Geometric Abstractions Using Medial Axis Transformation

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Abstract

The most important concerns in the design of a product are to ensure that the product serves the purpose it is intended for, and to maintain the producibility of the product at a reasonable cost. Ideally, in design environments, these two concerns should be taken into account. As a result, analyses should be performed from the viewpoint of these concerns. Some of these analyses are purely mathematical in nature, some are entirely heuristic, but quite often they require a treatment which is a mixture of both. In modern computer-aided design (CAD) environments, the products are usually represented by their indiscriminated geometry, such as faces, edges, vertices, primitive solids, etc. Such a mathematical representation is not convenient for heuristic analysis. Heuristic reasoning is expressed in terms of high level abstractions. One formalism to capture these abstractions is to view the products in terms of its features. Features are a way of representing patterns in form or in function. In essence, they are meant to facilitate the practice of design and reasoning about the designed object. Therefore, features emerge as the link to bridge the gap between the geometry and the heuristic analysis.

However, to recognize features directly from geometry is rather difficult But if an object can be expressed in terms of its skeletal model, then the recognition task might be easier. The features, then, would be viewed as patterns in the skeletal model. Recognizing features in this fashion may turn out to be a much simpler task. The medial axis transformation (MAT) provides one way of getting a skeletal form from the geometry. By incorporating MAT, one can expect the recognition of features from the designed object to become less cumbersome.

This article describes a representation of form features using medial axis transformation. The MAT is defined, its properties are studied and its computation is investigated. Furthermore, the MAT data base is established and some examples of extracting features will be given.

1. Introduction

One of the primary aims of introducing geometric modeling is to provide a computer shape model which is as useful in design, analysis and manufacture as a physical prototype. However, contemporary geometric modelers, which emphasize the basic geometric entities, have no explicit representation of a higher levels of abstraction dealing with features (a way of representing patterns in form or in function). Because of the mismatch in abstraction levels, they are unsuitable for mechanical design.

Form features have aroused considerable interest. Part of the reason is the manufacturing process analyses need information in the form of features. Features for manufacturing applications can be defined as "areas or regions of a part having certain manufacturing importance". For example, in three-dimensional (3D) mechanical design, these features might be holes, fillets, slots, keyways, bosses and pockets. Features represent geometric forms and associated attributes that convey more abstract and functional specification to a designer than the detailed and indiscriminated geometric descriptions. Therefore, the process of identifying features from a geometric modeler plays an important role in automated applications.

However, to extract features directly from geometry is rather difficult because it involves recognizing high level abstractions from the set of lower level entities in a geometric model. Furthermore, the nature of the features are highly context dependent. Different features are required if the part is to be manufactured by different processes, for example, machining, casting, forging or stamping. Recently, feature recognition and extraction has become an active research concern in the CAD/CAM community. Many ideas has been proposed to perform feature extraction from a solid modeler [1] [2] [3] [4]. However, they are either restricted to a few predefined features, or fall short of providing a generic means to extract more general categories of form features. Another research activity tries to apply the syntactic pattern recognition technique and feature grammar to recognize shape features in geometric modeling system [5] [6] [7]. Syntactic pattern recognition borrows most of its analysis methods from formal language theory. In those approach, a pattern description language describes a set of patterns in terms of its structural forms [8]. However, the list of required patterns tends to grow quickly. Consequently, the feature recognition techniques tend to grow exponentially in computation time with an increase in feature types and part complexity.

If an object can be expressed in terms of its skeletal model, then the recognition task might be easier. The features, then, would be viewed as patterns in the skeletal model. Recognizing features in this fashion may turn out to be a much simpler task. For example, in injection-molded parts, ribs are defined as elements that are oriented 90 degree to the thin-shell elements. In Figure 1-1, the task is to recognize a rib sitting on a base of a simplified injection part. The feature rib can be recognized rather straight forward by identifying the two simplified faces join together in the skeleton model. But, in a conventional CAD system this task would not be so simple, because a feature recognizer would need to perform an expensive search in the geometric representation in order to find a specific (geometric) relation that matches a predefined rule.

