Accounting for Intangible, Non-Marketed and Sunk Capital

Robert D. Cairns*
Department of Economics and Cireq
McGill University
855 Sherbrooke St. W.
Montreal QC Canada H3A 2T7
robert.cairns@mcgill.ca
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Abstract. Studies of intangible and of environmental assets have become key vehicles for the advancement of economists’ understanding of capital. Each has features that diverge from those of “produced means of production”. The present paper addresses issues in capital accounting through an analysis based on an investment project. A project consists of various types of non-marketed capital that have no prices and of marketed capital that are sunk or irreversibly invested. While sunk, capital is valued at internal values that have only a limited relation to external, market prices. A method of accounting is proposed that puts all types into a comprehensive analytical framework. Each type earns the market rate of interest.

JEL codes: D24, D92

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1. INTRODUCTION

The theory of comprehensive national accounting (e.g. Weitzman 1976, 2003) stresses that, in addition to the traditional “produced means of production” an economy’s capital encompasses assets that do not have market prices. Environmental resources are tangible assets that may not have prices. Measurement entails dynamic accounting for multiple types of capital, in the sense of Dasgupta and Mäler (2000). Two pertinent distinctions are between tangible and intangible and between marketed and non-marketed capital.

Hall (2001) finds intangible capital to be empirically important; Corrado, Hulten and Sichel (2009) find it to be a major contributor to economic growth. Subsuming intangible capital into non-marketed capital, Abraham (2005: 10-15) finds that the latter is increasingly important in modern economies. Among accounting issues discussed in a volume edited by Corrado, Haltiwanger and Sichel (2005) are the lack of arm’s-length valuations, inter-relatedness of assets, the strength of theoretic and empirical assumptions, and the estimation of deterioration and decay. Since capital is an input that contributes to production over two or more periods, irreversibility is a fundamental property.

Tangible capital goods are physically measurable. They can deteriorate with use or through time. But they may not have a market price: “marketed” and “tangible” are not synonymous. Some intangible capital (e.g. patents, software) may have a price. A feature of much intangible capital, however, is that it is not marketed.

It is essential to account consistently for non-marketed assets, even if they do not have market prices. The present paper provides a method of economic accounting that is symmetrical among all types of capital in a project or firm. Mathematical requirements are light, amounting to manipulations of the formula for net present value of an investment project under certainty. As in some macro studies (e.g. Hall
2000, World Bank 2011), non-marketed capital can be expressed only as an aggregate and only as a residual. Moreover, between transactions, irreversibility makes the components of the marketed capital stock in some ways comparable to non-marketed capital.

Accounting values are ascribed for both the marketed items and the non-marketed residual. They are internal to the project. The development leads to a reconsideration of some salient accounting issues including depreciation, investment timing, the internal rate of return, Tobin’s $q$, and the roles of shadow prices.

2. A SIMPLE INVESTMENT PROJECT

Capital is organized into projects. The view of a project in this section is consonant with the irreversible investment of capital of multiple types.

Definition 1. A project is a composite of distinct types of capital and other inputs that produces output.

Doms (1996: 80, n. 7) notes that much investment is lumpy, not convex. The simplest case, of investments at a single point of time, is fundamental.

Irving Fisher’s separation theorem (cf. Hirschleifer 1958, Becker 2009) expresses a dichotomy between investment and consumption decisions: market wealth is first maximized and then allocated to consumption over time. The theorem provides a link between welfare and capital through its focus on flows of financial value.

A Marxian perspective is that financial capital $M$ is first transformed into physical capital $K$ and then to financial capital $M'$, represented as $M \rightarrow K \rightarrow M'$, with $M' - M$ deemed to be surplus value accruing to the capitalist. A less consolidated view is that there are sources of the surplus. To be an asset a good must contribute to the project’s cash flows.

Definition 2. Any stock that contributes to future cash flow is a type of asset.
A surplus or rent that accrues in the future can be capitalized. Rent can be uncoupled from tangible assets by viewing a source of rent as a form of non-marketed capital. For example, entry barriers such as patents, copyrights, tariff walls, brands or trademarks underlie monopoly rent. Firms invest to enhance and to maintain their entry barriers. Entry barriers have all of the analytic properties of assets. They are not properties of other forms of capital.\(^1\)

Let the vector of marketed capital goods of a project at time \(t\) be represented by the \(k_t\)-vector \(K_t\) and the vector of non-marketed capital goods by the \(n_t\)-vector \(N_t\). Non-marketed capital includes entrepreneurship, effort, reputation, brand, some tangible environmental goods without property rights, etc. In the Marxian schema, it is not a part of \(M\) but it contributes to \(M'\). Some types of intangible capital are not mediated because they are qualitative and do not have natural units of measurement, such as organization (Lev and Radhakrishnan 2006: 74).

All the capital goods in concert, not severally, produce the value \(M'\). Like the atoms in a chemical compound, marketed and non-marketed capital are transformed and coalesced into the project. Aggregation must be in a common unit, the numeraire. So long as a capital good remains with its project, in many cases until it is scrapped, its contribution comes not solely on its own account but as a result of complementarity with other capital. It is not possible to disaggregate a project’s income into distinct contributions from individual assets.

Let the length of the project’s life be represented by \(T \leq \infty\). At time \(t < T\), the project \(P_t\) is identified with the vector of its future net cash flows, \((m_{t+1}, \ldots, m_T)\).

