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Accounting for Government Activities

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Accounting for Government Activities

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Abstract

We set out a three sector Solow growth model with the size of the government goods sector determined by "Samuelson" marginal benefit charges. Our attention is on two public goods consumed largely by firms rather than households: a flow public intermediate good (domestic security services (police, jails, courts, etc.)) and a capital intermediate good (public infrastructure (roads, bridges, etc.)). We analyse the solutions to the models and the representations of the solutions as national accounts.

- JEL Classification: H11; H41; H54; E01
- Key words: government functions; infrastructure charges; public good charges; national accounting

1 Introduction

The United States Consumer Price Index (CPI) is a price index that is based on the idea of a cost-of-living index. The U.S. Department of Labor’s Bureau of Labor Statistics (BLS) explains the difference:

"The CPI frequently is called a cost-of-living index, but it differs in important ways from a complete cost-of-living measure. BLS has for some time used a cost-of-living framework in making practical decisions about questions that arise in constructing the CPI. A cost-of-living index is a conceptual measurement goal, however, not a straightforward alternative to the CPI. A cost-of-living index would measure changes over time in the amount that consumers need to spend to reach a certain utility level or standard of living. Both the CPI and a cost-of-living index would reflect changes in the prices of goods and services, such as food and clothing that are directly purchased in the marketplace; but a complete cost-of-living index would go beyond this to also take into account changes in other governmental or environmental factors that affect consumers’ well-being. It is very difficult to determine the proper treatment of public goods, such as safety and education, and other broad concerns, such as health, water quality, and crime that would constitute a complete cost-of-living framework."
The OECD monograph (OECD (2009)) has a brief section on the distinction between public goods and individual goods. Table 5.2 (Currently Available Data) suggests indirectly the limited possibilities for testing theories of government activity. Samuelson’s famous article of 1954 on public goods is listed in the references though the long and equally famous article by Nordhaus and Tobin (1973) is not listed. In fact the distinction between government activity that is primarily intermediate rather than final is not taken up. The monograph’s emphasis is on government program i and the management of its delivery and the measurement of its effectiveness. Our approach takes most of the measurement, cost and effectiveness issues as solved. This is clearly a stretch but we still have much to say about "government activity" in the national economy.


We set out three variants of Solow’s neoclassical growth model with government production incorporated and the charging for consumption of government product being done in accord with marginal benefits to "consumers". One model involves pure public goods that are final goods as in Samuelson (1954) and the others involve public goods that are intermediate inputs, inputs to firms rather than households. We are interested in the national accounts associated with these models and also with the realization of a balanced growth path in the abstract for our models. The first intermediate good we have in mind is a produced flow good such as public order (local policing, the court system and regulation of worker-manager relations in firms). The value of most legal services, "consumed" by a firm gets embodied in the price of the good which the firm is producing and selling to households. Maintenance of domestic public

\footnote{Solow (1956). In the Solow approach, production takes place under constant returns to scale and savings behavior is governed by a constant savings rate for households. Our firms do not make intertemporal calculations and our households do not explicitly optimize across time periods, given the unchanging savings rate.}

\footnote{It turns out that there is a curious technical glitch rooted in charging households for public goods in accord with marginal benefits. Cobb-Douglas utility functions must be resorted to. Marginal benefit payment for public goods is not then in general "neutral".}

\footnote{The huge costs of our legal system create an average "litigation tax" of 2.5 percent on every product we buy. The tab for our out of control legal system comes to about $1,200 per person per year. This figure is much higher for some products, such as step-ladders (30 percent) and vaccines (95 percent) that tend to attract lawsuits. The cost of litigation doubles the price of a football helmet, will add $500 to the sticker price of your next new car, and pumps up the cost of a heart pacemaker by $3,000. Another analysis by The Economist has an estimate of an 8% markup on prices on average in the US to account for the costs of legal system."}
order and defense against invaders seem to be the two principal public goods of interest in a minimalist theory of government. National defense ultimately benefits households in a nation and is thus primarily a final good (Samuelson makes this case), whereas local policing, the court system and regulation of worker-manager relations in firms benefit firms considerably and households seemingly less so. These inputs to firms are of course "consumed" indirectly by households when they purchase the outputs of the firms. The services from government capital or infrastructure is the other intermediate good we take up. Here we have in mind roads, airports, etc. There is an unbroken line of thought descending from Hicks (1948) and Kuznets (1948) that identifies government services as largely intermediate in character. Nordhaus and Tobin (1973) are well known for making this case. They argued for a major re-working of government expenditure for the legal costs to firms of doing business (http://commitmentmatters.com/2011/08/08/world-economic-crisis-the-us-legal-system-is-a-target-for-reform/).

Meade (1952) introduced an analysis of generic "atmosphere" as a public good, largely intermediate. John MacMillan wrote extensively about these intermediate public goods. Local temperature in a model of global warming, affecting say farming, would be a negative public input in the Meade sense. Kamps (2006) presents his estimates of "public" capital stocks as percentages of GDP for his 22 countries. For 2000, Japan, New Zealand and Austria rank highest and Canada, Belgium and Ireland rank lowest. Sandmo (1972) mentions the services of a lighthouse to shippers as an example of a public input and Kaizuka (1965) mentions weather predictions for farmers and also results of publicly available research for the production activity of firms. Freely available weather announcements seem more like a flow of government product while the services of an automated lighthouse seem like the services of public capital.

Horz and Reich (1982) asked twenty experts how each of them would split up government production as defined in the 1968 UN System of National Accounts (127 categories) into final and intermediate. There was a lack of unanimity here and Horz and Reich reported that statistical agencies Sweden, the UK and the Federal Republic of Germany considered about 15% of government to be intermediate.

Vanoli (2005; p. 290), who was head of national accounting in France for many years, sees problems in defining intermediate goods: "Kuznets, Eisner, as well as Nordhaus and Tobin and many others tend to mix up the good or the service with the function, the end being pursued. Nordhaus and Tobin come then to write, "conceptually, the output of the defense effort is national security" (p. 8). "This is as true as saying that the output from doctors, hospital workers and pharmacists is the state of health, or the output from religious book industry is spiritual elevation. Production, consumption and the effects of the use of the products should not be confused, even if the products are normally designed in order to facilitate the achievement of the ends that are pursued. Ethical choices fall in the domain of ends." Vanoli (p. 257) actually seems to endorse the Nordhaus-Tobin approach: "non-market goods or services produced by government... of "three types": individualizable final use in actual consumption by households, individualizable intermediate use in intermediate consumption by producers... and finally collective use, non-individualizable by nature, and constituting a collective consumption by society as a whole." By individualizable one presumably one is dealing with private as opposed to public goods.
penditure in NNP in the national accounts with about 14\% to intermediate inputs to firms, 41\% to "regrettable necessities"\(^9\), also intermediate, and about 44\% to a new category, "current government investment".\(^10\) This would leave a small amount of government expenditure as "consumption" services flowing directly to households.\(^11\) We explore the place of "government product" in the national accounts, paying considerable attention to the possibility that much of government product may be intermediate, inputs to firms, rather than final, inputs to households.