The medial axis transformation (MAT) is a bijective mapping between the original object and the simplified medial axis. This mapping allows one to visit the transformed space, to perform recognition task and then travel back to the original object's space to modify the recognized geometry, see Figure 1-2.

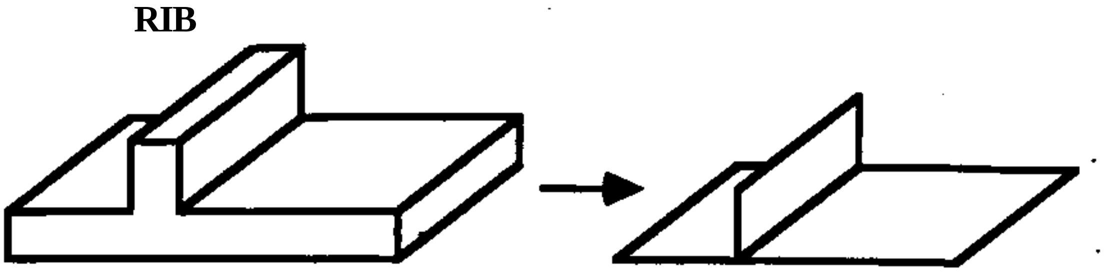

Figure 1-1: The skeleton model: an abstraction of the injection molding part

In doing this, the relevant geometric properties and topological relationships of a feature can be obtained easily. The MAT is introduced to serve as an intermediate abstraction to bridge the gap between the geometric representation and the feature based representation.

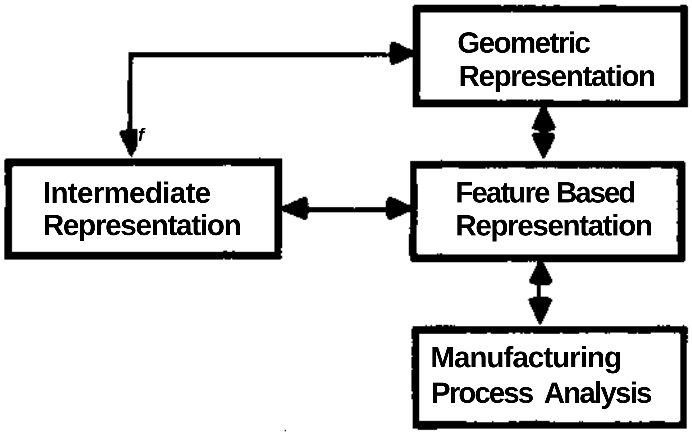

Figure 1-2: Recognition of features can be easier through the intermediate representation

In this article, the medial axis transformation is introduced as a means of getting the skeletal model. An important characteristic of the **MAT** is that it can be used to simplify the original object and still retaining the inherited object's information. For instance, the two-dimensional MAT defines a unique, coordinate-system-independent decomposition of an planar figure into lines; and the three-dimensional MAT simplifies an object into surface patches. The generated medial axis, called a skeleton, can be used effectively for shape description and feature recognition. In addition to the mentioned feature recognition application, the skeletal model also has numerous applications in many diverse areas such as computer vision [9], robot path planning [10] [11], machine cutting , wire layout, computer graphics and medical diagnostics [12].

2. A Brief Review of the MAT

The medial axis transformation is first proposed by Blum [13] as a means to describe a figure. Since then, there has been a number of attempts to find the medial axis; for example, Souza et. al [14] represented the medial axis by computer locating medial points through an array of mesh points approximation. Another research effort in the field of computational geometry, is to find the Voronoi diagram for certain elements. The Voronoi diagram on a set 5 of *n* points in the plane is a partition of the plane into *n* regions, one containing each point The region around point *peS is:*

$$V(p) = [q\ |\mathrm{Vr}_{€}\ \mathrm{S}- [p\}_9\ d(p4)< d(r_f q))$$

that is, the part of the plane closer to *p* than to any other point of 5. Example of the Voronoi diagram of a rectangular is shown in Figure 2-1.