\(^1\)Accounting aims to measure attained value, not potential value attainable through socially or even privately optimal behavior. Valuation of an entry barrier explicitly indicates the source of the gain. It does not attribute the gain to marketed capital. Although deadweight loss is not reported, the measurement indicates that there is a consequence of the restriction through the redistribution toward the project’s owner.
These flows may be from one or more products. The transformation from the initial components to the “omelet” of project capital $P_0$ is an investment at $t = 0$ in which, for cash outflows $M^i_0$, $i = 1, \ldots, k_0$, with $M_0 = \sum_{i=1}^{k_0} M^i_0$, and for variable profits or net cash flows $m_t$,

\[
(M^1_0, \ldots, M^{k_0}_0) \to (K^1_0, \ldots, K^{k_0}_0) = K_0;
\]

\[
(K_0, N_0) \to P_0 \equiv (m_1, \ldots, m_T).
\]

The full expenditure is incorporated in $M^i_0$, including any fixed charges, installation costs, etc. In $m_t$, there may be multiple products, bundling, non-linear prices, etc. Let the interest rate in period $t$ be denoted by $r_t$. Furthermore, let the present value of project $P$ be represented by $V(P)$. At $t = 0$,

\[
V(P_0) = \sum_{t=1}^{T} \frac{m_t}{\prod_{s=1}^{t} (1 + r_s)}.
\]

Hulten and Wyckoff (1996: 11) consider the price of a used asset to be both the present value of the income accruing to the asset and what a rational investor would pay for it. The value of the project, $V(P_t)$, has both properties since the remaining net present value can be realized in the capital market (Diewert 2009: 9-10).

Since they depend on complementarity with the other assets, marketed and non-marketed, the contributions of a non-marketed asset are specific to the project (cf. Oliner 1996: 69). The value of a non-marketed asset is not directly observed. If there is no unit, a marginal value is not defined. Non-marketed assets have incremental values, $I(N^j_0) \equiv V(P_0) - V(P_0 \setminus N^j_0)$, but in general, because of complementarity, $\sum_{j=1}^{n_0} I(N^j_0) \neq V(P_0) - M_0$. Even under certainty, the contributions of the non-marketed assets are confounded.

Definition 3. The value of the non-marketed capital at $t = 0$ is the discounted net cash flow minus the cost of the marketed capital,

\[
U(N_0) \equiv V(P_0) - M_0.
\]
Non-marketed capital is at the heart of capital valuation because the residual $U(N_0)$ imparts additivity over assets: $\sum M^i_0 + U(N_0) = V(P_0)$. In a project (\textit{a fortiori} in the aggregate), non-marketed assets provide a resolution of the controversy about whether wealth consists of discounted consumption or the value of assets: it is both. It obeys a double-entry condition.

The residual is defined by Hall (2000, 2001) and the World Bank (2011) as intangible capital. Comparable assets with market prices cannot be defined and used to reduce the residual to zero: an uncertainty principle applies. “Hall’s residual contains a diverse collection of factors that we will never fully understand” (Lamont 2000: 113).

Solow’s (1957) residual, for example, is the contribution of technical change attributed to the capitalized value. Empirical difficulties with it are in part the result of its being confounded with other non-marketed capital. Laffont and Tirole (1993) analyze the informational rents of a regulated firm. These rents are confounded with managerial skill and with efforts to limit cost. If the rents accruing to a firm are attributable in part to managerial effort or skill in reducing costs (if the costs are not known to the regulator because effort or skill is not known), the reported profit of the firm may include rents to a scarce input and not to the exercise of market power which is the object of regulation.

For a developed, producing petroleum reserve $R_t$ with net price of output $(p - c)_t$, the value of the combination of the reserve and the development investment is $V(P_t) \approx \frac{1}{2} (p - c)_t R_t$ (Adelman 1990). If the marketed capital required to exploit the reserve has value $M_0$ and the reserve is not priced in a market (because, say, it is developed by the firm that discovered it), then the value of the reserve is sometimes considered to be $V(P_0) - M_0$. Non-marked capital of various sorts is also vital in extraction. If, for example, the exploration firm sells the discovery to another firm that is more efficient at exploiting the reserve, there is an observed value of the reserve (it is marketed
capital), but there is also a rent of composition (Alchian 2008), the value net of the cash costs. The source of the greater efficiency is a non-marketed asset. If the more efficient firm discovers and exploits the reserve, the values of two non-marketed assets (the reserve and the organizational capital) are confounded in the residual.

If units of an intangible capital good can be devised and it has a price (e.g., a licenced patent or purchased software) then it can be classed among the marketed capital goods. If the good, i say, has price $M_i^0$, then

$$V \left( P_0 \setminus K_0^i \right) - \sum_{j \neq i} M_j^0 - M_i^0 > 0. \quad (3)$$

The LHS of condition (3) is a part of the value of the non-marketed asset. In this case, pricing the intangible capital is an exercise comparable to valuing specific, tangible capital (cf. Williamson 2005).

3. SUNK ASSETS AND VALUE SUNK

The theory of investment under uncertainty has highlighted the importance of irreversible investment and brought the implications of sunk capital into prominence. Dixit and Pindyck (1994: 211), for example, view a firm’s value as being largely the value of a set of options. Conditions derived are for making an investment or disinvestment — for the exercise of a real option.

Irreversibility also frames the valuation of capital during periods when an option is not exercised. Accounting would be trivial if there were observable prices at all times for all assets. Sometimes used capital goods are sold. For possibly long periods, however, they are not sold. Projects remain in operation because their assets are better deployed in the project than elsewhere.

