Accounting for Government Activities

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"... Both the CPI and a cost-of-living index would reflect changes in the prices of goods and services, such as food and clothing that are directly purchased in the marketplace; but a complete cost-of-living index would go beyond this to also take into account changes in other governmental or environmental factors that affect consumers’ well-being. It is very difficult to determine the proper treatment of public goods, such as safety and education, and other broad concerns, such as health, water quality and crime that would constitute a complete cost-of-living framework."

http://www.bls.gov/cpi/cpifaq.htm#Question_4 US Consumer Price Index FAQ question #4
The OECD monograph (OECD (2009)) has a brief section on the distinction between public goods and individual goods. Table 5.2 (Currently Available Data) suggests indirectly the limited possibilities for testing theories of government activity. Samuelson’s famous article of 1954 on public goods is listed in the references though the long and equally famous article by Nordhaus and Tobin (1973) is not listed. In fact the distinction between government activity that is primarily intermediate rather than final is not taken up. The OECD’s monograph’s emphasis is on government program i and the management of its delivery and the measurement of its effectiveness. Our approach takes most of the measurement, cost and effectiveness issues as solved.

We set out three variants of Solow’s neoclassical growth model\(^1\) with government production incorporated and the charging for consumption of government product being done in accord with marginal benefits to "consumers". One model involves pure public goods that are final goods as in Samuelson (1954) and the others involve public goods that are intermediate inputs, inputs to firms rather than households.

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\(^1\)Solow (1956). In the Solow approach, production takes place under constant returns to scale and savings behavior is governed by a constant savings rate for households. Our firms do not make intertemporal calculations and our households do not explicitly optimize across time periods, given the unchanging savings rate.
There is an unbroken line of thought descending from Hicks (1948) and Kuznets (1948) that identifies government services as largely intermediate in character. Nordhaus and Tobin (1973) are well known for making this case. They argued for a major re-working of government expenditure in NNP in the national accounts with about 14% to intermediate inputs to firms, 41% to "regrettable necessities", also intermediate, and about 44% to a new category, "current government investment". This leaves a small amount of government expenditure as "consumption" services flowing directly to households.
We employ social accounting matrices (SAMs) in organizing our analysis. The accounting approach forces us to keep track of income and expenditures (the "product" side of the account) simultaneously. Our abstract framework makes benefit taxation (charges based on marginal benefits) a natural way for government to pay for its product in theory; we recognize that benefit taxation is generally impractical to implement. We gloss over the complicated matter of how to elicit the true preference for government product by a "consumer". Once one sees the entries for benefit charges in an accounts matrix, one is drawn to the idea that such charges could in fact be arrived at in an approximate fashion and get set by a taxing authority.
The 1993 System of National Accounts (as well as the 2008 revisions (Chapter 22)) considers "government units" as "making three different kinds of final outlays" (p. 101). "The first group consists of actual or imputed expenditures on the free provision to the community of collective services such as public administration, defence, law enforcement, public health, etc. which, as a result of market failure, have to be organized collectively by government and financed out of general taxation or other government income." A bit further along: "Even in the case of most collective services, or so-called "public goods", for which there is market failure, governments are obliged only to assume responsibility for organizing and financing their production. They are not obliged to produce them." In brief "government activity" in the accounts is covering off "public goods" provision as one central mandate. The specific collective goods listed, namely public administration, defence, law enforcement, public health, do I think conform to our idea of fairly pure public goods in the Samuelson (1954) sense.
The problem of defining the appropriate boundary between public and private goods comes up a few sentences later: "Apart from some collective services such as public administration and defence, it is therefore difficult to categorize certain types of production, such as the production of education or health services, as intrinsically governmental, even though they are often produced by government units."