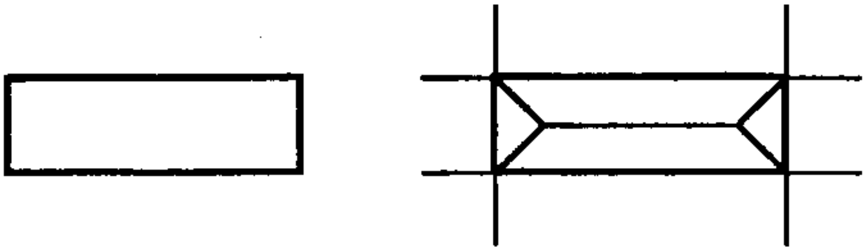

Figure 2-1: An example of the Voronoi diagram

In spite of the fact that the medial axis is a graphical subset of the Voronoi diagram, they are two different entities. Mathematically, the medial axis transformation is not formally deriveable from the Voronoi diagram; particularly, the forward transform or inverse mapping do not exist

The 3D MAT was first mentioned by Nackman and Pizer [15]. They proposed a theory to describe a 3D shape using the symmetric axis transform. Their classification theory can be used to address the shape variation. However, in their paper, no algorithm was mentioned about deriving the 3D symmetrical axis transformation. Later, Nackman [16] extended Bookstein's 2D line-skeleton [17] to approximate the 3D medial axis. But, the resulting line-skeleton of Bookstein's method is not the symmetric axis of a polygon. It has neither parabolic arcs nor segments contacting non-reentrant vertices. Thus, the discrete approximation in Bookstein's method will generate at best a linear approximation of the corresponding polygon. Therefore, the extension work to 3D resulted in losing the generic meaning of the true medial surfaces.

2.1. Characteristics of MAT

The most interesting and important characteristics of MAT are as follow:

- Decomposition
    - 2D MAT transforms planar figures into lines.
    - 3D MAT decomposes an object into surface patches.

- The medial axis is uniquely defined and coordinate system independent.
- The original object can be reconstructed from the MAT.

2J. Definition of MAT

The MAT is defined as follows: given an object represented, say by a polygon *Q*, the medial axis is the set of medial points internal to *Q* such that there exists at least two points on the object's boundary that are equidistant from the medial point and are closest to the medial point Associated with the medial axis is a radius function, which defines for each point on the axis its distance to the boundary of the object. The medial axis (MA) of a figure can be defined as the locus of centers of all the maximal disks, inside the figure but contained in no other disk. The medial axis transformation consists of the MA and the radius function. The MA of a figure is also called the skeleton or the symmetric axis of the figure. With the MA and the radius function one can reconstruct the figure by taking the union of all circles centered on the points comprising the MA, each with a radius given by the radius function.

A medial point can be classified into one of three types (i.e. end point, normal point, and branch point) depending on the order of the point End points are of order one, nonnal points of order two, and branch points of order three or more, corresponding to the number of neighboring medial point(s), as in Figure 2-2.

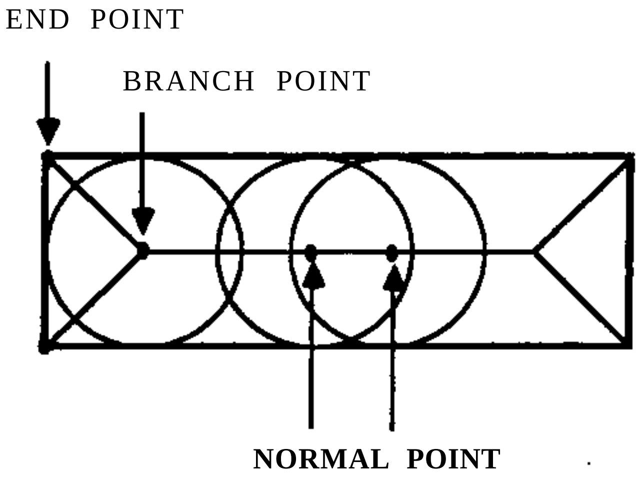

Figure 2-2: Medial axis point types

The symmetric surface of a three-dimensional object can also be viewed as the collection of the internal quench points of a grassfire started on the object's surface. Analogous to the 2D case, one can identify maximal spheres as those which touch the boundary at two or more points. The symmetric surface is then the surface of the centers of all the maximal internal spheres, those sphe: inside the object which cannot be wholly contained by any other internal spheres.