Since irreversibility is a key feature of capital and gives rise to a sunk cost (Dixit and Pindyck 1994), in economic accounting it is important to have an idea of what the term “sunk” entails and how much is sunk. In static analysis, what is sunk is the
difference between the cost of installing the capital and its net realizable value in the capital market. A natural generalization of the concept, for $t > 0$ and for capital that deteriorates, is the difference between accounting value and installed value: sunk cost is the value forgone by taking the net realizable value.

At any time $t < T$, the marketed, tangible capital of type $i$ has deteriorated from $K^i_0$ through use in the project. Good $i$ remains with the project at time $t$ if its productivity in the project, represented by $u^i_t K^i_0$, $u^i_t \leq 1$, contributes to the project more than its net realizable value $v^i_t (K^i_t)$ (its market value net of transactions or adjustment costs), where $K^i_t$ is not necessarily equal to $u^i_t K^i_0$:

$$V (P_t) > V (P_t \mid u^i_t K^i_0) + v^i_t (K^i_t).$$  \hspace{1cm} (4)

If condition (4) holds, good $i$ is not sold, not removed from the project.

Definition 4. Marketed capital is sunk if its incremental contribution to its project exceeds its net realizable value.

A new car famously loses a significant portion of its value as soon as it is driven off the dealer’s lot. Citing Akerlof (1970), some textbook writers and others attribute the decline in value to its being a “lemon”, having defects that the owner can perceive but the buyer cannot. An informational asymmetry applies to used cars. However, in view of the time an owner has owned a new car (hardly time to obtain specific information), of new-car warranties (Akerlof mentions guarantees) and of computer diagnostics, an alternative view is that the decline in price represents the inability to realize in a thin market the values of unpriced, irreversible inputs that increase the value of the car to a particular buyer. These assets include options (even color) and the dealer’s services, including making a market. They are sunk. A buyer who

\footnote{New-car brokers’ prices for the “same” car are significantly lower than dealers’. Brokers make a market, for cars which they can obtain, but provide fewer other services. As a thought experiment suppose that a consumer contracts for the delivery of a new car in a month’s time but decides the}
does not sell immediately continues to benefit from the inputs, only some of which are priced. The example suggests that specificity to the user is important in the valuation of capital and that depreciation of value is not the same as the value of physical deterioration. Because of unpriced inputs the value of the tangible asset is not necessarily the market value.

More generally, the values of different units of type $i$ are not necessarily equal to the value of the marginal unit of type $i$, nor, if sold, are necessarily sold at once. Some molecules of water evaporate from a lake. Others do not. A different value is implied if not all of the units of a given type of capital are being considered for possible sale. (For a fraction of the capital $f \in (0, 1)$, the relevant external or net realizable value $v^i_t (fK^i_t)$ may not be equal to $fv^i_t (K^i_t)$.) Depending on what comparison is made, there are a multitude of values sunk. Moreover, through complementarity, non-marketed capital contributes to value sunk.

There is an option to use or to sell marketed capital at any time. If the better choice is to keep the asset in the project, the external price is the price of a sub-optimal option. So long as value sunk is positive, the value of the outside option is zero and the level of the external price does not affect decisions. As with a non-binding constraint, whose shadow value is zero, the value of the outside opportunity may as well be zero. For purposes of accounting, the outside opportunity is not taken into account. There is no need to record the external value.

The net external value $v^i_t (K^i_t)$ would be realized by selling the capital of type $i$. If a transaction is the best option at time $s$, the option is exercised. Accounting for the capital is done on $[0, s]$. Even so, if the inequality in equation (4) is reversed, the asset still may not necessarily be sold: the firm will wait until $t$ for which $[V (P_t \backslash u^i_t K^i_0) + v^i_t (K^i_t)] / \prod_{s=1}^{t} (1 + r)^s$ is maximized. Until then, the capital remains car is not appropriate after two weeks and lists it for sale. Would the consumer expect to receive the full price? Gross of transactions costs?
with the project. The external value is accounted at the time at which it is sold. Over the interval $[0, t]$, depreciation $p_0^i(K_0^i) - v_i^t(K_t^i)$ is recorded at the point when the option to sell the capital is realized. Again, there may be a myriad such options at any time.

If a capital good is sunk, then, its full contribution to its project cannot be realized in the capital market. The productivity of capital good $i$ has links to the other periods of its life through the factors $u_i^t$.

Definition 5. Value sunk in a project is the difference between the value of the project and what can be realized by selling some or all of its marketed constituents at the most opportune time.

The above considerations imply the following.

**Proposition 1** The value sunk in a project is the value of the option to continue with the project rather than to sell some or all of the marketed capital at the most opportune time.

An option may be to rent out sunk capital for less than its remaining life. One might suggest that the potential flow of rent puts a floor on the user cost of a given type of capital at time $t$. But the opportunity of obtaining that rent does not affect the realized cash flow from the project at any time (including $t$) and does not affect the choice of the capital stock. Renting the asset out is another sub-optimal option.

**Proposition 2** The value of an outside option, such as net realizable or replacement value, is not used in accounting for sunk capital.

Once an asset is sunk it acquires some properties of a non-marketed asset, even if it is feasible to trade it in a second-hand market. Doms (1996: 80, n. 6) questions “whether the types of assets that are sold in second-hand markets are representative
of assets in place”. The price in the second-hand market is an upper bound for the value in the previous use and a lower bound in the new use. Use of a second-hand price is akin to benefits transfer in cost-benefit analysis. The transfer is an approximation sometimes used for want of better information.