The output of the government sector is complicated because as Samuelson emphasized, much of it should be viewed as being consumed jointly by end-users, i.e. as being a public rather than a private good. Each "consumer" is viewed as consuming the same physical flow of a government product, though in general she will value the same physical flow differently. Hence a need for personalized charges or taxes tailored to the valuation person or firm i puts on the physical flow. We follow Samuelson below and treat government product as pure "public good"; i.e. with each consuming unit receiving the same total physical flow of product currently. We limit attention to three representative public goods: national defense, domestic law and order and the court system, and certain types of infrastructure (types of government capital). Government is responsible for much capital in an economy; e.g. roads, bridges, airports, schools, hospitals in many cases, etc. Observers regularly reflect on whether the rate of return implicit on investment by government is similar to that on current private investment. We pay attention to this issue.

We employ social accounting matrices (SAMs) in organizing our analysis. The accounting approach forces us to keep track of income and expenditures

\(^9\)In classifying defense costs – or police protection or public health expenditures – as regrettable and instrumental, we certainly do not deny the possibility that given the unfavorable circumstances that prompt these expenditures consumers will ultimately be better off with them than without them... The only judgment we make is that these expenditures yield no direct satisfactions." (Nordhaus and Tobin (1973; p. 516)

\(^10\)In the Nordhaus-Tobin re-working of the national accounts, the traditional Samuelson category of household consumption of public goods ended up with just 1\% of government product (mail service and leisure activities).

\(^11\)When Nordhaus and Tobin suggest much of government production is "intermediate", they do not distinguish between essentially private intermediate (a local bus line) and public intermediate ("regulation" of worker-manager relations), though they do discuss national defence in some detail.
(the "product" side of the account) simultaneously. Our abstract framework makes benefit taxation (charges based on marginal benefits) a natural way for government to pay for its product in theory; we recognize that benefit taxation is generally impractical to implement. We gloss over the complicated matter of how to elicit the true preference for government product by a "consumer". Once one sees the entries for benefit charges in an accounts matrix, one is drawn to the idea that such charges could in fact be arrived at in an approximate fashion and get set by a taxing authority.

The 1993 System of National Accounts (as well as the 2008 revisions (Chapter 22)) considers "government units" as "making three different kinds of final outlays" (p. 101). "The first group consists of actual or imputed expenditures on the free provision to the community of collective services such as public administration, defence, law enforcement, public health, etc. which, as a result of market failure, have to be organized collectively by government and financed out of general taxation or other government income." A bit further along: "Even in the case of most collective services, or so-called "public goods", for which there is market failure, governments are obliged only to assume responsibility for organizing and financing their production. They are not obliged to produce them." In brief "government activity" in the accounts is covering off "public goods" provision as one central mandate. The specific collective goods listed, namely public administration, defence, law enforcement, public health, do I think conform to our idea of fairly pure public goods in the Samuelson (1954) sense. The problem of defining the appropriate boundary between public and private goods comes up a few sentences later: "Apart from some collective services such as public administration and defence, it is therefore difficult to categorize certain types of production, such as the production of education or health services, as intrinsically governmental, even though they are often produced by government units."12

"The second group of expenditures on the provision of goods or services free, or at prices that are not economically significant, to individual households. These expenditures are deliberately incurred and financed out of taxation or

12OECD (2009) is a recent monograph that does not refer to Kuznets, Hicks, Horz and Reich or Samnuelson. It takes a public administration approach to the issue of "government production" in the national accounts.
other income by government in the pursuit of its social or political objectives, even though individuals could be charged according to their usage." Here we have "government activity" in the accounts providing some goods for final consumption by households at heavily subsidized "prices". Decision-makers at the center select certain goods that they choose to provide from the center rather than let the private sector do the job. Government "may incur expenditures on the provision of services, such as education or health, primarily for the benefit of individual households." We have recognition that some goods provided from the center are intrinsically private goods, usually provided at very low "prices". There appear then to be government goods that are impurely public in the sense that a "consumer" could buy them like loaves of bread but may end up under-consuming them. Education and health care are two that come to mind. Primary education may often be supplied by the public sector because the public deems such education to be very useful for all citizens and yet such education might be under-consumed when privately provided because lower income households could not afford the fees being charged. Noticeable here is the fact that there is no mention of "government" providing services to private firms, though government may supply "non-market goods or services to other government units for purposes of intermediate consumption or gross fixed capital formation: for example, transport agencies, computer or communications agencies, etc." or related production entities. It is strange that "government" is not specified as a supplier of services to private firms (the "intermediate goods" issue as we see things).

There is a curious emphasis in these official guidelines on the government providing essentially private goods at very low prices. Consider the provision of opera for example. A large fraction of the cost of attendance is covered by government but attendance is by no means almost free. Thus the concept of appropriate boundary for "government activity" is ill-defined here.\(^{13}\) Later there is the recommendation that "units such as municipal theatres, museums, swimming pools, etc., which supply goods or services on a market basis should be treated as quasi-corporations whenever appropriate."(p. 104) Here it is spelled

\(^{13}\)In the SNA-2008 much attention is devoted to how to dissect and deal with the accounts of non-market producers.
out that these activities should be considered not government activity because the output is provided "on a market basis". Why government should be active in the production of private intermediate goods appears to involve the implicit subsidization of production of certain goods in favor of low income groups. Public transportation is a case in point. A very extensive system of local bus service could get commuters to employment at charges that cover costs. Instead most municipal governments become involved in faster and more commodious commuter services (eg. subways and related systems with dedicated rights of way) and end up charging for the use of the facilities at well below cost. And many municipalities provide recreational places such as swimming pools and charge users a fee well below cost. Again we are observing a system of providing services in a subsidized manner mostly to lower income groups.