Therefore, we can define the medial axis transformation, O, such that the following equations hold for any point *X* on the boundary of the object that has an unique nonnal.

$$\Phi[X_i(x,y,z)] = \Phi[X_j(x,y,z)] = C(x,y,z) \qquad (1)$$

$$|C - X_i| = |C - X_j| \qquad (2)$$

where *C (x,y,z)* is the center of a disc tangented to point X_i *(x,yj)* and point *Xj* (x,y_tz) on object's boundary, as in Figure 2-3. In this article, the MAT and the forward transform were referred as being interchangeable. The inverse transform Q^{*1} associates at least two points on the object and can be defined as

$$\Phi^{-1}[C] = X_i = X_j \qquad (3)$$

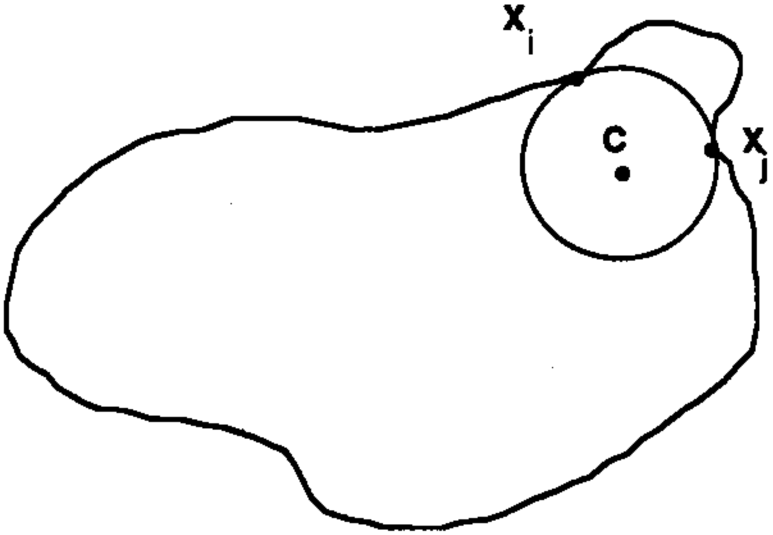

Figure 2-3: Graphical meaning of the medial point *C*

3. A Methodology For Calculating MAT

The formulation of finding the medial point for a given point on the object's boundary are illustrated in the following sections.

3.1. Two-Dimensional case

A simple drawing, see Figure 3-1, is used to illustrate of how to determine the 2D MAT. Points X_t and X_j are on different edges having the unit normal vector n_i and n; respectively. (Assumed that the direction of the normal vector always points inside the polygon Q.) The 2D MAT can be viewed as: given *{* X_{it} n_{it} $^{n}j\}>$ ***j*> *j* = 1» 2,3, • • • *k*. *k*= number of edges; find the corresponding medial point *C* (i.e. the center of a maximum circle with radius r) satisfies the following algebra equations.