For tangible capital, physical deterioration \((1 - u_i^t)\) is only one factor that affects depreciation. It is central to consistent accounting that depreciation not be identified as deterioration multiplied by some external price. Moreover, depreciation applies to all types of capital, even non-marketed capital. The method proposed in the next section uses prices in observed transactions for marketed assets to derive economic-accounting values that are internal to the project. Additivity is satisfied through the definition of the residual for non-marketed capital.

4. ECONOMIC ACCOUNTING FOR NON-PRICED ASSETS

Neither vintage-asset nor vintage-rental prices exist for most assets, certainly not for non-marketed capital. Even if there is a rental market, as for some buildings, vehicles and tools, the capital good is sunk to the firm that owns it. This section proposes a methodology of measuring the value of an asset over time when there is no market price for the good or when the market price is not appropriate.

The over-riding problem of capital measurement is projecting the stream of cash flows \(m_t\) (Diewert 2005: 535, n. 68).\(^3\) This problem is resolved under the conditions of certainty assumed herein. The project has a unique net present value, which can be obtained in the market at any time (Diewert 2009: 9-10). The change in its net present value, its depreciation, is unique (Hotelling 1925, Samuelson 1937).

The proposed attribution of depreciation and user cost to each type of capital equates its cost and contribution to value. The method, which owes much to an analysis by Baumol, Panzar and Willig (1982: ch. X), relies on a constructive def-

\(^3\)The cash flows can be interpreted as deflated to \(t = 0\).
inition of sources of cash flow as types of comprehensive capital and then identifies rental rates or user costs.

Given the cash flows, for any \( t \geq 0 \) let the rental or user cost of marketed capital of types \( i = 1, \ldots, k_t \) be denoted by \( \kappa^i_t \) and of the non-marketed residual by \( \nu_t \). The following properties are taken as foundational.

**Axiom 1. Non-negativity.** *User cost at any time is non-negative.* For originally marketed capital good \( i \),

\[
\kappa^i_t \geq 0. \tag{5}
\]

Moreover, for the non-marketed capital,

\[
\nu_t \geq 0. \tag{6}
\]

**Axiom 2. Cross-Sectional Additivity.** *At any time the sum of the user costs over all of the comprehensive capital is the user cost of the project, or the variable profit:*

\[
\sum_{i=1}^{k} \kappa^i_t + \nu_t = m_t. \tag{7}
\]

**Axiom 3. Temporal Additivity.** *The discounted user cost of each marketed capital good \( i \) is equal to the original cost of the good:*

\[
\sum_{t=1}^{\infty} \frac{\kappa^i_t}{\Pi_{s=1}^{t} (1 + r_s)} = M^i_0. \tag{8}
\]

*The discounted user cost of the non-marketed capital is the project value net of investment in marketed capital:*

\[
\sum_{t=1}^{\infty} \frac{\nu_t}{\Pi_{s=1}^{t} (1 + r_s)} = V (P_0) - M_0. \tag{9}
\]

Conditions (5) and (6) imply that \( \kappa^i_t \leq m_t \) and \( \nu_t \leq m_t \). Conditions (5) through (9) assure that under certainty no asset value ever turns negative.

**Definition 6. Admissible schedule.** *A schedule of user costs or of depreciation is admissible if it satisfies conditions (5) through (9).*
The schedule of rentals or user costs of an asset is (a) non-unique and (b) internal to the project. It allows for the recovery of the original capital value of the asset from the proceeds of the project. A schedule of user costs gives rise to a depreciation schedule, equal to the change in discounted user cost (and vice versa). It bears stressing that the value of a project at time $t$, $V(P_t)$, its depreciation $V(P_t) - V(P_{t+1})$ and its return $r_tV(P_t)$ are all unique and valued externally.

If $T < \infty$, the scrap value of asset $i$ is $v^i_T(K^i_T)$ and $\sum_{t=1}^{\infty} \frac{\kappa^i_t}{(1+r_s)^s} + \frac{v^i_T(K^i_T)}{(1+r_s)^T} = M^i_0$. The *value of deterioration* at $t \leq T$ in an external valuation for asset $i$ is $M^i_0 - v^i_t(K^i_t)$. This value may or may not be an admissible value of (cumulated, internal) *depreciation*. Other conditions discussed below must hold as well.

This method of accounting for non-priced (sunk and non-marketed) capital is founded on economic valuation. Because in reality future cash flows and interest rates must be predicted, accountants consider the present value to be subjective. To deal with subjectivity, accountancy has developed a number of principles (Bernstein 1988, Pratt 2000). Market values are used where possible because of their objectivity. Although at $t = 0$ there is a coincidence between market and net present values, market values of sunk assets may be hard to determine (Pratt 2000: 134). Objectivity is the reason frequently given for using original cost in valuation. Herein, original cost $M^i_0$ is used not so much for its objectivity as that the investor–entrepreneur anticipates that the asset will contribute $M^i_0$ to net present value. For $t > 0$, if there is no market transaction, commonly used market proxies are not appropriate.

Diewert (2009) observes that the user *cost* of capital is also a user *benefit*. The observation illustrates the analytic power of double-entry bookkeeping and underpins the accounting principle of matching of revenues to the costs of producing them.

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4 Diewert and Huang (2011: 541) also find many degrees of freedom in the recovery schedule.

5 If there is a closing expense (e.g. Muehlenbachs 2014), it must be paid from the cash flows. The liability, $v^0_T$, say, requires that a notional sinking fund be set up. A balancing virtual asset $K^0_0$ with user costs $\kappa^0_t$, $t = 1, ..., T$, cumulates to $v^0_T$. 