"The third group consists of transfers... in order to redistribute income or wealth."\textsuperscript{14}

We are concerned here with the "first group" of government activities, those with a large component of joint consumption by multiple end users. Our analysis deals with pure public goods alone, ones with no explicit private goods component. Domestic "law and order" involves a public goods component (the courts, the police, arbitrators, etc.) as well as a private component (one's lawyer, one's private investigators, one's payment for "court costs", one's private security personnel, etc.). We acknowledge the difficulty in arriving at pure public goods in practice, but we restrict our analysis to the pure public good phenomenon (the pure shared price, same quantity case).

\textsuperscript{14}The 2008 revisions to the SNA recommended that military equipment (tanks, aircraft, etc.) be treated as government capital and be depreciated properly. Australia, Canada and the United States are currently making these changes to the national accounts. The work on this for the Canadian national accounts was reported by James Tebrake of Statistics Canada at the annual meeting of the Canadian Economics Association, June, 2011 ("A Review of the Historical Revision of the Canadian System of National Accounts"). In addition, clarification of the place of non-market producers in the accounts is proceeding. The vexing problem of a rate of return to government investment or a measure of the cost of government capital is being revisited by the experts working on revisions to the accepted "regulations" for doing the national accounts.
2 Samuelson Public Good with N Household-consumers

We start with a static model with a classical public goods formulation: households alone consume the public good as a final good. This approach dates at least from Samuelson (1954). Static maximization of the utility sum $NU((q_C/N), q_G)$ for $N$ persons yields these 4 equilibrium conditions. ($f(K_C, N_C)$ is current production of the private good, with input levels $K_C$ and $N_C$. $h(K - K_C, N - N_C)$ is current production of the government good, with input levels $K - K_C$ ($=K_G$) and $N - N_C$ ($=N_G$). We assume that our production functions $f(.)$ and $g(.)$ exhibit constant returns to scale and the price of the consumption good is unity.)

\[
\begin{align*}
\frac{NU_{q_G}}{U_{q_C/N}} &= f_{N_C}/h_{N_G}, \\
q_C &= f(K_C, N_C) \\
q_G &= h(K - K_C, N - N_C) \\
f_C/h_{K_G} &= f_{N_C}/h_{N_G}
\end{align*}
\]

$U_{q_C/N}$ is a person’s marginal utility for her consumption of the private good, and $f_{K_C}$ is the marginal product of $K$ in the production of the private good. $\frac{U_{q_G}}{U_{q_C/N}}$ is the personalized price that each person is required to pay respectively for a unit of the public good. $h_{K_G}$ is a marginal amount of the public or government good. Similarly with $h_{N_G}$. The above system is 4 equations in $K_C$, $N_C$, $q_C$, and $q_G$.

We endogenize prices $p_G$, $r$, and $w$ with the following relations (3 equations):

\[
\begin{align*}
r &= w \frac{f_{K_C}}{f_{N_C}}, \\
q_C &= rK_C + wN_C, \\
\end{align*}
\]

$p_C$ is set at unity. Our Samuelsonian system is 7 equations in 7 unknowns. It is static: $K$ and $N$ are unchanging parameters. Each person must pay $[U_{q_G}/U_{q_C/N}]q_G$ in order that the cost of the public good is covered off. $p_Gq_G$ is
the cost of producing level $q_G$ of the public or government good. It follows that

$$N[U_{q_G}/U_{[q_C/N]}] = p_G.$$  

This is the well-known Samuelson condition for households consuming a pure public good. With identical households, we have each household being charged $p_G/N$ per unit of $q_G$.

We set the above model in a balanced growth framework by introducing an investment goods sector and a savings relation. $I$ indicates the investment goods sector. The model becomes 11 equations in 11 unknowns ($q_C, K_C, N_C,$ $q_G, p_I, p_G, r, w, q_I, K_I,$ and $N_I$.) $s$ is the exogenous savings rate and $p_C$ is the numeraire.

\[
\begin{align*}
NU_{q_G} & = f_{N_C}/h_{N_G}, \\
\frac{f_{K_C}}{f_{N_C}} & = h_K, \\
q_C & = f(K_C, N_C) \\
q_G & = h(K - K_C - K_I, N - N_C - N_I) \\
qu & = g(K_I, N_I), \\
\frac{f_{K_C}}{f_C} & = g_{K_I} \\
r & = w\frac{f_{K_C}}{f_{N_C}}, \\
q_C & = rK_C + wN_C, \\
p_Gq_G & = r[K - K_C - K_I] + w[N - N_C - N_I], \\
p_Iq_I & = rK_I + wN_I, \\
s[q_C + p_Gq_G + p_Iq_I] & = p_Iq_I.
\end{align*}
\]

$U_{[q_C/N]}$ is a household’s marginal utility for the private good and $f_{K_C}$ is the marginal product of $K$ in the production of the private good. The last equation involves the constant savings rate $s$ and the equality of the demand and supply of investment goods. We assume no depreciation and thus $K_{t+1} - K_t = q_I$.

This model admits the fairly standard national accounts statement. A national account set out in the matrix below (Table 1).
Table 1: Accounting Matrix

<table>
<thead>
<tr>
<th></th>
<th>( rK_C )</th>
<th>( wN_C )</th>
<th>( = q_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( rK_I )</td>
<td>( wN_I )</td>
<td>( = p_I q_I )</td>
<td></td>
</tr>
<tr>
<td>( r[K - K_C - K_I] )</td>
<td>( w[N - N_C - N_I] )</td>
<td>( = p_G q_G ) ( (\frac{NU_G}{U_{ac/N}}) )</td>
<td></td>
</tr>
<tr>
<td>( = rK )</td>
<td>( = wN )</td>
<td>( = )</td>
<td></td>
</tr>
</tbody>
</table>

Each row entry in Table 1 is part of the corresponding value-sum on the right. Each column entry is part of the corresponding value-sum at the bottom row. The sum of entries in the right column is net national product and the sum of the entries in the bottom row is national income. These two sums are usually the same, when production takes place under constant returns to scale. Government product is accounted for here by costs of inputs: \( r[K - K_C - K_I] + w[N - N_C - N_I] \). These costs sum to \( p_G q_G \). The corresponding revenue is \( q_G \frac{NU_G}{U_{ac/N}} \), where \( q_G \frac{U_{ac/N}}{U_{ac/N}} \) is the payment (tax charge) by a single household for the flow of services, \( q_G \).\(^{15}\) Heterogeneity of households would of course be associated with an array of household-specific charges for the same service flow \( q_G \).