"The second group of expenditures on the provision of goods or services free, or at prices that are not economically significant, to individual households. These expenditures are deliberately incurred and financed out of taxation or other income by government in the pursuit of its social or political objectives, even though individuals could be charged according to their usage." Here we have "government activity" in the accounts providing some goods for final consumption by households at heavily subsidized "prices".
Decision-makers at the center select certain goods that they choose to provide from the center rather than let the private sector do the job. Government "may incur expenditures on the provision of services, such as education or health, primarily for the benefit of individual households." We have recognition that some goods provided from the center are intrinsically private goods, usually provided at very low "prices". There appear then to be government goods that are impurely public in the sense that a "consumer" could buy them like loaves of bread but may end up under-consuming them. Education and health care are two that come to mind. Primary education may often be supplied by the public sector because the public deems such education to be very useful for all citizens and yet such education might be under-consumed when privately provided because lower income households could not afford the fees being charged.
Of interest here is the fact that there is no mention of "government" providing services to private firms, though government may supply "non-market goods or services to other government units for purposes of intermediate consumption or gross fixed capital formation: for example, transport agencies, computer or communications agencies, etc." or related production entities. It is strange that "government" is not specified as a supplier of services to private firms (the "intermediate goods" issue as we see things).
There is a curious emphasis in these official guidelines on the government providing essentially private goods at very low prices. Consider the provision of opera for example. A large fraction of the cost of attendance is covered by government but attendance is by no means almost free. Thus the concept of appropriate boundary for "government activity" is ill-defined here. Later there is the recommendation that "units such as municipal theatres, museums, swimming pools, etc., which supply goods or services on a market basis should be treated as quasi-corporations whenever appropriate."(p. 104) Here it is spelled out that these activities should be considered not government activity because the output is provided "on a market basis". Why government should be active in the production of private intermediate goods appears to involve the implicit subsidization of production of certain goods in favor of low income groups. Public transportation is a case in point. And many municipalities provide recreational places such as swimming pools and charge users a fee well below cost. Again we are observing a system of providing services in a subsidized manner mostly to lower income groups.
"The third group consists of transfers... in order to redistribute income or wealth."

We are concerned here with the "first group" of government activities, those with a large component of joint consumption by multiple end users. Our analysis deals with pure public goods alone, ones with no explicit private goods component. Domestic "law and order" involves a public goods component (the courts, the police, arbitrators, etc.) as well as a private component (one’s lawyer, one’s private investigators, one’s payment for "court costs", one’s private security personnel, etc.). We acknowledge the difficulty in arriving at pure public goods in practice, but we restrict our analysis to the pure public good phenomenon (the pure shared price, same quantity case).
Table 1: Accounting Matrix (public goods only final (Samuelson))

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$rK_C$</td>
<td>$wN_C$</td>
<td>$= q_C$</td>
</tr>
<tr>
<td>$rK_I$</td>
<td>$wN_I$</td>
<td>$= p_I q_I$</td>
</tr>
<tr>
<td>$r[K - K_C - K_I]$</td>
<td>$w[N - N_C - N_I]$</td>
<td>$= p_G q_G \left(= q_G \frac{NU}{U_{q_C/N}} \right)$</td>
</tr>
<tr>
<td>$= rK$</td>
<td>$= wN$</td>
<td></td>
</tr>
</tbody>
</table>

Each row entry in Table 1 is part of the corresponding value-sum on the right. Each column entry is part of the corresponding value-sum at the bottom row. The sum of entries in the right column is net national product and the sum of the entries in the bottom row is national income. These two sums are usually the same, when production takes place under constant returns to scale. Government product is accounted for here by costs of inputs: $r[K - K_C - K_I] + w[N - N_C - N_I]$. These costs sum to $p_G q_G$. The corresponding revenue is $q_G \frac{NU}{U_{q_C/N}}$, where $q_G \frac{U}{U_{q_C/N}}$ is the payment (tax charge) by a single household for the flow of services, $q_G$. 
The new pricing equation includes a marginal benefit charge per firm:

\[
\frac{NUq_G}{U_{q_C/N}} + MP^C_G + p_I MP^I_G = p_G,
\]

where \( MP^C_G \) is the marginal product of the government input to a consumer goods firm.

Table 2: National Account (both flow intermediate to firms and final to households)

<table>
<thead>
<tr>
<th>( rK_C )</th>
<th>( wN_C )</th>
<th>( q_G MP^C_G )</th>
<th>=</th>
<th>( q_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.848)</td>
<td>(4.24)</td>
<td>(3.393)</td>
<td>=</td>
<td>(8.482)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( rK_I )</th>
<th>( wN_I )</th>
<th>( q_G p_I MP^I_G )</th>
<th>=</th>
<th>( p_I q_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.181)</td>
<td>(0.454)</td>
<td>(0.9087)</td>
<td>=</td>
<td>(4.544)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( r[K-K_C-K_I] )</th>
<th>( w[N-N_C-N_I] )</th>
<th>( -[MP^C_G + p_I MP^I_G]q_G )</th>
<th>=</th>
<th>( q_G U_{q_C/N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.981)</td>
<td>(2.44)</td>
<td>(−4.302)</td>
<td>=</td>
<td>(2.120)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( rK )</th>
<th>( wN )</th>
<th>=</th>
<th>=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8.01)</td>
<td>(7.136)</td>
<td>=0</td>
<td>=0</td>
</tr>
</tbody>
</table>

The negative term \( -[MP^C_G + p_I MP^I_G]q_G \) is revenue from the sale of intermediate public goods.
Table 3: National Account (firms use government capital as an input; line 3 is current production of new government capital)