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In the economic theory of a competitive asset (e.g. Hulten and Wyckoff 1996) the match is unique. With comprehensive capital viewed as a composite, accounting for a project also has a unique match of benefits and costs of capital (Cairns 2013). In this case, the matches are of capital costs to net cash flows, which are the entrepreneur’s objective in making the investments. For component assets, however, equations (5) through (9) are formulas for admissible matches that are not unique.

A third principle is the appropriate timing of revenue recognition. Equation (7) provides that the user costs sum to the net cash flow. The levels and timing of net cash flows are objective.  

The method also meets the principle of consistency, across time periods and across accounting entities (projects). Conservatism, the final principle, can lead to manipulation (Pratt 2000: 139). Herein, conservatism consists of recognizing the futility of trying to measure individual values for the class of non-marketed assets and the conceptual importance of this class as forming a single residual.

5. DEVELOPMENT OF THE METHODOLOGY

For an admissible schedule $\kappa_s^i|_{s=0}^{\infty}$, the economic book value of originally marketed capital good $i$ any $t > 0$ is

$$B_t^i = M_0^i - \sum_{s=1}^{t} \frac{\kappa_s^i}{\Pi_{k=1}^s (1 + r_{t+k})} = \sum_{s=1}^{\infty} \frac{\kappa_t^i}{\Pi_{k=1}^s (1 + r_{t+k})},$$

so that

$$(1 + r_{t+1}) B_t^i = B_{t+1}^i + \kappa_{t+1}^i.$$ 

The (economic) depreciation of good $i$ at time $t$ is

$$\Delta_{t+1}^i = B_t^i - B_{t+1}^i,$$

Bernstein (1988: 125) observes that cash is “the starting point and the finish line of the accounting cycle,” the point at which management has maximum discretion. This discretion corresponds with the “long run” in economics.
and (consistently with financial accounting)

$$\sum_{t=0}^{\infty} \Delta_{t+1}^{i} = M_{0}^{i}. \tag{11}$$

The rate of depreciation of good $i$ at time $t$ is

$$\delta_{t+1}^{i} = \frac{\Delta_{t+1}^{i}}{B_{t}^{i}} = \frac{B_{t}^{i} - B_{t+1}^{i}}{B_{t}^{i}}. \tag{12}$$

Therefore,

$$\kappa_{t+1}^{i} = \kappa_{t}^{i} B_{t}^{i} + \left( B_{t}^{i} - B_{t+1}^{i} \right) = \left( r_{t+1} + \delta_{t+1}^{i} \right) B_{t}^{i}. \tag{12}$$

Even though arbitrage is not envisioned because the asset is sunk, the rental schedule $\{\kappa_{t}^{i}\}$ satisfies a no-arbitrage condition. The only role of the productivity outside the project, $K_{t}^{i}$, or of the market value $v_{t}^{i}(K_{t}^{i})$, if there is one, is to check condition (4).7

The only role of deterioration, measured by $1 - u_{t}^{i}$, is in influencing the project’s net cash flow, and hence admissible values of $\kappa_{t}^{i}$ through equation (7).

**Proposition 3** The user cost, the economic book or accounting value and the depreciation of any asset are not unique. Relative values of sunk capital goods are not uniquely defined.

It is worthy of stress that economic depreciation is a provision for repayment of capital cost, not a value of deterioration. In prospective, economic accounting, satisfying conditions (7), (11) and (8) is a participation constraint.

In accountancy and in economics it is considered important to break changes in value down into price and quantity components, $p_{t}^{i}$ and $q_{t}^{i}$. For a sunk asset, an external price or a marginal value is frequently used for $p_{t}^{i}$. If one of these is not available, sometimes an average price $p_{t}^{i} = \kappa_{t}^{i}/q_{t}^{i}$ is used. Since $\kappa_{t}^{i}$ is not unique, $p_{t}^{i}$ is not unique.

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7 If the external price is not acted upon through a transaction at time $t$, it is neglected as not pertinent at $t$. If the inequality is reversed at $t$, so that value is not sunk, then the project is redefined at the initial date as incorporating the sale of the asset at $t$. 
Book value is an assigned value with certain properties. The book value or the rental schedule does not involve an approximation or a statistical estimate of a price (to be multiplied by the number of units). A price does not exist for sunk capital; what does not exist cannot be estimated or approximated. For some intangible capital there are no physical units. In practice, depreciation is expressed as a value but not as a product of a price and a quantity. Because there is no price, in the definition of the rental there is no term for a capital gain or loss. Price and quantity indices should be measured using transactions values.

**Proposition 4** Accounting value is not decomposed into accounting price and quantity.

A price also does not exist for non-marketed capital. A book value can be defined to be

\[ B^N_t \equiv V(P_t) - \sum_{i=1}^{k_s} B^i_t \left( u^i_t K^i_0 \right) = \sum_{s=1}^{\infty} \frac{\nu_{t+s}}{\Pi_{k=1}^{s} (1 + r_{t+k})}, \]  

so that its rate of depreciation is given by

\[ \gamma_{t+1} = \frac{B^N_t - B^N_{t+1}}{B^N_t} \]

and its user cost by

\[ \nu_{t+1} = (r_{t+1} + \gamma_{t+1}) B^N_t. \]  

A constraint on depreciation schedules follows from “adding-up” condition (7).

**Proposition 5** Current user costs of other assets affect the current user cost that can be charged for any asset. Consequently, the depreciation that can be charged for asset is constrained by the depreciation of other assets.