Given initial values for \( K \) and \( N \), and forms for production functions and the utility function, we can solve the 11 equation system. For the utility function homogeneous of degree unity in its two arguments and the production functions exhibiting constant returns to scale, we assume that when we re-solve with \( \tilde{K} = (1 + \delta)K \) and \( \tilde{N} = (1 + \delta)N \) prices will be unchanged and quantities will have increased by \( \delta \) for \( \delta > 0 \). Such a realization is referred to as balanced growth. However when we attempt this exercise, we observe that balanced growth only obtains for the case of the utility function of the Cobb-Douglas form. We turn to an example.

Consider the utility function of the CES form in \( [\alpha(\frac{q_G}{N})^{-\beta} + (1 - \alpha)q_G^{-\beta}]^{-1/\beta} \) for \( \sigma = 1/(1 + \beta) \). Then the equilibrium condition for the household becomes

\[
\left[ \frac{\alpha}{1 - \alpha} \right] \left( \frac{N q_G}{q_C} \right)^{1/\sigma} = \frac{1}{(p_G/N)}.
\]

When \( \sigma = 1 \) (the Cobb-Douglas case), \( N \) cancels out on both sides. When we solve the 11 equations system, a balanced growth solution can be obtained.

\(^{15}\) Manning, and McMillan, (1979a) developed an optimal savings model with public goods. One obtains a dynamic public goods pricing condition in this framework.
We solve for the case of $\sigma = 2$ in Appendix 1. We specify $q_C = \{0.4[K_C]^{0.5} + 0.6[N_C]^{0.5}\}^2$ for $\beta = -0.5$, $q_I = [K_I]^{0.7}[N_I]^{0.3}$, $q_G = [K - K_C - K_I]^{0.5}[N - N_C - N_I]^{0.5}$. The savings rate is 0.2 and our base case endowments are $N=10$ and $K = 7$. See Appendix 1 and the Matlab program, nlef. When $\sigma \neq 1$, then $N$ does not cancel out and we observe in our numerical runs with nlef that a balanced growth solution cannot be obtained. Hence the well-behavedness of simple growth model with pure public final goods (with charges in accord with marginal benefits) requires that the utility function of a household be of the Cobb-Douglas form.

3 The Public Good both Intermediate for Firms and Final for Households

We turn to the case of the public good being both Samuelsonian ("consumed" as a final good by households) and intermediate ("consumed" as a pure intermediate public good by firms). This calls for a seeming minor re-write of our system above, namely inserting $q_G$ as an input to "firms" in the consumer goods and investment goods sectors, and expanding the public good "pricing equation" to incorporate unit charges to firms for their use of the public good. The new pricing equation includes a marginal benefit charge per firm:

$$\frac{NU_{q_G}}{U_{q_G} / N} + M_P^C + p_I M_P^I = p_G,$$

where $M_P^C$ is the marginal product of the government input to a consumer goods firm.\footnote{These conditions for public inputs to firms appear in Kaizuka (1965) and Sandmo (1972). Sandmo takes up relating these conditions to market prices. Manning, Markusen and McMillan (1985) point out cases of public inputs that do not satisfy Lindahl pricing and present an alternative pricing scheme.} Recall that we have not broken out distinctive firms, given that we are working with constant returns to scale a the level of the industry. Hence our use of the fiction that there is one firm in the consumption goods sector and one firm in the investment goods sector. The other novelty here is that each firm’s zero profit condition involves it paying its own "price" per unit of $q_G$.

The equations for an equilibrium given, $K$ and $N$ follow.
\[
\frac{NU_{qG}}{U_{qC/N}} + MP^G_{qC} + p_I MP^I_{qG} = p_G,
\]
\[
f_KC = \frac{h_{K_G}}{h_{N_C}},
\]
\[
f_C = f(K_C, N_C, q_G)
\]
\[
q_G = h(K - K_G - K_I, N - N_C - N_I)
\]
\[
q_I = g(K_I, N_I, q_G),
\]
\[
f_KC = \frac{g_K_I}{g_N_I}
\]
\[
r = w f_KC,
\]
\[
q_C = r K_C + w N_C,
\]
\[
p_G q_G = r[K - K_C - K_I] + w[N - N_C - N_I],
\]
\[
p_I q_I = r K_I + w N_I,
\]
\[
s[q_C + p_I q_I + q_G \frac{NU_{qG}}{U_{qC/N}}] = p_I q_I.
\]

Note that the savings rate is multiplied by the value of final demand (sum of entries in the right-hand column of the accounts matrix in Table 2 below). The value of intermediate goods flows are not part of final demand. The above is eleven equations in \(q_C, q_I, q_G, K_C, N_C, K_I, N_I, r, w, p_I, p_G\). We solved this system numerically with Matlab (Appendix 2). We have \(u = [q_C/N]^{0.8}[q_G]^{0.2}\).

We took the production functions for our three goods to be Cobb-Douglas and exhibiting constant returns to scale: \(q_C = [K_C]^{0.1}[N_C]^{0.5}[q_G]^{0.4}, q_I = [K_I]^{0.7}[N_I]^{0.1}[q_G]^{0.2}, q_G = [K - K_C - K_I]^{0.02}[N - N_C - N_I]^{0.38}\). The savings rate was 0.3. See Appendix 2 for details. Numerical outputs are

\[
q_C=8.4817, q_I=6.1641, q_G=8.0790, K_C=1.9059, N_C=11.8866, K_I=7.1472, N_I=1.2736, r=0.4450, w=0.3568, p_I=0.7371, p_G=0.7949.
\]

Table 2 is the national account for this model economy. We have inserted the numerical values of entries based on our numerical solution below the corresponding "theoretical" entry. There is some rounding of the values entered in the Table. NNP is the sum of entries in the right column.
Table 2: National Account

<table>
<thead>
<tr>
<th></th>
<th>$r_KC$</th>
<th>$w_NG$</th>
<th>$qGMP_G$</th>
<th>$= qG$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_KC$</td>
<td>$(0.848)$</td>
<td>$(4.24)$</td>
<td>$(3.393)$</td>
<td>$(8.482)$</td>
</tr>
<tr>
<td>$w_NG$</td>
<td>$(0.454)$</td>
<td>$(-2.44)$</td>
<td>$(0.9087)$</td>
<td>$(4.44)$</td>
</tr>
<tr>
<td>$qGMP_G$</td>
<td>$(-4.302)$</td>
<td>$(-2.120)$</td>
<td>$= 0$</td>
<td></td>
</tr>
</tbody>
</table>

In Table 2, the Samuelsonian portion of the value of the public good is present in NNP in the right hand column (value: 2.120 numeraire units). That is, 2.12 dollars of the public good is being paid for by household. In addition, however, there is a non-trivial value of the public good that is flowing to firms as an intermediate input (value: 4.302 numeraire units). The essential message is that households should be paying something for the flows of the public good which they are "consuming" AND firms should be paying non-trivial amounts for the flows of the public good which they are "consuming". Recall that the flow public good we have in mind here is domestic "public security" (the courts, jails, police as well as institutions for maintaining tranquility within firms). This model implies that a natural base for taxation is the income of firms. Presumably this approach did not evolve because authorities based revenue-raising on (a) ease or low-cost of collection and (b) a sizeable, stable base for taxation. Import duties served for many decades as a principal flow of revenues to governments. Such levies presumably made little sense if marginal benefit was the basis for charging for government services.