<table>
<thead>
<tr>
<th>$rK_C$</th>
<th>$wN_C$</th>
<th>$K^G MP^C_{Kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rK_I$</td>
<td>$wN_I$</td>
<td>$K^G p_I MP^I_{Kg}$</td>
</tr>
<tr>
<td>$r[K - K_C - K_I]$</td>
<td>$w[N - N_C - N_I]$</td>
<td>$K^G p_I^G MP^I_{Kg}$</td>
</tr>
<tr>
<td>=$rK$</td>
<td>=$wN$</td>
<td>=$[MP^C_{Kg} + p_I MP^I_{Kg} + p_I^G MP^I_{Kg}] K^G$</td>
</tr>
</tbody>
</table>

$K^G$ might be roads and the charges for same might be the fuel taxes which truckers must pay. Our table suggests that each of the consumption goods, investment goods and government goods sectors are indirectly consuming trucking services and the payment is in fact only the "road user charges" set out in column three.
APPENDIX 1: PUBLIC GOODS FINAL
This program provided a test for price neutrality during growth, with different values of elasticity, sigma.

function f=nlef(x)
% govt is intermediate flow AND SAMUELSON pub good as well.
% CES utilities (zi and dl) and CobbDouglas prod functions
ack=0.2;aik=0.7;ag=0.5;s=0.2;N=10*(1+.2);K=7*(1+.2);
zi=0.4;sig=2;
qc=x(1);
qi=x(2);
qg=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
% system is 3 real eqns
\[ f(1) = q_c - K_c^{\text{ack}} N_c^{(1-\text{ack})}; \]
\[ f(2) = q_i - K_i^{\text{aik}} N_i^{(1-\text{aik})}; \]
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
\[ f(3) = q_g - (K-K_i-K_c)^{\text{ag}} (N-N_c-N_i)^{(1-\text{ag})}; \]
% re-working
\[ f(4) = r - w \left( \frac{\text{ack}}{1-\text{ack}} \right) \left( \frac{N_c}{K_c} \right); \]
% two efficiency ratios
\[ f(5) = \left( \frac{\text{ack}}{1-\text{ack}} \right) \left( \frac{N_c}{K_c} \right) - \left( \frac{\text{aik}}{1-\text{aik}} \right) \left( \frac{N_i}{K_i} \right); \]
\[ f(6) = \left( \frac{\text{ack}}{1-\text{ack}} \right) \left( \frac{N_c}{K_c} \right) - \left( \frac{\text{ag}}{1-\text{ag}} \right) \frac{(N-N_c-N_i)}{(K-K_i-K_i)}; \]
% REDO: this is the Samuelson pub goods eqn for govt as input to production!!
\[ s_B = \left( \frac{(1-\text{zi})N}{\text{zi}} \right) / \left( \left( \frac{N*q_g}{q_c} \right)^{(1/\text{sig})} \right); \]
\[ f(7) = p_g - s_B; \]
% note savings is out of FINAL demand (intermediates excluded)
\[ f(8) = s^* \left( q_c + p_i q_i + q_g s_B \right) - p_i q_i; \]
APPENDIX 2: Public Good Flow to Households and Firms Simultaneously

function f=nlesAg(x)
% March 13... PRICE NEUTRALITY WITH PUBLIC INPUTS...
% govt is intermediate flow AND SAMUELSON pub good as well.
% Cobb Douglas utilities (zi) and CobbDouglas prod functions
ack=0.1;acn=0.5;aik=0.7;ain=0.1;ag=0.62;s=0.3;N=20*1.00;K=18*1.00;
zi=0.8;
qc=x(1);
qi=x(2);
qg=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
(9)
% system is 3 real eqns
f(1) = qc - Kc * ack * Nc * acn * qg^(1-ack-acn);
f(2) = qi - Ki * aik * Ni * ain * qg^(1-aik-ain);
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
f(3) = qg - (K - Ki - Kc) * ag * ( N - Nc - Ni )^(1-ag);
% re-working
f(4) = r - w * (ack/acn) * (Nc/Kc);
% two efficiency ratios
f(5) = (ack/acn) * (Nc/Kc) - (aik/ain) * (Ni/Ki);
f(6) = (ack/acn) * (Nc/Kc) - (ag/(1-ag)) * (N - Nc - Ni) / (K - Kc - Ki);
% this is the Samuelson pub goods eqn for govt (into prod and cons)
sB = ((1-zi)/zi) * (qc/qg);
f(7) = pg - (1-ack-acn) * qc/qg - pi * (1-aik-ain) * qi/qg - sB;
% note savings is out of FINAL demand (intermediates excluded)
f(8) = s * (qc + pi * qi + qg * sB) - pi * qi;
% three price-value equations
APPENDIX 3: Balanced Growth for Above static case