The non-uniqueness of depreciation and user cost is vital in understanding the limitations of accounting. When used-asset prices — market \( p_t^i(K_t^i) \) or net realizable...
$v_i^t(K_i^t)$ — are used to estimate the depreciation of marketed capital, a particular schedule is being applied. It is admissible if and only if it satisfies conditions (5) through (9). It is no better than a standard accounting schedule if both fit (or neither fits) the conditions. If there is a substantial value of non-marketed capital, in practice the traditional financial-accounting and economic measures may satisfy the conditions, but not always under uncertainty; capital sometimes has to be written off.

A portion of the production of a composite asset (project) is not attributed to individual assets; the total return is attributed to the composite. For individual assets the following applies.

**Proposition 6** When capital is comprehensively defined, an asset does not have an own rate of return. The rate of return to an asset is expressed in terms of the numeraire and is equal to the market rate of interest. It is applied to the book value of the asset.

In an admissible schedule, the internal rate of return to any asset (and to the project itself) is equal to the market rate. By defining capital comprehensively the analysis dispenses with this controversial measure. An investor whose marketed investment appears to be earning more than the market rate is reaping gains to non-marketed capital. Observation of an internal rate of return greater than the market rate indicates that one or more assets have been neglected. What is neglected may vary from project to project.

**Proposition 7** The internal rate of return to any asset, or to the project, is redundant.

In the methodology developed using conditions (5) through (9), the portion of cash flow that is attributed to a given asset is not unique and the book value is not unique.
Therefore, their quotient, the accounting rate of return, is not unique. Moreover, the accounting rate of return neglects non-marketed capital.

**Proposition 8** *The accounting rate of return, limited to marketed capital, is neither unique nor meaningful.*

When an irreversible investment is being contemplated, *even under certainty*, there is an optimal strike or stopping time, for which the net present value of the investment option or opportunity is maximized (Dixit and Pindyck 1994). Optimal timing is determined by a stopping condition.

A marginal condition in quantities is also a form of optimal stopping condition. Militating against the use of marginal or shadow prices to value capital are (a) non-constant returns to scale and non-convexity, (b) non-optimality and (c) non-marketed and sunk capital. Irreversibility entails a non-convexity that undermines the assumption of constant returns to scale and the validity of marginal values as estimates of value. Projects need not be optimal to warrant valuation. Marginal valuation is yet less tenable because some non-marketed capital does not have physical units and consequently does not have a marginal value.

**Proposition 9** *Marginal values or marginal shadow prices apply only to tangible capital. They are inputs to stopping conditions in quantity space. They are not prices to be used in accounting for capital.*

An important type of non-marketed capital is the entry barrier that gives rise to a monopoly rent. It is not valued using Lerner’s (1934) *index*, the ratio of price net of marginal cost to price, nor Lerner’s *monopoly revenue*, the difference between price and marginal revenue. These measures are derived from stopping conditions in quantity space. To make the enterprise viable, it may be necessary to exploit some inelasticity of demand, through part or all of the life of the enterprise: compare the
model of static monopolistic competition. In that case, the monopoly rent is lower, possibly much lower, than the discounted value of the price net of marginal cost or the divergence between price and marginal revenue.

The discussion leaves some leeway in choosing depreciation schedules. In particular, depreciation of marketed capital is not necessarily proportional to its physical deterioration. Indeed, intangible capital does not deteriorate measurably, for it has only a pecuniary metric. It can only depreciate.

6. FIRMS

The stylized investment project studied in this paper has provided a context for the examination of the role of capital and the evaluation of its value and of its depreciation. All types of capital contribute in concert to the cash flows of a project. The project discussed in the present paper has been invested, and the formulas for user cost above assume that no further investment is contemplated.

A concern in the measurement of depreciation has been replacement of the asset. In reality, conditions are not stationary. Replacement is not necessarily an optimal choice after the end of the usefulness of a capital good to a project. Indeed, some types of capital cannot be replaced. More generally, projects are organized into firms.

Over time, a firm may change its capital stock or its projects.

Definition 7. A multi-project firm is a combination or collection of projects.

Firms invest and divest capital, refit, merge, etc. The firm consists of realized projects as well as real options that the firm may realize in the future. The options contribute to the value of the firm. They are a further form of intangible, non-marketed capital.

Each project in a multi-project firm, including in a vertically or horizontally integrated firm, has a production and a cost function. Properties of the production or
cost function, including marginal product, pertain to the project. There is no single
meaning of the marginal product of capital of a multi-project firm. For example, a
pipeline company may be composed of several pipelines with different origins and
destinations, initiated at different dates.

Organizational capital, a non-marketed asset, links the projects. Other types of
capital may also be common to more than one project. Conditions (5) through (9)
provide limits for legitimate transfer pricing within the firm.

Constant returns to scale are sometimes justified by the assumption that an invest-
ment can be exactly replicated. It bears stress that, for present purposes, projects,
and hence assets, must be physically replicable. In the case of environmental re-
sources (land, mines, etc.), for example, mathematical properties may be retained
but physical replicability does not hold.

Since economic conditions may not be stationary, even under certainty there is
an optimal strike or stopping time, for which the net present value of the investment
option or opportunity is maximized (cf. Cairns and Davis 2007). Thus, an investment
cannot be replicated: investment at a time other than the optimal time reduces
value. Good timing is a part of entrepreneurship or management. In general, projects
initiated at different times may involve different quantities (in physical and value
terms) of each type of capital. Nor is the counter-factual investment observable, even
in principle. An extreme example is that a mine cannot be developed with a year-old
mine shaft. But the reasoning holds for a factory, a university, etc.