We verified in numerical runs that prices in our solution did not change when we made a small proportional change in our endowments of $K$ and $N$.

In Appendix 3, the model is expressed in per capita terms with an exogenous population growth rate, $n$.

Given a CES utility function in place of the Cobb-Douglas form above, the equilibrium condition for a household is now

$$\left[\frac{\alpha}{1-\alpha}\right] \left(\frac{Nq_G}{q_C}\right)^{1/\sigma} = \frac{1}{(p_G - MP_G^C - p_I MP_I^G)/N)}.$$ 

As we observed above, $N$ cancels when $\sigma = 1$ (the Cobb-Douglas case) and fails to cancel otherwise. This leads to the result that balanced growth only obtains for the case of $\sigma = 1$. Hence again we infer that the public goods charging
"formula" has a peculiar impact on the issue of obtaining a balanced growth solution in a simple neo-classical growth model.

4 Infrastructure as a Public Input

Above we considered a flow intermediate good, services of the government currently produced and "sold" mostly to firms. We turn now to the services of a government stock, such as a road, being provided as an intermediate good. Infrastructure is what we have in mind; roads, bridges, airports, ports, etc. Firms jointly consume the services of this capital, along with households but we leave the latter out as "consumers" in order to avoid clutter in our presentation. There is a substantial literature on government capital and its return relative to capital in the private sector and on government investment in the public sector as well. Nordhaus and Tobin (1973) emphasized that much of government activity was intermediate and of a capital goods nature. Barro (1991) placed the government sector into a growth model and analyzed the contribution of government activity to growth.\textsuperscript{17} Many subsequent analyses by others followed. The international System of National Accounts (SNA-1993) recommended splitting government expenditure into consumption and investment categories and the US proceeded to implement this approach in the 1990's. These matters were reflected on in the US National Academy of Sciences report, "Measuring the Government Sector in the Economic Accounts" (Slater and David (1998)).\textsuperscript{18}

Treating some of government expenditure as current investment implies that

\textsuperscript{17}Barro (1989), distinguished between government investment and consumption in his empirical analysis of the factors affecting growth rates. In his analysis of factors influencing growth rates of 95 nations, Barro (1991) separates the Summers-Heston datum "government consumption" into a pure consumption component ("government consumption" minus "nominal spending on education and defense" (p.430)) and investment component. *The idea is that expenditures on education and defense are more like public investment than public consumption; in particular, these expenditures are likely to affect private sector productivity or property rights, which matter for private investment.* (p. 430). Later Barro makes essential use of a variable "which includes public investment one-to-one with private investment". His government consumption "residual" contributed negatively to growth in his cross-section of nations. Aschauer (1989) and Aschauer (1995) initiated a debate about the contribution of government capital to a nation's growth performance. His estimate for this effect was relatively large. Kamps (2006) returned to this issue and also provided new estimates of the size of government net capital stocks.

\textsuperscript{18}The SNA 2008 "revisions" recommended treating military equipment (tanks and aircraft) as durable capital and also returned to the matter of a rate of return for investment by government.
cumulative government capital will be yielding services in each period. Income from government capital needs to be accounted for and the issue emerges as to how much of these services from government capital should be considered final and how much intermediate. Is imputation necessary for arriving at the appropriate rental stream on the income side of the national accounts? Certainly rentals on government capital should be registered in some way on the income side of the national accounts and we do such registering below. And treating much of government expenditure as intermediate input flows to firms involves the question of whether tax payments by firms can be viewed as payments for such services. We explore these matters in our abstract national accounting framework (in our accounts matrix). We treat the services of government capital as being "consumed" by all firms in the economy jointly, and we break out a distinct sector producing investment goods for the government capital. Hence there is "standard" private capital in our model with its own flow of investment and public capital with its own flow of investment. This latter capital yields a flow of services that each producing sector (households excluded) "consumes" in the same current quantity. Hence we can speak of government capital as a public good, an intermediate public input. We are addressing then (a) government capital as distinct from private capital and government capital as a public input providing a flow of services, mostly to firms (b) government investment as distinct from investment in the private sector and (c) a rate of return to government capital.

19 Boskin, Robinson and Huber (1989) remark of their empirical work: "The depreciation estimates generate internally consistent capital stock and imputed rent series." (p. 289) and later: "In addition to imputing rental flow from government capital as current consumption and developing improved estimates of depreciation to estimate net investment and the accrued capital stock, we also make corresponding adjustments for consumer durable purchases." (p. 289) We treat service flows from government capital as intermediate inputs to firms and infer that no part of rentals on government capital becomes part of "current consumption" in NNP directly. All rental payments for government capital, on the product side of the accounts, end up embodied in the prices of goods forming NNP.

20 Eisner (1989) in his comment on Boskin, Robinson and Huber (1989) notes: "They define this ("government consumption") as total government expenditures for goods and services minus government investment in tangible assets plus the imputed services of government tangible capital.... But much of government spending relates essentially to intermediate product or "regrettables" such as defense, police, and transportation, which contribute to final product, if they do, of investment as well as consumption. And Boskin, Robinson and Huber's government "consumption" very considerably involves services that go to the production of human and other intangible capital that are rather of the nature of investment than consumption." (p. 355).
Infrastructure is taken here to be simply a capital good $K^G$ owned and supplied by government. In addition there are investment goods produced by the government $q_{tg}$ that get added to the current stock of government capital in each period under consideration. The national account for this new situation is in Table 3. Novel is the fact that government capital or infrastructure is a public input and hence the rental rate is the sum of the marginal valuations of all users (firms) in the economy. $K^G$ is the public input with its rental $r_{Kg}$ "shared" by the three "firms" in the consumption goods sector, the investment goods sector and the sector producing government investment goods. That is, we have $r_{Kg} = [MP_{Kg}^C + p_I MP_{Kg}^I + p_{Ig} G MP_{Kg}^{Ig}]$ for $p_I MP_{Kg}^I$ the value of the marginal product of government capital in the production of private sector investment goods. New also is the savings rate $s_2$ "creating" investment flows for the government capital (infrastructure) sector. Given endowments, $K$, $K^G$ and $N$, at a point in time our economy is the solution to the following 12 equations system.