function f=nlesAg(x)
% March 13 SUCCESS... PRICE NEUTRALITY WITH PUBLIC INPUTS...
% govt is intermediate flow AND SAMUELSON pub good as well.
% Cobb Douglas utilities (zi and dl) and CobbDouglas prod functions
ack=0.1;acn=0.5;aik=0.7;ain=0.1;ag=0.62;s=0.3;N=1.00;K=18/20;
zi=0.8;n=6.1641/18;
qc=x(1);
qi=x(2);
qg=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
pi = x(10);
pg = x(11);
KP = x(12);

% system is 3 real eqns
f(1) = qc - Kc \times \text{ack} \times Nc \times \text{acn} \times qg \times (1 - \text{ack} - \text{acn});
f(2) = qi - Ki \times \text{aik} \times Ni \times \text{ain} \times qg \times (1 - \text{aik} - \text{ain});
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
f(3) = qg - (K - Ki - Kc) \times \text{ag} \times (N - Nc - Ni) \times (1 - \text{ag});
% re-working
f(4) = r - w \times (\text{ack} / \text{acn}) \times (Nc / Kc);
% two efficiency ratios
f(5) = (\text{ack} / \text{acn}) \times (Nc / Kc) - (\text{aik} / \text{ain}) \times (Ni / Ki);
f(6) = (\text{ack} / \text{acn}) \times (Nc / Kc) - (\text{ag} / (1 - \text{ag})) \times (N - Nc - Ni) / (K - Kc - Ki);
% REDO: this is the Samuelson pub goods eqn for govt as input to production!!
sB = ((1 - zi) / zi) \times (qc / qg);
function f=infr(x)
% March 13 SUCCESS... PRICE NEUTRALITY WITH PUBLIC INPUTS...
% govt is intermediate flow AND SAMUELSON pub good as well.
% Cobb Douglas utilities (zi and dl) and CobbDouglas prod functions
% ack=0.1;acn=0.5;aik=0.7;ain=0.1;ag=0.6;s=0.3;N=20*(1);K=18*(1);
ack=0.1;acn=0.5;aik=0.8;ain=0.1;agk=0.6;agn=0.1;s1=0.2;s2=.1;N=12*(1+.1);
kig=7*(1+.1);
qc=x(1);
qi=x(2);
qig=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
w=x(9);
pi=x(10);
pig=x(11);
rg=x(12);
% kig=7;
% system is 3 real eqns
f(1)=qc-Kc^ack*Nc^acn*kig^(1-ack-acn);
f(2)=qi-Ki^aik*Ni^ain*kig^(1-aik-ain);
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
f(3)=qig-(K-Ki-Kc)^agk*(N-Nc-Ni)^(agn)*kig^(1-agk-agn);
% re-working
f(4)=r-w*(ack/acn)*(Nc/Kc);
% two efficiency ratios
f(5)=(ack/acn)*(Nc/Kc)-(aik/ain)*(Ni/Ki);
f(6)=(ack/acn)*(Nc/Kc)-(agk/(agn))*(N-Nc-Ni)/(K-Kc-Ki);
% REDO: this is the Samuelson pub goods eqn for govt as input to production!!
sB=((1-zi)*N/zi)/(N*qg/qc)^(1/sig);
f(7)=rg-pi*(1-aik-ain)*qi/kig-(1-ack-acn)*qc/kig-pig*(1-agk-agn)*qig/kig;
% note savings is out of FINAL demand (intermediates excluded)
f(8)=s1*(qc+pi*qi+pig*qig)-pi*qi;
f(9)=s2*(qc+pi*qi+pig*qig)-pig*qig;
% three price-value equations
f(10)=qc-r*Kc-w*Nc-kig*(1-ack-acn)*qc/kig;
f(11)=qi*pi-r*Ki-w*Ni-kig*pi*(1-aik-ain)*qi/kig;
f(12)=qig*pig-r*(K-Kc-Ki)-w*(N-Nc-Ni)-kig*pig*(1-agk-agn)*qig/kig;
% f(13)=r-ack*qc/Kc;


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CLASSIC ARTICLES AND BOOK CHAPTERS ON NATIONAL ACCOUNTING (and growth) Edward Elgar, 2015, selected by John M. Hartwick, with an introduction with the above material included

the volume includes 3 articles by Erwin Diewert, 4 by Dale Jorgenson, also contributions of Solow, Stone, Dasgupta, Arrow, Chas Jones, Bert Balk, Asheim, Weitzman, Boskin, etc.