Decomposition of the values of tangible capital goods into “cross-sectional” and
“time-series” components is typically accomplished through the analysis of replicas-
tions of an investment using older capital goods (cross-sectional depreciation is the
difference in productivity of assets of different vintages in the same employment) and
at a different time. Diewert (2009: 8, 9) asks whether cross-sectional depreciation
can be uniquely defined. He notes that time-series depreciation has been used by
accountants since the beginnings of accountancy but cross-sectional depreciation has never been used.

The decomposition that forms cross-sectional depreciation is conceivable only if capital is not sunk and only if projects are physically replicable. For sunk capital, the decomposition is inconsistent with optimization, observability or irreversibility. For example, older capital must be taken from some reversible investment elsewhere. In this case the uses of the capital are qualitatively different. Non-marketed capital cannot have a unique “cross-sectional” component. More generally, it is not possible to identify the contribution of a single asset uniquely.

**Proposition 10** Cross-sectional depreciation is not defined. Depreciation is measured as a time series.

The net present value of a firm at any date incorporates the values of the net investments in comprehensive capital at that date. When new capital is added, it is folded into the project or firm, so that condition (4) holds incrementally. There is no revaluation of existing capital to the price of capital newly invested: the new capital is in a different use. The book value of the firm is updated recursively. If capital is sold or scrapped its market value is subtracted from the project value at that time. Under certainty, that value is foreseen in the rental schedule.

Conditions (5) through (9) must be respected for individual projects and for the firm as a whole. Since the formulas are additive and it is not essential to depreciate in a way that coincides with an asset’s life, a firm’s capital can be aggregated according to the convenience of the analyst. Consistently with accounting practice, items of particular types of marketed capital, even of different vintages, such as telecommunications switches, or several types of the capital, such as all marketed capital in a telecommunications firm, can be aggregated and the aggregate depreciated according to the conditions. Additivity is a weaker condition than linearity, interpreted to mean
valuation at marginal or shadow prices, which by Proposition 9 is not required and may not be admissible.

7. SOME COMPARISONS

Both financial accounting and economic theory attribute the returns from non-marketed capital to marketed capital. The attribution is becoming less plausible as the role of non-marketed capital increases in importance. The present approach parallels Hall’s (2000, 2001) use of the values of financial securities to measure capital, including intangible capital, in the aggregate: the value of firms’ securities is a projection of the present values of net cash flows. Hall abstracts from some of the questions stressed herein, such as the treatment of depreciation, fixity, irreversibility, the variety of capital, and rents of various sorts. As parts of the residual, rents are treated herein as components of capital.

In Hall’s equation for the value of capital, \( v = qk \), the shadow value of capital, Tobin’s \( q \), is a pure number, and \( v \) and \( k \) are expressed in terms of the numeraire. Intangible capital is not measurable; it is treated as a residual, the difference between the value of the firms and the value of the tangible capital stock.

Tobin’s \( q \) is the ratio of a firm’s market value to the replacement value of its tangible assets, \( V(P_t) / \left( \sum_i p_i^t (K_i^t) K_i^t \right) \). The ratio involves an assumed price \( p_i^t (K_i^t) \) that can serve as a replacement value. Non-marketed capital is not considered. The ratio indicates whether it would be worthwhile to rebuild the project if, say, it burned to the ground but the non-marketed capital remained intact. The comparison is to an option that is not generally available. In discussing Hall (2000), Lamont (2000) observes that investment regressions using \( q \) do not work well and that \( q \) “varies wildly”; in the general discussion of Hall’s paper (p. 114), Gregory Mankiw is quoted as observing that there is a large literature rejecting \( q \) theory.

An alternative measure of \( q \) uses the net realizable value of the tangible (or mar-
ketable) assets: \( \tilde{q} \equiv V(P_t)/\sum_i v_t^i(K^i_t) \). This ratio pertains to the option to wind up the project. A finding that \( \tilde{q} > 1 \) means that some capital is sunk or is not marketed. As extreme examples, the realizable value of a mineshaft, access road, dam, dredging of a harbor, etc. is zero.

If for some asset(s) \( i \), \( p_t^i(K^i_t) \) or \( v_t^i(K^i_t) \) cannot be estimated, sometimes \( q \) is estimated using reported book values. Since the present value \( V(P_t) \) is well defined but the book value of tangible (or marketable) assets \( \sum B_t^i \) is not unique, the ratio \( V(P_t)/\sum B_t^i \) is not unique.

**Proposition 11** Tobin’s \( q \). (i) A comparison of market value to replacement cost is a comparison to an option that may not be available. (ii) A comparison of market value to net realizable value indicates the presence of sunk or non-marketed capital. (iii) When capital is comprehensively defined and correctly valued, Tobin’s \( q \) is identically equal to one.

Therefore, when \( q \) is traditionally measured as confined to marketed capital, one should expect to find \( q > 1 \). This finding is different from that of Dixit and Pindyck (1994:146), who discuss an option premium that makes marginal (to the project) \( q \) exceed one under uncertainty. As discussed above, even under certainty, the value of sunk, marketable assets to the firm may be bracketed by net realizable value and replacement value so that there is a spread of values for which not acting on sunk assets is optimal. Again, the marginal value is a stopping value that is not the value applicable all units of identical assets. Proposition 11 is based on the value of non-marketed capital to the firm, and applies to average \( q \), as used by Hall and other empiricists, in equilibrium. Non-marketed capital necessarily makes \( q > 1 \), if it is properly measured.