$$r_{Kg} = [MP_{Kg}^C + p_I MP_{Kg}^I + p_{Ig} G MP_{Kg}^{Ig}], \quad (3)$$

$$f_{KC} = h_{Kg}, \quad \frac{f_{KC}}{h_{Kg}}, \quad (4)$$

$$q_C = f(K_C, N_C, q_G)$$

$$q_{tg} = h(K - K_C - K_I, N - N_C - N_I)$$

$$q_I = g(K_I, N_I, q_C),$$

$$f_{KC} = g_{Kg}, \quad \frac{f_{KC}}{g_{Kg}},$$

$$r = w \frac{f_{KC}}{f_{N_C}}$$

$$q_C = r K_C + w N_C + K^G MP_{Kg}^C,$$

$$p_G q_G = r[K - K_C - K_I] + w[N - N_C - N_I] + K^G p_{Ig} G MP_{Kg}^{Ig},$$

$$p_I q_I = r K_I + w N_I + K^G p_I MP_{Kg}^I,$$

$s_1[q_C + p_I q_I + p_{Ig} q_{Ig}] = p_I q_I$

$s_2[q_C + p_I q_I + p_{Ig} q_{Ig}] = p_{Ig} q_{Ig}. \quad (5)$

We took the production functions for our three goods to be Cobb-Douglas and exhibiting constant returns to scale: $q_C = [K_C]^{0.4}[N_C]^{0.5}[K^G]^{0.4}, q_I =$
$[K_i]^{0.7}[N_i]^{0.1}[K^G]^{0.2}$, $q_{IG} = [K - K_C - K_i]^{0.6}[N - N_C - N_i]^{0.4}$. We also have $s_1 = 0.2$ and $s_2 = 0.1$. Endowments are $N = 12$, $K = 5$, and $K^G = 7$. See Appendix 4. We verified that proportional shifts in the three endowments leaves the prices unchanged.

The national account is set out in Table 3. Since $K^G$ is a public input, each user is "consuming" the same amount of services from $K^G$ in the current period.

Table 3: National Account

<table>
<thead>
<tr>
<th>$rK_C$</th>
<th>$wNC$</th>
<th>$K^GMP_{Kg}^C$</th>
<th>$=q_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$rK_I$</td>
<td>$wN_I$</td>
<td>$K^Gp_I^MMP_{Kg}^I$</td>
<td>$=p_Iq_I$</td>
</tr>
<tr>
<td>$r[K - K_C - K_I]$</td>
<td>$w[N - N_C - N_I]$</td>
<td>$K^Gp_I^MP_{Kg}^I$</td>
<td>$=p_I^Gq_{IG}$</td>
</tr>
<tr>
<td>$=rK$</td>
<td>$=wN$</td>
<td>$=[MP_{Kg}^C + p_I^MMP_{Kg}^I + p_I^GMP_{Kg}^I]K^G$</td>
<td></td>
</tr>
</tbody>
</table>

The sum of entries in the right-hand column is net national product and the sum of entries in the bottom row is national income. Under constant returns to scale in production, these two sums are equal. $K^G$ might be roads and the charges for same might be the fuel taxes which truckers must pay. Our table suggests that each of the consumption goods, investment goods and government goods sectors are indirectly consuming trucking services and the payment is in fact only the "road user charges" set out in column three.

We infer that it is perfectly reasonable then to discuss the concept of "government capital" and investment in "government capital" and to quantify these entities in dollars. Of great interest is the relationship between government and private capital stocks and government and private investment flows. What mechanisms or forces contribute to the determination of the relative sizes of these entities and is there a tendency for the rate of return on the two types of capital to be approximately equal over many periods? Clearly the rates of return depend on the magnitude of the two rates of savings in the model. The basic run in Appendix 4 has a private rate of return ($r/p_I$) equal to 0.758 and a rate of return in the government sector ($r_{Kg}/p_I^G$) equal to 0.769. These rates are very similar, presumably by chance. When we increase the savings rate $s_2$ from 0.1 to 0.12 and re-solve the model, we observe that indeed the rate of return in the government sector declines to 0.7103 and the private rate rises to 0.7644.
5 Concluding Remarks

We have argued, with others, that a substantial portion of "government production" is consumed directly by firms (much of the cost of the maintenance of the legal system) and thus, firms in the private sector should be viewed as proper units for being charged or taxed in order to support government production. We set out the charges under the assumption that the inputs were pure public goods. In addition to flow intermediate public inputs, there are services of public or government capital. We set out a model of these goods, along with an investment sector linked to the production of government capital. Again we based charges for the services of government capital on marginal benefits to "consumers", in our case firms rather than households. We were able to shed some light on the issue of the rate of return on private capital being different from that on public capital. The linking of national accounting to various versions of a baseline neoclassical growth model sheds much light on long-standing criticisms of the way government is treated in the national accounts. Our accounting work raises indirectly questions about the proper charging for government services in modern economies. Revenues in the public sector are often very weakly linked to specific services of government. This raises question about a "correct government agenda of activities" and a "correct level" of various government services. Here we have made tight links between services and charges. Novel questions about the role of government in the economy are implicit and of course are of much interest.
APPENDIX 1: PUBLIC GOODS FINAL

This program provided a test for price neutrality during growth, with different values of elasticity, sigma.