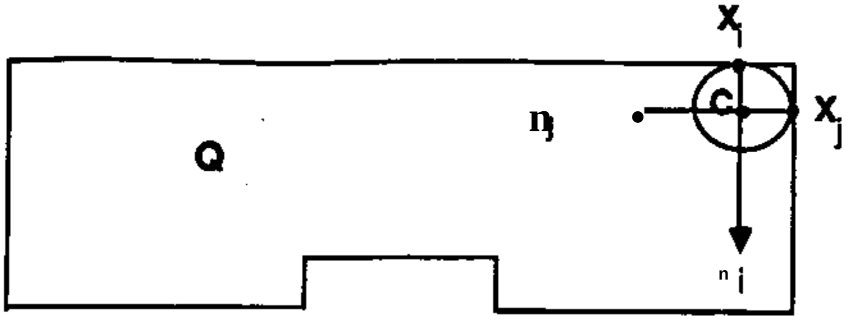

Figure 3-1: Illustration of determining the 2D MAT

$$C = X_i + r\,n_i \tag{4}$$

$$r = |C - X_j| \tag{5}$$

$$(C - X_j) \times n_j = 0 \tag{6}$$

3.2. Three-Dimensional case

Likewise, the 3D MAT can be viewed as : given a point X_i on the boundary of *Q*, determine the corresponding medial point *C* (i.e. the center of a maximum sphere of radius r)that satisfies Eq(4) to Eq(6). As illustrated in Figure 3-2, *Q* is a polyhedra representing a 3D object X_i and *Xj* are points on the object's boundary. The unit normal vectors of faces *i, j* are also drawn (assumed that the direction of a normal vector points inside 0.

4. A Prototype Implementation Of 2D MAT

An approach to automatically generate the MAT of 2D linear objects has been developed with a geometrical modeler. The approach can be applied to simply-connected polygons as well as multiply-connected domains.

The system first calculates the medial points from the set of discretized boundary. The generated medial points are stored and need for further classification to form the medial axis. The classification scheme takes neighboring medial points and inspects their inverse mapping. If their inverse mappings share same edges of the boundary, then they will be grouped together as in the same medial axis, otherwise, they belong to different medial axis. The topological relationship of the medial points derives directly from the original object without resort to other heuristic means.

In the data structure, the medial axis are represented by a set of "segments". Each *segment* is a subset of the medial axis. Segments consist of medial points and interconnect by *junctions.* The *junction* joins two or more segments. Junctions and segments form a graph, called the junction graph or skeleton graph, to represent the medial axis and the original object Figure 4-1 is used as an example to show the junction graph corresponding to a rectangular plate. Features, then, can be described and recognized using the junction graph.