In a standard financial or economic account, let \( \Delta_t \) denote the total, assigned depreciation (a portion of \( M_0 \)) of tangible, marketed capital at \( t \) and let the book
value be denoted by \( B_t \equiv M_0 - \sum_{s=0}^{t} \Delta_s \). The computed (accounting or economic) rate of return \( \rho_t \) is such that

\[
(r_t + \delta_t) V_t = m_t = \rho_t B_t + \Delta_t,
\]

so that \( \rho_t = \frac{[(r_t + \delta_t) V_t - \Delta_t]}{B_t} \).

In financial accounting, \( B_t \) is an internal value that satisfies accounting formulas. In economic theory, it is assumed that \( \delta_t V_t = \Delta_t \) and that, under competition, \( B_t = V_t \) (that the internal value is equal to the external value), so that \( \rho_t = r_t \). Only if there is a profit are \( B_t < V_t \) and \( \rho_t > r_t \).

Goodwill is an attempt in financial accounting to deal with that part of the non-marketed capital that is transferred with a project or firm upon sale.\(^8\) It is an aggregate of some of the unpriced assets associated with the project. Its value is a residual, endogenous to the project. In an uncertain reality, its value can be impaired as a reverse of a rent of composition or else because of a random event, miscalculation or moral hazard; all are ruled out under certainty. (The value of marketed, tangible capital, especially specific capital, can also be impaired for these same reasons.) It is argued above that non-marketed capital earns a rate of return equal to the market rate of interest and should be depreciated like marketed, sunk capital.

Uncertainty complicates accounting. It warrants a separate treatment. One can say, however, that it cannot entail the use of marginal values to value assets, nor using as the measure of depreciation the product of an external price and deterioration that is equal across uses, nor using external prices to value capital assets, nor assigning to marketed capital goods rents attributable to non-marketed assets. Under uncertainty one must deal with situations in which consistency with conditions (5) through (9) is not achieved because of unanticipated realizations. An accounting loss may then be

\(^8\)Organizational capital, for one thing, is usually not transferable (Lev and Radhakrishnan 2006). A problem for financial accounting is determining the expiration of goodwill. In the method of the present paper, the expiration date has much reduced import.
declared or assets written down. Subject to rents of composition resulting from the extinguishing of goodwill, conditions (5) through (9) can be used to evaluate whether a firm that is no longer viable should be broken up or continued.

8. CONCLUSION

Because of the diversity of types of capital, there are limitations to what can be measured or accounted that are not apparent when only a single type of capital, representing a project, is considered. The limitations are particularly evident for non-marketed capital. Techniques of non-market valuation cannot provide unique economic-accounting prices or other magnitudes. Accounting for irreversibly invested, tangible, marketed capital is also subject to limitations.

The paper provides a foundation for economic accounting for capital. The treatment of all types of capital, based on conditions (5) through (9), is symmetric. It provides a benchmark for understanding retrospective valuation as in standard accounting. The key considerations are diversity and irreversibility. The assumption of certainty is important in demonstrating that the conditions apply even in micro-theoretic models with perfect information. Under uncertainty, the underlying difficulty is the projection of net cash flows. That difficulty is shared with traditional theory. More importantly, in one way or another the projection must be done by the investor. Uncertainty, which is important in practical valuation, does not undermine the theory but does require a generalization, which is left for future work.

Investment subsumes capital goods into a project, a composite asset. Its distinct, component assets cannot be aggregated physically into a scalar.

The state of being marketed, for the tangible and intangible capital in that state, is infrequent and ephemeral. Between market transactions, marketed capital is sunk and has no price. If an asset $i$ is bought for $M_i^0$ and sold or scrapped for $M_i^t$, cumulated depreciation over the interval $(0, t)$ is $M_i^0 - M_i^t$. Only cumulated depreciation over
the life of the asset in the project can be measured uniquely or objectively. Non-unique, internal values satisfying conditions (5) and (8) and anchored in end-point realizations apply between the exercise of options to buy or to sell. External prices multiplied by quantities are candidates for accounting values only if, over the life of the project, they satisfy conditions (5) through (9). An asset is added to a going concern only if its incremental contribution to net present value is greater than its price. The valuation of newly purchased or sold units of capital of type \(i\) does not apply to existing units.

It is not possible to measure sunk, marketed capital without reference to non-marketed capital. As a stylization, non-marketed capital has no natural measure, no marginal value, no price. All types are confounded into a residual value. Corrado et al. (2009) argue that financial (or corporate) accounting neglects intangible (non-marketed) capital because the values are not verifiable. Financial accounting thereby confuses concepts and estimation, possibly as a result of there being no formal theory of accounting for assets. The present paper contributes to a theory of economic accounting and provides insights to the practice of financial accounting.

The various capital items contribute to the project but do not provide “own” cash flows or returns. Income from the project is market interest on its present value. Income from the assets is market interest on their book values. Depreciation of the component parts of an enterprise is a consistent, internal recovery of original value from the net cash flows of enterprise. Values are additive but not linear. The internal values derived for the assets and their depreciation do not affect decisions in the project but instead are determined by the outcomes of the decisions. If an external price is high enough, the corresponding option is exercised. For non-trivial periods of time the options are not exercised.

The large number of governing conditions may make the methodology seem cumbersome. The method applies irrespective of market structure (rents are capitalized
into non-marketed capital) or whether any price is linear or non-linear. Marginal values do not have to be estimated. Apart from the recognition of capital’s multiplicity, measurement is simplified.

REFERENCES