..............

function f=nlef(x)
% March 13 SUCCESS... PRICE NEUTRALITY WITH PUBLIC INPUTS...
% govt is intermediate ‡ ow AND SAMUELSON pub good as well.
% CES utilities (zi and dl) and CobbDouglas prod functions
ack=0.2;aik=0.7;ag=0.5;s=0.2;N=10*(1+.2);K=7*(1+.2);
zi=0.4;sig=2;
qc=x(1);
qi=x(2);
qg=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
w=x(9);
pi=x(10);
pg=x(11);
% system is 3 real eqns
f(1)=qc-Kc^ack*Nc^(1-ack);
f(2)=qi-Ki^aik*Ni^(1-aik);
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
f(3)=ag-(K-Ki-Kc)^ag*(N-Nc-Ni)/(1-ag);
% re-working
f(4)=r-w*(ack/(1-ack))*(Nc/Kc);
% two efficiency ratios
f(5)=(ack/(1-ack))*(Nc/Kc)-(aik/(1-aik))*(Ni/Ki);
f(6)=(ack/(1-ack))*(Nc/Kc)-(ag/(1-ag))*(N-Nc-Ni)/(K-Kc-Ki);
% REDO: this is the Samuelson pub goods eqn for govt as input to production!!

\[ sB = \frac{(1-\frac{z_1}{N})N/z_1}{((Nq_g/q_c)\frac{1}{\sigma_1})} \]

\[ f(7) = pg - sB \]

% note savings is out of FINAL demand (intermediates excluded)

\[ f(8) = s(q_c + p_1q_i + q_g sB) - p_i q_i \]

% three price-value equations

\[ f(9) = q_c - rK_c - \omega N_c \]

\[ f(10) = q_i - p_i rK_i - \omega N_i \]

\[ f(11) = q_g - p_g r(K-K_c-K_i) - \omega(N-N_c-N_i) \]

% \sigma = 1.0

\[
\begin{array}{cccccccc}
3.3987 & 1.7808 & 4.0414 & 1.0090 & 4.6043 & 2.2072 & 1.0791 \\
0.6737 & 0.5905 & 1.1928 & 1.2615 \\
\end{array}
\]

% test for price invariance, endowments*(1.1)... \sigma = 1... YES.. price invariance

\[
\begin{array}{cccccccc}
3.7386 & 1.9589 & 4.4555 & 1.1099 & 5.0647 & 2.4279 & 1.1871 \\
0.6737 & 0.5905 & 1.1928 & 1.2615 \\
\end{array}
\]

% \sigma = 2.0

\[
\begin{array}{cccccccc}
0.5438 & 1.6591 & 6.3118 & 0.1241 & 0.7869 & 1.8631 & 1.2659 \\
0.8766 & 0.5529 & 1.4062 & 1.3923 \\
\end{array}
\]

% \sigma = 2 and *(1.1)... NO PRICE INVARANCE

\[
\begin{array}{cccccccc}
0.5483 & 1.8235 & 6.9803 & 0.1246 & 0.7941 & 2.0448 & 1.3959 \\
0.8799 & 0.5524 & 1.4095 & 1.3943 \\
\end{array}
\]

% \sigma = 2 and *(1.2)

\[
\begin{array}{cccccccc}
0.5521 & 1.9879 & 7.6492 & 0.1251 & 0.8003 & 2.2264 & 1.5261 \\
0.8827 & 0.5519 & 1.4124 & 1.3960 \\
\end{array}
\]

////////

APPENDIX 2: Public Good Flow to Households and Firms Simultaneously

function f=nlesAg(x)

% March 13 SUCCESS... PRICE NEUTRALITY WITH PUBLIC INPUTS...

20
% govt is intermediate flow AND SAMUELSON pub good as well.
% Cobb Douglas utilities (zi) and CobbDouglas prod functions
ack=0.1;acn=0.5;aik=0.7;ain=0.1;ag=0.62;s=0.3;N=20*1.00;K=18*1.00;
zi=0.8;
qc=x(1);
qi=x(2);
qg=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
w=x(9);
pi=x(10);
pg=x(11);
% system is 3 real eqns
f(1)=qc-Kc^ack*Nc^acn*qg^(1-ack-acn);
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
f(2)=qi-Ki^aik*Ni^ain*qg^(1-aik-ain);
% re-working
f(4)=r-w*(ack/acn)*(Nc/Kc);
% two efficiency ratios
f(5)=(ack/acn)*(Nc/Kc)-(aik/ain)*(Ni/Ki);
f(6)=(ack/acn)*(Nc/Kc)-(ag/(1-ag))*(N-Nc-Ni)/(K-Kc-Ki);
% this is the Samuelson pub goods eqn for govt (into prod and cons)
sB=((1-zi)/zi)*(qc/qg);
% note savings is out of FINAL demand (intermediates excluded)
f(7)=pg-(1-ack-acn)*qc/qg-pi*(1-aik-ain)*qi/qg-sB;
% three price-value equations
f(8)=s*(qc+pi*qi+qg*sB)-pi*qi;
f(9)=qc-r*Kc-w*Nc-qg*(1-ack-acn)*qc/qg;
\[ f(10) = q_i \cdot r \cdot K_i - w \cdot N_i - p_i \cdot q_g \cdot (1 - a_{ik} - a_{in}) \cdot q_i / q_g; \]
\[ f(11) = q_g \cdot p_g \cdot r \cdot (K_c - K_i) - w \cdot (N_c - N_i); \]

% Wed., March 13, with N demanders for G
% \( q_C = 8.4817 \) \( q_I = 6.1641 \) \( q_G = 8.0790 \) \( K_c = 1.9059 \) \( N_c = 11.8866 \) \( K_i = 7.1472 \) \( N_i = 1.2736 \)
% \( r = 0.4450 \) \( w = 0.3568 \) \( p_I = 0.7371 \) \( p_G = 0.7949 \)
% RESOLVED WITH SMALL PROPORTIONATE CHANGE IN K and N:
Same prices....

APPENDIX 3: Balanced Growth for Above static case

............

function f=nlesAg(x)
% March 13 SUCCESS... PRICE NEUTRALITY WITH PUBLIC IN-
PUTS...
% govt is intermediate ow AND SAMUELSON pub good as well.
% Cobb Douglas utilities (zi and dl) and CobbDouglas prod functions
ack=0.1;acn=0.5;aik=0.7;ain=0.1;ag=0.62;s=0.3;N=1.00;K=18/20;
zi=0.8;n=6.1641/18;
qc=x(1);
qi=x(2);
qg=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
w=x(9);
pi=x(10);
pg=x(11);
KP=x(12);
% system is 3 real eqns
f(1)=qc-Kc^{-ack} \cdot Nc^{-acn} \cdot qg^{-} \cdot (1-ack-acn);
f(2)=qi-Ki^{-aik} \cdot Ni^{-ain} \cdot qg^{-} \cdot (1-aik-ain);
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production

\[ f(3) = q_g \cdot (K - Ki - K_c)^{\alpha_{g}} \cdot (Nc - Ni - Nc - Ni)^{(1 - \alpha_{g})}; \]

% re-working

\[ f(4) = r - w \cdot (\alpha_{ck} / \alpha_{cn}) \cdot (Nc / Kc); \]