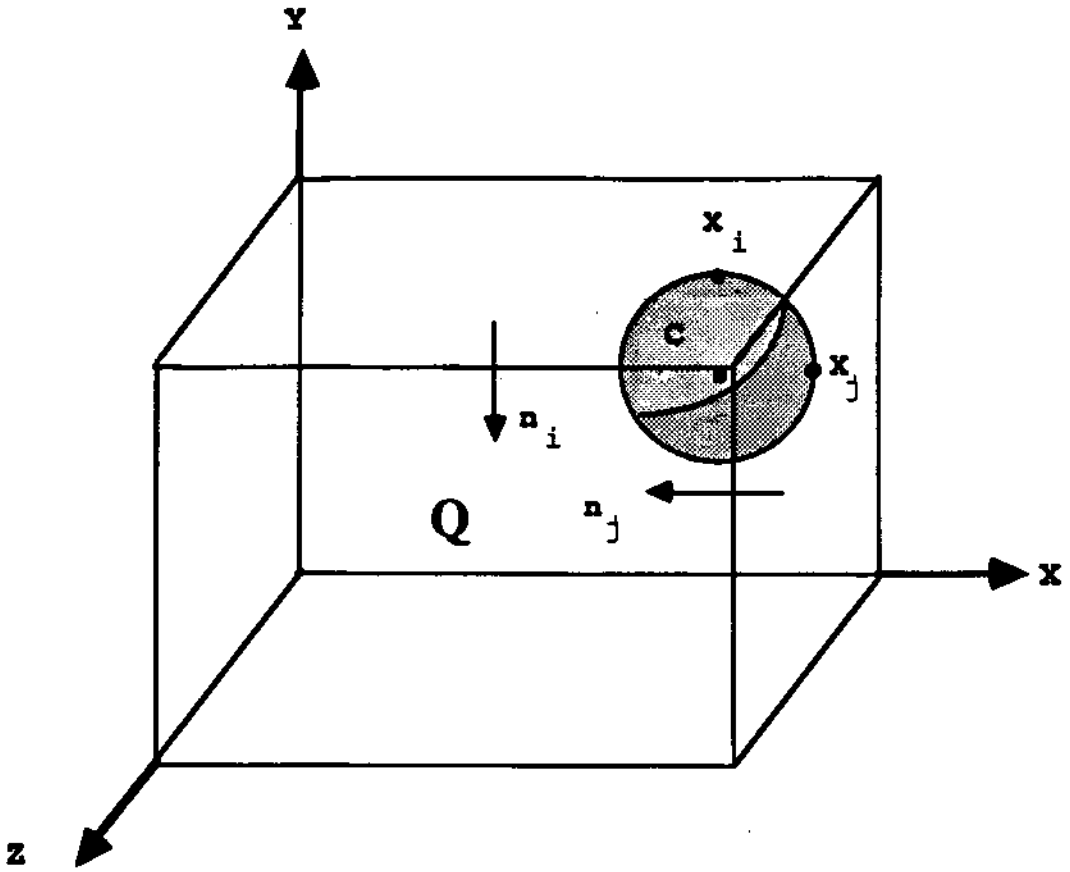

Figure 3-2: Illustration of finding the 3D MAT

Each segment is parametrized into polynomial(s) in terms of an independent variable in the interval 0 to 1. The shape of a segment is either a straight line or a parabolic curve. As a result, the parametric equation is of first order if the segment is a straight line; and a second order if it is a parabola . Having this parametric representation, a segment becomes a continuous curve.

5. Examples of Two-Dimensional Medial Axes and Application

Examples of different parts and features, taken from stamping and injection molding processes, are presented to illustrate how the calculated MAT can be used in some real-world applications. Figure 5-0 shows a rectangular plate and its corresponding MA. The representation of the plate is simplified into several straight lines as a result of the medial axis transformation. The feature corresponding to this skeleton can be further deduced to a line.

Figure 5-2 represents the MA of a plate having a hole inside. The shape of MA is a closed loop, this tells that the original plate has a inside hole, again the distance between the hole and outside plate is explicitly recorded in the corresponding radius function. Figure 5-3 shows a stamped piece for "E" shape transformer core. Evaluation of this part design requires to recognize part key features. In this particular

