0] + \left[M - b - E[p_i]\left(b - \frac{b}{N}\right)\right] \cdot \left[1 - \frac{bN}{L}\right] \\
&= \left[M - \left(\frac{bN}{2L}\right)\left(b + \frac{L-b}{N}\right)\right] \cdot \left[\frac{bN}{L}\right] + \left[M - b - \left(\frac{bN}{2L} + \frac{1}{2}\right)\left(b - \frac{b}{N}\right)\right] \cdot \left[1 - \frac{bN}{L}\right] \\
&= M - \frac{b^2N^2}{2L^2}\left(b - \frac{b}{N}\right) - \frac{b^2N}{2L} - b\left(1 - \frac{bN}{L}\right) - \frac{bN}{2L}\left(b - \frac{b}{N}\right)\left(1 - \frac{bN}{L}\right) - \frac{1}{2}\left(b - \frac{b}{N}\right)\left(1 - \frac{bN}{L}\right) \\
&= M - \frac{b^2N^2}{2L^2}\left(b - \frac{b}{N}\right) - \frac{b^2N}{2L} - b + \frac{b^2N}{L} - \frac{bN}{2L}\left(b - \frac{b}{N}\right) + \frac{b^2N^2}{2L^2}\left(b - \frac{b}{N}\right) - \frac{1}{2}\left(b - \frac{b}{N}\right) + \frac{bN}{2L}\left(b - \frac{b}{N}\right) \\
&= M + \frac{b^2N}{2L} - b - \frac{1}{2}\left(b - \frac{b}{N}\right) \\
&= M - b - \frac{1}{2}\left(b - \frac{b}{N}\right) + \frac{b^2N}{2L}
\end{aligned}
$$

Case (TI6):

$$
\begin{aligned}
&\text{Payoff}[passivity] \cdot Pr[passivity] + \text{Payoff}[insurance] \cdot Pr[insurance] + \text{Payoff}[protection] \cdot Pr[protection] \\
&= \left[M - E[p_i]\left(b + \frac{L-b}{N}\right)\right] \cdot [1] + [M-c] \cdot [0] + \left[M - b - E[p_i]\left(b - \frac{b}{N}\right)\right] \cdot [0] \\
&= M - \frac{1}{2}\left(b + \frac{L-b}{N}\right)
\end{aligned}
$$

Total effort security game. Derivations for total expected game payoffs, not conditioned on other players: The following derivation refers to Table 15. For the total effort game, the dependence on other players is noted in terms of the integer K, the number of players other than player i who choose protection. To remove dependence on this K we must compute an appropriate expected value. To begin we rewrite each of the case expressions as a linear constraint on K. After doing this it becomes clear that cases TC2 through TC4 are mutually exclusive and exhaustive in terms of K, and similarly for cases TC5 and TC6. We define case TC2-4 to be the union of cases TC2, TC3, and TC4. similarly, we define case TC5-6 to be the union of cases TC5 and TC6. Now to compute an expected payoff for case TC2-4, we take the sum, over all possible values k for K, of the probability that exactly k players protect, times the payoff for this k (considering the case TC2, TC3, or TC4, that such a choice of $K = k$ determines). We proceed similarly to compute the expected payoff for case TC5-6.

To obtain the expected payoff for TC2-4 we compute:

Case (TC2-4):

$$\sum_{k=0}^{N-1} Pr[k] \cdot \text{ Payoff assuming TC2-4 and that } K = k$$

$$= \sum_{k=0}^{\lfloor N-\frac{c}{b}\rfloor} Pr[k] \cdot \left(M - c + \frac{c^2}{2L\left(1-\frac{k}{N}\right)} \right)$$

$$+ \sum_{k=\lfloor N-\frac{c}{b}+1\rfloor}^{\lfloor N-1-\frac{N}{L}(c-b)\rfloor} Pr[k] \cdot \left(M - c + \frac{b^2 N}{2L} + \frac{(c-b)^2}{2L\left(1-\frac{k+1}{N}\right)} \right)$$

$$+ \sum_{k=\lfloor N-\frac{N}{L}(c-b)\rfloor}^{N-1} Pr[k] \cdot \left(M - b - \frac{L}{2}\left(1 - \frac{k+1}{N}\right) + \frac{b^2 N}{2L} \right)$$

and to obtain the expected payoff for TC5-6 we compute:

Case (TC5-6):

$$\sum_{k=0}^{N-1} Pr[k] \cdot \text{ Payoff assuming TC5-6 and that } K = k$$

$$= \sum_{k=0}^{\lfloor N-\frac{cN}{L}\rfloor} Pr[k] \cdot \left(M - c + \frac{c^2}{2L\left(1-\frac{k}{N}\right)} \right)$$

$$+ \sum_{k=\lfloor N-\frac{cN}{L}+1\rfloor}^{N-1} Pr[k] \cdot \left(M - \frac{L}{2N}(N-k) \right)$$

where as before, $Pr[k] = \binom{N-1}{k}\left(1-\frac{b}{L}\right)^k \left(\frac{b}{L}\right)^{N-1-k}$ is the probability that exactly k players other than player i choose protection.