% two efficiency ratios

\[ f(5) = (\alpha_{ck} / \alpha_{cn}) \cdot (Nc / Kc) - (\alpha_{ik} / \alpha_{in}) \cdot (Ni / Ki); \]

\[ f(6) = (\alpha_{ck} / \alpha_{cn}) \cdot (Nc / Kc) - (\alpha_{g} / (1 - \alpha_{g})) \cdot (Nc - Ni - Ni / (K_c - K_i)); \]

% REDO: this is the Samuelson pub goods eqn for govt as input to production!!

\[ sB = ((1 - zi) / zi) \cdot (qc / qg); \]

\[ f(7) = pg - (1 - \alpha_{ck} - \alpha_{cn}) \cdot qc / qg - p_i \cdot (1 - \alpha_{ik} - \alpha_{in}) \cdot qi / qg - sB; \]

% note savings is out of FINAL demand (intermediates excluded)

\[ f(8) = s \cdot (qc + p_i \cdot qi + qg \cdot sB) - p_i \cdot qi; \]

% three price-value equations

\[ f(9) = qc - r \cdot K_c - w \cdot Nc - q_g \cdot (1 - \alpha_{ck} - \alpha_{cn}) \cdot qc / qg; \]

\[ f(10) = qi - p_i \cdot r \cdot K_i - w \cdot Ni - p_i \cdot qg \cdot (1 - \alpha_{ik} - \alpha_{in}) \cdot qi / qg; \]

\[ f(11) = q_g - pg \cdot r \cdot (K_c - K_i) - w \cdot (Nc - Ni); \]

\[ f(12) = KP - qi - K_i + n \cdot K; \]

% Wed., March 13, with N demanders for G

% qC = 8.4817 qI = 6.1641 qG = 8.0790 Kc = 1.9059 Nc = 11.8866 Ki = 7.1472 Ni = 1.2736
% r = 0.4450 w = 0.3568 pI = 0.7371 pG = 0.7949

% RESOLVED WITH SMALL PROPORTIONATE CHANGE IN K and N:

Same prices....

% Dynamics (bal-growth) Output:

% 0.4241 0.3082 0.4039 0.0953 0.5943 0.3574 0.0637
% 0.4450 0.3568 0.7371 0.7949 (K/N)t+1 = 0.9000
% prices are reproduced and \((K/N)_{t+1}=(K/N)_{t}\). Quantities are each divided by
\% \(N=20\).

\begin{verbatim}
function f=infr(x)
% March 13 SUCCESS... PRICE NEUTRALITY WITH PUBLIC INPUTS...
% govt is intermediate flow AND SAMUELSON pub good as well.
% Cobb Douglas utilities (zi and dl) and CobbDouglas prod functions
% ack=0.1;acn=0.5;aik=0.7;ain=0.1;ag=0.6;s=0.3;N=20*(1);K=18*(1);
ack=0.1;acn=0.5;aik=0.8;ain=0.1;agk=0.6;agn=0.1;s1=0.2;s2=.1;N=12*(1+.1);K=5*(1+.1);
kig=7*(1+.1);
qc=x(1);
qi=x(2);
qig=x(3);
Kc=x(4);
Nc=x(5);
Ki=x(6);
Ni=x(7);
r=x(8);
w=x(9);
pi=x(10);
pig=x(11);
rg=x(12);
% kig=7;
% system is 3 real eqns
f(1)=qc-Kcˆack*Ncˆacn*kigˆ(1-ack-acn);
f(2)=qi-Kiˆaik*Niˆain*kigˆ(1-aik-ain);
% THIS IS GOVT FLOW PRODUCTION... input into C good and I good production
\end{verbatim}
\( f(3) = q_{ig} - (K-K_i-K_c)^{ag_k} (N-N_c-N_i)^{agn} k_{ig}^{-1-ag_k-ag_n}; \)
% re-working
\( f(4) = r - w \cdot \frac{ack}{ac_n} \cdot \frac{N_c}{K_c}; \)
% two efficiency ratios
\( f(5) = \frac{ack}{ac_n} \cdot \frac{N_c}{K_c} - \frac{a_{ik}}{a_{in}} \cdot \frac{N_i}{K_i}; \)
\( f(6) = \frac{ack}{ac_n} \cdot \frac{N_c}{K_c} - \frac{a_{ig}}{a_{gn}} \cdot \frac{N-N_c-N_i}{K-K_c-K_i}; \)
% REDO: this is the Samuelson pub goods eqn for govt as input to production!!
\( s_B = \frac{(1-zi) \cdot N/zi}{(N*q_{g}/q_{c})} \cdot \frac{1}{\text{sig}}; \)
\( f(7) = r_{g} - \pi \cdot (1-a_{ik}-a_{in}) \cdot q_{i}/k_{ig} - (1-ack-ac_n) \cdot q_{c}/k_{ig} - p_{ig} \cdot (1-ag_k-ag_n) \cdot q_{ig}/k_{ig}; \)
% note savings is out of FINAL demand (intermediates excluded)
\( f(8) = s_{1} \cdot (q_{c} + \pi \cdot q_{i} + p_{ig} \cdot q_{ig}) - \pi \cdot q_{i}; \)
\( f(9) = s_{2} \cdot (q_{c} + \pi \cdot q_{i} + p_{ig} \cdot q_{ig}) - p_{ig} \cdot q_{ig}; \)
% three price-value equations
\( f(10) = q_{c} - r \cdot \frac{K_c - w \cdot N_c - k_{ig} \cdot (1-ack-ac_n)}{q_{c}/k_{ig}}; \)
\( f(11) = q_{i} - r \cdot \frac{K_i - w \cdot N_i - k_{ig} \cdot (1-aik-a_{in})}{q_{i}/k_{ig}}; \)
\( f(12) = q_{ig} \cdot p_{ig} - r \cdot (K-K_c-K_i) - w \cdot (N-N_c-N_i) - k_{ig} \cdot p_{ig} \cdot (1-ag_k-ag_n) \cdot q_{ig}/k_{ig}; \)
% f(13) = r - ack \cdot q_{c}/K_{c};
% monday...
% 7.3780 2.6128 1.6304 1.2069 11.0526 2.7586 0.6316 0.6113
% 0.3338 0.8068 0.6465 0.4969
% with
% x0 = [8.4817, 6.1641, 8.0790, 1.9059, 11.8866, 7.1472, 1.2736, 0.4450, 0.3568, 0.7371, 0.7949, 1];
% check for price neutrality...*(1+.1)... YES.
% 8.1158 2.8741 1.7935 1.3276 12.1579 3.0345 0.6947 0.6113
% 0.3338 0.8068 0.6465 0.4969

.............

25
References