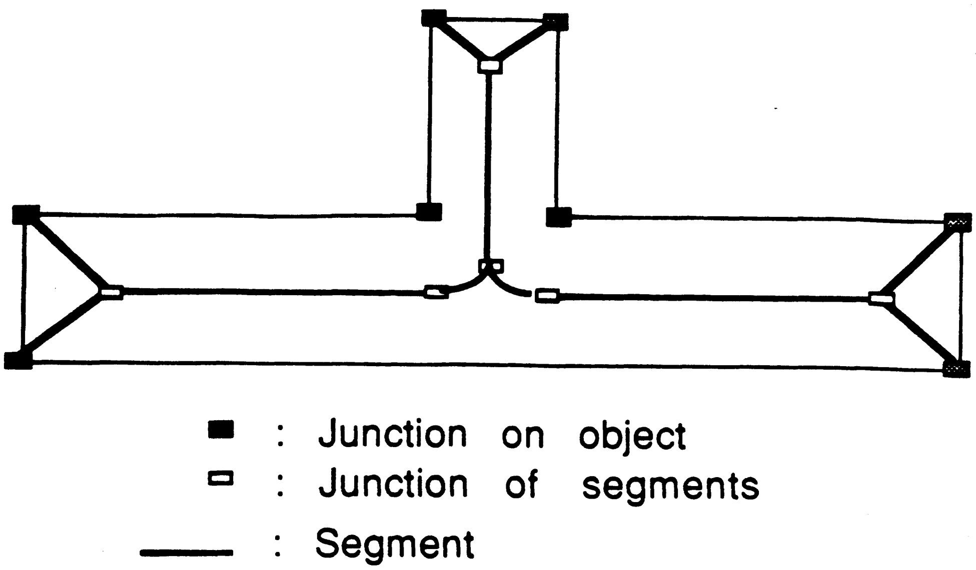

Figure 4-1: Junction Graph: the MAT Representation

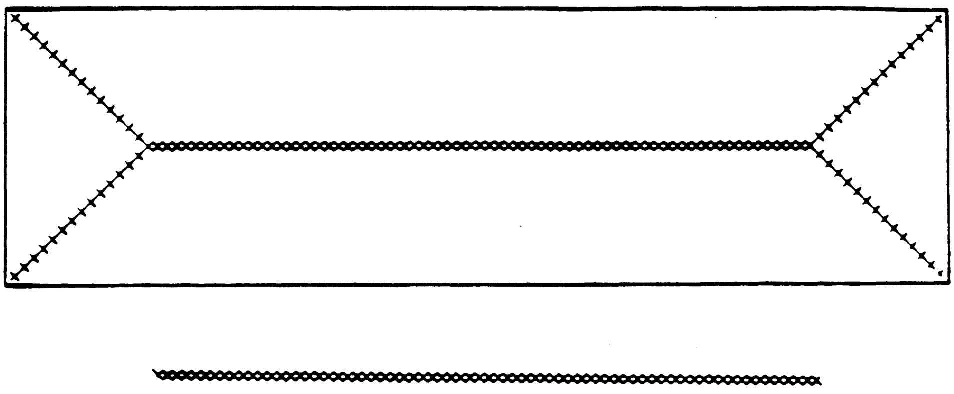

Figure 5-1: A rectangular plate, its MAT and simplified feature

example, features, namely three fingers and two slots, are found by matching different templates (templates are feature descriptors) in the MAT "junction" and "segment" abstraction representation. The geometrical and topological characteristics are essential to the knowledge base to perform interrogation and evaluation in the part design. For example, the domain knowledge of design for stampability will acquire the dimensions of the related features to determine whether one particular feature (such as finger and slot) can be stamped without causing fracture either to the part or to the stamping tool, or the part simply cannot be stamped.

Interactions between the knowledge base and the MAT representation can be seen in the following questions (using the previous example).

1. What is the ratio of the height of the transformer core to the finger width?

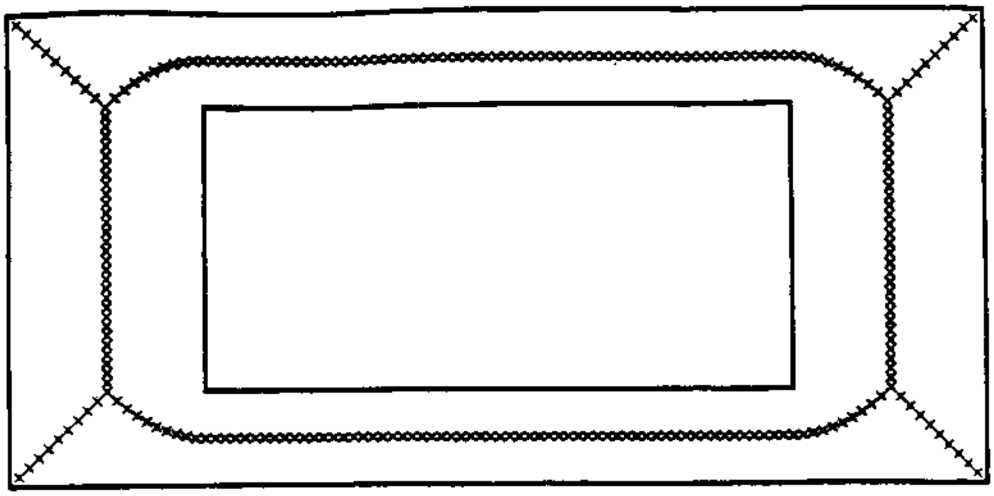

Figure 5-2: A rectangular with a hole inside and the computed MAT

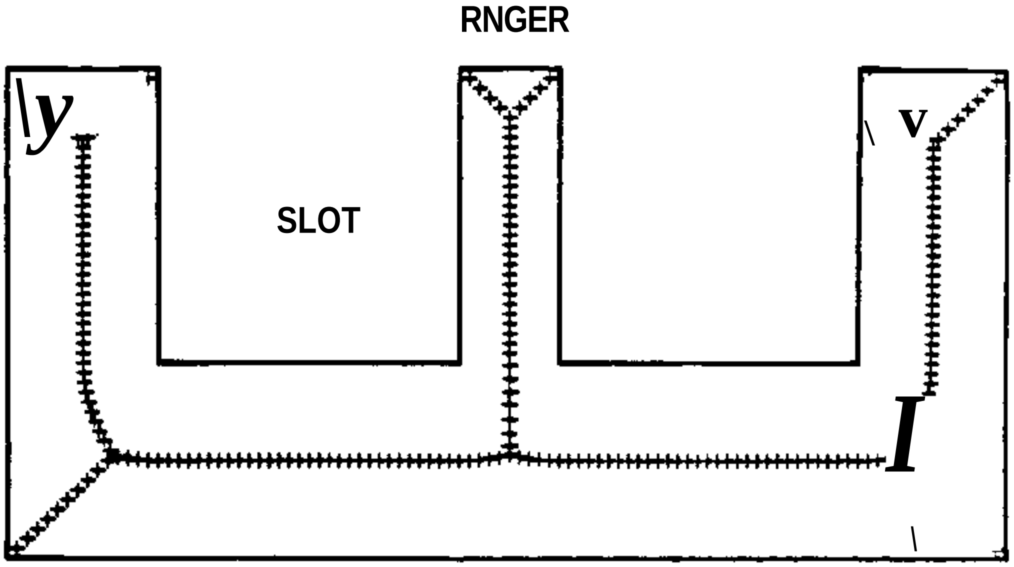

Figure 5-3: A transformer core part and its medial axes

2. **How many fingers and slots are there** in this part?
3. **How many holes exist in this part?**
4. **Are the sizes of fingers equal?**

Finger **and** *slot* **have different segments** joining together *at junction.* Using the MAT representation, the **manner in which answers to the above questions are provided below.**

Features of *finger* and *core* can be identified by recognizing different *segment-junction* pairs that matches the finger's template and core's template accordingly. By doing this, three fingers and two slots are recognized. The recognition task is trivial by using the MAT representation. However, the recognition process was not possible or too tedious to be implemented by using surface or boundary representations. The dimensions of a finger or a core can be seen from the recognized *segment* by the radius function and the length of the segment, which was hardly obtained by geometrical reasoning. Another useful recognition is: there is no hole inside the part since there is no closed loop formed by the segments. This deduction came along with the MAT representation, that many key topological relations are inherented to this representation. Therefore, answers to the above questions can be found in the MAT data structure either explicitly or implicitly. It shows by adopting the MAT as an intermediate representation, the effort of recognition process can be cut substantially.

6. Summary and Future Work

This article describes research in progress on the representation in recognizing features in order to assist CAD/CAM systems. The ability of CAD systems to reason about form features can contribute to major improvements in design for manufacturability and manufacturing process analyses. To develop this ability, representation in terms of features is needed.

This research work is different from other approaches in feature recognition because features are viewed as patterns in the skeletal model. Furthermore, the medial axis transformation was introduced as a means to obtain the skeletal model. A simple methodology have been implemented to compute the MAT in 2D. This method can be applied for finding the MAT of the simply-connected as well as of the multiply-connected polygonal objects. Using this method, recognition and extraction of features from the geometry of 2D objects was demonstrated. The significance of the approach is the ability to communicate with a geometric modeler at a "feature" level and capture information not explicitly stored in the CAD data base. Furthermore, the processes of feature recognition and evaluation emerge as a single task by adopting the skeletal model.

The next step of this research is to construct the skeleton grammer to describe and recognize features in the skeletal model. In the mean time, the extension of this work towards a 3D MAT is currently being implemented. For this, the forward transform and inverse mapping have already been worked out. However, some difficulty arises from the complexity of grouping the medial points into corresponding medial surfaces. Further enhancements will be made to complete the 3D medial axis transformation in the near future.

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